Iterative Reconstruction of Magnetic Induction using Lorentz Transmission Electron Tomography

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Abstract

Intense ongoing research on complex nanomagnetic structures requires a fundamental understanding of the 3D magnetization and the stray fields around the nano-objects. 3D visualization of such fields offers the best way to achieve this. Lorentz transmission electron microscopy provides a suitable combination of high resolution and ability to quantitatively visualize the magnetization vectors using phase retrieval methods. In this paper, we present a formalism to represent the magnetic phase shift of electrons as a Radon transform of the magnetic induction of the sample. Using this formalism, we then present the application of common tomographic methods particularly the iterative methods, to reconstruct the 3D components of the vector field. We present an analysis of the effect of missing wedge and the limited angular sampling as well as reconstruction of complex 3D magnetization in a nanowire using simulations.

Keywords: electron tomography, three dimensional vector fields, transmission electron microscopy, phase retrieval

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1. Introduction

There is a growing interest in nanomagnetic structures that exhibit complex topological spin states. The research is both fundamental to understand the underlying spin textures and the physical phenomena as well as applied to ensure the stability of the state and ability to control it. For example, skyrmions exhibit three-dimensional (3D) variation of spin texture in chiral magnetic thin films that are stabilized by the Dzyalshinskii-Moriya interaction [1, 2]. They are often present at specific points in arrays in an otherwise uniform magnetization pattern. They are topologically protected so that they cannot unwind or alter their magnetization continuously. As a result, they are being studied intensively for next generation information storage [3]. Towards achieving this understanding, there is a need to quantitatively map and visualize the entire 3D magnetization in such materials. Not only is 3D information necessary but also at a high spatial resolution. Lorentz transmission electron microscopy (LTEM) provides an ideal combination of high spatial resolution and ability to quantitatively map the magnetization in nanostructures [4, 5].

It has been shown experimentally that combining LTEM with tomographic methods, it is possible to reconstruct the 3D magnetic vector potential of a thin Permalloy (NiFe) islands [6]. The approach used acquisition of four tilt series about two mutually orthogonal axis in the plane of the sample. For each axis, two tilt series were acquired with the sample in as-is orientation and flipped upside down in order to separate the magnetic component of the phase shift from the electrostatic component. The magnetic

25 phase shift tilt series data was then used for reconstructing the individual
 26 vector components using the vectorial filtered backprojection algorithm [7, 8].
 27 Although the reconstruction showed a correct representation of the spatial
 28 variation of the vector components, it suffered from the usual artifacts of
 29 limited tilt tomography, such as blurring, and streaking [9]. Additionally the
 30 resolution of the components along the z direction is not the same as that in
 31 the x and y directions.

32 In other tomographic techniques such as X-ray computed tomography,
 33 the weighted (or filtered) backprojection method is typically not used or
 34 only used as a starting point for the 3D reconstruction. The initial estimate
 35 obtained using this method is then improved using various forms of iterative
 36 reconstruction methods such as algebraic reconstruction technique (ART) or
 37 simultaneous iterative reconstruction technique (SIRT) [10]. These methods
 38 rely on posing the reconstruction as an inverse problem and rely on solving
 39 a set of equations using linear algebra methods. In order to be able to do so,
 40 a projector matrix needs to be established that connects the projection data
 41 with the data to be reconstructed.

42 In this paper, we present a formalism to represent the magnetic phase shift
 43 data as a Radon transform of the magnetic induction components directly
 44 such that we are able to define a projector matrix. We will then introduce
 45 briefly the four basic reconstruction methods: (1) Weighted Backprojection
 46 Method (WBP), (2) Regridding reconstruction method (Gridrec), (3) ART,
 47 and (4) SIRT. These methods are commonly available in freeware and com-
 48 mercial softwares and are used for standard scalar tomography [11, 12]. These
 49 methods are then be applied to the reconstruction of vector field components

of a uniformly magnetized nanosphere since its 3D magnetic field is analytically known. We perform a detailed analysis of the effect of missing wedge, effect of limited angular sampling on the reconstructed components using the four methods. Finally we show the application of these methods to reconstruct the complex 3D magnetization in a magnetic nanowire with circular cross-section.

