A comprehensive analysis of the electric field distribution in an electrodynamic screen

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Abstract

The electrodynamic screen, or EDS, has shown promising results in mitigation of dust accumulation losses in solar energy harvesting systems. In this paper, the electric field distributions in two EDS configurations have been thoroughly investigated. The analytical solutions for the electric potential and electric field distribution in the first EDS configuration have been provided and corroborated numerically using finite element analysis (FEA) software. The electrostatic model of second EDS configuration has been developed in the FEA software and its electric field distribution has been analyzed numerically. A comparison has been made between the two configurations regarding dust removal efficiency.

Index Terms

Self-cleaning, Electrostatics, Dust, Electrodynamic

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1. INTRODUCTION

Dust accumulation on the optical surfaces of solar energy harvesting devices such as flat-plate photovoltaic (PV) panels, concentrated solar mirrors, and concentrated photovoltaics (CPV) systems has an adverse impact on their performance by scattering and absorption of the incident sunlight. A literature survey on the dust accumulation problem by Sayyah *et al* [1] has shown that dust deposition can cause up to 1% loss in the daily output power in flat-plate PV installations located in arid and semi-arid areas of the world, where the average of annual solar insolation is more than 4.5 kWh/m²/day. Accumulation of carbon particulates, caused by incomplete combustion of fuels in motor vehicles and industrial plants also deteriorates the performance of the PV installation in urban areas [2], [3]. The dust accumulation problem is more detrimental for the performance of concentrated solar harvesting systems as they are sensitive to direct normal irradiance (DNI) while flat-plate PV panels are capable of using scattered sunlight in power generation [4].

In order to restore the efficiency of the solar energy harvesting systems, use of high-pressure water jets, sometimes mixed with detergents is the most commonly-practiced manual cleaning method in large-scale solar plants. This method, however, requires significant amounts of distilled or demineralized water, which is scant in arid zones

and also is labor-intensive as it requires a team of trained personnel to perform the cleaning task. Furthermore, water cleaning use is not scalable, i.e., lack of it will impede growth of CSP and PV plants. Recently, there have been some advancements in utilizing robotic devices in cleaning flat-plate PV panels [5] yet they are still in the developmental stage, require water resources for cleaning, and their scalability in large-scale solar plants has not been well established.

In order to cope with the shortcomings of commonly-practiced cleaning methods, the concept of automatic dust removal using electrostatic forces is in developmental stages and on a commercialization path. The general idea of automatic dust removal was initially introduced by Tatom *et al.* [6] at NASA and then developed by Masuda *et al.* [7] [8] at the University of Tokyo in the 1970s for transporting charged aerosol particles. Although the concept has found a niche application for dust removal on solar modules to be used in Mars exploration missions [9], [10], [11], [12], there have been significant endeavors to adopt this technology to mitigate dust accumulation losses in solar energy harvesting plants [13], [14]. Electrodynamic screen (EDS) technology represents a viable promising solution as it does not require water resources or mechanical movement in removing dust particles, and it is an extremely low-power technology that can be fed from the harvesting device itself and does not need external power source. The EDS consists of inter-digitated electrodes deposited on a glass substrate, encapsulated by one or more transparent dielectric coating(s) that protect the electrodes from direct exposure to the outdoor environment. The EDS can be easily integrated onto a PV panel or the reflecting mirror in concentrated solar power (CSP) applications. The electrodes are connected to a single-phase or a multi-phase power supply that energizes the electrodes with a specified waveform pattern and frequency. Once the deposited dust particles are charged sufficiently, they are removed by the Coulomb force from the EDS surface.

Electric field distribution model plays a decisive role in performance evaluation of the EDS and how to increase its dust removal efficiency by manipulating the design parameters as well as material selection. This paper deals with the analysis of electric field distributions in two EDS configurations. The organization of this paper is as follows: the analytical solutions for the electric potential and electric field components for the first EDS configuration are presented in Section 2. The corroboration of analytical solutions has been addressed in Section 3. Section 4 provides the simulation results for the electric field distribution in the second EDS configuration and a comparison between two configurations. Finally, Section 5 provides the conclusions of this paper.

2. ANALYTICAL SOLUTION FOR THE ELECTRIC FIELD

In this section, we provide closed-form solutions for the electric field and electric potential in the first EDS configuration. The analytical solutions show explicitly how various design parameters play role in the electric field distribution and provides valuable insight through the physical model of the problem. The solutions can be easily implemented for the dust particle trajectory studies.

A. First EDS Configuration

Figure 1 depicts the configuration of the EDS in the xy -plane. The electrodes are deposited on a glass substrate while their height is neglected in the calculations. In Fig. 1, the width of the electrodes and inter-electrode spacing are denoted as w and g , respectively. The electrodes are encapsulated by two layers of transparent dielectric materials 1 and 2, with relative permittivities of ϵ_{d1} and ϵ_{d2} , respectively. As an example in the practice, dielectric 1 is optically clear adhesive (OCA) and dielectric 2 is a very thin borosilicate glass. The electrodes have the same size and inter-electrode spacing and are connected to a three-phase power supply that provides traveling-waves to repel and convey dust particles. As the output of the power supply for energizing the electrodes is known at any given time t , regardless of the sinusoidal, rectangular, saw tooth, etc., the electrodes' voltages are known at each time instant and are represented as a function of time.

Fig. 1. Schematic of the EDS configuration with two stacked layers of transparent dielectric coatings. The solar module could be a photovoltaic cell or a reflecting mirror film in a concentrated solar power application. The dust particles, not depicted in the sketch, deposit on the dielectric layer 2. The electrodes are connected to a three-phase power supply. The problem is assumed to be infinite in the z direction (normal to the page).

Since the electrodes are connected to a three-phase power supply, the fundamental spatial period, denoted as T_s , is expressed as

$$T_s = 3(w + g). \quad (1)$$

We assume that the electric field distribution is not affected by the charge of deposited dust particles on the EDS surface, i.e., they carry no net charge. Hence, in the absence of net electric charge in the region above the EDS surface, we can write Laplace's equation for the electric potential in the air, denoted as $\phi_a(x, y)$ as follows:

$$\frac{\partial^2 \phi_a(x, y)}{\partial x^2} + \frac{\partial^2 \phi_a(x, y)}{\partial y^2} = 0, \quad 0 \leq x \leq T_s, \quad \delta_1 + \delta_2 \leq y < \infty \quad (2)$$

We have added the subscript “a” that highlights the electric potential in the air. Similarly, the subscripts “d1” and “d2” denote the potentials in the dielectric layers 1 and 2, respectively. The goal is to find the electric potential and the electric field components in the two layers of the dielectric materials and air. The electric field components in the air are the ones exerted on the dust particles and effective in the dust removal process. However, the electric field solutions within two dielectric layers can be used for further analysis of electric stress and dielectric breakdown.

