Quantitative 3D electromagnetic field determination of 1D nanostructures from single projection

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Abstract

One-dimensional (1D) nanostructures have been regarded as the most promising building blocks for nanoelectronics and nanocomposite material systems as well as for alternative energy applications. Although they result in confinement of a material, their properties and interactions with other nanostructures are still very much three-dimensional (3D) in nature. In this work, we present a novel method for quantitative determination of the 3D electromagnetic fields in and around 1D nanostructures using a single electron wave phase image, thereby eliminating the cumbersome acquisition of tomographic data. Using symmetry arguments, we have reconstructed the 3D magnetic field of a nickel nanowire as well as the 3D electric field around a carbon nanotube field emitter, from one single projection. The accuracy of quantitative values determined here is shown to be a better fit to the physics at play than the value obtained by conventional analysis. Moreover the 3D reconstructions can then directly be visualized and used in the design of functional 3D

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architectures built using 1D nanostructures.

Keywords: 3D electromagnetic fields, 1D nanostructures, phase reconstruction, transmission electron microscopy

1. Introduction

Nanoscale structures, such as ultra-thin films, nanoparticles, and nanowires, have been intensively investigated over the past decade as a result of their novel size-dependant behavior. Amongst them, one dimensional (1D) nanostructures such as nanowires and nanotubes are of significant importance as they exhibit molecular scale properties and behavior such as density of states singularities, and emergent charge transport properties [1, 2, 3]. These structures also have direct applications as functional nanostructures in nanoelectronics and nanophotonics. The recent progress in fabrication methods have enabled 1D nanostructures to be made from a wide variety of materials including ferromagnetic [4], ferroelectric [5], thermoelectric [6] and semiconductor [7]. In all these 1D nanostructures, it is necessary to quantitatively measure the local properties so that their behavior can be better understood. Moreover, enhancement of some properties such as local electromagnetic fields in 1D nanostructures needs to be quantified, so that it can be controlled for required applications.

In this work we focus on two particular 1D nanostructures: (1) ferromagnetic nanowires, and (2) electrostatically charged carbon nanotips. Ferromagnetic nanowires form an integral part of existing and novel applications in magnetic storage [8] and sensing [9]. Magnetic nanowires with circular cross-section, are of utmost importance from theoretical and technological

22 aspects. Their high aspect ratio and cylindrical symmetry leads to novel
23 domain wall behavior such as suppression of Walker breakdown, which is of
24 interest in magnetic logic circuits and domain wall memory [10, 11]. They
25 also have potential applications in single cell bio-sensing [12, 13] as well as
26 in nanocomposites for exchange spring magnets [14]. They can be fabricated
27 using a variety of methods such as lithography based templating [15], wet
28 chemical methods and electro deposition [16]. 1D carbon-based nanostruc-
29 tures such as carbon nanotubes are amongst the best candidates for field
30 emission displays [17] and new high-brightness electron sources [18]. Since
31 their discovery by Iijima [19], carbon nanotubes have been extensively stud-
32 ied and are now used in a wide variety applications [20, 21]. One such ap-
33 plication is field emitters because they provide a unique combination of high
34 conductivity, high aspect ratio and robustness [22]. A carbon cone nanotip
35 (CCnT) was recently successfully used in a high-voltage electron gun [23].

36 The confinement effects in 1D nanostructures can alter their properties
37 and subsequently their behavior significantly. Hence it is necessary to under-
38 stand the strong effect of their size on their three-dimensional (3D) properties
39 such as the magnetic and electric fields associated with nanowires and nan-
40 otubes completely before they can be used in applications. There are a vari-
41 ety of methods to study electromagnetic fields at the nanometer length scale
42 such as scanning tunneling microscopy [4], scanning electron microscopy with
43 polarization analysis (SEMPA) [24], and transmission electron microscopy
44 (TEM) [25, 26, 27, 28]. TEM offers the best approach because it can be
45 used to determine their physical/crystal structure, composition as well as to
46 image the electromagnetic field at a high spatial resolution. In particular for

47 Lorentz TEM, the electromagnetic information of the sample is encoded in
 48 the phase shift of the electron wave, ϕ , and can be written as a combination
 49 of projections of the 3D magnetic vector potential, $\mathbf{A}$, and 3D electrostatic
 50 potential, V , given as [29]:

$$\begin{aligned}
 \phi(x, y) &= \phi_e(x, y) - \phi_m(x, y) \\
 &= C_E \int_{-\infty}^{+\infty} V(x, y, z) dz - C_B \int_{-\infty}^{+\infty} A_z(x, y, z) dz
 \end{aligned} \tag{1}$$