2. Methods

2.1. Projection equations for 3D vector field reconstruction

In order to perform 3D reconstruction of the vector field components using various algorithms including iterative methods, it is essential to establish the correct projection equations i.e. forward projection and backward projection for the magnetic phase shift of electrons in terms of the components of magnetic induction of the sample. The magnetic phase shift of the electrons in the presence of a vector potential $\mathbf{A}$, of the sample can be written from the Aharonov-Bohm relation as [13]:

$$\phi_m(\mathbf{r}_\perp) = -\frac{e}{\hbar} \int \mathbf{A}(\mathbf{r}) \cdot d\mathbf{l}; \quad (1)$$

where $\mathbf{r}_\perp$ represents the 2D position vector in the projection plane, which is perpendicular to the direction of projection given by $\mathbf{l}$, and $\mathbf{A}(\mathbf{r})$ represents the vector potential. Considering the direction of projection to be along z axis, we have $d\mathbf{l} = dz \hat{\mathbf{e}}_z$. Substituting this into the above equation and simplifying, we get:

$$\phi_m(\mathbf{r}_\perp) = -\frac{e}{\hbar} \int A_z(\mathbf{r}) dz; \quad (2)$$

70 Taking the derivative with respect to y , we get:

$$\frac{\partial \phi_m}{\partial y} = -\frac{e}{\hbar} \int \frac{\partial A_z}{\partial y} dz; \quad (3)$$

71 where y is defined in the projection plane as $\mathbf{r}_\perp = x \hat{\mathbf{e}}_x + y \hat{\mathbf{e}}_y$. From the
72 equation of curl of $\mathbf{A}$, considering only the x component of the curl, we have

$$B_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \quad (4)$$

73 Using equation (3) and equation (4), we get

$$\begin{aligned} \frac{\partial \phi_m}{\partial y} &= -\frac{e}{\hbar} \int \left(B_x + \frac{\partial A_y}{\partial z} \right) dz; \\ &= -\frac{e}{\hbar} \left(\int B_x dz - \int \frac{\partial A_y}{\partial z} dz \right); \end{aligned} \quad (5)$$

74 The second term of the above equation becomes zero, since the vector
75 potential is assumed to be zero at infinity. Hence the equation relating the
76 magnetic phase shift with the magnetic induction can be written as:

$$\frac{\partial}{\partial y} \phi_m(x, y) = -\left(\frac{e}{\hbar}\right) \int_{-\infty}^{\infty} B_x(x, y, z) dz; \quad (6)$$

77 Similar equation can be derived for the x derivative of the magnetic phase
78 shift as:

$$\frac{\partial}{\partial x} \phi_m(x, y) = \left(\frac{e}{\hbar}\right) \int_{-\infty}^{\infty} B_y(x, y, z) dz; \quad (7)$$

79 The above equations are similar to the projection equations of a scalar quan-
80 tity. In fact they are the real space representation of the equations de-
81 rived previously by Phatak et. al. [8]. Considering a tilt series around
82 the x -axis, for a given value of x , in the $y - z$ plane, the vector compo-
83 nent B_x remains invariant as a function of the tilting of the sample by θ

84 or equivalently tilting the beam by $-\theta$. Thus the projection along any line
 85 in the (y, z) plane can be parametrized using the relation $(y(\rho, \xi), z(\rho, \xi)) =$
 86 $((\rho \cos \theta + \xi \sin \theta), (\rho \sin \theta - \xi \cos \theta))$. Here ρ gives the distance of the line
 87 from the origin. This is shown schematically in Fig. 1. Equation (6) can now
 88 be rewritten for any tilt angle θ about the x -axis as:

$$\frac{\partial}{\partial y} \phi_m(x, \rho, \theta) = - \left(\frac{e}{\hbar} \right) \int_{-\infty}^{\infty} B_x(x, \rho \cos \theta + \xi \sin \theta, \rho \sin \theta - \xi \cos \theta) d\xi; \quad (8)$$

89 If we compare this equation with the definition of Radon transform:

$$R(\theta, \rho) = \int_{-\infty}^{\infty} A(\rho \cos \theta - s \sin \theta, \rho \sin \theta + s \cos \theta) ds, \quad (9)$$

90 we can see that the Radon transform of a 2D slice of B_x in (y, z) plane for
 91 a given value of x , gives the respective sinogram at that x in the projec-
 92 tion array. The projections thus obtained are essentially the derivative of
 93 the magnetic phase shift with respect to y . This establishes the forward
 94 projection equation that can be used for computing the projections from
 95 the vector field component. The backward projection equations can then be
 96 simply computed using the inverse Radon transform as:

$$B_x(x, y, z) = \int_0^{\pi} \nabla_y \phi_m(x, y \cos \theta + z \sin \theta, \theta) d\theta. \quad (10)$$