B. Electric Potential

The electric potentials at $y = 0$, $y = \delta_1$, and $y = \delta_1 + \delta_2$ (air/dielectric 2 boundary) in Fig. 1 are functions of x and are respectively expressed as:

$$v_m(x) = \sum_{k=0}^{\infty} a_k \cos(k\Omega_0 x) + b_k \sin(k\Omega_0 x), \quad (3)$$

$$u_m(x) = \sum_{k=0}^{\infty} g_k \cos(k\Omega_0 x) + h_k \sin(k\Omega_0 x), \quad (4)$$

$$w_m(x) = \sum_{k=0}^{\infty} p_k \cos(k\Omega_0 x) + q_k \sin(k\Omega_0 x), \quad (5)$$

where a_k , b_k , g_k , h_k , p_k , and q_k are the Fourier coefficients, and $\Omega_0 = \frac{2\pi}{T_s}$. The Fourier coefficients a_k and b_k are known at each time instant as the electric potential applied to the electrodes is known, and g_k , h_k , p_k , and q_k are unknown Fourier coefficients that need to be calculated. We can assume that the electric potential varies linearly between adjacent electrodes at $y = 0$. Because applied voltages are time varying, the introduced Fourier coefficients are also functions of time; for instance $a_k(t)$ and $b_k(t)$. For the ease of representation, however, we drop the dependency upon “t”. We may write the electric potential inside the dielectric materials in such a way that both boundary conditions at their boundaries are satisfied. We assume the potential inside the first dielectric is a summation of two functions, one a function of v_m and the other a function of u_m , as defined in Eqs. 3 and 4, respectively:

$$\phi_{d1}(x, y) = f_1(v_m) + f_2(u_m), \quad 0 \leq x \leq T_s, \quad 0 \leq y \leq \delta_1 \quad (6)$$

in which the functions f_1 and f_2 are variables of both x and y . The function $f_1(v_m)$ is written as:

$$f_1(v_m) = v_m(A_1 e^{-k\Omega_0 y} + A_2 e^{k\Omega_0 y}), \quad (7)$$

where the constants A_1 and A_2 need to be obtained. Since we must have the boundary conditions at $y = 0$ and

$y = \delta_1$ satisfied, we must have $f_1(v_m) = v_m$ and $f_1(v_m) = 0$, respectively. Hence,

$$\begin{cases} A_1 + A_2 = 1, & \text{if } y = 0 \\ A_1 e^{-k\Omega_0 \delta_1} + A_2 e^{k\Omega_0 \delta_1} = 0 & \text{if } y = \delta_1 \end{cases} \quad (8)$$

The constants A_1 and A_2 in the Eq. (8) are obtained as:

$$A_1 = \frac{1}{2} e^{k\Omega_0 \delta_1} \operatorname{csch}(k\Omega_0 \delta_1), \quad (9)$$

$$A_2 = -\frac{1}{2} e^{-k\Omega_0 \delta_1} \operatorname{csch}(k\Omega_0 \delta_1). \quad (10)$$

By looking into coefficients A_1 and A_2 obtained for $f_1(v_m)$, we can write the function $f_2(u_m)$ as:

$$f_2(u_m) = u_m \frac{e^{k\Omega_0 y} - e^{-k\Omega_0 y}}{e^{k\Omega_0 \delta_1} - e^{-k\Omega_0 \delta_1}}, \quad (11)$$

in which $f_2(u_m) = 0$ if $y = 0$ and $f_2(u_m) = u_m$ at $y = \delta_1$. Therefore,

$$f_2(u_m) = u_m \frac{\sinh(k\Omega_0 y)}{\sinh(k\Omega_0 \delta_1)}. \quad (12)$$

Using the expressions obtained for $f_1(v_m)$ and $f_2(u_m)$, the electric potential inside the first dielectric layer is

expressed as:

$$\phi_{d1}(x, y) = -v_m \frac{\sinh[k\Omega_0(y - \delta_1)]}{\sinh(k\Omega_0 \delta_1)} + u_m \frac{\sinh(k\Omega_0 y)}{\sinh(k\Omega_0 \delta_1)}, \quad 0 \leq x \leq T_s, \quad 0 \leq y \leq \delta_1 \quad (13)$$

Substitution of the Fourier series representation introduced for the v_m and u_m in Eqs. 3 and 4, respectively,

yields the electric potential in the first dielectric material:

$$\begin{aligned} \phi_{d1}(x, y) = & - \sum_k \frac{\sinh[k\Omega_0(y - \delta_1)]}{\sinh(k\Omega_0\delta_1)} [a_k \cos(k\Omega_0x) + b_k \sin(k\Omega_0x)] \\ & + \sum_k \frac{\sinh(k\Omega_0y)}{\sinh(k\Omega_0\delta_1)} [g_k \cos(k\Omega_0x) + h_k \sin(k\Omega_0x)], \quad 0 \leq x \leq T_s, \quad 0 \leq y \leq \delta_1 \end{aligned} \quad (14)$$

A similar approach can be applied to the second dielectric layer, yielding the electric potential in the latter:

$$\phi_{d2}(x, y) = f_3(u_m) + f_4(w_m), \quad 0 \leq x \leq T_s, \quad \delta_1 \leq y \leq \delta_1 + \delta_2 \quad (15)$$

in which the functions f_3 and f_4 are variables of both x and y . The function $f_3(u_m)$ is expressed as

$$f_3(u_m) = u_m(A_3 e^{-k\Omega_0 y} + A_4 e^{k\Omega_0 y}), \quad (16)$$

where the two constants A_3 and A_4 are unknown. The two boundary conditions in the second dielectric layer impose $f_3(u_m) = u_m$ and $f_3(u_m) = 0$ at $y = \delta_1$ and $y = \delta_1 + \delta_2$, respectively. Hence,

$$\begin{cases} A_3 e^{-k\Omega_0 \delta_1} + A_4 e^{k\Omega_0 \delta_1} = 1, & \text{if } y = \delta_1 \\ A_3 e^{-k\Omega_0 (\delta_1 + \delta_2)} + A_4 e^{k\Omega_0 (\delta_1 + \delta_2)} = 0 & \text{if } y = \delta_1 + \delta_2 \end{cases} \quad (17)$$

The constants A_3 and A_4 are the solution set for the Eqs. 17:

$$A_3 = \frac{e^{k\Omega_0 (\delta_1 + \delta_2)}}{e^{k\Omega_0 \delta_2} - e^{-k\Omega_0 \delta_2}}, \quad (18)$$

$$A_4 = -\frac{e^{-k\Omega_0 (\delta_1 + \delta_2)}}{e^{k\Omega_0 \delta_2} - e^{-k\Omega_0 \delta_2}}. \quad (19)$$

For the second term in Eq. 15, i.e. $f_4(w_m)$, we would like to have $f_4(w_m) = 0$ and $f_4(w_m) = w_m$ for $y = \delta_1$ and $y = \delta_1 + \delta_2$, respectively. Hence, we can write $f_4(w_m)$ as

$$f_4(w_m) = w_m \frac{e^{k\Omega_0 (y - \delta_1)} - e^{-k\Omega_0 (y - \delta_1)}}{e^{k\Omega_0 \delta_2} - e^{-k\Omega_0 \delta_2}} \quad (20)$$

