Effects of wall roughness on drag and lift forces of a particle at finite Reynolds number

Hyungoo Lee^a, S. Balachandar^{b,*}

^a*Power Reactor Development Division, Korea Atomic Energy Research Institute, Daedeok-daero 989, Yuseong-gu, Daejeon 305-353, S. Korea*

^b*Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville, FL 32611, USA*

Abstract

In order to investigate drag and lift forces exerted on a spherical particle induced by a uniform shear flow bounded by rough wall, we have conducted numerical simulations at finite shear Reynolds number in the range of $2 \leq Re \leq 100$. The gap between the particle and the rough bed ($\tilde{\delta}$) is the second important parameter, which was also varied from the limiting case of particle sitting on the bed ($\tilde{\delta} = 0$) to the particle being more than one diameter away from the bed. A fully developed undisturbed ambient shear flow over a rough bed, made of hemispherical elements on a flat wall, was first obtained without the spherical particle, whose purpose is to serve as reference and provide the undisturbed flow. A spherical particle was then located near this rough bed and subjected to the ambient shear flow. Drag and lift forces exerted on the particle located over the rough bed were computed and compared with those for a particle similarly located over a smooth bed. The wall roughness in general increased the drag and lift coefficients over their smooth-wall values. But, at lower Reynolds numbers of $Re \leq 5$, the lift coefficient was observed to decrease substantially as the particle location approached the wall ($\tilde{\delta} \rightarrow 0$). When the particle is located away from the bed, the hydrodynamic forces over the rough bed are close to those over a smooth bed. In general, at moderate Reynolds numbers, we observe the drag and lift correlations developed by Zeng et al. (2009) for the smooth wall can be used for the rough wall as well, provided the particle Reynolds number and δ are appropriately defined.

*Corresponding author

Email address: bala1s@ufl.edu (S. Balachandar)

1. Introduction

Hydrodynamic forces acting on a particle sitting in the boundary layer of a wall and the resulting motion of the particle are greatly influenced by wall roughness. Here we are interested in the case where the roughness of the wall is of the same order as the mobile particle. This is the scenario encountered in problems of particle resuspension, where particle being mobilized and resuspended into the flow initially sits on a bed made of similar particles.

The influence of the rough wall can be considered in steps. First, the nature of boundary layer turbulence can be strongly influenced by wall turbulence. The key parameter is the roughness Reynolds number defined as $k_+ = \tilde{k}\tilde{u}_*/\nu$, where $\tilde{k}$ is the roughness size, which in the case of a bottom bed of particles is related to the particle diameter, $\tilde{u}_*$ is the friction velocity and ν is kinematic viscosity. For $k_+ > 5$ the bed roughness will influence the structure of wall turbulence (Raupach et al., 1991) and the flow statistics, such as the mean and rms velocity, at any point within the flow will be influenced by roughness. The time history of hydrodynamic drag and lift forces on a particle sitting on the rough wall will clearly depend on the details of the oncoming turbulent flow approaching the particle. In addition, the hydrodynamic forces will also depend on the geometry of the pocket within which the particle is sitting. For example, the drag force on a fully exposed particle will be substantially larger than a particle that is deeply submerged within a pocket. The hiding effect can preclude the entrainment of particles that are finer than the roughness element (Niño et al., 2003). Finally, whether the hydrodynamic force is sufficient to mobilize the particle out of the pocket and transport it along the bed or entrain into the flow as part of suspended sediment will also depend on the details of the geometry (Buffington and Montgomery, 1997; Garcia, 2008; Lee et al., 2012).

The effect of the bottom boundary is the strongest when the particle is in contact with the boundary. There has been substantial advancement in our understanding and modeling of hydrodynamic forces on a particle in motion close to a flat wall. The earliest relevant work goes back to the lubrication analysis of Goldman et al. (1967a,b) who considered the problem of a stationary particle close to a flat boundary in a linear shear flow and the problem of a

translating-rotating particle close to a flat boundary in an otherwise quiescent fluid. Their analysis was in the Stokes limit and they observed substantial increase in the drag force due to the presence of the nearby boundary, which persisted for separations between the particle and the wall of the order of several particle diameters.

The particle also experiences a lift force directed perpendicular to the boundary and therefore perpendicular to the oncoming mean flow. The lift force is due to inertial effects and therefore is absent in the lubrication limit. Starting from the pioneering works of Rubinow and Keller (1961) and Saffman (1965) there have been many theoretical, experimental and computational efforts to quantify the lift force on a particle located close to a flat boundary (Cox and Hsu, 1977; McLaughlin, 1993; Cherukat and McLaughlin, 1994; Takemura et al., 2002; Takemura and Magnaudet, 2003; Magnaudet, 2003; Zeng et al., 2005, 2009). These efforts considered either the case of a stationary particle in a wall-bounded linear shear flow or the case of a particle moving parallel to a flat boundary in a quiescent fluid. Three different length scales can be identified: the particle diameter ($\tilde{d}$), the gap between the flat boundary and the bottom of the particle ($\tilde{\delta}$), and the Stokes length scale ($\nu/\tilde{U}$), where $\tilde{U}$ is the velocity of the oncoming fluid or the particle, depending on the case considered. The theoretical efforts focused on limiting values of these length scale ratios, such as the flat boundary being in the inner Stokes layer or far outside.

Of particular importance are the theoretical investigations of Leighton and Acrivos (1985) and Krishnan and Leighton (1995), who showed that at a finite, but small Reynolds number, the drag and lift forces on a particle close to a flat boundary can be separated into six contributions. Using fully-resolved simulations Lee and Balachandar (2010) extended these contributions to finite Reynolds numbers of the order of few hundred. The validity of these force expressions have been tested against experiment and direct numerical simulations (King and Leighton, 1995; Lee et al., 2011). Here we consider a spherical particle sitting on a rough bottom boundary made up of a bed of same-sized particles. Our interest is to examine how well the drag and lift correlations developed for a particle sitting on a smooth wall apply to the present case of a particle located on a rough wall. In this investigation attention will be limited to the case of a stationary particle subjected to a nominally linear

shear flow. In the case of a smooth boundary, the non-dimensional drag and lift forces have been parameterized in terms of nondimensional distance from the boundary ($\tilde{\delta}/\tilde{d}$) and the particle Reynolds number defined as $\text{Re}_p = \tilde{u}_p \tilde{d}/\nu$, where $\tilde{u}_p$ is the velocity of the oncoming shear flow at the elevation of particle center. In the case of a smooth boundary, the mean streamwise velocity profile of a shear flow varies only along the wall-normal direction. In particular, if the oncoming flow is of uniform shear magnitude $\tilde{G}$, then the velocity can be simply expressed as $\tilde{u}_p = \tilde{d}\tilde{G}/2$. In the case of a rough wall, with the roughness height of the same order as particle size, the oncoming flow as seen by the particle, even in the limit of a steady laminar flow, is highly complex and three-dimensional. For example, the shear magnitude will vary over the size of the particle and thus we must develop an appropriate definition of effective shear (or effective shear Reynolds number) that can be employed in the shear-induced lift correlation. Furthermore, quantities such as the gap between the bottom wall and the particle ($\tilde{\delta}$) can be precisely defined in case of a flat boundary. In the context of a rough boundary, there are different possibilities for defining the location of the nominal bed and thus the gap between the bed and the particle.

Most prior investigations of particle motion on a rough bed pertain to sediment migration in the context of geomorphology (see for example, Bagnold (1966), Buffington and Montgomery (1997), Garcia (2008)). Most of these investigations have been at the macroscale concerning relations between net flux of particle transport, either along the bed as bedload transport or into the fluid as resuspension flux, and the turbulent stress at the bottom rough boundary. Although only a few, there have been recent studies at the microscale investigating the motion of individual particles on a rough bed. Diplas and coworkers introduced the concept of a critical impulse, and argued that the net impulse on the particle must exceed a critical value for the particle to escape out of the pocket and migrate downstream (Diplas et al. (2008); Celik et al. (2010); Valyrakis et al. (2010)). This line of argument has led to the definition of a work-based criterion for the migration of a particle out of the pocket in which it is located in a rough bed. Based on the work criterion Lee et al. (2012) have computed the bed load transport in a turbulent boundary layer with excellent agreement with prior experimental results and empirical correlations. Experiments of a moving particle

along a rough wall in a viscous shear flow were conducted by Charru et al. (2007). As in the present study, they considered that the size of the particle and rough wall to be identical and the driving force on the particle is imposed by a shearing flow. It was found that the particle mean velocity scaled by the settling velocity depends on the Shields parameter for small particle Reynolds number. Recently, Chan-Braun et al. (2010, 2011, 2013) conducted direct numerical simulations of many particles over a rough bed in a turbulent boundary layer.

