Modeling and Experimental Investigations on the Relationship between Wear Debris Concentration and Wear Rate in Lubrication Systems

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Abstract: The aim of this work is to map the wear debris concentration with wear rate. A wear debris attenuation function is proposed to depict quantitatively the removal of wear debris. The total residual mass of wear debris in lubrication systems can be stated as the convolution of the wear rate and the wear debris attenuation function. Additionally, the dynamic responses of the wear debris concentration under different wear rate excitations are also presented. To simulate and verify how the debris concentration changes, an oil lubrication system test rig is built, and on-line visual ferrograph (OLVF) is used to monitor the debris concentration. Two experimental results indicate that the variation of the wear debris concentration is consistent with the model result.

Keyword: Wear debris monitoring; Wear rate; Wear debris removal; Lubrication system

Nomenclature

t	Time of machine running (min)
$m(t)$	Wear rate (mg min^{-1})
m_0, m_s	Wear rate at the initial moment and steady-state stage, respectively (mg min^{-1})
$W_R'(t)$	White noise which obeys a Gauss distribution at the running-in stage
$W_S'(t)$	White noise which obeys a Gauss distribution at the steady-state wear stage
$W_F'(t)$	White noise which obeys a Gauss distribution at the severe wear stage
$r(\Delta t)$	Wear debris attenuation function (mg min^{-1})
k	The attenuation coefficient ($k = [(1 - 1/\beta_x)Q + \psi]/V_0$)
ψ	Debris loss factor by other removal factors (e.g. sedimentation, comminution and sticking to surfaces, etc)
β_x	The Beta ratio for describing a filter's performance where x is the micron rating of the filter element
Q	The nominal oil flow through the filter ($\text{L} \cdot \text{min}^{-1}$)
$M(u_i)$	Mass of wear debris generated over the time interval $[u_i, u_{i+1}]$ (mg)
$MR(u_i)$	Residual mass of debris in the oil over the time interval $[u_i, u_{i+1}]$ (mg)
$MR(t - u_i)$	Residual mass of wear debris generated at time u_i after a time period of $t - u_i$ (mg)
$TMR(t)$	Over the time $[0, t]$, the total residual mass of debris in a lubrication system (mg)
V_0	Initial oil volume in a lubrication system (L)
$V(t)$	Volume of oil vs. t (L)
V_q	Oil loss rate ($\text{L} \cdot \text{min}^{-1}$)

V_r	Fresh oil replenishment rate ($L \cdot min^{-1}$)
$C(t)$	Wear debris concentration (ppm)
C_0	Initial debris concentration in an oil lubrication system
$E\{C(t)\}$	Mean function of $C(t)$ (ppm)
C_s	Stationary concentration of wear debris (ppm)
ΔT	Sampling oil interval of OLVF (s)
L	Residence time of wear debris in a lubrication system (s)
C_{add}	High concentration of oil contaminated wear debris in the beaker (ppm)
q	Flow rate of the wriggle pump ($ml \cdot min^{-1}$)
t_α	The time when a debris concentration of αC_s is reached after oil change, where α is close to one (s).

1 Introduction

With the development of predictive maintenance, a variety of machine condition monitoring techniques have been widely used by the industrial enterprises [1-4]. Wear debris monitoring is an effective approach for revealing the wear condition of a machine without disassembly [5-7]. This method can provide important symptom to decision makers at an early stage, eliminate the need for costly machine shutdowns for inspection, and avoid the possibility of catastrophic component failure in service [8, 9]. Currently, off-line wear debris monitoring technique has been widely used in many engineering applications [10, 11]. However, due to complex operations, low efficiency and inevitable information lagging in the process of off-line analysis, on-line wear debris monitoring is attracting more and more

attention.