Therefore, the electric potential inside the second dielectric material after some algebraic simplifications is expressed as

$$\phi_{d2}(x, y) = -u_m \frac{\sinh[k\Omega_0(y - (\delta_1 + \delta_2))]}{\sinh(k\Omega_0\delta_2)} + w_m \frac{\sinh[k\Omega_0(y - \delta_1)]}{\sinh(k\Omega_0\delta_2)} \quad (21)$$

Using the expression introduced for the u_m and w_m in Eqs. 4 and 5, the electric potential in the second dielectric layer is

$$\begin{aligned} \phi_{d2}(x, y) = & - \sum_k \frac{\sinh[k\Omega_0[y - (\delta_1 + \delta_2)]]}{\sinh(k\Omega_0\delta_2)} [g_k \cos(k\Omega_0x) + h_k \sin(k\Omega_0x)] + \\ & \sum_k \frac{\sinh[k\Omega_0(y - \delta_1)]}{\sinh(k\Omega_0\delta_2)} [p_k \cos(k\Omega_0x) + q_k \sin(k\Omega_0x)] \quad 0 \leq x \leq T_s, \quad \delta_1 \leq y \leq \delta_1 + \delta_2 \quad (22) \end{aligned}$$

Horenstein *et al.* [15] have provided the electric field solution for an EDS with no dielectric coating by solving Laplace's equation, where the deposited electrodes are exposed to the air. The electrodes are always coated so as to be protected from outdoor exposure. However, their solution can be adopted herein as a representation of the electric field in the air. The solution for the electric potential for the current configuration as in Fig. 1 is written as

$$\phi_a(x, y) = \sum_k e^{-k\Omega_0[y - (\delta_1 + \delta_2)]} [p_k \cos(k\Omega_0x) + q_k \sin(k\Omega_0x)], \quad 0 \leq x \leq T_s, \quad \delta_1 + \delta_2 \leq y < \infty \quad (23)$$

It should be noted that the electric field potential in the air satisfies the following boundary conditions:

- 1) At $y = \delta_1 + \delta_2$: $\phi_a(x, \delta_1 + \delta_2) = \sum_k [p_k \cos(k\Omega_0x) + q_k \sin(k\Omega_0x)] = w_m$
- 2) When $y \rightarrow \infty$: $\phi_a(x, y) = 0$

The electric potentials in the dielectric 1, dielectric 2, and air are obtained via the Eqs. 14, 22, and 23, respectively.

C. Electric Field

Because we are able to write the electric potentials in the three dielectric media, the electric field components can also be calculated as follows. For the dielectric layer 1, the electric field $\mathbf{E}_{d1}(x, y)$ is expressed as:

$$\mathbf{E}_{d1}(x, y) = E_{d1,x} \mathbf{a}_x + E_{d1,y} \mathbf{a}_y, \quad (24)$$

in which $E_{d1,x}$ and $E_{d1,y}$ are the electric field components in the x and y directions, respectively. The $\mathbf{a}_x$ and $\mathbf{a}_y$ are the unit vectors in the x and y directions, respectively. Similarly, for the electric fields in the dielectric layer 2 and air denoted as $\mathbf{E}_{d2}$ and $\mathbf{E}_a$, respectively, we have:

$$\mathbf{E}_{d2}(x, y) = E_{d2,x}\mathbf{a}_x + E_{d2,y}\mathbf{a}_y, \quad (25)$$

and

$$\mathbf{E}_a(x, y) = E_{a,x}\mathbf{a}_x + E_{a,y}\mathbf{a}_y. \quad (26)$$

Given that $\mathbf{E} = -\nabla\phi$, the electric potential (Eq. 14) results in electric field components of:

$$\begin{aligned} E_{d1,x} &= -\frac{\partial\phi_{d1}(x, y)}{\partial x} \\ &= \Omega_0 \sum_k \frac{\sinh[k\Omega_0(y - \delta_1)]}{\sinh(k\Omega_0\delta_1)} [-ka_k \sin(k\Omega_0x) + kb_k \cos(k\Omega_0x)] \\ &\quad - \Omega_0 \sum_k \frac{\sinh(k\Omega_0y)}{\sinh(k\Omega_0\delta_1)} [-kg_k \sin(k\Omega_0x) + kh_k \cos(k\Omega_0x)] \end{aligned} \quad (27)$$

and

$$\begin{aligned} E_{d1,y} &= -\frac{\partial\phi_{d1}(x, y)}{\partial y} \\ &= \Omega_0 \sum_k \frac{\cosh[k\Omega_0(y - \delta_1)]}{\sinh(k\Omega_0\delta_1)} [ka_k \cos(k\Omega_0x) + kb_k \sin(k\Omega_0x)] \\ &\quad - \Omega_0 \sum_k \frac{\cosh(k\Omega_0y)}{\sinh(k\Omega_0\delta_1)} [kg_k \cos(k\Omega_0x) + kh_k \sin(k\Omega_0x)] \end{aligned} \quad (28)$$

Based on the potential in the dielectric layer 2 expressed in Eq. 22, the x and y electric field components are:

$$\begin{aligned} E_{d2,x} &= -\frac{\partial\phi_{d2}(x, y)}{\partial x} \\ &= \Omega_0 \sum_k \frac{\sinh[k\Omega_0[y - (\delta_1 + \delta_2)]]}{\sinh(k\Omega_0\delta_2)} [-kg_k \sin(k\Omega_0x) + kh_k \cos(k\Omega_0x)] \\ &\quad - \Omega_0 \sum_k \frac{\sinh[k\Omega_0(y - \delta_1)]}{\sinh(k\Omega_0\delta_2)} [-kp_k \sin(k\Omega_0x) + kq_k \cos(k\Omega_0x)], \end{aligned} \quad (29)$$

and

$$\begin{aligned}
E_{d2,y} &= -\frac{\partial \phi_{d2}(x, y)}{\partial y} \\
&= \Omega_0 \sum_k \frac{\cosh[k\Omega_0[y - (\delta_1 + \delta_2)]]}{\sinh(k\Omega_0\delta_2)} [kg_k \cos(k\Omega_0x) + kh_k \sin(k\Omega_0x)] \\
&\quad - \Omega_0 \sum_k \frac{\cosh[k\Omega_0(y - \delta_1)]}{\sinh(k\Omega_0\delta_2)} [kp_k \cos(k\Omega_0x) + kq_k \sin(k\Omega_0x)].
\end{aligned} \tag{30}$$

