Pressure-Viscosity Response in the Inlet Zone for Quantitative Elastohydrodynamics

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Abstract

The descriptions employed in classical elastohydrodynamic lubrication (EHL) of the piezoviscous effect at low pressures characteristic of the inlet zone are simply inaccurate for low viscosity liquids. This article offers the 1952 McEwen equation for accurately describing the pressure-viscosity effect at low inlet pressures for those liquids with sufficiently high inflection pressure that the faster-than-exponential response does not significantly affect the viscosity in the inlet. The parameters of this model have significance for the classical EHL film thickness formulas and these parameters are listed for several low viscosity liquids including a refrigerant.

Key Words: elastohydrodynamic; film thickness; rheology; pressure-viscosity coefficient

1. INTRODUCTION

In a letter to the editor of a tribology journal [1] seven coauthors suggested six years ago that the field of elastohydrodynamic lubrication (EHL or EHD) undertake a change in philosophy. At that time, the value of the viscosity employed for the analysis of EHL was considered a matter of personal choice, which was not to be influenced by accurate measurements in a viscometer. Indeed, the use of the exponential pressure-viscosity relation or the Roelands pressure-viscosity relation may indicate that the analysis was not based upon a realistic description of the pressure dependence [2]. There are numerous examples of comparisons of viscometer measurements with the viscosity used in classical EHL analysis [3-7] which emphasize these discrepancies.

Beginning with the first full EHL simulation employing real pressure and shear dependent viscosity, measured in viscometers [8], there have been revolutionary advances in the understanding of friction and film-forming. See [9-12] for examples. While it is apparent that the Newtonian film thickness solution is much less widely applicable than previously thought, Newtonian inlets do exist in practical problems and, even when the viscosity is shear dependent in the inlet, it remains necessary to describe the pressure dependence of the low-shear viscosity to apply any shear-thinning solution using superposition.

In spite of the recent progress, there seems to be poor understanding in general of the pressure-viscosity response in the inlet zone as evidenced by the continuing use of inaccurate descriptions of the pressure-viscosity effect in a text on the subject [13] and a recent article [14]. This article offers a framework for accurately describing the piezoviscous effect at low inlet pressures for those liquids with sufficiently high inflection pressure that the faster-than-exponential response does not significantly affect the viscosity in the inlet. These liquids encompass most low viscosity liquid lubricants with the exception of traction fluids.

2. PRESSURE-VISCOSITY RESPONSE IN THE INLET ZONE FOR CLASSICAL EHL

The pressure in the inlet zone is generally regarded as being much less than the average or maximum Hertz pressure, although the assumed pressure at the nip varies widely from 70MPa

[15] to 0.3 times the maximum Hertz pressure [16]. Blok [17] noted that the film must become almost parallel where the pressure, p , exceeds a value of about $3p_{ia}$, where the asymptotic isoviscous pressure is

$$p_{ia} = \int_0^\infty \frac{\mu(p=0)dp}{\mu(p)}. \quad (1)$$

If the response were to be exponential (Arrhenius) with

$$\mu = \mu_0 \exp(\alpha p) \quad (2)$$

then for Blok's criteria the inlet pressure reaches to $3/\alpha$.

The exponential law (2), which applies near the pressure versus log viscosity inflection, has seen wide use in classical EHL. It has been widely and mistakenly attributed to a paper on effective viscosity of flowing solid glue by Barus [18], most known for describing extrudate swell, the "Barus effect". The origin of the exponential law is not clear. It is not likely to have originated from a researcher familiar with accurate measurements. Unfortunately, except for high viscosity oils at low temperature or for some traction fluids and ionic liquids, it is not a good approximation to real lubricant behavior at low pressure.

Most classical EHL articles which include a section on piezoviscous response describe the dependence of viscosity on low pressures with a narrative similar to this quotation from reference [14]. "At the relatively low pressures present in EHD contacts inlets (up to 150 MPa), α is often constant with pressure, so simple integration yields the Barus equation: $\mu(p) = \mu_0 e^{\alpha p}$." To show the inaccuracy of such a statement for modern oils, consider the viscosities measured by Lowitz and coworkers [19] in 1959 for 9-n-octylheptadecane (trioctylmethane or TOM) a prototype polyalphaolefin (PAO), shown in Figure 1. The exponential relation has been fitted to pressure intervals of 0 to 40, 60 to 100 and 140 to 180 MPa giving values of α of 12, 9.5 and 7.8 GPa^{-1} , respectively. Similar data for this liquid, in good agreement, may be found in the 1953 ASME pressure-viscosity report [20]. Clearly, α is not constant in the interval 0 to 150 MPa.