Here we will perform fully resolved simulations of a particle sitting on a rough bed of same-sized particles. From the results of the direct numerical simulations we will examine the applicability of the smooth wall drag and lift correlations to the present case of a rough wall. The present fully resolved simulations of a particle on a rough bed are somewhat computationally intensive. A range of particle Reynolds numbers and gap between the particle and the rough bed are considered. But attention is limited to a steady nominally linear shear flow. Eventual application of interest is resuspension of particles from a rough bed in a turbulent flow (Reeks et al., 1988; Reeks and Hall, 2001). In a turbulent flow, strong fluctuations away from the mean play a significant role in the evaluation of the rate at which particles are mobilized and entrained into the flow (Lee et al., 2012; Amir et al., 2014; Celik et al., 2014). Furthermore, unsteady forces other than the quasi-steady drag and lift forces can strongly influence the force balance and the dynamics of the particles (Schmeeckle et al., 2007). Such additional complexities are ignored in the present study.

2. Problem description

A schematic of the problem set-up, geometric relations and the coordinate system employed are presented in Fig. 1. We consider a spherical rigid body of diameter $\tilde{d}$, which we will refer to as the *particle*, whose center is located at a distance $\tilde{L}$ from a *base level*. Here the base level is defined as the origin of the vertical coordinate used in the computations (i.e., $\tilde{y} = 0$). The rough bed is made up of a staggered array of hemispheres of diameter $\tilde{d}$ arranged as shown in Fig. 2. The centers of the hemispheres are located at the base level, and thus the roughness height from the base level is $\tilde{d}/2$. The streamwise $\tilde{x}$ coordinate is

aligned with the center of the particle. Hereinafter, the tilde denotes dimensional variable to distinguish it from dimensionless one. The particle is considered as a stationary object, which means that any motions (rolling, sliding or lifting) of the particle is not allowed in this study. The array of hemispherical elements are in contact with their nearest elements and thus are closely packed. The hemispherical elements are taken to sit on a flat plate. Clearly this is an idealization of a rough bed. Even in the context of a monodispersed bed of spherical elements, different arrangements of the roughness elements can be considered. Furthermore, the hemispherical elements considered here only represents a half-a-layer of elements. One could consider a monolayer of full spherical elements, or multiple layers of the spherical elements to define the rough bed. Such an arrangement would allow some flow through the porous bed. By considering a monolayer of hemispherical elements here we have simplified the problem and thereby reduced the computational cost. Due to the very small value of flow to be expected within the additional layers of spherical elements we can expect the results of the present study to be a good representation for the results from multiple layers of roughness elements.

Figure 2 shows a top-view of the array of hemispherical elements. We use $\tilde{d}$ as the length scale and in non-dimensional units the center of the $\{i, j\}^{\text{th}}$ individual hemisphere follows:

$$\begin{aligned} x_{ij} &= \begin{cases} x_o + j, & \text{for odd } i, \\ x_o + (j - \frac{1}{2}), & \text{for even } i, \end{cases} \\ y_{ij} &= 0, \\ z_{ij} &= z_o + \sin\left(\frac{\pi}{3}\right)(i - 1), \end{aligned} \tag{1}$$

where subscripts ‘ i ’ and ‘ j ’ are indicators of the Column (C) and Row (R) of the element, respectively. Note that the definition *row* and *column* are from the perspective of the oncoming flow. The streamwise velocity first encounters R1 hemispherical elements, then R2, and so on. The length $x_o (= 5.0)$ is introduced as an entrance region in front of the rough bed to prevent direct impaction from inflow to the hemispherical rough element. The value of z_o equals to $-5\sqrt{3}/4$ (≈ -2.165) which is the z -value of the 1st column of elements shown as C1 in Fig. 2.

Figure 3 shows the geometric details of a particle which lies within the deepest valley of hemispherical rough bed. When the particle is touching the bed along the deepest valley, the vertical distance L can be expressed as $L = \sqrt{d^2 - a^2}$, where in the $x - z$ plane two geometric lengths can be defined as $a = \sqrt{r^2 + b^2}$ and $b = r \tan(\pi/6)$. The resulting minimal vertical location of the particle center over the base line of the rough bed $L_{min} = 0.8165$ will be considered here.

In order to investigate the effect of the gap between the particle and the rough boundary, we need to first define the separation clearly. For a smooth wall, it is clear that the wall distance can be unambiguously defined as the vertical distance from the flat wall to the bottom of the particle. However, for the rough wall, the determination of the wall distance is subjective due to an uneven wall surface. First, we defined the *Base level*, as the plane on which the centers of the hemispheres are positioned as shown in Fig. 1. From which, we define a particle's bottom as the *bed level* when the particle is sitting on the rough bed at the lowest possible position inside the deepest pocket (see also Fig. 3).

We defined the height h_{BL} as vertical distance from the base level to the bed level. The dimensionless height h_{BL} had been commonly taken as 0.2~0.35 (Bridge and Bennett, 1992; Wu and Chou, 2003). van Rijn (1984), Cheng and Chiew (1998) and Wu and Chou (2003) had taken the height to be $h_{BL} = 0.25$ based on where the extrapolated velocity profile of the near-wall shear layer goes to zero, while Ling (1995) adopted the height $h_{BL} = 0.3165$ based on consideration of a two-dimensional geometric relationship. A schematic demonstrating these different definitions of h_{BL} and the gap between the particle and the bed δ are shown in Fig. 4. Also shown in frame (c) is the definition of gap as the shortest distance between the particle and the rough bed. With this later definition there is no need to define the bed level. For the smooth wall $h_{BL} = 0$, but for the present rough wall of hemispherical elements we take $h_{BL} = 0.3165$ and δ to be the standard definition of vertical distance from the bed level to the bottom of the particle.

Figure 5 shows the computational domain in which the *particle* (shown as dark-gray sphere) is taken to be sitting within a pocket in an array of hemispherical roughness elements positions in a six-column seven-row (C6×R7) arrangement (hemispherical elements

are shown as white-gray hemispheres). An entrance region, in which the bottom wall is smooth, is adopted for buffering between inflow section and the rough boundary.

An ambient flow approaching the rough wall is assumed to be a linear shear flow, whose only non-zero velocity component is along the x -axis parallel to the wall. The linear shear flow can be written as a function of vertical position, $\tilde{u}_i(\tilde{y}) = \tilde{G}_i \tilde{y}$, where $\tilde{G}_i = d\tilde{u}_i/d\tilde{y}$ is the dimensional shear rate of the ambient flow. Here subscript i indicates inflow. It can be expected that the shear flow over the rough bed may not perfectly preserve the linearity of the inflow, even under fully-developed condition. Thus, the shear flow over the rough wall cannot be simply described as a function of wall-normal direction like the flows over a smooth wall. This makes the rough wall problem more complicated than the smooth wall counterpart.

3. Numerical Methodology

We take particle diameter $\tilde{d}$ as a length scale. But there are different choices for the velocity scale: (i) undisturbed ambient shear flow velocity $\tilde{G}\tilde{d}$ and (ii) undisturbed streamwise fluid velocity at a particle center $\tilde{u}_p$. These two definitions are related in the case of linear shear flow over a smooth boundary. In the case of rough bed, since the local shear $\tilde{G}$ and the velocity $\tilde{u}_p$ vary with the particle location, there is no simple geometric relationship between the different choices, although they can be expected to be similar. It should be pointed out that there are other practical choices for the velocity scale. For example, in experiments of turbulent flow over a particle, since the instantaneous undisturbed flow velocity or shear at the particle location is unavailable, often the approaching flow velocity one or two diameters upstream of the particle is used as the velocity scale (Schmeeckle et al., 2007). In the present context, for each simulation with the particle, we have also performed companion simulations without the particle, from which we obtain the undisturbed flow velocity (that exists without the particle under otherwise identical conditions) at the particle location. Here we chose the inflow shear velocity $\tilde{G}_i\tilde{d}$ as a velocity scale for convenience. Note that the shear velocity can be obtained from the constant shear rate $\tilde{G}_i$ imposed at the inlet boundary or at the location of the particle over the rough bed. Here we choose the inlet constant shear

as the reference. The influence of the other choices will be discussed in the section 5. The pressure and time scales are defined accordingly.