51 where ϕ_e is the electrostatic phase shift, ϕ_m is the magnetic phase shift,
 52 $C_E = \pi/E\lambda$, E , the relativistic electron scattering potential and λ , the
 53 wavelength of electrons, $C_B = \pi/\Phi_0$, Φ_0 , the magnetic flux quantum and
 54 the direction of propagation of the electrons is assumed to be along z . The
 55 phase shift can be recovered experimentally using various techniques such as
 56 transport-of-intensity based methods or off-axis electron holography. How-
 57 ever, from the equations it can be seen that the recovered phase shifts are
 58 an integration of the signal over the electron path that yield only projected
 59 2D information. This means that the information from the sample itself is
 60 integrated along with the stray field, thereby leading to inaccurate measure-
 61 ments of electromagnetic fields. There are currently very few methods, which
 62 have the capability to visualize the complete 3D vector fields associated with
 63 nanowires. Nominally, determining the 3D magnetic field requires recording
 64 a series of phase shift images as the sample is tilted about its axis. This
 65 becomes a tedious and cumbersome process with various sources of error
 66 or requires specialized electron microscopes [30, 31]. In a 1D nanostructure
 67 such as a nanocylinder that is uniformly magnetized along its long axis, the
 68 magnetic field possesses cylindrical symmetry with respect to the long axis.

69 In cases, where the 1D nanostructure does not possess cylindrical symmetry,
70 the usual tilt based tomography must be performed to reconstruct the 3D
71 magnetic field [32]. Similarly, carbon nanotubes under applied bias exhibit
72 a cylindrically symmetric potential and electric field. Here we show that
73 the cylindrical symmetry can be exploited to enhance the quantification of
74 electromagnetic properties by recovering the entire 3D vectorial field in and
75 around nanowires and nanotubes from a single image.

76 **2. 3D magnetic field**

77 The magnetic information from the sample is contained in the second term
78 of equation (1), ϕ_m , which can be separated using time reversal symmetry,
79 i.e. by recording the phase shift additionally with the sample turned over by
80 180° and computing the difference between the two phase shifts [33]. Taking
81 the derivative of the magnetic phase shift with respect to x , and using the
82 relation between vector potential and magnetic induction ($\mathbf{B} = \nabla \times \mathbf{A}$) gives:

$$\frac{\partial}{\partial x} \phi(x, y) = C_B \int_{-\infty}^{\infty} B_y(x, y, z) \, dz. \quad (2)$$

83 This equation correlates the projection of one component of the magnetic
84 induction with the magnetic phase shift image. When there is a cylindrically-
85 symmetric magnetic induction present, using the inverse Abel transform [34],
86 the y component of the magnetic induction, B_y , can be reconstructed from
87 the phase shift image as:

$$B_y(\rho, y) = -\frac{1}{C_B \pi} \int_{\rho}^{\infty} \frac{\partial^2 \phi(x, y)}{\partial x^2} \frac{dx}{\sqrt{x^2 - \rho^2}}, \quad (3)$$

where $\rho^2 = x^2 + z^2$. A detailed derivation of the inverse Abel transform is presented in the supporting information. It should be noted that using the above equation provides the y component of all magnetization vectors in 3D. Furthermore, using the divergenceless condition $\nabla \cdot \mathbf{B} = 0$ in cylindrical coordinates, B_ρ can be calculated as:

$$B_\rho(\rho, y) = -\frac{1}{\rho} \int_{-\infty}^{\infty} \rho' \frac{\partial B_y(\rho', y)}{\partial y} d\rho', \quad (4)$$

the x and z components of the magnetic induction being obtained as $(B_x, B_z) = (B_\rho \cos \theta, B_\rho \sin \theta)$.