The actual pressure interval over which the exponential rule applies may be shown in Figure 2 to be in the region of the pressure versus log viscosity inflection, where

$d^2 \ln \mu / dp^2 = 0$. The liquid is a PAO-6 (6 centistokes at 100°C). The exponential relation has been fitted to pressure intervals of 0 to 70, 70 to 150 and 700 to 1100 MPa giving values of α of 18.5, 13.9 and 7.6 GPa⁻¹, respectively. Clearly, α , should only be regarded as constant in the vicinity of the inflection situated at $p \approx 865$ MPa.

The pressure-viscosity relation of Roelands [21] is in classical EHL often extolled as the preferred correlation (for example [22]) for the full range of EHL pressures.

$$\mu = \mu_p \left(\frac{\mu_0}{\mu_p} \right)^{\left(\frac{p_p - p}{p_p} \right)^Z} \quad (3)$$

This equation was derived from the concept of a pressure-viscosity pole at which all isotherms converge in the negative pressure (tensile) domain. Roelands suggested setting the pole parameters to universal values of $\mu_p = 6.31 \times 10^{-5}$ Pa·s and $p_p = -0.196$ GPa. He also set, as the upper limit of applicability, the inflection pressure, which he placed at 300-500 MPa [21]. Roelands [21] also warned that at low pressures, equation (3) understates the pressure dependence near ambient pressure. This is illustrated by the data in Figures 3 through 5 where equation (3) has been fitted to the data by minimizing the absolute average deviation. The liquids shown in Figures 3 through 5 are 9-n-octylheptadecane (TOM), a PAO-2 (Mobil SpectraSyn 2) and a jet engine oil meeting military specification L7808 grade 4, respectively.

There are conditions for which the Roelands model (3) fits data to experimental accuracy, for example, trimethylolpropane trioleate at 100°C and pressure to 1.3 GPa [23]. However, a measurement in a viscometer is necessary to determine whether this model is appropriate and then viscosity measurements are essential to determine the parameters.

3. PRESSURE-VISCOSITY RESPONSE IN THE INLET ZONE FOR QUANTITATIVE EHL

With so many possible shortcomings, it is surprising that the Roelands model has been and remains the most common pressure-viscosity law in classical EHL. It seems to have been chosen for numerical stability [24] rather than accuracy. This model has not found use outside of tribology. In the field of thermophysical properties, where there is interest in accurate

expressions for the pressure-viscosity effect and the bias of classical EHL [6] is absent, two correlations are the most widely used.

3.1 The McEwen Model for EHL

One is a simple polynomial in pressure, $\mu = \exp(a_0 + a_1 p + a_2 p^2 + a_3 p^3 \dots)$, [25] for example. The other is the Tait-like equation [26].

$$\mu = A \exp \left[B \ln \left(\frac{C + p}{C + p_R} \right) \right] \quad (4)$$

If the reference pressure, p_R , is zero, or small enough to neglect (as for ambient pressure), this expression is equivalent to the 1952 McEwen model [27].

$$\mu = \mu_0 \left(1 + \frac{\alpha^*}{q-1} p \right)^q, \quad q > 1 \quad (5)$$

For convenient application in EHL, it is written here in terms of the reciprocal asymptotic isoviscous pressure coefficient, $\alpha^* = 1/p_{ia}$.

This equation was used in lubrication and subsequently abandoned. Many researchers in lubrication have attributed this model to a later 1962 article by Chu and Cameron [28] which appeared in the same journal. This model reappeared in tribology once in 1987 [29].

It may be noticed that this expression is equivalent to the Munro, Block and Piermarini [30] model for pressure greater than the inflection pressure with $Q = -q$.

$$\mu = \mu_0 \left(\frac{p_\infty}{p_\infty - p} \right)^Q \quad (6)$$

Here, the viscosity is unbounded at $p = p_\infty = (1-q)/\alpha^*$ and since α^* and p_∞ are both greater than zero, $q < 1$. This model has not found widespread use since its pressure-Stickel function is

not typical [31]. As discussed later, the pressure-Stickel function for glass-formers plots as a straight, descending line.