The resulting dimensionless continuity and Navier-Stokes equations for incompressible flow can be written as:

$$\begin{aligned}\nabla \cdot \mathbf{u} &= 0, \\ \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} &= -\nabla p + \frac{1}{\text{Re}} \nabla^2 \mathbf{u} + \mathbf{f},\end{aligned}\tag{2}$$

where, the Reynolds number based on shear velocity can be defined as $\text{Re} = \tilde{G}_i \tilde{d}^2 / \nu$.

We employ an immersed boundary method in order to impose the no-slip velocity boundary conditions on the surface of both rough wall and the particle embedded within a fixed Cartesian grid system (Uhlmann, 2005a). The localized force field, $\mathbf{f}$, in Eq. (2) is applied within the volume around the particle in the computational domain. The force is identically zero away from the particle and the standard Navier-Stokes equation in the fluid is recovered.

The accuracy of the immersed boundary technique is improved by using multiple layers of Lagrangian forcing points (Uhlmann, 2005a,b). Although it adds to computational cost, we adopt 20 spherical layers covering the entire particle or the hemisphere in conjunction with a grid resolution of 40 points resolving one particle diameter in order to obtain accurate hydrodynamic forces and moments. The resulting number of Lagrangian forcing points used in the implementation of the immersed boundary technique for the particle is 36,077. Correspondingly half the number of points are used for each hemispherical roughness element.

A second-order accurate central difference scheme based on the finite volume method on non-staggered Cartesian grid system is employed in the present numerical computations. A fractional-step method is used for time advancement. In the advection-diffusion step, the nonlinear terms are treated explicitly using second-order Adams-Bashforth scheme and diffusion terms are treated implicitly with Crank-Nicolson scheme. The details of the numerical methodology and the immersed technique used in this simulations have been discussed in Lee and Balachandar (2010) and Lee et al. (2011) and will not be repeated here.

We impose the no-slip boundary conditions on the surface of both the particle and the rough/smooth wall. The inlet flow boundary condition is fixed as a linear shear flow. Also, Neumann boundary condition for all the variables is used at the domain exit ($x=L_x$). The lateral boundaries at $z = \pm L_z/2$ are treated as symmetry planes, and at top boundary ($y = L_y$) free-slip but no-penetration condition is applied.

A mixed uniform and non-uniform Cartesian grid system is employed in this simulations. In the immersed boundary region that embeds the sphere and hemispheres, a uniform Cartesian mesh of equal spacing along all three directions is used (i.e. $\Delta x=\Delta y=\Delta z$). This highly-resolved uniform grid region extends over $[4.5, 14.5] \times [0, 3] \times [-3, 3]$ along the stream-wise (x), wall-normal (y) and spanwise (z) directions, respectively. Outside this uniform grid region, a non-uniform grid with a geometric grid-stretching is employed. Each successive grid spacing is progressively increased by a multiplicative factor. The overall grid resolution used in this simulations is $587 \times 161 \times 349$ grid points and the computational domain employed extends over $[0, 30] \times [0, 7] \times [-5.8, 5.8]$ along all three directions. The validity of grid resolution and domain size for the case of a smooth wall were reported in Zeng et al. (2009), Lee and Balachandar (2010) and Lee et al. (2011). Unfortunately there is no clear guidance for resolution in the present case of a rough wall. Due to the laminar nature of the flow, here we use the resolution requirement for a spherical particle close to a smooth wall to apply in the present simulations for both the particle and the hemispherical roughness elements.

3.1. Validation of fully-developed rough bed flow

Highly-resolved direct numerical simulations around irregular shapes are computationally demanding, even with the immersed boundary methodology. The rough wall in the present study requires a number of the hemispheres to build the roughness. Therefore, one of the crucial factors to the computational cost is the number of hemispherical elements used to represent the rough boundary. This proportionately increases the number of Lagrangian forcing points to be employed in the immersed boundary technique for the imposition of the given boundary condition. Hence, we assume there exists a minimal array of the roughness

which successfully represents a rough wall, instead of fully-covering the rough bed over the whole computational domain.

We first explore the influence of the bed roughness on a surrounding flow without considering the particle. Numerical simulations without the presence of the particle, with only the rows and columns of hemispherical roughness elements were conducted. A rough bed of C6×R7 arrangement was considered for shear Reynolds numbers ranging from 2 to 100. As a flow goes downstream over the rough bed the development of the flow from the first row of roughness elements to the last (seventh) row of roughness elements will be monitored. This will allow us to establish a development region that consists of several rows of roughness elements. Figure 6 shows undisturbed streamwise velocity plotted as a function of the bed normal y direction at different x and z locations for $Re = 100$. An important point to emphasize is the flow remains laminar and steady at this Reynolds number and the steady-state velocity profiles are shown. In fact, for the present linear shear flow, the flow remains steady for all cases to be considered in this study, due to the modest values of Re being employed.

Frames (a) and (b) show the plots of streamwise velocity at two fixed spanwise locations for varying $x=6, 7, 8, 9, 10$. The two z locations are selected as follows: $z=0.144$ represents the position of the deepest valley between columns C3 and C4 of the rough elements, while $z = -0.433$ is across the topmost position in column 3. In both Figs. 6(a) and (b) we can observe that the flow over the rough bed is fully developed beyond $x \approx 8$, i.e., beyond the third row of roughness elements. For further investigation, at a fixed $x=9.0$, the u velocity profiles at three different spanwise locations representing the deepest valley locations are shown in Fig. 6(c). The different velocity profiles nearly collapse into a single curve. Similarly, the velocities at four selected topmost positions over the hemispherical elements for $x > 8$ are shown in Fig. 6(d). From which we can again assure that flow is invariant in the range $x > 8$. As a result, we choose $x=9$ (R4) and $z=0.144$ (between C3 and C4) to be the representative location of the fully developed flow, and will call it as position ‘P’ (9, 0.144) hereinafter. The u velocity at the topmost position satisfies the no-slip condition at $y = 0.5$ (see Figs. 6(b),(d)), while velocity at a valley is almost quiescent below $y \approx 0.25$

(see Figs. 6(a),(c)).

To fully establish the minimal arrangement of roughness necessary to reach fully developed conditions, we now perform simulations with the particle present on top of the rough bed. In the above discussion we established that the flow after the 4th row can be considered fully developed. However, the inclusion of the particle on top of the rough bed will generate a wake that will extend over downstream rows of roughness elements. Thus, it is important to establish that the hydrodynamic forces on the particle is invariant to the number of rows of roughness elements included in the simulations. Similarly, the adequacy of the spanwise columns of roughness elements must also be established, since they can influence the spanwise extent/confinement of the wave. Here we perform simulations where we vary the number of columns from 4 to 8 (C4, 6, 8) and number of rows from 5 to 8 (R5, 6, 7, 8). We monitor the drag and lift coefficients on the particle for varying streamwise and spanwise extent of bed roughness for $Re=100$ and both $L = 0.8165$ and 1.8165 . The dimensionless drag and lift coefficients of the particle are defined as follows,

$$C_D = \frac{\tilde{F}_x}{\frac{1}{2}\tilde{\rho}_f\tilde{u}_p^2\tilde{A}_p}, \quad C_L = \frac{\tilde{F}_y}{\frac{1}{2}\tilde{\rho}_f\tilde{u}_p^2\tilde{A}_p}, \quad (3)$$

where $\tilde{\rho}_f$ is fluid density, $\tilde{A}_p = \pi\tilde{d}^2/4$ is the cross-section area of the particle. The projected area of the particle will be slightly lower as the particle location approaches the bed. In the limit of the particle sitting on the rough bed within the deepest pocket ($L = 0.8165$), looking at the particle from upstream, from geometrical arguments it can be shown that the exposed area of the particle is 94.2% of the cross-sectional area. In the above definition of force coefficient $\tilde{u}_p$ is the undisturbed flow velocity at the particle center location, which can be obtained from the numerical simulation without the particle. Due to the roughness of the wall, this velocity is not equal to the specified inflow velocity of the linear shear flow at the z location of the particle center. In the present numerical simulations, the force field $\mathbf{f}$ in (2) is integrated around the particle to obtain the drag and lift forces.