97 From henceforth, the gradient of the phase shift data is referred to as the
 98 projection data. Similar equation can be derived for the y component of the
 99 magnetic induction using equation (7) and relating it to the tilt series about
 100 the y axis as follows:

$$B_y(x, y, z) = \int_0^{\pi} \nabla_x \phi_m(x \cos \gamma + z \sin \gamma, y, \gamma) d\gamma, \quad (11)$$

101 where γ is the tilt angle about the y axis. The third component of the
 102 magnetic induction, B_z , can then be computed using the zero-divergence
 103 condition, $\nabla \cdot \mathbf{B} = 0$, as:

$$B_z(x, y, z) = - \int \left(\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} \right) dz \quad (12)$$

104 These relations can now be used in various reconstruction algorithms
 105 including iterative methods to reconstruct the 3D individual x and y com-
 106 ponents of the magnetic induction. Next we will briefly describe the various
 107 methods that will be compared in the paper for reconstructing the x and y
 108 components of the magnetic induction.

109 2.2. *Weighted Backprojection Method - WBP*

110 Weighted backprojection is one of the standard methods used in tomo-
 111 graphic reconstruction of 3D datasets. The algorithm is derived by using the
 112 Fourier Slice Theorem [14] that relates the the measured projection data to
 113 the 2D Fourier transforms of the object cross section. According to Fourier
 114 Slice Theorem, the Fourier transform of each projection, samples a slice of
 115 the 2D Fourier space $F(u, v)$ of the original object, and the cross section of
 116 the object $f(x, y)$ can be recovered by a 2D simple inverse Fourier transform:

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) \exp[j2\pi(ux + vy)] du dv. \quad (13)$$

117 In polar coordinates (w, ϕ) , equation (13) can be written as:

$$f(x, y) = \int_0^{\pi} \int_{-\infty}^{\infty} F(w \cos \phi, w \sin \phi) \exp[j2\pi wt] |w| dw d\phi. \quad (14)$$

118 where $t = x \cos \phi + y \sin \phi$. Since the Fourier slice method inherently acts
 119 like a blurring function due to oversampling of lower spatial frequencies as

120 compared to higher spatial frequencies, a filtering operation (i.e., $|w|$) is
 121 used to improve the reconstruction. Typically this filter is a ramp filter
 122 in Fourier space. However using only a ramp filter amplifies high frequency
 123 noise, therefore a ramp filter multiplied with a Hanning window filter is more
 124 preferred.

125 *2.3. Regridding Reconstruction Method - Gridrec*

126 The regridding reconstruction method (Gridrec) is a viable alternative to
 127 the standard WBP which is derived similarly by using the Fourier Slice The-
 128 orem [15]. In Gridrec, the Fourier transform of the measured projection data
 129 is first convolved with a convolution kernel, $W(u, v)$, and then the convolu-
 130 tion output is evaluated at the points of a Cartesian grid. The corresponding
 131 convolution output $H(u, v)$ can be written as:

$$H(u, v) = \int_0^\pi \int_{-\infty}^\infty F(w \cos \phi, w \sin \phi) W(u - w \cos \phi, v - w \sin \phi) |w| dw d\phi. \quad (15)$$

132 In equation (13), the multiplication $F(w \cos \phi, w \sin \phi) |w|$ corresponds to the
 133 typical high-pass filtering operation in WBP. Then one can apply 2D Fast-
 134 Fourier Transform (FFT) to obtain object cross section similar to equation
 135 (13). The convolution kernel $W(u, v)$ is usually selected such that it acts as
 136 a low-pass filter with cutoff frequencies corresponding to a few spatial grid
 137 spacings in a concentrated region of interest. Typically Gridrec allows up
 138 to a 20-fold increase in computation speed compared to WBP due to the
 139 utilization of FFTs on a regular grid [16].