3.2 Pressure-viscosity coefficients derived from the McEwen Model

The structure of the McEwen model (5) allows for the ready calculation of two more of the several definitions of pressure-viscosity coefficient. The conventional pressure-viscosity coefficient [32] is

$$\alpha_0 = \left[\frac{d \ln \mu}{dp} \right]_{p=0} = \frac{\alpha^*}{\left(1 - \frac{1}{q} \right)}, \quad q > 1 \quad (7)$$

Another definition of pressure-viscosity coefficient was developed [33] to provide a parameter, α_{film} , that quantifies the piezoviscous strength in such a way that the central film thickness would be equal for all Newtonian liquids with the same compressibility at the same contact conditions for liquids with the same values of α_{film} and μ_0 , respectively. The isoviscous pressure is

$$p_i(p) = \int_0^p \frac{\mu(p=0) d\hat{p}}{\mu(\hat{p})} \quad (8)$$

The pressure-viscosity coefficient for film thickness with general pressure-viscosity behavior [33] is

$$\alpha_{film} = \frac{1 - \exp(-3)}{p_i(3 / \alpha^*)} \quad (9)$$

when Blok's integral in equation (1) converges. In terms of the McEwen parameters it is

$$\alpha_{film} = \frac{\alpha^* (1 - \exp(-3))}{1 - \left[1 + \frac{3}{q-1} \right]^{1-q}} \quad (10)$$

The McEwen model has been fitted to the data in Figures 3 through 5 and the parameters are listed in Table 1 along with the three pressure-viscosity coefficients. The improved fit of this model over the Roelands equation (3) should be clear from the figures. In the case of TOM, the average absolute deviation was 0.6% for McEwen and 4.5% for Roelands.

3.3 Limitations of the McEwen Model

The viscosity of a reference liquid, squalane, has been extensively measured to high pressure in various laboratories [34-37]. A correlation using the McEwen model in reference [34] yielded parameters at 65°C of $\mu_0 = 6.82$ mPa·s, $q = 6.88$ and $\alpha^* = 14.77$ GPa⁻¹. The deviations of this correlation from data of three publications [35-37] are plotted in Figure 6, showing systematic departures as pressure increases above about 325 MPa. This deviation results from the pressure-viscosity inflection discussed above.

The pressure-viscosity response may be correlated across the inflection with a hybrid of the McEwen and Paluch equations [38].

$$\mu = \mu_0 \left(1 + \frac{\alpha^*}{q-1} p \right)^q \exp \left(\frac{C_F p}{p_\infty - p} \right) \quad (11)$$

However, the meanings of the pressure-viscosity coefficients are then lost unless the inflection pressure is many times greater than $3/\alpha^*$. For example, applying the hybrid equation (11) to data for squalane at 65°C to 1.2 GPa gives $\alpha^* = 11.59$ GPa⁻¹ versus 14.77 GPa⁻¹ for equation (5). The inflection pressure for squalane at 65°C is about 1100 MPa and $3/\alpha^* \approx 200$ MPa.

There exists a powerful method to test temperature-viscosity correlations and pressure-viscosity correlations at high pressures. The Stickel analysis plots the Stickel function, $S(X) = [d \ln(\mu)/dX]^{-1/2}$ versus X , where $X = 1/T$ [39] or $X = p$ [31]. The Vogel, Tammann and Fulcher temperature-viscosity equation [40] plots $S(1/T)$ versus $1/T$ as a descending straight line and the Johari and Whalley [41] pressure-viscosity equation plots $S(p)$ versus p also as a descending straight line.

A modified Stickel function was recently introduced [38] as a test for the McEwen equation (5).

$$S^*(p) = \left[d \ln(\mu) / dp \right]^{-1} \quad (12)$$

The McEwen equation plots $S^*(p)$ versus p as an ascending straight line. This has been done for 9-n-Octylheptadecane in Figure 7 at 38 and 115°C indicating the appropriate behavior to $p=300$ MPa. The PAO-6 is shown to depart from a straight line for pressure above about 400 MPa at 57°C in Figure 8. Here the McEwen model was fitted to PAO-6 for pressure up to 450 MPa.

Concentrated contacts within refrigeration compressors are lubricated by the process liquid which contains refrigerant [42]. The film thickness generated in these contacts must be influenced by the pressure-viscosity response of the refrigerant and such data are scarce. The viscosities reported in reference [43] were used to plot the modified Stickel function for the refrigerant, R134a, in Figure 8 and the parameters are listed in Table 1. The response at 40°C is shown to be appropriate for the McEwen model. For higher temperatures, there is another type of inflection in the pressure-viscosity response where $d^2\mu/dp^2 = 0$ at experimental pressure. The McEwen model cannot capture this behavior and thermodynamic scaling is more appropriate here [43].