On-line monitoring sensors are the tool of obtaining wear debris information. In many literatures, a variety of on-line/in-line wear debris sensors such as electrostatic sensing, optics, acoustic emission, capacitance sensing, and bulk inductive sensing have been reported [12-18]. The wear debris in lubrication systems can be real-time detected at present. Real time results are not as detailed and accurate as off-line results, but they reduce the problem of misrepresentative samples. In lubrication systems, the wear debris concentration is usually a primary indicator of on-line/in-line oil monitoring. It is used to quantify the wear extent and wear rate progression [19, 20]. However, the wear debris concentration in a practical lubrication system is influenced by many factors, such as wear debris removal, oil consumption, oil replenishment and harsh working conditions, etc. The wear debris concentration is mixed with some “noise” caused by the interferences mentioned above [6]. This leads to some issues that on-line monitoring data is difficult to be explained, and the mapping relationship between the evolution process of machine wear and the real time monitoring data is hard to be established. Therefore, on-line dynamic identification of machine wear state is becoming a bottleneck.

Wear occurs in tiny gaps between contact surfaces. The wear rate is a time-varying parameter, which can quantify the wear extent and wear state progression. But it is difficult to real-time measure wear rate without changing contact states of tribo-pairs. Comparatively, the wear debris in oil is products of the tribo-pairs wear. On-line monitoring wear debris concentration becomes relatively easy due to the development of on-line/in-line wear debris sensors. Therefore, determining the relationship between wear debris concentration and wear

rate becomes crucial to the practical application of oil monitoring.

Efforts have been made to research the mapping relationship. During the early 1980's, most research pointed out that the wear debris concentration from the ferrography can reach a dynamic equilibrium. At first Anderson and Driver [21] described mathematically the arrival of an equilibrium concentration of debris through the summation of a geometric series, and calculate the time to the equilibrium concentration under a constant wear rate. Kjer [22] built a differential relationship between wear debris concentration and wear rate from running-in to steady state for unspecified small and large debris. Szymczyk [23] improved the Kjer' model by introducing the exponentially increase wear rate prior to machine failure. Subsequently, Liu et al. [24] introduced stochastic noise to the wear rate equation, and put forward the stationary concentration of wear debris. On this basis, Henneberg et al. [25, 26] introduced a array of debris sizes to characterize the concentrations of wear debris with different sizes, and expanded to a dual filter model. Furthermore, taking into account the oil consumption and replenishment effects, Macian V et al. [19] developed an analytical approach for the wear rate of internal combustion engine based on spectrometric oil analysis. Feng et al. [20] built a model of the wear debris concentration from ferrography to quantify a wear process of a gearbox in a full-life cycle gear wear based on Kragelsky's method. However, filtering effect and random factors were ignored in the two models.

In these models mentioned above, the wear debris removal usually is described by a simple debris loss factor. The influence of those factors such as filtration, sedimentation, comminution and sticking to surfaces, etc on the debris concentration are blurred by this way. However, the explicit expression for the wear debris removal can be not provided. In addition,

the models need to solve the complex differential equation, in which wear rate is assumed to be given. But if the removal of wear debris, replenishment and consumption of lube oil are all considered, an analytic solution of the equation can be hardly obtained. Moreover, in practice, the wear rate is usually unknown due to the complex structure and poor working conditions of machines. Based on the models mentioned above, it is very difficult to explain and analyze the wear debris concentration time-series data from a machine in service, and also to quantify and predict the wear process of the machine.

In this work, to overcome this drawback, a wear debris attenuation function is proposed to depict quantitatively the removal of wear debris, in which the filtration and other removal factors are considered separately. A general mathematical model is developed to map the wear debris concentration with wear rate. Moreover, to illustrate the evolution of the wear debris concentration in lubrication system of machines, the dynamic responses of the wear debris concentration under different wear rate excitations are also presented based on proposed model. An oil lubrication system test rig is built, and an on-line visual ferrograph (OLVF) is used to on-line monitor the debris concentration. The proposed model is verified experimentally by two simulation tests. The study provides a new approach for the explanation of on-line wear debris concentration data and the evaluation of wear state.

2 Modeling wear debris concentration in the lubrication system of machines

In any machine with oil cycle system, the oil is always in the condition of cycle flow and stir. Moreover, the wear debris is generated and removed continuously in the oil lubrication system. A simplified lubrication system of machines is shown in Fig. 1. For establishing the relationship between wear debris concentration and wear rate, some previous assumptions are

made:

- Wear debris produced are instantaneous and homogeneously mixed in oil [19].
- Oil top-up can be assumed as a continuous process [19].
- The history of a machine is from the beginning of its lifetime.
- The wear rate is equal to the wear debris produced rate.
- Sampling location is set in the oil tank.