140 2.4. Algebraic Reconstruction Technique - ART

141 The second set of reconstruction methods formulate the problem in a
142 linear algebra framework, thus attempting to iteratively solve a large linear
143 system of equations of the form, $Ax = b$, where b is the projection data, A is
144 the projection operator, and x is the unknown 3D object to be determined.
145 The standard algebraic reconstruction technique also known as the Kaczmarz
146 method [17] solves the set of equations in an iterative fashion. In this method,
147 one iteration consists of updates to the solution for each angle using the
148 following equation:

$$x^{k+1} = x^k + \lambda_k \frac{b_i - a_i x^k}{||a_i||^2} a_i, \quad (16)$$

149 where k is the iteration number, i represents each row of matrix b which in
150 turn corresponds to a particular angle in the tilt series, and λ_k is the relax-
151 ation parameter with a value between $(0, 2)$. In the case of reconstruction
152 of magnetic induction, b is the projection data i.e., the gradient of the mag-
153 netic phase shift, A is the projection/Radon operator and x is either B_x or
154 B_y depending on the tilt series. In typical implementation of this method,
155 the relaxation parameter λ_k is fixed to be 1. However, the parameter can
156 be adjusted or optimized for each iteration, that is, usually by progressively
157 decreasing it with increased number of iterations. This usually improves the
158 imaging quality, at the expense of convergence.

159 2.5. Simultaneous Iterative Reconstruction Technique - SIRT

160 A variation of the ART methods is the simultaneous iterative reconstruc-
161 tion technique in which the updates to the solution are computed by taking
162 into account all the tilt angles simultaneously in one iteration. This can be

163 written as:

$$x^{k+1} = x^k + \lambda_k A^T(b - Ax^k), \quad (17)$$

164 Similar to the ART method, the relaxation parameter λ_k has a value between
165 $(0, 2)$. The advantage of SIRT is that it typically results in higher contrast-
166 to-noise ratio in the reconstructions as compared to ART, and is more robust
167 to outliers in measurement data.

168 In the presence of measurement errors, it is not trivial to make an accu-
169 rate estimation on parameters for iterative algorithms (e.g., stopping crite-
170 rion, relaxation parameter, etc.) that provides the best final image quality.
171 Usually the difference between the measured and simulated projection data,
172 or difference in image between consecutive iterations, can be used to ob-
173 tain a general idea on the convergence behavior. Unfortunately in many
174 cases this can be misleading as the solution can diverge from the true image,
175 while the difference measures show convergence. Especially for the case of
176 ART-like algorithms that update the images individually for each tilt angle,
177 salt-and-pepper, or checkerboard pattern artifacts starts to build-up after
178 some iterations. SIRT is more robust to similar artifacts due to the simulta-
179 neous updating strategy. In this work, we heuristically selected the stopping
180 criteria that provided the best image quality for both algorithms.

181 **3. Results**

182 *3.1. Simulation for uniformly magnetized sphere*

183 The magnetic induction of a uniformly magnetized sphere can be cal-
184 culated analytically and hence it serves as a good test object for evaluat-
185 ing the various methods for reconstruction of the 3D magnetic induction.

186 The magnetic induction of a sphere of radius a with the magnetization
 187 $\mathbf{M} = [M_x, M_y, M_z]$ is given as [18]:

$$\begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{cases} \frac{2\mu_0}{3} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} & r \leq a \\ \frac{\mu_0 a^3}{3r^5} \begin{pmatrix} 3x(yM_y + zM_z) + M_x(2x^2 - y^2 - z^2) \\ 3y(xM_x + zM_z) - M_y(x^2 - 2y^2 + z^2) \\ 3z(xM_x + yM_y) - M_z(x^2 + y^2 - 2z^2) \end{pmatrix} & r > a \end{cases} \quad (18)$$

188 Similarly the magnetic phase shift of such a sphere can also be calculated
 189 analytically and can be written as a function of the tilt angle θ about the x
 190 axis as [19]:

$$\phi_m(x, y, \theta) = \left(\frac{e}{\hbar}\right) \frac{2\mu_0 a^3}{3} \times \left(\frac{x(M_y \cos \theta - M_z \sin \theta) - yM_x}{x^2 + y^2} \right) \begin{cases} \left(1 - \left(1 - \frac{x^2 + y^2}{a^2}\right)^{1.5}\right) & r \leq a \\ 1 & r > a \end{cases} \quad (19)$$

191 A similar equation can be written for the magnetic phase shift as a function
 192 of tilt angle γ about the y axis. These equations were implemented in IDL
 193 programming language to compute the tilt series data. The unit vector of
 194 magnetization of the sphere was taken to be $[\cos(\pi/6), \sin(\pi/6), 0]$. Fig. 2(a)
 195 shows the phase shift computed for a tilt angle of 0° , and (b) shows the
 196 schematic of the projection data which is a function of (x, y) co-ordinates
 197 of the projection plane and the tilt angle. Here the projection data consists
 198 of the individual components of the gradient of the phase shift. For e.g.,