Figure 7 shows the drag and lift coefficients obtained from different simulations employing varying streamwise and spanwise extent of roughness elements. Frame (a) is for a particle in near-contact with the rough bed (i.e., $L = 0.8165$ and the particle center is located at (9.0,

0.8165, 0.144)) and frame (b) is for a particle located one diameter above the bed level (i.e., $L = 1.8165$ and the particle center is located at (9.0, 1.8165, 0.144)). From the figure it can be concluded that the C6×R7 configuration is adequate to be considered as a minimal rough bed arrangement for the parametric range to be considered here. Hereinafter we will consider the arrangement (C6×R7) of hemispherical elements as the rough bed.

4. Theoretical background

In the Stokes limit, the drag force on a particle translating parallel to a wall within the boundary layer can be considered as a linear superposition of shear-induced, translation-induced and rotation-induced contributions Goldman et al. (1967b). The shear-induced contribution to drag depends on shear Reynolds number and non-dimensional distance from the wall. The translation-induced contribution depends on translation Reynolds number and non-dimensional distance from the wall and rotation-induced contribution depends on rotational Reynolds number and non-dimensional distance from the wall. In the Stokes limit there is no lift force on the particle.

At small, but finite Reynolds numbers, Leighton and Acrivos (1985) and Krishnan and Leighton (1995) have shown that the above superposition can be extended to lift force. Since the shear-induced, translation-induced and rotation-induced lift forces are quadratic effects arising from the respective mechanisms, we must also consider their binary interactions as shear-translation, shear-rotation and translation-rotation coupling terms. For small Reynolds numbers such binary coupling effects are small and can be ignored for the drag force. This superposition has been extended to finite Reynolds numbers of the order of few hundred by Lee and Balachandar (2010) where the binary coupling terms are observed to be important both for lift and drag forces.

Of particular interest here are the shear-induced drag and lift forces on a stationary particle. Clearly, as the particle is allowed to move the additional mechanisms will come into play. For a stationary particle in a wall-bounded linear shear flow we will use the drag and lift forces models of Zeng et al. (2009). These force models were developed for a smooth wall and they cover a wide-range of two important parameters: the separation distance δ

and the particle Reynolds number Re_p . Their drag coefficient model for a smooth wall is,

$$C_{Ds} = \frac{24}{\text{Re}_p} \left(1 + 0.138 \exp(-2\delta) + \frac{9}{16(1+2\delta)} \right) (1 + \alpha_D \text{Re}_p^{\beta_D}), \quad (4)$$

where

$$\alpha_D = 0.15 - 0.046(1 - 0.16\delta^2) \exp(-0.7\delta) \quad \text{and} \quad \beta_D = 0.687 + 0.066(1 - 0.76\delta^2) \exp(-\delta^{0.9}). \quad (5)$$

Here the particle Reynolds number is defined in terms of the fluid velocity at the particle center as $\text{Re}_p = \tilde{u}_p \tilde{d} / \nu$. The corresponding model for the lift coefficient is,

$$C_{Ls} = \frac{3.663}{(\text{Re}_p^2 + 0.1173)^{0.22}} \exp \left(-0.5\delta \left(\frac{\text{Re}_p}{250} \right)^{4/3} \right) [\exp(\alpha_L(\text{Re}_p)\delta^{\beta_L(\text{Re}_p)}) - \lambda_L(\delta, \text{Re}_p)], \quad (6)$$

where

$$\alpha_L = -\exp(-0.3 + 0.025\text{Re}_p), \beta_L = 0.8 + 0.01\text{Re}_p \quad \text{and} \quad \lambda_L = (1 - \exp(-\delta)) \left(\frac{\text{Re}_p}{250} \right)^{5/2}. \quad (7)$$

We will apply the above correlations (4) and (6) to evaluate and predict the drag and lift forces on a particle near a rough wall and compare with the actual forces obtained from the numerical simulations. An important question to be addressed is what are the appropriate definition of δ and Re_p in the context of a rough wall, which when used with the above correlations offers the best prediction of drag and lift forces for a rough wall. Such an exercise will reveal over what parametric range can the smooth-wall correlations can be used for rough wall force calculation.

5. Results

In the case of a linear shear flow over a smooth-wall, the non-dimensional shear G across the particle is uniform and correspondingly the velocity across the particle, from the bottom to the top, linearly increases. This regular structure is destroyed in the presence of roughness elements. We first evaluate the extent to which the roughness elements modify the undisturbed flow that would exist in the absence of the particle. To further investigate

the details of the undisturbed flow features at the position ‘P’, we plot bed-normal profiles of u and the shear rate ($G = \partial u / \partial y$) for the different Reynolds numbers between 2 to 100 in Fig. 8. Since the vertical line at the horizontal location ‘P’ passes through a valley over the rough bed, the u velocities are nearly zero below $y \approx 0.25$ for all Reynolds numbers considered. Above $y \approx 0.25$ the streamwise velocity rapidly increases and approaches the linear velocity profile. The rate of this approach is faster at higher Re. It is interesting to note that below $y \approx 2.0$ all the velocity profiles are lower than the shear flow velocity specified at the inlet, due to the presence of the roughness elements. Whereas, above $y \approx 2.0$ the velocities are higher than the inflow condition, to compensate the velocity deficit seen closer to the rough boundary. The departure from the linear profile is most significant for $\text{Re} = 2$. The effect of the rough boundary can also be seen in the shear magnitude. Due to the rapid approach to the linear velocity profile the magnitude of shear is higher for the higher Reynolds number. For $\text{Re} = 100$ the maximum value of shear occurs at $y = 0.7$ with $G = 1.59$. But the increased shear persists only for a short distance from the rough wall and the $\text{Re} = 100$ velocity profile quickly approaches the linear velocity profile. In contrast at $\text{Re} = 2$ the effect of the rough wall penetrates deeper away from the boundary and remains significant even at $y = 4$.

5.1. Flow features

Flow structures induced by particle-wall interactions are observed by plotting streamlines, vortical structures and pressure contours around the particle. Figure 9 shows isometric view of streamline patterns for four selected cases. The insets are side-views of the streamlines. The streamlines are generated by releasing 16 stream tracers on the plane $z=0.144$ from $(5, 0.2, 0.144)$ to $(5, 3.2, 0.144)$ with an uniform y -spacing of $\Delta y=0.2$. Clear signatures of strong recirculation in the wake of the particle can be seen in the two higher Reynolds number cases. The flow however remains steady in all the cases considered. This is consistent with the fact that the critical Reynolds number for the onset of unsteady vortex shedding for a sphere close to a flat wall is above 250 (Zeng et al., 2005).

The vortical structures corresponding to Fig. 9 are shown in Fig. 10. Here iso-surface of

swirling strength (λ_{ci}) is plotted, where swirling strength is defined as the imaginary part of the complex eigenvalues of the local velocity gradient tensor and it is zero in regions when all three eigenvalues are real (see Zhou et al. (1999) and Chakraborty et al. (2005)). The swirling strength has been widely used to identify vortical regions. It can be seen that the vortex structure at the lower Reynolds number is a vortex ring around the particle. But only the top half of the ring is present in the case of the particle being nearly in contact with the bed (Frame a). As the particle is located away from the boundary more of the vortex ring appears in the larger gap between the particle and the rough boundary. Even in this case, the vortex structure is highly non-axisymmetric due to the strong influence of the boundary. As can be seen from the side view, it is not the rough boundary itself directly interferes with the vortex structure in the gap region, but the effect of the boundary in altering the local flow field that influences the vortex structure.