Fig. 1 is here

2.1 Quantifying the wear debris removal capacity of the lubrication system

Due to the impact of filtration, sedimentation, comminution and sticking to surfaces, etc, factors, wear debris suspended in oil was continuously removed or lost. In this work, the wear debris attenuation function is defined to characterize quantitatively the removal capacity of the oil lubrication system for the wear debris. Its meaning is the residual percentage of wear debris generated at t_0 after a time period of $t-t_0$, which can be approximately expressed as

$$r(\Delta t) = \frac{MR(\Delta t)}{M(t_0)} * 100\% \quad (1)$$

where $r(\Delta t)$ ($0 \leq r(\Delta t) \leq 1$, $\Delta t = t - t_0$) is the attenuation function of wear debris (mg/min).

$M(t_0)$ is mass of wear debris generated at time t_0 ; $MR(\Delta t)$ is residual mass of wear debris generated at time t_0 after a time period of $t-t_0$.

For the filtration, a Beta ratio (β_x) is now the commonly accepted method for describing a filter's performance in terms of retaining particles [27]. The subscript x refers to the micron rating of the filter element. In this work, the Beta ratio is used to characterize quantitatively the filtration. Moreover, the debris size smaller than the micron rating of the filter are

assumed not be removed by normal oil filters. Besides, a simple debris loss factor ψ ($\psi > 0$) is adopted to characterize other removal factors (e.g. sedimentation, comminution and sticking to surfaces, etc). The removal mass causing by other removal factors is assumed to be proportional to the concentration of debris.

As shown in Fig. 1, in the oil lubrication system, the removal mass of wear debris per unit time equal to the loss of wear debris in oil. On this basis, when the wear rate is equal to zero, the concentration of wear debris can be expressed as follows:

$$\frac{dC(t)}{dt} + \frac{(1 - 1/\beta_x)QC(t)}{V_0} + \frac{\psi C(t)}{V_0} = 0 \quad (2)$$

where $C(t)$ is the wear debris concentration; Q is the nominal oil flow through the filter; V_0 is the initial volume of lube oil in the tank.

After integration, we have

$$C(t) = A \exp\left\{-\frac{(1 - 1/\beta_x)Q + \psi}{V_0}(t - t_0)\right\} \quad (3)$$

where A is the integration constant and t_0 is the initial time.

When $t=t_0=0$, $A=C_0$ (initial debris concentration at the time t_0). Then Eq. (3) becomes

$$C(t) = C_0 \exp\left(-\frac{(1 - 1/\beta_x)Q + \psi}{V_0}t\right) \quad (4)$$

Therefore, in Fig. 1, the wear debris attenuation function can be expressed as follows:

$$r(t) = \frac{C(t)}{C_0} = \exp(-kt) * 100\% \quad (5)$$

where k is the attenuation coefficient ($k = [(1 - 1/\beta_x)Q + \psi]/V_0$).

From Eq. (5), the attenuation of wear debris can be stated as an exponential decrease function. The attenuation coefficient is related to both the size range of wear debris and the

basic parameters of lubrication system of machines.

2.2 Determining the relationship among wear debris concentration, wear debris attenuation and wear rate

The debris in oil is produced and removed continuously as the machine is running. Therefore, the wear debris concentration in oil lubrication system is related to the wear rate and the wear debris attenuation.

Assuming a machine has been run continuously for some period of time. The running time t (min) is divided into N identical intervals (see Fig. 2). Over the time interval $[u_i, u_{i+1}]$, the mass of wear debris produced $M(u_i)$ can be approximately expressed as

$$M(u_i) \approx m(u_i)\Delta u \quad (6)$$

where $m(u_i)$ is wear rate at time u_i ; Δu ($\Delta u = u_{i+1} - u_i$, $i = 0, 1, 2, \dots, N$) is the time interval.