The numerical implementation of the reconstruction procedure was done using the Interactive Data Language (IDL) and was tested by reconstructing the magnetic induction of a uniformly magnetized spherical nanoparticle as the analytical expression for this case is well known. The simulations were performed using a 256×256 pixel grid with a resolution of 1 nm per pixel. The magnetic induction of such a sphere has cylindrical symmetry about the axis of the magnetization vector [35], which here is assumed to be lying in the $x - y$ plane. The computations were performed for a uniformly magnetized sphere of radius, $R = 16$ nm, $B_0 = 1.0$ T, and magnetization unit vector given by $[0, 1, 0]$. The magnetic phase shift of such a sphere was numerically computed using the analytical expression given in the paper by De Graef et. al. [36]. Figure 1(a) shows the simulated magnetic phase shift image of the spherical nanoparticle. Figure 1(b) shows the derivative of the magnetic phase shift with respect to the x axis (horizontal axis of the image) required for reconstruction (equation (2)). The symmetry about the y axis of the image (vertical axis) in the derivative image can be clearly observed. This

111 image was then used for numerical implementation of equation (3) to recon-
 112 struct the complete 3D magnetic induction inside and outside the spherical
 113 nanoparticle shown in Figure 1(c). The inverse Abel transform was com-
 114 puted using the modified Fourier-Hankel algorithm described by S. Ma *et al.*
 115 [37]. The B_ρ component was then calculated by numerically implementing
 116 equation (4) in polar coordinates. The numerical integration was performed
 117 using the 5-point Newton-Cotes integration formula. The vector quantities,
 118 B_y and B_ρ as well as the scalar potential, V were then interpolated from po-
 119 lar coordinates to Cartesian coordinates. The color of the arrows indicates
 120 the magnitude of the y component of the magnetic induction with respect
 121 to the color bar, in units of T. The main advantage of this method, as illus-
 122 trated in Figure 1, is that it can be directly used to quantitatively map the
 123 3D magnetic induction, which leads to a better quantification of magnetic
 124 induction within the nanoparticle by eliminating the effect of surrounding
 125 stray fields. This can yield a more thorough understanding of the strength
 126 of magnetic interactions between spherical magnetic nanoparticles that can
 127 be useful for the self-assembly process. Additional comparisons as well as
 128 systematic errors analysis can be found in the supplementary material.

129 **3. 3D electric field**

130 The electrostatic information from the sample is similarly contained in
 131 the first term of equation (1), ϕ_e . This term contains the contribution due
 132 to the charge density in the sample resulting from the applied bias as well as
 133 the mean inner potential (MIP) of the sample. The MIP can be assumed to
 134 be constant inside the sample, and hence its contribution to the electrostatic

135 phase shift depends only on the sample's thickness variation. This contri-
 136 bution can be eliminated by computing the difference between phase shifts
 137 of unbiased and biased samples. Resulting phase shift, $\phi_{e,q}$, due to only the
 138 charge density can then be written as:

$$\phi_{e,q}(x, y) = C_E \int_{-\infty}^{\infty} V_q(x, y, z) dz, \quad (5)$$

139 where V_q represents the potential due to the charge density in the biased
 140 sample. For a sample with cylindrically-symmetric potential, and assuming
 141 that the sample is oriented such that the axis of symmetry is along the y
 142 axis, inverse Abel transform can be used to reconstruct the 3D electrostatic
 143 potential as:

$$V_q(\rho, y) = -\frac{1}{C_E \pi} \int_{\rho}^{\infty} \left(\frac{\partial \phi_{e,q}(x, y)}{\partial x} \right) \frac{dx}{\sqrt{x^2 - \rho^2}}. \quad (6)$$

144 The electric field of the sample can then be computed numerically using the
 145 relation $\mathbf{E}(\mathbf{r}) = -\vec{\nabla} V$, where $\vec{\nabla}$ is the gradient operator in 3D.

146 4. Results

147 **Magnetic nanowire.** Magnetic nanowires with circular cross section
 148 which are uniformly magnetized along their long axis also posses cylindri-
 149 cally symmetric magnetic induction. Similar to the case of the uniformly
 150 magnetized nanosphere, it is possible to compute the complete 3D magnetic
 151 field of such a nanowire using a single phase image. Experiments were per-
 152 formed on a polycrystalline nickel (Ni) nanowire of 75 nm diameter [16]. The
 153 nickel nanowires were grown by electro deposition in commercial 6 μm thick

154 polycarbonate membranes. The membranes were then dissolved in CH_2Cl_2 ,
 155 and the Ni nanowires were recovered on a carbon foil for TEM imaging.
 156 Only the region near the extremity of the nanowire was used for reconstruc-
 157 tion such that information about the magnetic field within the nanowire can
 158 be acquired along with the stray fields.