199 the projection data from tilt series about x axis consists of the y component
 200 of the gradient of the phase shift and vice-versa for the tilt series about y
 201 axis. Fig. 2(c) shows the 2D sections along plane 1 indicated in (b) for tilt
 202 series about x axis and y axis. The top row shows the gradient components
 203 computed from the phase shift (left side of equation (8) and the bottom row
 204 shows the forward projection computed from 3D B_x and B_y (right side of
 205 equation (8)) components respectively. Fig. 2(d) top row shows the projec-
 206 tion data as a sinogram representation at $x = 0$ for x tilt series and $y = 0$
 207 for y tilt series respectively (indicated by plane 2 in Fig. 2). Fig. 2(d) bot-
 208 tom row shows the sinogram computed using the Radon transform of the 3D
 209 magnetic induction along $x = 0$ and $y = 0$ of B_x and B_y . All the images are
 210 scaled to the same intensity levels. Thus, Fig. 2(c) shows that the projection
 211 equations are valid for a given angle for all (x, y) in the phase shift data, and
 212 Fig. 2(d) shows it is valid for all angles for a given x or y coordinate of the
 213 phase shift data.

214 Next we will test the reconstruction of magnetic induction components
 215 using the 4 different methods mentioned previously and analyze the various
 216 effects and limits of experimental tomography such as the effect of the miss-
 217 ing wedge, and angular sampling of the tilt series. The reconstructions using
 218 WBP and SIRT methods were performed using the TomoJ plugin for Im-
 219 ageJ [12] and Gridrec and ART methods were performed using the TomoPy
 220 package [20]. The performance of the reconstruction methods was compared
 221 by computing the root mean square error (RMSE) separately inside and out-
 222 side the object. This error was calculated in this manner because inside the
 223 object, the reconstructed vector field represents the magnetization of the ob-

ject and outside represents the stray field from the object. Thus it helps in understanding and analyzing the error. The RMSE was calculated as follows:

$$\mu_F \equiv \left[\frac{1}{N_D} \sum_{i,j,k} (\bar{F}(i,j,k) - F(i,j,k))^2 D(i,j,k) \right]^{1/2}; \quad (20)$$

where F represents the individual reconstructed component, $\bar{F}$ represents the analytical value, $D(i,j,k)$ represents the region inside or outside the object, and N_D represents the total number of pixels in the region $D(i,j,k)$.

3.2. Effect of missing wedge

In typical TEM tomography due to the limitation of the sample shape/size or the limitation of the instrument (holder, pole piece gap), an entire tilt series over $\pm 90^\circ$ is difficult to record. As a result, there is a missing wedge of information from the tilt range over which no data is recorded. The effect of this missing wedge is well established and well studied for scalar tomography in TEM such as HAADF-STEM tomography. Here we discuss the effect of the missing wedge of information on the vector field tomographic reconstruction. A projection tilt series data within the range of $\pm 80^\circ$ and $\pm 70^\circ$ with tilt steps of 2° was used for reconstructing individual B_x and B_y components of the magnetic induction. The iterative reconstruction using ART was performed with 20 iterations while SIRT was performed with 40 iterations and a fixed relaxation parameter, $\lambda_k = 1$. Fig. 3 shows the comparison of the reconstructed components for all the 4 methods. The reconstructed components are displayed 2D images along various planes. The orientation of the planes and the object is schematically shown in the top-right corner inset. All the images are scaled to the same intensity range indicated by the