Electric field in the air is calculated based on the electric potential in Eq. 23:

$$\begin{aligned}
E_{a,x} &= -\frac{\partial \phi_a(x, y)}{\partial x} \\
&= \Omega_0 \sum_k e^{-k\Omega_0[y - (\delta_1 + \delta_2)]} [kp_k \sin(k\Omega_0x) - kq_k \cos(k\Omega_0x)],
\end{aligned} \tag{31}$$

and

$$\begin{aligned}
E_{a,y} &= -\frac{\partial \phi_a(x, y)}{\partial y} \\
&= \Omega_0 \sum_k e^{-k\Omega_0[y - (\delta_1 + \delta_2)]} [kp_k \cos(k\Omega_0x) + kq_k \sin(k\Omega_0x)].
\end{aligned} \tag{32}$$

The electric field components are evaluated at the interface of the two dielectric materials, i.e. $y = \delta_1$ and the interface of dielectric 2 and air, i.e. $y = \delta_1 + \delta_2$, in order to match the boundary conditions in the next step. The electric field components for the first dielectric material at $y = \delta_1$ are:

$$E_{d1,x}(x, \delta_1) = \Omega_0 \sum_k [kg_k \sin(k\Omega_0x) - kh_k \cos(k\Omega_0x)] \tag{33}$$

$$\begin{aligned}
E_{d1,y}(x, \delta_1) &= \Omega_0 \sum_k \operatorname{csch}(k\Omega_0\delta_1) [ka_k \cos(k\Omega_0x) + kb_k \sin(k\Omega_0x)] \\
&\quad - \Omega_0 \sum_k \coth(k\Omega_0\delta_1) [kg_k \cos(k\Omega_0x) + kh_k \sin(k\Omega_0x)]
\end{aligned} \tag{34}$$

At $y = \delta_1$, the electric field components for the second dielectric are:

$$E_{d2,x}(x, \delta_1) = \Omega_0 \sum_k [kg_k \sin(k\Omega_0 x) - kh_k \cos(k\Omega_0 x)] \quad (35)$$

$$E_{d2,y}(x, \delta_1) = \Omega_0 \sum_k \coth[k\Omega_0 \delta_2] [kg_k \cos(k\Omega_0 x) + kh_k \sin(k\Omega_0 x)] - \Omega_0 \sum_k \operatorname{csch}(k\Omega_0 \delta_2) [kp_k \cos(k\Omega_0 x) + kq_k \sin(k\Omega_0 x)] \quad (36)$$

At $y = \delta_1 + \delta_2$, the x and y electric field components in the second dielectric layer and air are obtained, respectively, as follows:

$$E_{d2,x}(x, \delta_1 + \delta_2) = \Omega_0 \sum_k [kp_k \sin(k\Omega_0 x) - kq_k \cos(k\Omega_0 x)], \quad (37)$$

$$E_{d2,y}(x, \delta_1 + \delta_2) = \Omega_0 \sum_k \operatorname{csch}(k\Omega_0 \delta_2) [kg_k \cos(k\Omega_0 x) + kh_k \sin(k\Omega_0 x)] - \Omega_0 \sum_k \coth(k\Omega_0 \delta_2) [kp_k \cos(k\Omega_0 x) + kq_k \sin(k\Omega_0 x)], \quad (38)$$

and

$$E_{a,x}(x, \delta_1 + \delta_2) = \Omega_0 \sum_k [kp_k \sin(k\Omega_0 x) - kq_k \cos(k\Omega_0 x)], \quad (39)$$

$$E_{a,y}(x, \delta_1 + \delta_2) = \Omega_0 \sum_k [kp_k \cos(k\Omega_0 x) + kq_k \sin(k\Omega_0 x)]. \quad (40)$$

At this point, since the electric field components are available, we can match the boundary conditions at the two boundaries $y = \delta_1$ and $y = \delta_1 + \delta_2$. At $y = \delta_1$, the tangential components must satisfy $E_{d1,x} = E_{d2,x}$. By looking into the expressions provided for $E_{d1,x}$ and $E_{d2,x}$ in Eqs. 33 and 35, respectively, it is clear that the boundary condition for the tangential components is automatically satisfied. The boundary condition for the normal components at $y = \delta_1$ must satisfy $\epsilon_0 \epsilon_{d1} E_{d1,y} = \epsilon_0 \epsilon_{d2} E_{d2,y}$. Therefore,

$$\epsilon_{d1} \left\{ \sum_k \operatorname{csch}(k\Omega_0 \delta_1) [ka_k \cos(k\Omega_0 x) + kb_k \sin(k\Omega_0 x)] - \sum_k \coth(k\Omega_0 \delta_1) [kg_k \cos(k\Omega_0 x) + kh_k \sin(k\Omega_0 x)] \right\} = \epsilon_{d2} \left\{ \sum_k \coth(k\Omega_0 \delta_2) [kg_k \cos(k\Omega_0 x) + kh_k \sin(k\Omega_0 x)] - \sum_k \operatorname{csch}(k\Omega_0 \delta_2) [kp_k \cos(k\Omega_0 x) + kq_k \sin(k\Omega_0 x)] \right\} \quad (41)$$

Re-arranging the expressions for the $\cos(\cdot)$ and $\sin(\cdot)$ terms in Eq. 41, we have:

$$\begin{aligned} \epsilon_{d1} \left\{ \sum_k [\operatorname{csch}(k\Omega_0\delta_1)ka_k - \coth(k\Omega_0\delta_1)kg_k] \cos(k\Omega_0x) + [\operatorname{csch}(k\Omega_0\delta_1)kb_k - \coth(k\Omega_0\delta_2)kh_k] \sin(k\Omega_0x) \right\} = \\ \epsilon_{d2} \left\{ \sum_k [\coth(k\Omega_0\delta_2)kg_k - \operatorname{csch}(k\Omega_0\delta_2)kp_k] \cos(k\Omega_0x) + [\coth(k\Omega_0\delta_2)kh_k - \operatorname{csch}(k\Omega_0\delta_2)kq_k] \sin(k\Omega_0x) \right\} \end{aligned} \quad (42)$$

Equating the coefficients for the $\cos(\cdot)$ terms in Eq. 42 yields:

$$\epsilon_{d1} [\operatorname{csch}(k\Omega_0\delta_1)ka_k - \coth(k\Omega_0\delta_1)kg_k] = \epsilon_{d2} [\coth(k\Omega_0\delta_2)kg_k - \operatorname{csch}(k\Omega_0\delta_2)kp_k] \quad (43)$$

Similarly, for the coefficients for the $\sin(\cdot)$ terms in Eq. 42, we have:

$$\epsilon_{d1} [\operatorname{csch}(k\Omega_0\delta_1)kb_k - \coth(k\Omega_0\delta_2)kh_k] = \epsilon_{d2} [\coth(k\Omega_0\delta_2)kh_k - \operatorname{csch}(k\Omega_0\delta_2)kq_k] \quad (44)$$