4. CONCLUSION

The analysis of film-forming in classical EHL has been hindered by an inaccurate description of the piezoviscous response at the low pressures representative of the inlet zone. Newtonian inlets do exist in practical problems and, even when the viscosity is shear dependent in the inlet, it remains necessary to describe the pressure dependence of the low-shear viscosity to apply any shear-thinning solution using superposition. The 1952 McEwen equation accurately describes the pressure-viscosity effect at low inlet pressures for low viscosity liquids. The parameters of this model have significance for the classical EHL film thickness formulas and these parameters are listed for several low viscosity liquids including a refrigerant.

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NOMENCLATURE

C_F	fragility parameter
p	pressure (Pa)
p_∞	divergence pressure (Pa)
p_{ia}	asymptotic isoviscous pressure (Pa)
p_p	pole pressure (Pa)
p_R	reference pressure (Pa)
q, Q	exponents
$S(X)$	Stickel function ($\text{Pa}^{1/2}$ or $\text{K}^{-1/2}$)
$S^*(p)$	modified Stickel function (Pa)
T	temperature (K or °C)
α	Arrhenius (exponential) pressure-viscosity coefficient (Pa^{-1})
α_0	conventional or initial pressure-viscosity coefficient (Pa^{-1})
α^*	reciprocal asymptotic isoviscous pressure coefficient (Pa^{-1})
α_{film}	pressure-viscosity coefficient for general piezoviscous response (Pa^{-1})
μ	limiting low-shear viscosity at local pressure ($\text{Pa}\cdot\text{s}$)
μ_o	low-shear viscosity at $p=0$ ($\text{Pa}\cdot\text{s}$)
μ_p	pole viscosity ($\text{Pa}\cdot\text{s}$)

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Table 1. Parameters of the McEwen model and derived pressure-viscosity coefficients

Material:	TOM	TOM	TOM	TOM	TOM	PAO-2	PAO-2	L7808	L7808	PAO-6	R134a	4cs grp3	4cs grp3
Temperature / °C:	37.8	60.0	79.4	98.9	115	40	150	100	150	57	40	140	180
μ_0 / mPa·s	7.08	3.92	2.61	1.874	1.483	3.78	0.662	3.67	1.708	20.06	0.1838	1.70	1.09
α^* / GPa ⁻¹	12.17	10.76	9.84	8.81	8.30	12.38	6.52	9.57	7.83	16.7	2.81	9.33	7.96
q	5.59	4.45	4.11	3.58	3.21	4.43	1.94	4.01	3.33	5.47	1.293	3.25	2.605
α_0 / GPa ⁻¹	14.8	13.9	13.0	12.2	12.1	16.0	13.5	12.7	11.2	20.4	12.4	13.5	12.9
α_{film} / GPa ⁻¹	12.8	11.6	10.7	9.7	9.3	13.3	8.4	10.4	8.7	17.6	5.3	10.4	9.3

Figure 1. The viscosity of 9-n-Octylheptadecane from Lowitz, et al. [19]. The exponential relation has been fitted to pressure intervals giving values of α : 0 to 40 MPa, 12 GPa⁻¹; 60 to 100 MPa, 9.5 GPa⁻¹; 140 to 180 MPa, 7.8 GPa⁻¹.

Figure 2. Viscosity of a PAO-6 measured in the author's laboratory. The exponential relation has been fitted to pressure intervals giving values of α : 0 to 70 MPa, 18.5 GPa^{-1} ; 70 to 150 MPa, 13.9 GPa^{-1} ; 700 to 1100 MPa, 7.6 GPa^{-1} .

Figure 3. The viscosity of 9-n-Octylheptadecane from Lowitz, et al. [19] comparing Roelands with McEwen.

Figure 4. The viscosity of a PAO-2 (Mobil SpectraSyn 2) comparing Roelands with McEwen.

Figure 5. The viscosity of a jet engine oil meeting military specification L7808 grade 4, comparing Roelands with McEwen.

Figure 6. Deviation of the McEwen model for increasing pressure from data of three laboratories for squalane at 65°C.

Figure 7. The modified Stickel function for 9-n-Octylheptadecane showing ascending straight line behavior.

Figure 8. The modified Stickel function for the PAO-6 and a refrigerant, R134a.