A debris attenuation function mentioned above is used to express the removal rules. After $t - u_i$, the residual mass $MR(t - u_i)$ of wear debris produced at time u_i is:

$$MR(t - u_i) \approx M(u_i)r(t - u_i) \quad (7)$$

Accordingly, at time t , the total residual mass of wear debris in lube oil $TMR(t)$ can be approximately expressed as

$$TMR(t) = \sum_{i=0}^{i=N} MR(t - u_i) = \sum_{i=0}^{i=N} m(u_i)\Delta u r(t - u_i) \quad (8)$$

Let $u_\Delta \rightarrow 0$, then Eq. (8) can be simplified as

$$TMR(t) = \int_0^t m(u) \cdot r(t - u) du = m(t) * r(t) \quad (9)$$

Consequently, the total residual mass of wear debris in the lubrication system is the convolution of wear rate and attenuation function.

Fig. 2 is here

Taking into account oil consumption and replenishment, the volume of oil $V(t)$ is written as

$$V(t) = V_0 + V_r \cdot t - V_q \cdot t \quad (10)$$

where V_r is the fresh oil replenishment rate; V_q is the oil loss rate; V_0 is the initial oil volume.

The wear debris concentration $C(t)$ in oil lubrication system is expressed as

$$C(t) = \frac{TMR(t)}{V(t)} = \frac{m(t) \cdot r(t)}{V_0 + V_r \cdot t - V_q \cdot t} \quad (11)$$

From Eq. (11), the general relationship between the wear debris concentration and the wear rate in lubrication system of machines has been determined. In practice, the evolution of the wear debris concentration can be on-line monitored. Once $r(t)$ and $V(t)$ are given, the wear process of a machine can be quantified and predicted.

3 Dynamic responses of wear debris concentration under different wear rate excitations

A full-life wear process often has three stages, that is, running-in, normal (steady-state) and severe wear. The different stages of wear are characterized by different wear rate. Based on the model mentioned above, the dynamic responses of the wear debris concentration in lubrication system of machines are investigated under different wear rate excitations.

3.1 Dynamic responses of debris concentration under running-in wear rate excitation

For a running-in of a new machine, initially the wear rate is high which reduces exponentially with time and reaches steady-state after some time [20, 28, 29]. The wear rate can be expressed as follows:

$$m(t) = m_s + (m_0 - m_s)e^{-bt} + W_R'(t) \quad (12)$$

where m_0 and m_s are wear rate at the initial moment and steady-state stage, respectively; the

coefficient $b(b > 0)$ has inverse relation with running-in period, which is usually influenced by the material of tribo-pairs, surface roughness and lube oil, load and rotational speed, etc.

$W_R'(t)$ is white Gaussian noise in the running-in stage.

Substituting Eq. (5) and (12) into Eq. (9), in the running-in stage, the total residual mass of wear debris in an oil lubrication system $TMR(t)$ is obtained:

$$TMR(t) = \frac{m_s}{k}(1 - e^{-kt}) + \frac{m_0 - m_s}{k - b}(e^{-bt} - e^{-kt}) + \int_0^t e^{-k(t-u)} dW_R(u) \quad (13)$$

Substituting Eq. (13) into Eq. (11), the wear debris concentration is as follows:

$$C(t) = \frac{TMR(t)}{V(t)} = \frac{\frac{m_s}{k}(1 - e^{-kt}) + \frac{m_0 - m_s}{k - b}(e^{-bt} - e^{-kt}) + \int_0^t e^{-k(t-u)} dW_R(u)}{V_0 + V_r \cdot t - V_q \cdot t} \quad (14)$$

The mean function of $C(t)$ is

$$E\{C(t)\} = \frac{\frac{m_s}{k}(1 - e^{-kt}) + \frac{m_0 - m_s}{k - b}(e^{-bt} - e^{-kt})}{V_0 + V_r \cdot t - V_q \cdot t} \quad (15)$$

If the quantity of oil replenishment is equal to the oil consumption exactly, i.e., $V_r = V_q$,

Eq. (15) becomes

$$E\{C(t)\} = \frac{\frac{m_s}{k}(1 - e^{-kt}) + \frac{m_0 - m_s}{k - b}(e^{-bt} - e^{-kt})}{kV_0} \quad (16)$$

The responses of the wear debris concentration with different size ranges under running-in wear rate excitation are shown in Fig. 3.

Fig. 3 is here

3.2 Dynamic responses of debris concentration under steady-state wear rate excitation

During a steady state, the contact condition of the friction pair remains relatively steady.