246 color bar. For each method, the top row shows the reconstruction for the
 247 tilt range of $\pm 80^\circ$ and the bottom row for the tilt range of $\pm 70^\circ$. As with
 248 scalar tomography, as the tilting range is reduced, the elongation artifacts
 249 can be observed. In scalar tomography such as reconstruction of shape or
 250 morphology of the object, the object is typically a discrete object in space.
 251 This is a major difference as compared to vector field tomography, wherein
 252 the components of vector field are not discrete i.e., each component varies
 253 continuously in and around the object except at the boundary of the object.
 254 As a result, even with only a slightly reduced tilt range of $\pm 80^\circ$, there is
 255 significant smearing of the components outside of the object. Fig. 4 shows
 256 a comparison of line plots of the B_x component of the vector field along the
 257 white line indicated in Fig. 3 for all the 4 methods. It can be seen that the
 258 WBP method suffers the most from missing wedge artifacts, and the mag-
 259 nitude of the magnetization inside the nanosphere as well as the stray field
 260 outside varies significantly with reduced tilt range. Interestingly the Gridrec
 261 method suffers significantly less from the missing wedge of information par-
 262 ticularly for magnetization inside the nanosphere, which does not show any
 263 effect of missing information. The stray field reconstructed using this method
 264 varies significantly as compared to the theoretical value. The reconstruction
 265 using ART shows the most promising result for reconstructing magnetization
 266 as well as stray fields with limited tilt range. However, the reconstructions
 267 have more noise as seen from the line plots. The reconstruction using SIRT
 268 reduces the noise significantly as compared to ART and shows the best re-
 269 construction result for tilt range of $\pm 80^\circ$. However, the magnitude of the
 270 reconstructed components is slightly lower as compared to the theoretical

value for the tilt range of $\pm 70^\circ$. This can be further improved by increasing the number of iterations. The reconstructed components outside the sphere (the stray field) show some directional influence. The reconstruction along the x direction for B_x (y for B_y) is better than that along the z direction. Table 1 shows the RMSE inside (subscript i) and outside (subscript o) the nanosphere for the reconstructed x and y components. The reconstructed B_z component is not compared here since that component is not directly reconstructed using any of the tomographic reconstruction methods. However, it should be noted that since B_z is computed using the B_x and B_y components, improved reconstruction of these components should directly also improve the reconstruction of the B_z component.

Table 1: RMS errors in the reconstructed components as a function of limited tilt range.

	$B_{x,i}$		$B_{x,o}$		$B_{y,i}$		$B_{y,o}$	
	$\pm 80^\circ$	$\pm 70^\circ$	$\pm 80^\circ$	$\pm 70^\circ$	$\pm 80^\circ$	$\pm 70^\circ$	$\pm 80^\circ$	$\pm 70^\circ$
WBP	0.2228	0.2856	0.0262	0.0350	0.1541	0.1875	0.0174	0.0220
Gridrec	0.1668	0.1813	0.0274	0.0420	0.0926	0.1049	0.0160	0.0255
ART	0.1730	0.1815	0.0186	0.0246	0.0966	0.1051	0.0109	0.0146
SIRT	0.1105	0.1646	0.0169	0.0295	0.0747	0.1011	0.0105	0.0173

281

282 3.3. Effect of angular sampling

283 The reconstruction of vector fields inherently requires at least two tilt
284 series acquisitions to be performed to obtain two components. Additionally
285 to separate the magnetic phase shift from the total phase shift, the sample
286 has to be inverted and the tilt series need to be repeated. This can lead
287 to extremely cumbersome experiments as well as large amounts of data to

288 be generated. In order to reduce the data as well as experimental time, we
 289 have analyzed the effect of angular sampling during tilt series acquisition.
 290 By varying the tilt steps, the total number of images recorded can be re-
 291 duced. For an ideal reconstruction, one would record images every 1° or 0.5°
 292 tilt step. Here we have tested the reconstruction of B_x and B_y components
 293 using the tilt series data within a range of ± 72 with tilt steps of 4° and 8° .
 294 Fig. 5 shows the comparison of the reconstructed components as compared
 295 to the theoretical values along various planes similar to Fig. 3. All the im-
 296 ages are scaled to the same intensity range indicated by the color bar. For
 297 each method, the top row shows the reconstruction for a tilt step of 4° and
 298 the bottom row for 8° . Fig. 6 shows a comparison of line plots of the B_x
 299 component of the vector field along the white line indicated in Fig. 6 for all
 300 the 4 methods. As before, the WBP method suffers the most due to limited
 301 angular sampling. This also directly affects the resolution along z axis. In
 302 spite of severe undersampling, the Gridrec method can be seen to reconstruct
 303 the magnetic field inside the nanosphere with very low error. This is clearly
 304 evident also from the line plots in Fig. 6. The reconstructions using ART
 305 and SIRT show similar variations in the magnetic field inside and outside the
 306 nanosphere and are able to reconstruct the values fairly close to the theo-
 307 retical value. As anticipated, SIRT method shows less noise as compared to
 308 ART method. This shows that even with large under sampling of tilt images
 309 recorded every 8° , the magnetic field inside the object can be reconstructed
 310 very faithfully. But to reconstruct the stray fields (magnetic field outside the
 311 object), one has to sample at least every 4° and use iterative methods such
 312 as ART/SIRT to obtain meaningful results.