159 Figure 2(a) and (b) shows the experimental electrostatic and magnetic
 160 phase shifts of the nanowire. The phase shift was recovered using off-axis
 161 electron holography in a FEI Tecnai F20 TEM operating at 200 kV using
 162 the first transfer lens of the Cs-corrector as a Lorentz lens. The magnetic
 163 phase shift was obtained from the Ni nanowire by recording the phase shift
 164 images with the sample as-is and rotating it by 180° about its axis and
 165 then subtracting the two phase shifts. The mean inner potential (MIP)
 166 contribution to the phase shift gives an overview of the shape of the wire.
 167 Moreover, a weak electric field in vacuum can be seen due to charging effect
 168 on the nanowire giving us high confidence into the separation of the two phase
 169 shifts. The magnetic phase image was then used to perform the numerical
 170 reconstruction of the 3D magnetic field. The experimental reconstructions
 171 were performed on a 256^3 grid. Figure 2(c) shows the conventional approach
 172 of estimating magnetic induction in the nanowire using the relation, $B_{proj} =$
 173 $\nabla\phi_m/C_B t$, computed from the gradient of the phase shift and dividing by the
 174 thickness, t of the sample, derived from the MIP. With the same phase data,
 175 now we can reconstruct the entire 3D magnetic field. A vectorial view of the
 176 reconstruction can thus be displayed(Figure 2(e)) to emphasis the vectorial
 177 3D nature of the new dataset. Moreover we can visualize and measure the
 178 magnetic induction in the $x - y$ plane in the middle of the nanowire as shown

179 in Figure 2(d).

180 Such slicing capability is the main advantage of our method as we can
181 obtain more precise estimate of the magnetic saturation in the nanowire
182 as shown in Figure 3. The conventional integrative approach (Figure 2(c))
183 typically does not account for the integration of the stray fields outside the
184 nanowire leading to an inaccurate quantification of the magnetic saturation
185 of the material. This is schematically shown in Figure 3(a) where the stray
186 fields are shown in the plane above the nanowire, the true magnetic induction
187 of the sample is measured in the mid-plane of the nanowire, and the measured
188 magnetic induction in the projected plane is the integration of these values.
189 The $\mu_0.M_s$ is now measured as 0.7 ± 0.1 T as compared to the 0.5 ± 0.1 T
190 value obtained from the conventional phase gradient method as shown by
191 the lineplots in Figure 3(b). It worth noting that this new experimental
192 value for magnetic saturation of Ni was the one that was found to best fit
193 micromagnetic simulations performed for domain wall studies in Ni nanowires
194 [16]. The underestimation due to stray field contribution is eliminated in our
195 measurements. It is however limited to the extent of the reconstruction
196 volume. The strong decrease of the field away from the nanowire ($1/r^3$)
197 versus the size of the reconstruction volume ($\sim 286 \text{ nm}^3$) is sufficient to
198 assume the complete removal of the stray field versus our sensitivity. The
199 only remaining source of error in the quantification comes from the perturbed
200 reference wave used for performing holographic phase reconstruction. The
201 experimental induction map suffers slightly from a noisy phase reconstruction
202 which is caused due to the strong diffraction contrast from the polycrystalline
203 Ni nanowire. A Savitzky-Golay differentiation filter [38] was used to minimize

204 the effect of this noise.

205 **Charged Nanotube** There is intense interest in exploring carbon nanos-
206 tructures for high efficiency field emitter applications: in particular carbon
207 nanotubes and their derivatives. When an electrical bias is applied to a
208 carbon nanotube, charge builds up on the surface, which is proportional to
209 the applied bias until field emission occurs. The cylindrical geometry of the
210 nanotubes results in a cylindrically-symmetric charge distribution and po-
211 tential. It is possible to measure the effect of such a potential by measuring
212 the resulting phase shift using off-axis electron holography. Here we have
213 used the single image method developed in this work to determine the 3D
214 potential around the charge distribution as well as to obtain the 3D electric
215 field around the carbon nanotube.

216 Experiments were performed on carbon cone nanotip (CCnT) using an
217 in-situ TEM holder (Nanofactory Instruments), in which the CCnT was ex-
218 posed to an applied bias and the phase shift was measured using off-axis
219 holography [39] (Figure 4(a)). The carbon cone nanotips (CCnTs) were pre-
220 pared by deposition of pyrolytic carbon onto a multi-wall carbon nanotube.
221 The CCnTs were then mounted on a tungsten tip using a dual Focused Ion
222 Beam/FEG Scanning Electron Microscope (FIB/SEM). Figure 4(b) shows
223 the phase image of the CCnT under an applied bias of 60 V. The contours
224 are drawn every π radians. Figure 4(c) shows the 3D distribution of the re-
225 constructed potential along with contours drawn every 10 V associated with
226 the CCnT (shown in gray). The color bar shows the magnitude of the poten-
227 tial in volts. The reconstructed 3D potential from the CCnT was then used
228 to compute the 3D electric field as shown in Figure 5.