The tangential components of the electric fields in the dielectric 2 and air at $y = \delta_1 + \delta_2$ must satisfy

$$E_{a,x}(x, \delta_1 + \delta_2) = E_{d2,x}(x, \delta_1 + \delta_2), \quad (45)$$

in which the terms $E_{d2,x}(x, \delta_1 + \delta_2)$ and $E_{a,x}(x, \delta_1 + \delta_2)$ are provided in Eqs. 37 and 39, respectively. It is

clear that the condition is satisfied automatically. For the normal components at $y = \delta_1 + \delta_2$, we must have:

$\epsilon_0 E_{a,y} = \epsilon_0 \epsilon_{d2} E_{d2,y}$ or $E_{a,y} = \epsilon_{d2} E_{d2,y}$. Hence,

$$\begin{aligned} \sum_k [kp_k \cos(k\Omega_0x) + kq_k \sin(k\Omega_0x)] = \epsilon_{d2} \cdot \left\{ \sum_k \operatorname{csch}(k\Omega_0\delta_2) [kg_k \cos(k\Omega_0x) + kh_k \sin(k\Omega_0x)] - \right. \\ \left. \sum_k \coth(k\Omega_0\delta_2) [kp_k \cos(k\Omega_0x) + kq_k \sin(k\Omega_0x)] \right\} \end{aligned} \quad (46)$$

By re-arranging the terms for the $\cos(\cdot)$ and $\sin(\cdot)$ terms in Eq. 46, we arrive at the following expressions for

p_k and q_k :

$$p_k = \frac{\epsilon_{d2}}{\sinh(k\Omega_0\delta_2) + \epsilon_{d2} \cosh(k\Omega_0\delta_2)} g_k \quad (47)$$

and

$$q_k = \frac{\epsilon_{d2}}{\sinh(k\Omega_0\delta_2) + \epsilon_{d2} \cosh(k\Omega_0\delta_2)} h_k. \quad (48)$$

By defining the coefficient C_1 as

$$C_1 = \frac{\epsilon_{d2}}{\sinh(k\Omega_0\delta_2) + \epsilon_{d2} \cosh(k\Omega_0\delta_2)}, \quad (49)$$

we have $p_k = C_1 g_k$ and $q_k = C_1 h_k$. Also, we can define the coefficients C_2 to C_4 as:

$$C_2 = \epsilon_{d1} \operatorname{csch}(k\Omega_0\delta_1) \quad (50)$$

$$C_3 = \epsilon_{d2} \operatorname{csch}(k\Omega_0\delta_2) \quad (51)$$

$$C_4 = \epsilon_{d2} \coth(k\Omega_0\delta_2) + \epsilon_{d1} \coth(k\Omega_0\delta_1) \quad (52)$$

Based on the definitions provided for the coefficients C_2 to C_4 in Eqs. 50 to 52, Eqs. 43 and 44 can be updated

as follows:

$$C_2 a_k + C_3 p_k = C_4 g_k, \quad (53)$$

$$C_2 b_k + C_3 q_k = C_4 h_k. \quad (54)$$

Since the Fourier coefficients a_k and b_k are known, we have the unknown Fourier coefficients g_k , h_k , p_k , and

q_k , respectively, as:

$$g_k = \frac{C_2}{C_4 - C_1 C_3} a_k \quad (55)$$

$$h_k = \frac{C_2}{C_4 - C_1 C_3} b_k \quad (56)$$

$$p_k = \frac{C_1 C_2}{C_4 - C_1 C_3} a_k \quad (57)$$

$$q_k = \frac{C_1 C_2}{C_4 - C_1 C_3} b_k \quad (58)$$

in which the coefficients C_1 to C_4 have been provided in Eqs. 49 to 52, respectively.

By noting that $\Omega_0 = \frac{2\pi}{3(w+g)}$, it is obvious that the electric potential and the electric field components discussed above are dependent upon the geometric parameters w , g , δ_1 , and δ_2 and the electrical permittivities of the two dielectric coatings ϵ_{d1} and ϵ_{d2} . In the next section, we have provided expressions based upon the pulse patterns of the three-phase power supply to show how the Fourier coefficients a_k and b_k are calculated.

3. NUMERICAL SOLUTION

A. Verification of Analytical Approach

In order to verify the closed-form solutions provided in Section 2, the EDS configuration presented in Fig. 1 has been developed in the COMSOL Multiphysics[®] finite element analysis (FEA) software. We are interested to compare the solutions from the two methods to see which one is more useful in what circumstances. The results have been shown in Fig. 2 for the electric field norm, defined as $|\mathbf{E}| = (E_x^2 + E_y^2)^{0.5}$ and the electric potential when the phases A, B, and C have the voltages 1 kV, 0, and 1 kV, respectively. As can be expected and also is shown in Fig. 2, the electric field intensity reaches its maximum in the vicinity of the electrodes tips. Although studying the electric field intensity in the dielectric layers and its dependency upon the geometric as well as operational parameters will provide valuable information on the dielectric breakdown and stress, it is not in the scope of this study. In this paper, we are mainly investigating the electric field distribution on the EDS, where dust particles are deposited and exposed to different attracting and repelling forces.

The electrodes of the EDS are connected to a three-phase power supply that energizes the electrodes with rectangular pulses at 50% duty cycle. Figure 3 shows these waveforms over a single period T_t . As can be seen in Fig. 3, there are a total 6 different voltage combinations in which each phase has 0 or V volts. Suppose the voltages of the first, second, and third electrode in one fundamental spatial period T_s are denoted as V_1 , V_2 , and V_3 , respectively, in which they switch between 0 and V , as illustrated in Fig. 3. As shown in previous section, the analytical solution is dependent upon the Fourier coefficients a_k and b_k . To obtain these coefficients, we need to have the electric potential at $y = 0$ (Fig. 1). The electric potential at the $y = 0$ with the assumption of linear voltage change between two adjacent electrodes can be expressed as:

Fig. 2. Numerical simulation results using finite element analysis software for the (a) electric field norm ($|\mathbf{E}|$) and (b) electric potential of the EDS model in one fundamental spatial period T_s . The unit of the vertical and horizontal axes is meter. The height of the electrodes have been considered to be $5 \mu\text{m}$ and the maximum of the electric field norm reaches 3.2298×10^7 V/m. The voltages of phases A, B, and C with Chromium electrodes are 1 kV, 0, and 1 kV, respectively. The electrode width and inter-electrode spacing are considered to be $w = 50 \mu\text{m}$ and $g = 200 \mu\text{m}$, respectively. The thicknesses of the first and second user-defined dielectric layers are $\delta_1 = 50 \mu\text{m}$ and $\delta_2 = 100 \mu\text{m}$, respectively, and their relative permittivities are $\epsilon_{d1} = 5.14$ and $\epsilon_{d2} = 6.7$.

$$\phi_{d1}(x, 0) = \begin{cases} V_1 & 0 \leq x \leq w \\ \frac{V_2 - V_1}{g}(x - w) + V_1 & w \leq x \leq w + g \\ V_2 & w + g \leq x \leq 2w + g \\ \frac{V_3 - V_2}{g}[x - (2w + g)] + V_2 & 2w + g \leq x \leq 2(w + g) \\ V_3 & 2(w + g) \leq x \leq 3w + 2g \\ \frac{V_1 - V_3}{g}[x - (3w + 2g)] + V_3 & 3w + 2g \leq x \leq T \end{cases} \quad (59)$$