At the higher Reynolds number the vortex structure is quite different for the case of particle in contact with the boundary versus the case of particle sufficiently away from the boundary with $L = 1.8165$. In the later case the familiar double threaded wake can be observed, which appears for $Re > 182$ even in the case of a particle in an unbounded uniform flow, due to natural symmetry breaking. Here, due to the presence of the bottom boundary the symmetry is broken at all Reynolds numbers and an earlier onset of the double threaded wake can be observed. The vortex structures seen here are qualitatively similar to those observed for a smooth wall by Zeng et al. (2009).

5.2. Hydrodynamic forces

5.2.1. Drag coefficient

It is well established that when a particle is close to a wall in a shear flow the drag force increases. For instance, in the Stokes flow regime, drag coefficient (C_D) increases nearly 70% in the limit of $\delta \ll 1$ when compared with the standard drag law given by:

$$C_D = \frac{24}{Re_p} (1 + 0.15 Re_p^{0.687}) . \quad (8)$$

The low Re_p theoretical prediction of 70% increase in drag force above the standard drag law due to the wall remains appropriate even at moderate Reynolds number, though there

Table 1: Drag and lift coefficients for the rough wall and the smooth wall.

Re	δ	$\langle u_p \rangle$	$\langle \text{Re}_p \rangle$	$C_D[\text{DNS}]$	$C_D[\text{Eq.}(4)]$	$C_L[\text{DNS}]$	$C_L[\text{Eq.}(6)]$
2	0.005	0.539	1.1	43.894	41.858	0.645	3.434
	0.1	0.659	1.3	33.157	32.582	2.050	2.842
	0.5	1.160	2.3	16.236	16.782	1.536	1.616
	1.0	1.762	3.5	10.340	10.786	0.941	0.935
10	0.005	0.557	5.6	11.018	10.069	1.882	1.704
	0.1	0.682	6.8	8.462	8.057	1.572	1.396
	0.5	1.195	11.9	4.530	4.565	0.764	0.725
	1.0	1.785	17.8	3.163	3.185	0.304	0.323
100	0.005	0.664	66.4	2.158	2.111	0.731	0.577
	0.1	0.803	80.3	1.763	1.801	0.565	0.461
	0.5	1.290	129.0	1.146	1.214	0.040	-0.027
	1.0	1.813	181.3	0.918	0.926	-0.062	-0.101

exists some Reynolds number dependence (Goldman et al., 1967a; Zeng et al., 2009). As the particle is located away from the wall, the wall effect on C_D vanishes and the standard drag law is approached. Figure 11 shows the variation of C_D as a function of separation distance δ for different shear Reynolds numbers from 2 to 100 (also see Table 1). Note that following (Zeng et al., 2009) we have chosen the very small gap of $\delta = 0.005$ to correspond to a particle nearly sitting on the wall, (which indirectly corresponds to some roughness of the particle and the bed). Unlike in the body-fitted methodology used by (Zeng et al., 2009), in the present immersed boundary methodology, the precise value of this gap is accurate to only two grid spacings.

In this figure, the symbols are the C_D obtained from the present numerical simulations with the rough wall and the lines are C_D estimated from Eq. (4) taken from the smooth wall results. In estimating C_D using (4), the particle Reynolds number was computed using the local undisturbed fluid velocity $\tilde{u}_p$ at the particle center obtained from the present simulation. Also as defined before, the gap between the particle and the rough bed is taken to be between the bed level and the bottom of the particle (see Fig. 1). Similar to the case of the smooth wall, C_D over the rough wall increases as the separation distance δ decreases. When δ is small ($\delta < 0.05 \sim 0.3$) C_D over the rough wall is slightly larger (up to 10%) than

that for the smooth wall. For larger separation distances ($\delta > 0.05 \sim 0.3$) C_D over the rough wall is slightly smaller (up to 5%) than that for smooth wall. A possible explanation for this small discrepancy lies in the dependence of the drag coefficient on Reynolds number. As observed in Fig. 8, at location closer to the bed the undisturbed flow velocity is lower than that of uniform shear flow. The resulting slightly lower Reynolds number can be expected to result in a slightly high C_D . In contrast, as the particle is located away from the bottom boundary, the velocity (and the Reynolds number) seen by the particle is slightly higher and thus the corresponding C_D is lower. When the separation distance increases, for example when δ is approximately 1, the drag coefficients for both smooth and rough wall are almost identical. It means that the wall roughness effect on the drag force is negligible beyond $y_p > 1$, at least in the Re range considered in this study. Oppositely, when the particle is approaching the wall, the discrepancy in the drag force between a smooth and rough wall is slightly increased.

5.2.2. Lift coefficient

The lift coefficient (C_L) over a smooth wall has been observed to rapidly increase when the particle is located close to the bottom boundary. Understandably this increase primarily due to the increased pressure difference between the bottom and the top of the particle. With the rough boundary as $\delta \rightarrow 0$ the bottom of the particle can be shielded within the pocket. Based on the above line of argument, it can be anticipated that the effect of roughness is likely to be stronger on the lift force than on drag. Therefore, it is of interest to evaluate the role of wall roughness on the lift force on the particle. Similar to drag, the lift coefficient over the rough wall is compared with that over the smooth wall. Figure 12 shows the lift coefficient as a function of δ for different Re. Again the present DNS results are plotted as symbols and the prediction using the lift correlation (6) for the smooth wall is also shown as continuous lines. It can be readily observed that the lift coefficient over the rough wall is slightly larger in the range $\text{Re} \geq 10$. For lower Reynolds numbers of $\text{Re} = 2$ and 5, the lift coefficients over the rough wall decrease when the particle is located close to the rough wall. This reduction in C_L with decreasing δ for $\text{Re} < 10$ is distinctly different behavior

than what was observed for the case of smooth wall. In particular, at $Re = 2$ as $\delta \rightarrow 0$ over rough bed the lift coefficient approaches nearly zero. It means that a spherical particle sitting over a rough bed in a viscosity dominating flow may not experience a positive lift force. Thus its mobilization is likely to be entirely due to the drag force rolling the particle out of the pocket.

This result is not entirely surprising, since such a reduction of lift force over a rough wall were reported by Watters and Rao (1971); Davies and Samad (1978); Marsh et al. (2004). In fact, they have even reported negative lift forces over rough bed were observed at low Reynolds number. For example, Davies and Samad (1978) claimed that a significant volume of flow under the particle through the rough wall packing may be a cause of the decrease of C_L . Marsh et al. (2004) compiled available experimental results on lift coefficient in the low Reynolds number regime, and therein observed a clear trend of negative or downward lift force, although some scatter in the data was noticed. For such low Reynolds number, they pointed that the negative or small value of C_L is due to Stokes flow conditions within the pocket in which viscosity is dominated and wake flow past a particle is not generated. At low Reynolds number, the significance of viscous component against pressure component on total lift force is well described in Zeng et al. (2009).

The velocity vector plot around the particle at $Re = 100$ and $L = 0.8165$ is shown in Fig. 13(a) and it is extracted from the $z = 0.144$ plane. The low velocity regions (marked by blue vectors) can be seen upstream of the particle close to the bed, in between the hemispheres that make up the bed, and in the wake of the particle. It is clear that the flow accelerates over the particle creating a low pressure region on top of the particle, which can be seen in Fig. 13(b). This difference in pressure between the top and the bottom of the sphere is responsible for the strong lift force in the high Re case. The situation is quite different at low Re where there is much less pronounced pressure difference between the top and the bottom of the sphere, contributing to a much lower lift.

5.2.3. Lift on a particle sitting on the wall

The lift force on a particle in the limiting case of it sitting on the bed is of considerable interest. Lift force on a particle sitting on a flat wall in laminar and turbulent boundary layers has been considered experimentally by several researchers including Hall (1988), Mollinger and Nieuwstadt (1996) and Muthanna et al. (2005). In case of a particle in an unbounded ambient shear flow, the inertial lift force on the particle depends both on the ambient shear and on relative velocity. In other words, the coefficient of lift will depend on both shear Reynolds number ($\text{Re} = \tilde{G}\tilde{d}^2/\nu$) and particle Reynolds number ($\text{Re}_p = \tilde{u}_p\tilde{d}/\nu$), where $\tilde{u}_p$ is the undisturbed ambient flow velocity as seen by the particle (Saffman, 1965). In the case of a linear shear flow, for a particle sitting on the wall, the two Reynolds numbers are related (i.e., $\text{Re}_p = \text{Re}/2$) and as a result in Eq. (7) as $\delta \rightarrow 0$ the lift coefficient depends only on Re . Thus, the smooth wall drag and lift correlations as presented in Eqs. (4) and (6) exploit the structure of the linear shear flow. In the context of a rough wall, whose roughness scale is of the order of the particle size, the ambient flow velocity and shear seen by the particle has considerable spatial variation on the scale of the particle, even under steady laminar conditions. Here we re-parameterize the lift coefficient in terms of both Re and Re_p , where the ambient shear $\tilde{G}$ and streamwise velocity $\tilde{u}_p$ of the oncoming flow at the elevation of particle center are used to calculate the Reynolds numbers.