229 Figure 5(a) shows the conventional approach for estimating the electric
 230 field from the phase shift, similar to that for magnetic induction by com-
 231 puting the gradient of the phase shift. This method again represents an
 232 integrated measurement of the electric field that leads in that case to an
 233 overestimation of the measurement due to stray fields. Figure 5(b) shows
 234 the reconstructed electric field in the $x - y$ plane in the middle of the CCnT
 235 obtained using our method. Figure 5(c) shows the quantitative comparison
 236 of the electric field as measured from the projection and in the middle of the
 237 CCnT, with the vertical dashed line showing the edge of the CCnT. Similar
 238 to the magnetic saturation measurement, here too, we can see that we have
 239 more accurate estimate of the electric field at the apex of the CCnT since
 240 we are removing the effect of field outside the CCnT. Figure 5(d) shows the
 241 3D vector field plot of the electric field in and around the CCnT, with the
 242 arrows colored according to the magnitude of the electric field in units of
 243 V/nm as shown by the colorbar. The quantitative analysis of such a biased
 244 CCnT field emission process is rather easy as a direct local electric field can
 245 be instantly measured with better accuracy. We were thus able to measure
 246 an electric field of $2 \pm 0.5 \text{ V.nm}^{-1}$ at the extremity of the CCnT. Accurate
 247 measurement of this value is of importance as it enables the estimation of the
 248 field enhancement factor [40] that is used for Fowler-Nordheim plot analysis
 249 and work function estimation of carbon nanotubes. Knowing this value as
 250 well as the applied electrical bias and anode-CCnT distance, one can esti-
 251 mate a field enhancement value of $\gamma = 23 \pm 6$ which is consistent with the one
 252 estimated using finite element modelling [39]. The estimation here is once
 253 again only limited by the disturbed reference wave that causes a lowering of

254 the measured electric potential in the image.

255 5. Discussion

256 The primary requirement for this method is that the field should be cylin-
257 drically symmetric. This necessitates that the axis of symmetry in the phase
258 image is accurately determined prior to the 3D reconstruction of the field.
259 However, experimental limitations, such as low spatial resolution or a mag-
260 netization distribution that is not completely symmetric, may prevent the
261 accurate determination of the axis of symmetry. Furthermore, other experi-
262 mental limitations such as the biprism orientation being not exactly perpen-
263 dicular to the long axis of the 1D nanostructure can also introduce error in
264 preserving cylindrical symmetry in the reconstruction. It is also true for the
265 determination of the 3D electric potential and field. For example, the mag-
266 netization of a uniformly magnetized nanosphere can have a z -component
267 but depending on the direction of projection, the phase shift may appear
268 to be cylindrically symmetric. Another possibility is that the magnetization
269 can be in a ‘S’ or ‘C’ state such that at the edge of the particle, it rotates
270 in plane, thus leading to incomplete cylindrical symmetry. In order to as-
271 sess the effect of variation in symmetry and possible errors in determining
272 the symmetric axis, the magnetization of the spherical nanoparticle was ro-
273 tated by an angle θ varying from 0° to 20° about the y axis in the $x - y$
274 plane. The phase shift was then computed for the corresponding magneti-
275 zation. However, while implementing and performing the 3D reconstruction
276 of the magnetic induction, the symmetric axis was assumed to be along y
277 axis. The 3D magnetic induction was then reconstructed using the single