The Fourier coefficients a_k and b_k , addressed in Eq. 3, are obtained using the following expressions:

$$a_k = \frac{2}{T_s} \int_{T_s} \phi_{d1}(x, 0) \cos(\Omega x) dx, \quad (60)$$

$$b_k = \frac{2}{T_s} \int_{T_s} \phi_{d1}(x, 0) \sin(\Omega x) dx, \quad (61)$$

in which $\Omega = k\Omega_0$ and $\phi_{d1}(x, 0)$ is provided in Eq. 59. By substituting Eq. 59 into Eqs. 60 and 61, we have the expressions for a_k and b_k , respectively, as follows:

$$a_k = \frac{1}{\pi g \Omega_0 k^2} \left\{ (V_2 - V_1) [\cos(k\Omega_0(w + g)) - \cos(k\Omega_0 w)] + \right. \\ \left. (V_3 - V_2) [\cos(2k\Omega_0(w + g)) - \cos(k\Omega_0(2w + g))] + (V_1 - V_3) [1 - \cos(k\Omega_0(3w + 2g))] \right\} \quad (62)$$

$$b_k = \frac{1}{\pi g \Omega_0 k^2} \left\{ (V_2 - V_1) [\sin(k\Omega_0(w + g)) - \sin(k\Omega_0 w)] + \right. \\ \left. (V_3 - V_2) [\sin(2k\Omega_0(w + g)) - \sin(k\Omega_0(2w + g))] - (V_1 - V_3) \sin(k\Omega_0(3w + 2g)) \right\} \quad (63)$$

Fig. 3. Three-phase rectangular voltage pulses with 50% duty cycle in one period T_t .

In order to verify the analytical results obtained in previous section, Fig. 4 has been provided, in which the electric field norm on the EDS surface (i.e. $y = \delta_1 + \delta_2$) is calculated from two approaches: analytical (ANA) and finite element analysis (FEA) for six different voltage combinations introduced in Fig. 3. To obtain the electric field

components from the ANA method, the Fourier coefficients a_k and b_k were initially calculated for each case of voltage combination from Eqs. 62 and 63. Then, Fourier coefficients g_k , h_k , p_k , and q_k are calculated according to Eq. 55 to 58, respectively. The expressions to calculate coefficients C_1 to C_4 are provided in Eqs. 49 to 52, respectively. The electric field components on the EDS surface (i.e. $y = \delta_1 + \delta_2$) can be calculated using the expressions for the electric field distribution in the air as provided in Eqs. 39 and 40 or using the electric field components in the second dielectric coating at $y = \delta_1 + \delta_2$ provided in Eqs. 37 and 38. As can be seen in Fig. 4, the electric field norm calculated from analytical approach does not fully follow the one provided by FEA method. To examine the accuracy of the analytical solutions, we set the FEA method as the base for comparison and define the parameter “error percentage” as:

$$\text{Error Percentage} = \frac{|\text{FEA} - \text{ANA}|}{\text{FEA}} \times 100\%. \quad (64)$$

The electric field norms provided in Fig. 4 have been compared with each other using Eq. 64. The maximum value of error percentage between the two methods shown in Fig. 4, calculated using Eq. 64 is 4.27%, 10.29%, 4.25%, 10.44%, 4.06%, 10.56%, for cases 1 through 6, respectively. Also, the average of maximum values of error percentages for the six cases is 6.81%. The maximum value of E_y component on the EDS surface in cases 2, 4, and 6 is 2.09×10^6 V/m while the maximum of E_y is 1.47×10^6 in cases 1, 3, and 5. The E_x component reaches its maximum value of 1.88×10^6 V/m in all six cases. The maximum and minimum values of electric field norm in cases 1, 3, and 5 are 1.41×10^6 and 2.04×10^6 V/m, respectively. In cases 2, 4, and 6, the $|\mathbf{E}|$ attains its maximum and minimum values of 2.12×10^6 and 1.33×10^6 V/m, respectively.

It should be pointed out that the difference between the results from the two methods, as can be observed in Fig. 4, is attributable to the fact that the voltage change between adjacent electrodes has been considered to be linear in the analytical approach while FEA method provides a more complex voltage profile. Figure 5 depicts the electric potential at $y = 0$, where the electrodes are located, for the two ANA and FEA methods. In Fig. 5, when one of the two adjacent electrodes is at zero electric potential and the other at 1000 volts, linear voltage profile used in the analytical calculations is merely an approximation of the profile that FEA method provides. The most significant deviation between the two voltage profiles occurs when adjacent electrodes are both at zero potential or at 1 kV. For instance, when both phases A and B are at 1 kV, Fig. 5 case 3, FEA method provides a voltage profile that decreases down to 905 volts while the ANA method maintains the voltage at 1000 volts between the electrodes. Therefore, linear approximation introduces an error up to 10.49% in the electric potential in this case.

Similarly, in case 6 of Fig. 5 when both A and B phases are at zero electric potential, the voltage profile obtained from FEA method experiences an increase up to 85.5 volts along the x axis while linear approximation assumes the electric potential remains at 0 between the phases. The nonlinear behavior of the electric potential between two adjacent equipotential electrodes in the two aforementioned cases and other similar cases, is attributable to the neighboring electrodes. In case 3 of Fig. 5, for instance, the electric potential between phases A and B which are at 1 kV, is affected by the phase C in current and previous spatial periods that are at zero electric potential. Along similar lines, in case 6 of Fig. 5, voltage profile between phases A and B, which are maintained at zero potential, is affected by the presence of phase C electrodes of previous and current spatial period. Analytical models have been derived for the electric potential between two cylindrical conductors, like [16] or two-wire transmission line [17], that can be potentially used to provide better approximations for the voltage profile closer to what FEA method provides, rather than a linear one. Detailed analytical model of the voltage profile is out of the scope of this paper and is left for further studies.

It was mentioned that the cases 2, 4, and 6 in Fig. 4 experience the maximum error percentage around 10% while cases 1, 3, and 5 have an error percentage around 4%. The maximum deviation between the two methods in cases 2, 4, and 6 in Fig. 4 occur at 350, 550, and 150 μm , respectively, which are the middle points between two equipotential electrodes that are at zero electric potential. In cases 2 and 6 of Fig. 4, these two equipotential electrodes belong to the shown spatial period while in case 4, phase C electrode of the current spatial period and phase A electrode from the next period are at zero electric potential. By comparing Figs. 4 and 5, it reveals that the maximum deviation between the two methods are at the points that the voltage profile provided by FEA method reaches its maximum on a semi-circular curve while the ANA method assumes zero electric potential between grounded adjacent electrodes.