The theoretical lift force expressions of Saffman (1965), Leighton and Acrivos (1985) and the numerical simulation results of Zeng et al. (2009) can be expressed as,

$$C_L = \begin{cases} \frac{6.46}{\pi} \left(\frac{\text{Re}^{3/2}}{\text{Re}_p^2} \right) & : \text{Saffman,} \\ \frac{4.61}{\pi} \left(\frac{\text{Re}^2}{\text{Re}_p^2} \right) & : \text{Leighton and Acrivos,} \\ \frac{3.90}{\pi} \left(\frac{\text{Re}^{1.56}}{\text{Re}_p^2} \right) & : \text{Zeng et al.} \end{cases} \quad (9)$$

It can be seen that while the dependence on Re_p remains the same the dependence on Re varies and the numerical results are closer to the power-law of Saffman (1965). In Fig. 14 we present the computed values of lift force for a particle sitting on a rough wall (plotted

as symbols) along with predictions made with the above three lift expressions (plotted as lines). In all three predictions Re and Re_p were computed based on local values at the particle center.

In the case of particle in an unbounded shear flow, at finite Reynolds number the lift force on the particle is substantially lower than the theoretical prediction by Saffman(1965) (McLaughlin (1991), Kurose and Komori (1999) and Bagchi and Balachandar (2002)). In the presence of a bounding wall the lift force substantially increases, but as can be seen from (9) and from Fig. 14, the wall-bounded shear-induced lift force for a flat-boundary (blue curve) still remains lower than Saffman lift (black curve). At $Re > 5$ the effect of wall roughness is to further increase the lift force and most interestingly the lift force on a particle sitting on a rough wall is well approximated by the Saffman lift force. However this agreement is to be interpreted as coincidence.

5.3. Effects of Faxén correction and local shear

In the above presented results the drag and lift forces were normalized by the undisturbed flow velocity at the center of the particle (i.e., by $\tilde{u}_p$ in (3)). Due to the ambient shear, the undisturbed flow substantially varies on the scale of the particle. Here we will examine the leading order effect of this spatial variation of the ambient flow by including the Faxén correction. According to Faxén theorem, in a spatially varying ambient flow the quasi steady drag force must be evaluated in terms of the undisturbed flow velocity averaged over the surface of the particle. If the length scale of spatial variation of undisturbed ambient flow is much larger than the particle diameter, the surface average can be approximated as

$$\overline{\tilde{u}_p}^S \approx \tilde{u}_p^F = \tilde{u}_p + \frac{\tilde{d}^2}{24} \nabla^2 \tilde{u}_p. \quad (10)$$

where $\overline{(\cdot)}^S$ denotes an average over the surface of the particle.

The advantage of the above approximation is that both $\tilde{u}_p$ and $\nabla^2 \tilde{u}_p$ are evaluated at the center of the particle. As pointed out earlier for each simulation with the particle, we also performed companion simulations without the particle. The later simulation is the undisturbed flow that the particle sees and $\tilde{u}_p$ and $\nabla^2 \tilde{u}_p$ are evaluated from the companion simulation. In a perfectly linear ambient flow the surface-averaged velocity is the same as

the velocity at the particle center, and $\nabla^2 \tilde{u}_P = 0$. However, in the present case of a rough wall, the linear inflow velocity profile is modified by the surface roughness and contributes to non-zero $\nabla^2 \tilde{u}_P$. Here we will test $\tilde{u}_P^F$ as the undisturbed flow velocity seen by the particle. The corresponding Faxén corrected drag and lift coefficients evaluated from the simulation results can be redefined as

$$C_D^F = \frac{\tilde{F}_x}{\frac{1}{2}\tilde{\rho}_f(\tilde{u}_P^F)^2\tilde{A}_p}, C_L^F = \frac{\tilde{F}_y}{\frac{1}{2}\tilde{\rho}_f(\tilde{u}_P^F)^2\tilde{A}_p} \quad (11)$$

We note that the Faxén theorem is strictly valid only in the Stokes limit and for the drag force. Here we are testing it even at finite Re and for lift force.

Figures 15 and 16 show C_D^F and C_L^F computed based on Faxén-corrected undisturbed flow along with the drag and lift coefficients obtained from (3) without the Faxén correction. At low Reynolds numbers below $Re \approx 25$ the difference between the two is quite small. But the difference in drag and lift coefficients evaluated with and without Faxén correction increases at higher Reynolds number. This difference is mainly due to the fact that at higher Re the second term of (10) is negative and non-negligible, and as a result the approximate surface-averaged undisturbed velocity is lower than the undisturbed velocity at the center of the particle. This reduction in the effective flow velocity seen by the particle contributes to an increase in the drag and lift coefficients as computed with the Faxén correction.

We wish to accurately predict the computed drag and lift coefficients using the smooth-wall correlations given in (4) and (6). The two parameters of these correlations are Re_p and δ . Therefore we address the question how should Re_p and δ be defined in the case of rough wall so that when used in (4) and (6) we obtain a good estimation of the actual drag and lift forces. We fix δ to be the distance between the bottom of the particle to the bed level.

For the Reynolds number to be used in (4) and (6) we consider four different choices

$$\begin{aligned}
\text{Re}_{p1} &= \frac{\tilde{u}_p \tilde{d}}{\nu} &= u_p \text{Re} \\
\text{Re}_{p2} &= \frac{\tilde{u}_p^F \tilde{d}}{\nu} &= u_p^F \text{Re} \\
\text{Re}_{p3} &= u_p \text{Re} \frac{\tilde{G}_p}{\tilde{G}_i} \\
\text{Re}_{p4} &= u_p^F \text{Re} \frac{\tilde{G}_p}{\tilde{G}_i}
\end{aligned} \tag{12}$$

The first one is the definition that was used in evaluating the drag and lift coefficients plotted in Figs. 11 and 12. This definition uses the undisturbed flow velocity at the particle center. In Re_{p2} the leading order Faxén correction is included in the definition of undisturbed flow velocity. In the definitions Re_{p3} and Re_{p4} we make adjustment for the fact that the shear magnitude at the particle center may not be that prescribed at the inflow. Clearly, there are other choices of particle Reynolds number definitions. But the above definitions should be sufficient to illustrate the main points.

Also plotted in Figs. 15 and 16 are the drag and lift coefficients estimated using the smooth-wall correlations (4) and (6) with the above four definitions of particle Reynolds number. The blue and red curves should be compared against the blue symbols as they are without the Faxén correction. The green and black curves are with Faxén correction and they should be compared with the red symbols. Several conclusions can be drawn. First, the local values of shear used in the definitions of Re_{p3} and Re_{p4} increase the level of under-prediction of the actual drag and lift forces. While the drag and lift coefficients predicted with Faxén-corrected Reynolds number in the correlations (4) and (6) is slightly higher than those predicted without the Faxén correction. But the difference is far smaller than the difference between the actual drag and lift coefficients computed with and without the Faxén correction. In summary, one may conclude that it is not easy to improve upon the smooth-wall drag and lift corrections (4) and (6) used with the definition of Re_{p1} . Furthermore, in the present case, simple implementation of Faxén correction does not appear to improve the prediction.