278 image method and compared with the true 3D magnetic induction calcu-
 279 lated analytically. Of particular importance and relevance is the axial field
 280 magnitude (B_y) that was compared numerically: for example the theoretical
 281 and reconstructed value of axial field at the tip of the sphere is 0.66 T and
 282 0.63 T for $\theta = 0$ and decreases to 0.62 T and 0.59 T for $\theta = 20^\circ$ respectively.
 283 This corresponds to only about 5% error. Similarly the error in the angle of
 284 the magnetization vector was also compared. The comparison showed that
 285 the error in determination of the angle of the magnetization vector is smaller
 286 inside the nanosphere than outside. This can be related to the fact that
 287 outside the nanosphere, the magnetization vector changes direction rapidly
 288 as compared to the magnetization vector inside the sphere. Additional de-
 289 tails about the calculation of errors and a rigorous quantitative comparison
 290 of errors is given in the supporting information. The final source of error in
 291 the quantitative measurement of the electromagnetic fields using our method
 292 is due to the perturbed reference wave. However, it has been shown previ-
 293 ously that the reference wave perturbation at the length scale of the overlap
 294 area is not significant with error below 10%.[39, 41].This demonstrates that
 295 although the condition of cylindrical symmetry is a fairly strict criterion for
 296 applicability of this method, for experimental purposes, this method can still
 297 reliably give quantitative values of magnetic induction or electric field in and
 298 around the nanostructures in 3D.

299 Finally, other approaches that quantitatively measure electromagnetic
 300 fields using phase reconstructed electron microscopy apart from tomographic
 301 methods, rely on building models of the sample either through finite ele-
 302 ment simulations or analytical models and then compare the experimental

303 2D measurements with the results obtained from the models. Recent work by
304 Beleggia *et. al.* have used this approach to show that using off-axis holog-
305 raphy combined with theoretical modeling, it is possible to quantitatively
306 measure the electric field around the tip of a biased needle and accurately
307 take into account the shape of the tip [42]. However, the method presented in
308 this work is unique in that it relies on experimental measurements to directly
309 reconstruct the 3D electromagnetic fields.

310 6. Conclusion

311 Here we have successfully demonstrated that by exploiting the cylindrical
312 symmetry of 1D nanostructures and the associated electromagnetic fields, it
313 is indeed possible to reconstruct the entire 3D electromagnetic field from a
314 single TEM phase image. This significantly reduces the time required to
315 acquire multiple tilt series images as well as reducing the electron dose to the
316 sample that is conventionally required for 3D reconstruction. The method
317 was numerically implemented to reconstruct the 3D magnetic induction in
318 and around a ferromagnetic nickel nanowire as well as the electric field in and
319 around a carbon cone nanotip under applied bias. In both the cases, we have
320 shown that using the information obtained in 3D, we can get more precise
321 measurement of relevant physical quantities such as magnetic saturation, or
322 field enhancement factors as compared to conventional approaches. This
323 work opens new paths for quantitative analysis of the 3D electromagnetic
324 interactions for technologically important one-dimensional nanostructures.

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Figure 1: (a) shows the magnetic phase shift image of a uniformly magnetized spherical nanoparticle, indicated by the dashed line, and magnetization vector is indicated by the blue arrow, (b) the derivative of the magnetic phase shift with respect to x showing the symmetry about the vertical axis of the image, and (c) the 3D reconstructed magnetic induction inside and outside the spherical particle. The color bar shows the magnitude of the y component of the magnetic induction, B_y , in T.

Figure 2: (a) shows the electrostatic contribution to the phase shift displayed as $\cos(5\Delta\phi)$ of the nickel nanowire. The associated scale displays the amount of ϕ_e between dark lines (b) shows the magnetic contribution to the phase shift (ϕ_m). (c) shows the B_y component calculated from the gradient of magnetic phase shift, and (d) shows the B_y component in the $x-y$ plane in the middle of the nanowire. The color represents the strength of magnetic induction indicated by the colorbar. (e) shows the 3D vector plot of the reconstructed magnetic induction from the nanowire. The vectors are colored according to the y component of the induction indicated by the color bar (in units of T).

Figure 3: (a) shows a schematic to illustrate the advantage of our method by showing the magnetic induction that can be measured at various locations in 3D as opposed to the projected magnetic induction, and (b) shows the quantitative comparison of the magnetic induction at various locations represented in (a) from the white dashed region shown in Figure 2(d).

Figure 4: (a) shows the as acquired interference hologram, which shows the positions of the CCnT and the anode. (b) shows the experimentally measured phase shift from a carbon cone nanotip (CCnT) under an applied bias of 60 V, (c) shows the reconstructed 3D distribution of potential (in V) around the CCnT along with contours drawn every 10 V.

Figure 5: (a) shows reconstructed electric field in the projection plane computed from gradient of the phase shift, and (b) shows the reconstructed electric field in the $x-y$ plane in the middle of the CCnT. (c) shows the quantitative comparison of the reconstructed electric field in the projection plane and the $x-y$ plane in the middle of the CCnT. (d) shows the 3D vectorial view of the reconstructed electric field (in V/nm) around the CCnT.