The two significant features of the analytical solutions are

- 1) it can be easily embedded into the software programs for particle trajectory simulations,
- 2) it provides flexibility and generality in the study on the effect of different parameters in the behavior of the electric field with less computational burden.

B. Impact of Dielectric Coating

The electrodes in the EDS are always embedded within one or more layers of transparent dielectric coatings to be protected from direct exposure to the outdoor environment. The material characteristics of the dielectric material plays a pivotal role in the charging mechanism and the amount of charge acquired by the deposited dust

Fig. 4. Comparison of analytical (ANA) results with numerical simulations using finite element analysis (FEA) software for $w = g = 100 \mu\text{m}$ in one fundamental spatial period $T_s = 600 \mu\text{m}$ for six different voltage combinations for phases A, B, and C. The magnitude of the electric field (unit: [MV/m]) on the EDS surface (i.e. $y = \delta_1 + \delta_2$) is illustrated vs. x . The positions of the first, second, and third electrodes are at $[0, 100 \mu\text{m}]$, $[200 \mu\text{m}, 300 \mu\text{m}]$, and $[400 \mu\text{m}, 500 \mu\text{m}]$, respectively. The voltages of phases (A,B,C) for the cases (1) to (6) are respectively as follows: $(V, 0, V)$, $(V, 0, 0)$, $(V, V, 0)$, $(0, V, 0)$, $(0, V, V)$, and $(0, 0, V)$, in which $V = 1000$ volts. The thicknesses of the first and second dielectric coatings are 50 and $100 \mu\text{m}$, respectively, and their relative permittivities are 5.14 and 6.7 .

particles. As analytically provided in the preceding section, the electric field components in the dielectric materials and air are dependent upon the geometric parameters and relative permittivities of two transparent dielectric layers ϵ_{d1} and ϵ_{d2} . It is worth to study how the electric field distribution on the EDS surface, where the dust particles are deposited, behaves as a function of the dielectric coating thickness. For this purpose, we have run the FEA software for different values of the thickness of the first dielectric coating δ_1 to see its impact on the electric field distribution on the EDS surface. Figure 6 shows the simulation results of the normal electric field component E_y in one fundamental spatial period on the EDS surface when the electrode width and inter-electrode spacing are 50 and $700 \mu\text{m}$, respectively, for various values of the δ_1 in the range of $25 \mu\text{m}$ to $140 \mu\text{m}$, while the other parameters are fixed. Although several values for δ_1 in the aforementioned range were considered in the simulation study, only

Fig. 5. Comparison of the voltage profiles used in the analytical (ANA) approach and computed with numerical simulations using finite element analysis (FEA) software for $w = g = 100 \mu\text{m}$ in one fundamental spatial period $T_s = 600 \mu\text{m}$ for six different voltage combinations for phases A, B, and C. The electric potential at $y = 0$, where the electrodes are located is shown vs. x . The positions of the first, second, and third electrodes are at $[0, 100 \mu\text{m}]$, $[200 \mu\text{m}, 300 \mu\text{m}]$, and $[400 \mu\text{m}, 500 \mu\text{m}]$, respectively. The rest of the parameters are identical to Fig. 4.

four of them are illustrated in Fig. 6 for the sake of readability of the figure. As can be observed in Fig. 6, the E_y experiences a significant change when δ_1 changes from $50 \mu\text{m}$ to $63 \mu\text{m}$, while when δ_1 increases from $63 \mu\text{m}$ to $125 \mu\text{m}$, the E_y does not change drastically. The $\delta_1 = 63 \mu\text{m}$ provides the maximum of the E_y in the positive direction in all of the the six cases of the voltage combinations.

As mentioned earlier, the Coulomb force is the predominant force in dust particle removal process. Although the motion of dust particles in x and y - direction due to exerted forces are both important in detailed modeling of the particle trajectories, we pay significant attention to the vertical motion of the particle and the balance of repelling and attracting forces. In other words, we endeavor to ensure at each given point on the EDS surface, we maintain the required amount of electric field component in the positive y -direction to strengthen the Coulomb force among other forces, assuming the particle is positively charged. When the dust particle is sitting on the EDS surface, the

gravity and adhesion force are the attracting ones that push the particle into the EDS surface. The adhesion force is the summation of van der Waals and capillary forces [18]. As can be noticed in Fig. 6, the normal component of the electric field E_y attains negative values, which is not in our interest circle as not only it does not help in repelling the positively charged particle from the surface, but also pushes the particle into the EDS surface. The results obtained on how the E_y changes as a function of the δ_1 are beneficial in the optimization procedure of the EDS for a higher efficiency in dust removal. We must have the balance of the forces exerted on the charged dust particle in a fashion that the repelling forces dominate the attracting ones: the summation of the all the exerted forces points toward positive y -axis.

Fig. 6. The comparison between the E_y (unit: [MV/m]) components on the EDS surface in one fundamental spatial period for different values of the thickness of the first dielectric coating δ_1 . The results obtained from the COMSOL Multiphysics software. The positions of the first, second, and third electrodes are at $[0, 50 \mu\text{m}]$, $[750 \mu\text{m}, 800 \mu\text{m}]$, and $[1500 \mu\text{m}, 1550 \mu\text{m}]$, respectively, and the fundamental spatial period is $2250 \mu\text{m}$. The voltages of phases (A,B,C) for the cases (1) to (6) are respectively as follows: $(V, 0, V)$, $(V, 0, 0)$, $(V, V, 0)$, $(0, V, 0)$, $(0, V, V)$, and $(0, 0, V)$, in which $V = 1000$ volts. The thickness of the second dielectric coating is fixed at $100 \mu\text{m}$ and the relative permittivities of the first and second dielectric layers are 5.14 and 6.7, respectively.

4. SECOND EDS CONFIGURATION

A. Geometry

Figure 7 shows the second EDS configuration considered in this study. The configuration depicted in Fig. 7 is a viable candidate for dust removal from solar mirror in CSP application. As depicted in Fig. 7, EDS consists of three layers of dielectric coatings in which the electrodes are printed on the lower side of the third dielectric layer. Similar to Fig. 1, the electrode width and inter-electrode spacing are denoted as w and g , respectively. The thicknesses of transparent dielectric layers 1 to 3 are denoted as δ_1 , δ_2 , and δ_3 , respectively, and their relative permittivities highlighted as ϵ_{d1} , ϵ_{d2} , and ϵ_{d3} . In our laboratory work, the dielectric layers 1, 2, and 3 are considered to be acrylic, optically clear adhesive (OCA) [19], and Corning[®] Willow[®] Glass [20], respectively. The metal layer is usually a thin layer, in the order of 1-2 μm of Aluminum or Silver. The dielectric constants of acrylic, OCA, and Corning[®] Willow[®] Glass have been considered to be 3.5, 5.14, and 5.5, respectively, throughout the FEA study.

Fig. 7. The second EDS configuration, in which the electrodes are printed on the lower side of dielectric layer 3.