6. Conclusions

We have performed direct numerical simulations of a shear flow past a spherical particle sitting over a rough wall formed by hemispherical bumps. In this study the hemispherical bumps were taken to be the with diameter as the particle. The two key parameters are the Reynolds number based on inflow shear and particle diameter (Re), and the non-dimensional distance between the particle and the the rough bed (δ). A matrix of simulations for varying Re and δ were performed. For each parametric combination, two simulations were performed. One with the particle, which is used in evaluating the steady state drag and lift forces on the particle, and a companion second simulation without the particle (flow over a rough wall) which provided the undisturbed flow, defined to be the flow in the absence of the particle. This undisturbed flow is important in evaluating the drag and lift correlations which are based on undisturbed flow at the particle location.

We first conducted a test of convergence to ensure that the number of hemispherical elements that form the rough bed upstream, downstream and on the lateral directions are sufficient to produce a fully developed rough-wall shear flow as it approaches the particle. The drag and lift forces obtained from the simulations were established to be well converged both in terms of grid resolution and in terms of the number of roughness elements.

The results of the numerical simulations over a rough wall were compared with the drag and lift correlations for a particle over a smooth wall obtained by Zeng et al. (2009). These smooth-wall correlations given in (4) and (6) depend on the two parameters: particle Reynolds number Re_p and gas between the particle and the bed δ . We addressed the question how should Re_p and δ be defined in the case of a rough wall so that smooth-wall correlations remain applicable and accurate even in the rough wall case. We fixed δ to be the distance between the bottom of the particle to the bed level and considered four different definitions of particle Reynolds number that included Faxén correction and local estimation of shear. It was found that particle Reynolds number based on mean shear and local particle velocity at the center of particle was sufficiently accurate to predict the drag and lift coefficients.

The effect of the rough wall was to increase the lift coefficient at moderate Reynolds

numbers. Only at low Reynolds numbers of $Re = 2$ and 5 a substantial reduction in the lift force was observed for the rough wall. A plausible cause may be the non-zero flow under the particle through the rough bed, unlike that for the smooth bed. We also considered the special case of the particle sitting on the rough bed and compared the drag and lift forces to those obtained for the smooth bed.

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8. References

- Amir, M., Nikora, V., Stewart, M., 2014. Pressure forces on sediment particles in turbulent open-channel flow: a laboratory study. *J. Fluids Mech.* 757, 458–497.
- Bagchi, P., Balachandar, S., 2002. Shear versus vortex-induced lift force on a rigid sphere at moderate Re . *J. Fluid Mech.* 473, 379–388.
- Bagnold, R. A., 1966. An approach to the sediment transport problem from general physics. U.S. Geol. Surv. Prof. Pap. 422-I, 1–37.
- Bridge, J. S., Bennett, S. J., 1992. A model for the entrainment and transport of sediment grains of mixed sizes, shapes, and densities. *Water Resour. Res.* 28 (2), 337–363.
- Buffington, J. M., Montgomery, D. R., 1997. A systematic analysis of eight decades of incipient motion studies, with special reference to gravel-bedded rivers. *Water Resour. Res.* 33 (8), 1993–2029.
- Celik, A., Diplas, P., Dancey, C., 2014. Instantaneous pressure measurements on a spherical grain under threshold flow conditions. *J. Fluids Mech.* 741, 60–97.
- Celik, A. O., Diplas, P., Dancey, C. L., Valyrakis, M., 2010. Impulse and particle dislodgement under turbulent flow conditions. *Phys. Fluids* 22 (4), 046601.
- Chakraborty, P., Balachandar, S., Adrian, R. J., 2005. On the relationships between local vortex identification schemes. *J. Fluid Mech.* 535, 189–214.
- Chan-Braun, C., García-Villalba, M., Uhlmann, M., 2010. Numerical simulation of fully resolved particles in rough-wall turbulent open channel flow. 7th Int. Conf. Multiphase Flows Tampa, Florida, USA.
- Chan-Braun, C., García-Villalba, M., Uhlmann, M., 2011. Force and torque acting on particles in a rough wall open channel flow. *J. Fluid Mech.* 684, 441–474.
- Chan-Braun, C., García-Villalba, M., Uhlmann, M., 2013. Spatial and temporal scales of force and torque acting on wall-mounted spherical particles in open channel flow. *Phys. Fluids* 25, 075103.
- Charru, F., Larrieu, E., Dupont, J.-B., Zenit, R., 2007. Motion of a particle near a rough wall in a viscous shear flow. *J. Fluid Mech.* 570, 431–453.
- Cheng, N.-S., Chiew, Y.-M., 1998. Pickup probability for sediment entrainment. *J. Hydraul. Eng.* 124 (2), 232–235.
- Cherukat, P., McLaughlin, J. B., 1994. The inertial lift on a rigid sphere in a linear shear flow field near a flat wall. *J. Fluid Mech.* 263, 1–18.
- Cox, R. G., Hsu, S. K., 1977. The lateral migration of solid particles in a laminar flow near a plane. *Int. J. Multiphase flow* 3 (3), 201–222.
- Davies, T. R. H., Samad, M. F. A., 1978. Fluid dynamic lift on a bed particle. *J. Hydraul. Div., Am. Soc. Civ. Eng.* 104 (8), 1171–1182.
- Diplas, P., Dancey, C. L., Celik, A. O., Valyrakis, M., Greer, K., Akar, T., 2008. The role of impulse on the initiation of particle movement under turbulent flow conditions. *Science* 322 (5902), 717–720.

- Garcia, M. H., 2008. Sediment transport and morphodynamics. In: Sedimentation Engineering: processes, measurements, modeling, and practice, ASCE manual of practice 110. ASCE, Reston, Va., Ch. 2, pp. 21–163.
- Goldman, A., Cox, R. G., Brenner, H., 1967a. Slow viscous motion of a sphere parallel to a plane wall - I. Motion through a quiescent fluid. *Chem. Engng. Sci.* 22, 637–651.
- Goldman, A., Cox, R. G., Brenner, H., 1967b. Slow viscous motion of a sphere parallel to a plane wall - II. Couette flow. *Chem. Engng. Sci.* 22, 653–660.
- Hall, D., 1988. Measurements of the mean force on a particle near a boundary in turbulent flow. *J. Fluid Mech.* 187, 451–466.
- King, M. R., Leighton, D. T., 1995. Measurement of the inertial lift on a moving sphere in contact with a plane wall in a shear flow. *Phys. Fluids*. 7 (11), 2538–2545.
- Krishnan, G. P., Leighton, D. T., 1995. Inertial lift on a moving sphere in contact with a plane wall in a shear flow. *Phys. Fluids*. 7 (11), 2538–2545.
- Kurose, R., Komori, S., 1999. Drag and lift forces on a rotating sphere in a linear shear flow. *J. Fluid Mech.* 384, 183–206.
- Lee, H., Balachandar, S., 2010. Drag and lift forces on a spherical particle moving on a wall in a shear flow at finite Re. *J. Fluid Mech.* 657, 89–125.
- Lee, H., Ha, M. Y., Balachandar, S., 2011. Rolling/sliding of a particle on a flat wall in a linear shear flow at finite Re. *Int. J. Multiphase flow* 37, 108–124.
- Lee, H., Ha, M. Y., Balachandar, S., 2012. Work-based criterion for particle motion and implication for turbulent bed-load transport. *Phys. Fluids* 24 (11), 116604.
- Leighton, D. T., Acrivos, A., 1985. The lift on a small sphere touching a plane in the presence of a simple shear flow. *Z. Angew. Math. Phys.* 36, 174–178.
- Ling, C. H., 1995. Criteria for incipient motion of spherical sediment particles. *J. Hydraul. Eng.* 121 (6), 472–478.
- Magnaudet, J., 2003. Small inertial effects on a spherical bubble, drop or particle moving near a wall in a time-dependent linear flow. *J. Fluid Mech.* 485, 115–142.
- Marsh, N. A., Western, A. W., Grayson, R. B., 2004. Comparison of methods for predicting incipient motion for sand beds. *J. Hydraul. Eng.* 130, 616–621.
- McLaughlin, J. B., 1991. Inertial migration of a small sphere in linear shear flows. *J. Fluid Mech.* 224, 261–274.
- McLaughlin, J. B., 1993. The lift on a small sphere in wall-bounded linear shear flows. *J. Fluid Mech.* 246, 249–265.
- Mollinger, A. M., Nieuwstadt, F. T. M., 1996. Measurement of the lift force on a particle fixed to the wall in the viscous sublayer of a fully developed turbulent boundary layer. *J. Fluid Mech.* 316, 285–306.
- Muthanna, C., Nieuwstadt, F. T. M., Hunt, J. C. R., 2005. Measurement of the aerodynamic forces on a small particle attached to a wall. *Exp. Fluids* 39, 455–463.
- Niño, Y., Lopez, F., Garcia, M. H., 2003. Threshold for particle entrainment into suspension. *Sedimentology* 50, 247–263.
- Raupach, M. R., Antonia, R. A., Rajagopalan, S., 1991. Rough-wall turbulent boundary layers. *Appl. Mech. Rev.* 44 (1), 1–25.
- Reeks, M. W., Hall, D., 2001. Kinetic models for particle resuspension in turbulent flows: theory and measurement. *Journal of Aerosol Science* 32 (1), 1–31.
- Reeks, M. W., Reed, J., Hall, D., 1988. On the resuspension of small particles by a turbulent flow. *J. Phys. D: Appl. Phys.* 21 (4), 574–589.
- Rubinow, S., Keller, J. B., 1961. The transverse force on a spinning sphere moving in a viscous fluid. *J. Fluid Mech.* 11 (03), 447–459.
- Saffman, P. G., 1965. The lift on a small sphere in a slow shear flow. *J. Fluid Mech.* 22, 385–400, (Corrigendum 1968, 31, 624).
- Schmeeckle, M. W., Nelson, J. M., Shreve, R. L., 2007. Forces on stationary particles in near-bed turbulent flows. *J. Geophys. Res.: Earth Surf.* 112 (F2).