The wear rate can be expressed as follows

$$m(t) = m_s + W_s'(t) \quad (17)$$

where m_s is the wear rate in steady-state wear stage; $W_s'(t)$ is white Gaussian noise in steady-state wear stage.

In practice, to prolong the life of the machine, oil change is usually performed in running-in end. Therefore, we assume that the wear debris produced during the running-in period have been removed from the oil lubrication system. Similarly, substituting Eq. (5) and (17) into Eq. (11), the wear debris concentration is as follows:

$$C(t) = \frac{\frac{m_s}{k}(1 - e^{-kt}) + \int_0^t e^{-k(t-u)} dW_s(u)}{V_0 + V_r \cdot t - V_q \cdot t} \quad (18)$$

Similarly, if the quantity of oil replenishment is equal to the oil consumption exactly, i.e., $V_r = V_q$, the mean function of $C(t)$ is

$$E\{C(t)\} = \frac{m_s}{kV_0}(1 - e^{-kt}) \quad (19)$$

When $t \rightarrow \infty$, Eq. (19) becomes

$$C_s = \lim_{t \rightarrow \infty} E\{C(t)\} = \frac{m_s}{kV_0} = \frac{m_s}{(1 - 1/\beta_x)Q + \psi} \quad (20)$$

where C_s is the stationary concentration of wear debris.

In the formulas above, for the debris size larger than the micron rating of the filter, a dynamic equilibrium still exists in steady-state wear stage, unless the wear rate increases or filter failures occur. If k and m_s are constant, C_s is also constant. The responses of the wear debris concentration with different size ranges under steady-state wear rate excitation are shown in Fig. 4.

Fig. 4 is here

3.3 Dynamic responses of debris concentration under severe wear rate excitation

After a long time steady-state wear, the contact status of the friction pair gradually and irreversibly deteriorates. The wear rate $m(t)$ is arbitrarily assumed to increase exponentially with time in severe wear stage. The wear rate can be expressed as follows [20, 23]:

$$m(t) = m_s e^{c(t-t_1)} + W_F'(t) \quad (21)$$

where $c (c > 0)$ are related to wear mode in severe wear stage, different wear modes behave differently, for example, fatigue wear corresponds to a gradual wear rate change, but a dramatic growth of wear rate occurs in severe adhesive wear [20]; t_1 is the initiation time of abnormal wear; $W_F'(t)$ are white Gaussian noise in the severe wear stage.

Assuming the initial concentration of wear debris in an oil lubrication system equals to zero. Similarly, substituting Eq. (5) and (21) into Eq. (9), the total residual mass of wear debris in an oil lubrication system $TMR(t)$ is obtained:

$$TMR(t) = \frac{m_s}{c+k} (e^{c(t-t_1)} - e^{-k(t-t_1)}) + \int_{t_1}^t e^{-k(t-u)} dW_F(u) \quad (22)$$

Similarly, substituting Eq. (22) into Eq. (11), the wear debris concentration is as follows:

$$C(t) = \frac{\frac{m_s}{c+k} (e^{c(t-t_1)} - e^{-k(t-t_1)}) + \int_{t_1}^t e^{-k(t-u)} dW_F(u)}{V_0 + V_r \cdot t - V_q \cdot t} \quad (23)$$

If the quantity of oil replenishment is equal to the oil consumption exactly, i.e., $V_r = V_q$,

$C(t)$ becomes

$$C(t) = \frac{m_s}{(c+k)V_0} (e^{c(t-t_1)} - e^{-k(t-t_1)}) + \frac{1}{V_0} \int_{t_1}^t e^{-k(t-u)} dW_F(u) \quad (24)$$

The mean function of $C(t)$ is

$$E\{C(t)\} = \frac{m_s}{(c+k)V_0} (e^{c(t-t_1)} - e^{-k(t-t_1)}) \quad (25)$$

The responses of the wear debris concentration with different size ranges under severe

wear rate excitation are shown in Fig. 5.

Fig. 5 is here

Based on the analysis above, a full-life wear process of a machine can be modeled by the wear debris concentration. If the concentration of wear debris with different sizes are real-time monitored, the full-life wear process of a machine can also be quantified and predicted based on the proposed model.