313 3.4. *Effect of the relaxation parameter*

314 As mentioned earlier, it is not trivial to estimate the parameters such
315 as relaxation parameter, for iterative reconstruction methods. Here we have
316 tested various values of the relaxation parameter on the SIRT method. The
317 simulated phase shift data for the nanosphere with a tilt range of $\pm 72^\circ$ with
318 tilt steps of 8° was used to reconstruct the magnetic induction. Since these
319 methods are computationally expensive, we have only studied the effect on
320 the reconstruction of the x component of the magnetic induction. The re-
321 laxation paramter was varied from 0.05 to 2.0 while keeping the number of
322 iterations fixed. A successive error between two consecutive iterations was
323 computed and plotted against the iteration number to give an idea about
324 the convergence of the algorithm. Fig. 7(a) shows these plots for the various
325 values of lambda. It can be seen that the error decreases at a much slower
326 rate as the value of lambda is decreased. Fig. 7(b) shows the lineplot of
327 the x component of the vector field as shown previously. It is interesting to
328 note that with fixed number of iterations, varying the value of lambda does
329 not make significant difference on the reconstructed component of magneti-
330 zation. This is also evident from the RMS values which change from 0.033
331 for $\lambda = 2.0$ to 0.034 for $\lambda = 0.05$. This shows that the relaxation parameter
332 does not have a significant effect on the reconstructed components. However,
333 from the error decay, a value of $\lambda = 1.0$ can be seen to be an optimal value
334 for reconstructions. It should be noted here however, that the projection
335 images used for the SIRT reconstruction are simulated images and do not
336 have significant noise as compared to typical experimental images.

337 3.5. Domain wall in nanowires

338 From the previous section, it can be concluded that iterative methods can
339 significantly improve the reconstruction of magnetic field for limited tilt range
340 as well as under sampling. In particular, the magnetic field inside the object
341 can be reconstructed with very low error margin. Finally to demonstrate the
342 applicability of these methods to more complex magnetization patterns, we
343 performed simulations to reconstruct the complex 3D magnetization inside
344 a magnetic nanowire. In nanowires with circular cross section, there are two
345 types of domain walls that can exist: (1) Transverse domain walls, and (2)
346 Bloch point domain walls [21, 22]. Both these type of domain walls have a
347 complex magnetization pattern in 3D. Of particular interest are the Bloch
348 point walls as they are topologically protected, i.e. they are stable and cannot
349 deform under external perturbations. This is of significance for 3D magnetic
350 domain wall memory applications [23, 24].

351 We performed micromagnetic simulations using OOMMF to obtain the
352 complex magnetization pattern in such a nanowire. Fig. 8(a) shows the
353 schematic of the magnetization inside the nanowire and location of the do-
354 main wall. The magnetization thus obtained was used for computation of the
355 magnetic phase shift using a modified Mansuripur algorithm [25]. The phase
356 shift was computed as a function of varying angle of the incident electron
357 beam to simulate the tilt series about x and y axes. Fig. 8(b) shows the
358 magnetic phase shift computed for 0° tilt, (c) and (d) show the correspond-
359 ing simulated in-focus and under-focus Lorentz TEM images. The location
360 of the domain wall in the under focus image is indicated by the white arrow.
361 The tilt series data was simulated for a tilt range of $\pm 70^\circ$ with tilt steps of

362 2° . The reconstructions were then performed using all the 4 methods. Fig. 9
 363 shows a comparison of the reconstructed components in the $x - y$ plane for
 364 the four methods. The orientation of the nanowire and the planes is shown
 365 in the schematic. From the images, it is clear that there is significant error
 366 in the reconstructed components of the stray field (outside the nanowire).
 367 However, the reconstructed magnetization inside the nanowire matches very
 368 closely with that obtained from the micromagnetic simulations (indicated as
 369 theo). The B_z component was computed using the divergenceless condition as
 370 described previously in equation (12). It can be seen that with the improve-
 371 ment of reconstruction of B_x and B_y components particularly by the Gridrec
 372 method, the reconstruction of B_z component is also improved. Fig. 10 shows
 373 the 3D vector field plots of the magnetization inside the nanowire. The data
 374 is clipped along $y = 0$ to show the circular rotation of the magnetization in
 375 3D. Overall all the methods give better results as compared to WBP. Particu-
 376 larly the results obtained using Gridrec method show the best reconstruction
 377 of magnetization inside the nanowire. This demonstrates that with improved
 378 reconstruction methods, the complex 3D magnetization in nanoobjects such
 379 as nanowires can be faithfully reconstructed.