B. Numerical Results

The EDS configuration has been developed in the COMSOL Multiphysics simulation software and the results for the electric potential and electric field are shown in Figure 8, in which the electrode width and inter-electrode spacing are set to be 75 and 300 μm , respectively. In Fig. 8, the maximum of $|\mathbf{E}|$, E_y , and E_x reach 2.41×10^6 , 4.7×10^5 , and 2.38×10^6 V/m, respectively, on the EDS surface in the fundamental spatial period of 1125 μm . The height of the electrodes and the thickness of underlying of metal layer are much smaller than the dimensions of other parameters like electrode width and the thickness of dielectric layers: they have been considered to be 2 μm . In addition, we are interested to study the electric field distribution on the EDS more thoroughly rather

than the electric field intensity inside the dielectric layers or in the vicinity of the electrodes. Hence, by defining three different mesh patterns: (1) extremely coarse for conductors, (2) extra fine for the dielectric layers, and (3) extremely fine on the EDS surface, we were able to reduce the mesh generation time significantly yet having accurate computations.

Fig. 8. Numerical simulation results using finite element analysis software for the (a) electric field norm $|\mathbf{E}|$ (unit: V/m) and (b) electric potential (unit: V) of the second EDS configuration model in one fundamental spatial period T_s . The unit of the vertical and horizontal axes is meter. The height of the electrodes and the thickness of the metal layer have been considered both to be $2 \mu\text{m}$ and the maximum of the electric field norm reaches $1.8 \times 10^6 \text{ V/m}$ on the EDS surface. The voltages of phases A, B, and C are 1 kV, 1 kV, and 0, respectively. The electrode width and inter-electrode spacing are considered to be $w = 75 \mu\text{m}$ and $g = 300 \mu\text{m}$, respectively. The thicknesses of the first, second, and third user-defined dielectric layers are $\delta_1 = 100 \mu\text{m}$, $\delta_2 = 50 \mu\text{m}$, $\delta_3 = 100 \mu\text{m}$, respectively, and their relative permittivities are $\epsilon_{d1} = 3.5$, $\epsilon_{d2} = 5.14$, and $\epsilon_{d3} = 5.5$.

Thus far, we have analyzed the electric field distribution in two EDS configurations. In order to compare these two configuration to realize which one of these geometries performs better regarding dust removal, we consider

the normal electric field component E_y on the EDS surface in one fundamental spatial period, while it is tried to fix all the other parameters. Figure 9 shows the comparison between the E_y components for six different voltage combinations in one fundamental spatial period and on the EDS surface. For both configurations, the electrode width and inter-electrode spacing are considered to be 50 and 700 μm , respectively. For the first EDS configuration (Fig. 1), the thicknesses of the first and second dielectric layers are 50 and 100 μm , respectively, with relative permittivities of 5.14 and 6.7. For the second EDS configuration (Fig. 7), the thickness of the first, second, and third transparent dielectric layers are 100, 50, and 100 μm , respectively, with relative permittivities of 3.5, 5.14, and 5.5. The mentioned dimensions are typical for EDS.

As can be seen in Fig. 9, the E_y in both configurations attain their extrema (maxima and minima) at identical points along the x -direction in one fundamental spatial period. However, the second EDS configuration provides higher values for E_y at the maxima for all the cases. The percentage of increase in the E_y of second configuration at the maxima for the cases (1) through (6) are 63.86%, 16.66%, 66.36%, 15.41%, 62.98%, and 15.32%, respectively, compared to the first configuration. Also, it is interesting to note that what percent of one fundamental spatial period the values E_y of second configuration are greater than that of first configuration. In 40.98%, 56.12%, 41.16%, 55.68%, 40.49%, and 40.98% of the fundamental spatial period, the second EDS configuration yields higher values for E_y than the first configuration for the cases (1) through (6), respectively.

5. CONCLUSION

Electric field distribution model is an essential part in performance analysis of the electrodynamic screen (EDS) to achieve high dust removal rates. The ultimate goal is to ensure that at each given point on the EDS surface, the summation of repelling forces are greater than the attracting ones in the balance of exerted forces on the particle. In this paper, we provided the analytical solutions for the electric potential and electric field distribution in an EDS configuration with two stacked layers of transparent dielectric coatings. The dependency of the solutions on various design parameters as well as electrical properties of the dielectric materials was emphasized. Corroboration of the analytical solutions using the developed EDS model in the FEA software showed that the average of maximum error percentages between the solutions is less than 7% for six voltage combinations. By providing more accurate expressions for the voltage profile between adjacent electrodes in the analytical method, the error percentage can be reduced much further. The optimal value for the thickness of one of the dielectric layers in order to maximize the normal electric field component on the EDS surface was reported. Furthermore, the electric field analysis for the second EDS configuration was performed using FEA software. In order to evaluate the two configurations,

Fig. 9. The comparison between the E_y (unit: [MV/m]) components on the EDS surface in one fundamental spatial period for the two EDS configurations. The electrode width, inter-electrode spacing, and fundamental spatial period are 50, 700, and 2250 μm , respectively. The positions of the first, second, and third electrodes are at [0,50 μm], [750 μm ,800 μm], and [1500 μm ,1550 μm], respectively. The voltages of phases (A,B,C) for the cases (1) to (6) are respectively as follows: (V, 0, V), (V, 0, 0), (V, V, 0), (0, V, 0), (0, V, V), and (0, 0, V), where $V = 1000$ volts.

the normal electric field components on the EDS surfaces in two configurations was compared. It was found that when the design as well as operational parameters are kept identical, there is no significant distinction between two configurations. More criteria can be considered for further evaluation of the EDS configurations. The results of this paper are beneficial in detailed dust particle trajectory modeling and optimization procedure of the EDS for higher dust removal rates.

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NOMENCLATURE

404		
405	a_k, b_k	Fourier coefficients
406	f	locally defined electric potential function, V
407	g	inter-electrode spacing, m
408	g_k, h_k	Fourier coefficients
409	k	index of summation
410	p_k, q_k	Fourier coefficients
411	T_s	fundamental spatial period, m
412	T_t	time period, s
413	u_m	electric potential at $y = \delta_1$, V
414	v_m	electric potential at $y = 0$, V
415	w	electrode width, m
416	w_m	electric potential at $y = \delta_1 + \delta_2$, V
417	$\mathbf{a}_x$	unit vector in x direction, m
418	$\mathbf{a}_y$	unit vector in y direction, m
419	$\mathbf{E}$	electric field vector, V/m
420	E_x	tangential component of the electric field, V/m
421	E_y	normal component of the electric field, V/m
422		<i>Greek symbols</i>
423	δ	thickness of the transparent dielectric layer, m
424	ϵ	relative permittivity of the dielectric coating
425	ϵ_0	permittivity of free space, 8.854×10^{-12} F/m
426	ϕ	electric potential, V
427	Ω_0	spatial angular frequency, rad/m
428		<i>Subscript</i>
429	a	air
430	$d1$	dielectric 1
431	$d2$	dielectric 2
432	$d3$	dielectric 3

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