4 Experimental verification

4.1 Experimental setup

As shown in Fig. 6, an oil lubrication system test rig was built, which was mainly composed of an oil circuit, a simulated generator of wear debris and an OLVF. The lube oil pump drives the oil circulation of flow at a flow rate of 20 L/min. A coarse filter has a typical filter efficiency of 99.5% ($\beta_{25\mu\text{m}}=200$), and a fine filter has also a filter efficiency of 99.5% ($\beta_{10\mu\text{m}}=200$). The L-AN68 40[#] machine oil was used in the tests. The initial volume of lube oil in the tank was 40 L. The oil temperature was controlled at 40±2 °C during the tests.

In this work, the oil containing high concentration wear debris was prepared. During the tests, the oil containing wear debris was pumped into the oil tank by wriggle pump, whose flow rate was controlled to simulate the variation of the wear rate with time. The dispersion rate of wriggle pump flow rate is ±0.1 ml/min.

The OLVF was directly connected to the lubrication system for real-time monitoring the wear debris concentration. As shown in Fig. 6(b), the sampling location is set at the upper left area of suction filter. Moreover, the suction line inlet is at half of the oil level of the rig. The wear debris carried by the oil was drawn into the OLVF and deposited on the substrate by

activated magnetic force. The wear debris ferrogram was captured by the COMS unit. Subsequently, the magnetic force was released, and the flow channel was flushed after demagnetization. The oil was drained back into the lubrication system. A wear debris concentration indicator, namely the index of particle coverage area (IPCA), was extracted based on the ferrogram obtained [17, 30]. In the tests, the sampling parameters of the OLVF are listed in Table 1.

Fig. 6 is here

Table 1 is here

4.2 Test scheme

Different wear modes combined with wear debris size, as described by [25, 31, 32], are shown in Table 2. In the work, the experimental wear debris size range is similar to the ranges defined in Table 2.

Table 2 is here

In addition, the wear rate function is like a bathtub curve in the full-life cycle of machine. To simulate how the concentration of debris changes in full-life period, the oil containing wear debris was continuously drawn into oil tank by a wriggle pump, whose flow rate was controlled in accordance with the bathtub curve, as illustrated in Fig. 7. The simulation wear rate can be expressed as

$$m(t) = qC_{\text{add}} + W'(t) \quad (26)$$

where q (ml/min) is the flow rate of the wriggle pump; C_{add} (mg/ml) is the wear debris concentration of the added contaminated oil in the beaker (see Table 3) and $W'(t)$ is white Gaussian noise.

Fig. 7 is here

To illustrate how the concentration of debris with different size ranges changes in full-life period. Two tests were performed on the test rig, in which the debris size ranges were chosen to overlap the ranges defined in Table 2. The description and operating conditions of the two tests are listed Table 3. Moreover, to avoid the effect of residual particles in the simulation tests, oil change was performed when each test was over. Both fine filter and coarse filter worked continuously before each test. Not until the OLVF monitoring value was equal continuously to zero was the test performed in accordance with the corresponding test scheme.

Table 3 is here

5 Results

5.1 Results of test 1

To research how the concentration of debris ($>13\text{ }\mu\text{m}$) changes in full-life period, the test 1 was performed on the test rig. During the test 1, the fine filter only worked. The OLVF monitoring results of test 1 are shown in Fig. 8.

Fig. 8 is here

The results from test 1 show that the IPCA grows up rapidly in initial stage and then drops down slowly to reach stationary condition in about 60 min, and then begins to increase exponentially in about 240 min. Essentially, the IPCA sequence is random process. When entering the steady-state stage, it becomes a quasi-stationary process.

Based on the proposed model, the concentration of wear debris in full life-cycle period is numerical simulated. During the test 1, the oil loss and fresh oil replenishment did not occur,

i.e., $V_r = V_q = 0$. The added contaminated oil volume is ignored. In addition, compared with the filtration, other removal factors are a little effect for debris loss in short periods of time. Therefore those removal factors are not considered in the model, i.e. $\psi = 0$. The basic parameters of oil lubrication system test rig are given, i.e. $V_0 = 40$ L, $Q = 20$ L/min, $\beta_{10\mu m} = 200$. The attenuation coefficient of wear debris attenuation function on the simulation test rig can be determined, i.e. $k = [(1 - 1/\beta_{10\mu m})Q + \psi]/V_0 = 0.4975$. The numerical simulated results are also shown in Fig. 8. From running-in to steady-state wear, $C(t)$ grows up rapidly from starting and then drops down slowly, trends to a stationary concentration C_s . Additionally, $C(t)$ is also a quasi-stationary process in the steady-state wear stage. During the severe wear stage, $C(t)$ becomes a non-stationary process. The theoretical model simulated results are consistent with the changing trends of IPCA of the debris, whose size is larger than the micron rating of the filter.