380 4. Conclusions

381 We have shown in this work, that the projection data for magnetic vec-
 382 tor field tomography can be reformulated in terms of Radon projection of
 383 the magnetic induction of a nanostructure. This allows for implementing
 384 the reconstruction as a linear algebra problem such that various iterative
 385 algebraic reconstruction methods can be used. We have demonstrated the

386 application of 4 common methods to reconstruct the vector field components
387 of the magnetic induction of a uniformly magnetized nanosphere. From the
388 analysis reconstructions for missing wedge and limited angular sampling, we
389 can conclude that the improved methods such as Gridrec and SIRT per-
390 form better with lower errors than others. Overall, it was concluded that
391 the iterative methods are better at reconstructing the magnetization inside
392 the nano-object as compared to the stray field outside the object. This
393 was demonstrated by reconstructing a more complex magnetization pattern
394 around a domain wall in a magnetic nanowire.

395 However, these methods only reconstruct and improve the individual x
396 and y components of the magnetic induction. Recent work by De Graef et.
397 al. [26] has shown that the vector field reconstruction can be formulated in a
398 model based iterative reconstruction method, where all the three components
399 of magnetic induction can be reconstructed simultaneously. Additionally, the
400 methods presented in this paper do not use any prior information or enforce
401 any regularization conditions. Future work along this direction is in progress
402 to incorporate smoothness criterion and couple the reconstructions to further
403 improve the results.

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Figure 1: Schematic figure showing the orientation of projection geometry. The figure shows the $y - z$ plane at $x = 0$ and the projection direction $d\xi$ at an angle of θ and a distance ρ from the origin.

Figure 2: (a) Phase shift of a uniformly magnetized sphere (outline indicated by dashed red line) at 0° tilt with the magnetization direction indicated by the blue arrow, (b) schematic representation of the projection data cube, (c) Projection data along plane 1 obtained using gradient of phase shift (top row) and integration of 3D magnetic induction components (bottom row), and (d) Projection data along plane 2 represented as sinograms obtained using gradient of phase shift (top row) and Radon transform of 3D magnetic induction components (bottom row).

Figure 3: Comparison of reconstructed components of the vector field shown as 2D images along various planes. The orientation of the planes is indicated in the schematic. For each method, the top row shows reconstruction obtained using a tilt range of $\pm 80^\circ$ and the bottom row using $\pm 70^\circ$. All the images are scaled to the same intensity range indicated by the color bar.

Figure 4: Comparison of line plots of the reconstructed B_x component along the direction indicated as a white line in Fig. 3.

Figure 5: Comparison of reconstructed components of the vector field shown as 2D images along various planes. The orientation of the planes is indicated in the schematic. For each method, the top row shows reconstruction obtained using a tilt step of 4° and the bottom row using 8° . All the images are scaled to the same intensity range indicated by the color bar.

Figure 6: Comparison of line plots of the reconstructed B_x component along the direction indicated as a white line in Fig. 5.

Figure 7: (a) Plot showing the decay of error as a function of iteration number for SIRT method for varying values of the relaxation parameter, λ , and (b) comparison of line plots of the reconstructed B_x component along the direction indicated in Fig. 5.

Figure 8: (a) Schematic showing the magnetization setup in the nanowire and the location of the domain wall (DW), (b) shows the simulated magnetic phase shift from the magnetization obtained using micromagnetic simulations at 0° tilt, (c) and (d) show the simulated in-focus and under-focus LTEM images. The presence of domain wall is indicated by the white arrow in (d).

Figure 9: Comparison of reconstructed components of the vector field shown as 2D images along the $x - y$ plane. The orientation of the planes and the nanowire is indicated in the schematic. All the images are scaled to the same intensity range indicated by the color bar.

Figure 10: 3D vector field plots comparing the reconstructed magnetization in the nanowire for different methods. The data is clipped along the $x - z$ plane at $y = 2$ for clarity.