- Takemura, F., Magnaudet, J., 2003. The transverse force on clean and contaminated bubbles rising near a vertical wall at moderate Reynolds number. *J. Fluid Mech.* 495, 235–253.
- Takemura, F., Takagi, S., Magnaudet, J., Matsumoto, Y., 2002. Drag and lift forces on a bubble rising near a vertical wall in a viscous liquid. *J. Fluid Mech.* 461, 277–300.
- Uhlmann, M., 2005a. An immersed boundary method with direct forcing for the simulation of particulate flows. *J. Comput. Phys.* 209, 448–476.
- Uhlmann, M., 2005b. An improved fluid-solid coupling method for dns of particulate flow on a fixed mesh. In: *Proc. 11th Workshop Two-Phase Flow Predictions*, Merseburg, Germany.
- Valyrakis, M., Diplas, P., Dancey, C. L., Greer, K., Celik, A. O., 2010. Role of instantaneous force magnitude and duration on particle entrainment. *J. Geophys. Res.: Earth Surf.* 115 (F2).
- van Rijn, L. C., 1984. Sediment transport, part i: Bed load transport. *J. Hydraul. Eng.* 110, 1431–1456.
- Watters, G. Z., Rao, M. V. P., 1971. Hydrodynamic effects of seepage on bed particles. *J. Hydraul. Div., Am. Soc. Civ. Eng.* 97 (3), 421–439.
- Wu, F.-C., Chou, Y.-J., 2003. Rolling and lifting probabilities for sediment entrainment. *J. Hydraul. Eng.* 129 (2), 110–119.
- Zeng, L., Balachandar, S., Fisher, P., 2005. Wall-induced forces on a rigid sphere at finite Reynolds number. *J. Fluid Mech.* 536, 1–25.
- Zeng, L., Najjar, F., Balachandar, S., Fisher, P., 2009. Forces on a finite-sized particle located close to a wall in a linear shear flow. *Phys. Fluids* 21, 033302.
- Zhou, J., Adrian, R. J., Balachandar, S., Kendall, T. M., 1999. Mechanisms for generating coherent packets of hairpin vortices in channel flow. *J. Fluid Mech.* 387, 353–396.

Figure 1: Schematics of a particle near a hemispherical rough wall in a sheared ambient flow and its coordinate system.

Figure 2: Top-view of the staggered rough bed and the particle.

Figure 3: Sketch of a spherical particle touching hemispherical wall bed. (a) top view a particle and hemispheres consisting of rough wall bed, (solid circles) outlines of hemispheres, (dashed circle) outline of a spherical particle, (b) iso-view of a particle position over a rough bed.

Figure 4: Frames (a) and (b) show the two different definitions of bed level and its height h_{BL} from the base level. In this definitions the gap δ is defined as distance from the bottom of the particle to the bed level. Frame (c) offers a definition of gap in terms of the shortest distance between the particle and the bed and does not require the definition of a bed level.

Figure 5: Isometric-view of the computational domain; a particle and rough bed configuration.

Figure 6: Variations of undisturbed streamwise velocity u along y -direction at different locations ($Re_s=100$) . Varying $x = 6 \sim 10$ at two fixed z -locations (a) $z=0.144$, (b) $z=-0.433$. (c) Varying $z=-1.010, 0.144, 0.722$ at fixed x -location ($x=9.0$). (d) At four different topmost positions over hemispheres.

Figure 7: Results of convergence test for rough bed arrangement for $Re=100$. (a) $L=0.8165$, (b) $L=1.8165$.

Figure 8: Reynolds number effect on undisturbed flow at a specific location 'p' ($x=9$, $z=0.144$). (a) stream-wise flow velocity u , (b) flow shear rate $G = \partial u / \partial y$.

Figure 9: Streamlines around particle at $z = 0.144$. (a) $\text{Re}=2$, $L = 0.8165$ (b) $\text{Re}=2$, $L = 1.8165$ (c) $\text{Re}=100$, $L = 0.8165$ (d) $\text{Re}=100$, $L = 1.8165$. The insets represent the side-view of the streamlines.

Figure 10: Vortical structures ($\lambda_{ci}=0.75$) (a) $\text{Re}=2$, $L=0.8165$ (b) $\text{Re}=2$, $L=1.8165$ (c) $\text{Re}=100$, $L=0.8165$ (d) $\text{Re}=100$, $L=1.8165$.

Figure 11: Drag coefficient of the particle over both smooth and rough wall as a function of the separation distance δ for different Reynolds number (a) $Re=2$, (b) 5, (c) 10, (d) 25, (e) 50 and (f) 100. (Symbols) Results obtained from present numerical simulations over rough wall, (lines) from the relation of Zeng et al. (2009) for smooth wall.

Figure 12: Lift coefficient as a function of the separation distance δ for different Reynolds number (a) $Re=2$, (b) 5, (c) 10, (d) 25, (e) 50 and (f) 100. Symbols denote the present numerical data for the rough wall and lines are data from (6) for smooth wall.

Figure 13: (a) Velocity vector plot over a particle located above a rough bed with $L = 0.8165$ at $\text{Re} = 100$ ($z=0.144$ plane). Vectors are colored with velocity magnitude. (b) Shows the corresponding pressure distribution.

Figure 14: Lift coefficient C_L for a particle sitting on a wall in range of shear Reynolds number $2 \leq Re \leq 100$. Comparisons were conducted with theoretical expressions by Saffman (1965) and Leighton and Acrivos (1985) and empirical by Zeng et al. (2009).

Figure 15: Drag coefficient as a function of δ . The symbols are computed drag coefficient based on two different definitions of undisturbed velocity at the particle location (blue diamond: without Faxén correction, red square: with Faxén correction). The four different curves are predictions using correlation (4). Blue: Without Faxén correction for velocity and with mean shear; red: With Faxén correction for velocity and mean shear; Green: Without Faxén correction for velocity and with local shear; Black: With Faxén correction for velocity and local shear. (a) $Re = 10$, (b) $Re = 25$, (c) $Re = 50$, and (d) $Re = 100$.

Figure 16: Lift coefficient as a function of δ . The symbols are computed lift coefficient based on two different definitions of undisturbed velocity at the particle location (blue diamond: without Faxén correction, red square: with Faxén correction). The four different curves are predictions using correlation (6). Blue: Without Faxén correction for velocity and with mean shear; red: With Faxén correction for velocity and mean shear; Green: Without Faxén correction for velocity and with local shear; Black: With Faxén correction for velocity and local shear. (a) $Re = 10$, (b) $Re = 25$, (c) $Re = 50$, and (d) $Re = 100$.