5.2 Results of Test 2

For the debris size smaller than the micron rating of the fine filter, such size ranges of wear debris tend to stay suspended and build up over time. To illustrate how this small size range debris concentration changes in full-life period, the test 2 is performed in the rig. In addition, the debris size smaller than the micron rating of the filter are assumed not be removed by normal oil filters in the model. Therefore, both fine filter and coarse filter did not work during the test 2. The OLVF monitoring results from the test 2 are shown in Fig. 9.

Fig. 9 is here

A measure of small wear debris concentration $C(t)$ provides an integrated history of wear. From Fig. 9, the IPCA grows up rapidly in initial stage. Entering the steady-state wear stage,

1 IPCA presents linear increase trend, and then begins to increase exponentially in about 240
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 3 min. Additionally, the small wear debris concentration $C(t)$ is always a non-stationary process
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 6 in full-life period.
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 9 Similarly, the concentration of small wear debris in full life-cycle period is also
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 11 numerical simulated based on the proposed model. During the test 2, the oil loss and fresh oil
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 13 replenishment did not also occur, i.e., $V_r = V_q = 0$, and the added contaminated oil volume is
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 15 ignored. In addition, because both fine and coarse filter were off in the test 2, the impact of
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 17 filtration in the model is not considered. The attenuation coefficient of wear debris attenuation
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 19 function for the simulation test rig has been determined by a testing, i.e. $k = \psi / V_0 = 0.0001$.
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 21 The numerical simulated results are also shown in Fig. 9. The concentration of small wear
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 23 debris $C(t)$ is a non-stationary process in the full-life period. $C(t)$ grows up exponentially from
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 25 starting. Entering the steady-state wear stage, $C(t)$ becomes linear increase, and finally begins
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 27 to increase exponentially. The theoretical model simulated results are also consistent with the
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 29 changing trends of IPCA for the debris size smaller than the micron rating of the filter in the
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 31 three stages.
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42 **6 Discussions**

43 **6.1 Comparison between theoretical models and on-line wear debris monitoring results**

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 45 It can be seen from two tests that the presented method can map the wear debris
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 47 concentration with wear rate (wear debris produce rate), and explain the concentration of
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 49 wear debris time-series data well. Provided that the lubrication system is specified, and the
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 51 concentration of wear debris with different size ranges are real-time monitored, the wear rate
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 53 in different stages can be quantified and predicted. And a full-life wear process of a machine
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can be also modeled by the concentration of wear debris.

Additionally, when the variation of abnormal wear occurs but no failures happen, the state is called a “marginal state”. From Fig. 8, the “marginal state” may last for a short time. An instant growth of wear rate occurred in this stage, the steady condition was broken out rapidly. The numerical simulated results show that the mean function of $C(t)$ grows up rapidly in marginal state, and then increases exponentially. However, in this stage, a few data points of IPCA from the test 1 are contained, and the fluctuation of IPCA caused by measurement uncertainty hid the real trend information, so that the changing trend of wear debris concentration could be not so apparent in the IPCA curve. In the future work, this issue can be dealt with by increases on-line sampling oil frequency and accuracy of measurement. Moreover, in the modeling process, the assumption that wear debris produced are instantaneous and homogeneously mixed in oil. The added contaminated oil volume is ignored. Besides the filtration, other removal factors are also not considered. This simplified processing and assumptions may be also a reason to cause the difference between the model and the monitoring results.

6.2 Effects oil change for the wear debris concentration

In practice, oil change may occur due to the performance degradation of the lubricating oil. In this case, initially the concentrations of large and small debris will become near zero. Since the small wear debris concentration is an integrated history, the small wear debris concentration, after oil change, will not return to its original configuration but will resume the rate of increase previously established. Large wear debris, will quickly return to equilibrium (the stationary concentration of large wear debris C_s). Noted that the large wear debris refer

to its size larger than the micron rating of the filter.

Hence, define a quantity α which is close to one. t_α means the time when a debris concentration of αC_s is reached after oil change. The time to reach C_s can be evaluated approximately as follows:

$$\frac{E\{C(t_\alpha)\}}{C_s} = \frac{\frac{m_s}{kV_0}(1-e^{-kt_\alpha})}{\frac{m_s}{kV_0}} = \alpha \quad (27)$$

Solve the Eq. (27) for t_α :

$$t_\alpha = -\frac{\ln(1-\alpha)}{k} = -\frac{V_0 \ln(1-\alpha)}{(1-1/\beta_x)Q + \psi} \quad (28)$$

Essentially, the large wear debris concentration at time t is only related to the wear during the period from $t-L$ to t , where L is the residence time of wear debris. In the steady-state wear stage, the residence time of the large wear debris L is the time from changing oil to the instant at which a stationary concentration is reached. In Eq. (28), when α and k are specified, L ($L \approx t_\alpha$) can be evaluated approximately. Similarly, if α and L are specified, k can also be evaluated approximately.

The study provides a foundation for explanations of the on-line wear debris concentration time series and evaluation of wear state. In future work, it is interesting to quantitatively analyze the wear evolution of specific machines in service based on the method proposed.

7 Conclusions

According to the theoretical analysis and experimental checking so far, the main conclusions are drawn as follows:

(1) The study provides a new approach for explaining quantitative the on-line wear debris concentration. In lubrication systems, the total residual mass of wear debris is the convolution of wear rate function and debris attenuation function. The removal of wear debris can be stated as an exponential attenuation function. Additionally, the relationship between wear debris concentration and wear rate is also determined, as given by

$$C(t) = \frac{m(t) * r(t)}{V_0 + V_r \cdot t - V_q \cdot t}$$

(2) In an oil lubrication system, the dynamic responses of the wear debris concentration under different wear rate excitations are presented based on the proposed model. The two simulation experiment results show that the presented method can map the wear debris concentration with wear rate (wear debris produce rate) and explain the concentration of wear debris time-series data well. The evolution of the large and small debris concentration is also consistent with the modeled results.

(3) The debris size is larger than the micron rating of the filter, whose residence time in an oil lubrication system is equal to the time from changing oil to the instant at which a stationary concentration is reached. In addition, an easy-to-use formula is also proposed to evaluate approximately the residence time, as given by

$$L \approx t_\alpha = -\frac{V_0 \ln(1-\alpha)}{(1-1/\beta_x)Q + \psi}$$

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Figure 1

Figure 2

Figure 3(a)

Figure 3(b)

Figure 3(c)

Figure 4(a)

Figure 4(b)

Figure 4(c)

Figure 5(a)

Figure 5(b)

Figure 5(c)

Figure 6(a)

(a)

Figure 6(b)

(b)

Figure 7

Figure 8

Figure 9

Table 1 Sampling parameters of OLVF

Sampling Volume (mL)	Flow Rate (mL/min)	Magnetomotive Force (AT)	Sampling Interval ΔT (min)
2	4	800	2

Table 2 Wear modes, wear mechanism, wear debris types and wear debris sizes [25, 31, 32]

Wear mode	Wear mechanism	Wear debris type	Debris size (um)
Quasi-stationary wear	Abrasion	Rubbing	<15
Running in, Quasi-stationary wear	Abrasion	Sliding	20-50
Break-down	Fatigue	Laminar	20-50
Break-down	Adhesion/fatigue	Fracture	>50
Running in, Break-down	Adhesion	Severe sliding	>50

Table 3 Wear debris concentration simulation test description and operating conditions

Test number	Debris size ranges (μm)	Fine filter conditions	C_{add} (mg/ml)	V_0 (L)	$m(t)$ (mg/min)					Duration (min)
					m_s	m_0	b	c	t_1	
1	>13	on	12.5	40	25	62.5	0.07	0.028	240	300
2	$\leq$ 13	off	12.5	40	25	62.5	0.07	0.028	240	300