A Velocity-Slip Model for Analysis of the Fluid Film in the Cavitation Region of a Journal Bearing

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Abstract

The Reynolds equation is derived by using the continuity equation, Navier-Stokes equation and slip length equation within cavitation region. A finite difference method are used to obtain the static and dynamic parameters of journal bearing. A comparison with the available experimental results reveals that the load capacity is well predicted by the present model. The results also show that slip effects in the cavitation region have an influence on the oil film pressure, the load capacity, the dynamic characteristics coefficients and the stability of journal bearing. The good behavior of the proposed algorithm is further illustrated in one example of a dynamically loaded journal bearing, and evolution with time of the journal center is well obtained.

Key words: journal bearing; velocity-slip; cavitation; static and dynamic characteristic.

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Nomenclature

- C_{XX}, C_{YY} = direct damping coefficient (Ns/m)
 C_{XY}, C_{YX} = cross-coupled damping coefficients (Ns/m)
 h = local bearing clearance (m)
 h_0 = radical clearance (m)
 j, k = node along φ and z directions
 K_s = slip coefficient at gas-liquid interface
 K_{st} = the effective bearing stiffness (N/m)
 K_{XX}, K_{YY} = direct stiffness coefficient (N/m)
 K_{XY}, K_{YX} = cross-coupled stiffness coefficients (N/m)
 L = axial length of journal bearing (m)
 L_s = slip length (m)
 M = mass of the shaft or the mass of system (Kg)
 M_{cr} = the critical mass (Kg)
 M_x, M_y, M_z = volume flows in x, y , and z (m^3/s)
 n = the number of time step
 n_1, n_2 = number of grid cells along φ and z directions
 p = fluid film pressure (Pa)
 R = journal radius (m)
 t = time (s)
 Δt = time step (s)
 T_0 = working temperature ($^{\circ}C$)
 U = journal surface circumferential velocity component (m/s)
 v_x, v_y, v_z = x, y, z component of fluid velocity (m/s)
 w = applied load (N)
 w_X, w_Y = X, Y component of the applied load (N)
 W = load carrying capacity (N)
 W_X, W_Y = X, Y component of the load carrying capacity (N)
 x = circumferential coordinate (m)
 y = radial coordinate (m)
 z = axial coordinate (m)
 $\Delta\varphi, \Delta z$ = grid spacing along φ and z directions (rad, m)
 X, Y = X, Y coordinates of journal center (m)
 $\dot{X}, \dot{Y}$ = X, Y component of velocity perturbation of journal center (m/s)
 α = local fluid volume fraction
 β = local gas volume fraction
 μ_l = viscosity of liquid (Ns/m²)
 μ_a = viscosity of air (Ns/m²)
 δ = pure air layer of thickness (m)
 θ = attitude angle ($^{\circ}$)
 ξ = convergence tolerance
 ε = the eccentricity
 Ω = rotating speed (rpm)
 g = gravitational acceleration (m/s²)

γ_{st} = the instability whirl frequency (1/s)

φ = the dimensionless variables ($\varphi = x/R$)

$\bar{y}$ = the dimensionless variables ($\bar{y} = y/h$)

$\bar{z}$ = the dimensionless variables ($\bar{z} = 2z/L$)

$\bar{h}$ = the dimensionless variables ($\bar{h} = h/h_0$)

$\bar{p}$ = the dimensionless variables ($\bar{p} = h_0^2 p / (6\Omega\mu R^2)$)

T = the dimensionless variables ($T = Ut / (2\pi R)$)

1 Introduction

Cavitation phenomenon occurs currently in oil lubricated journal bearing for diesel engines of marine or power station applications and automotive engines. Because of fluctuation of radial force from crankshaft, instability of lubricant flow, variation of oil pressure can be sufficient to produce bubble inception and collapse within the oil film of journal bearing.

The treatment of cavitation phenomena is a key concern in the modeling of lubrication problems. Several numerical methods have been developed to deal with this problem. Reynolds condition [1] are the most widely used at present, whereas, oil flow cannot be determined because mass is not conserved within the cavitation region. To account for the film reformation and ensure mass continuity, Jakobsson, Floberg [2] and Olsson [3] proposed a JFO cavitation model, which are realistic but not easily implemented since the cavitation boundaries are not a priori known. The mass conserving Elrod–Adams model [4, 5, 6] is now commonly accepted as a universal cavitation algorithm for simulation in hydrodynamic lubrication involving cavitation. This model contains two unknown fields, the pressure p and the fluid fraction α , so that the equations describing the full film area and cavitated area could be combined into a unified general equation.

Among the above models, zero flow velocity is assumed at a wall of journal bearing, and a "no-slip" boundary condition is used to analyze continuum fluid flow within the whole oil film. However, molecular dynamics simulations have shown that the slippage can be conditioned through changes in fluid-surface interactions [7, 8]. Several reasons have been proposed for the slip at the solid-liquid interface: molecular slip [9], a decrease in the viscosity of a boundary layer [10], and the air trapped between the surface structures [11, 12]. Ruckenstein and Rajora [11] suggested that there may be a gas "gap" at the interface between the solid and the liquid caused by the different nature of the two materials. Depending on the characteristics of the liquid-solid interface, two different classes of superhydrophobic states are exhibited, namely the so-called Wenzel and Cassie states [13, 14]. In particular, gas gaps or nanobubbles at the interface between lubricant and bearing was addressed in the cavitation zone of journal bearing by Groper et al. [15] and Sun et al. [16].

Recently, some studies by Zhang et al. [17] and Rao et al. [18] have reported that, the boundary slippage occurs on the texturing surface of journal bearing or in a high speed condition. However, there are no reports available in the literature with regard to a slip effect in the cavitation zone of a journal bearing. The slip effect in the cavitation zone can change the cavitation boundary conditions of fluid lubrication, and influence the static and dynamic characteristics of the journal bearing as well as the stability of the rotor-bearing system. In the study, a new theoretical model is proposed to solve the velocity-slip problem in the cavitation zone of the journal bearing. The present work aims at evaluating static, dynamic characteristics and stability of the journal bearing when considering the slip flow over the cavitation zone. The study lets us to better understand the influence of a slip effect on the journal bearing, which in turn casts light on their potential engineering applications.

2 Theoretical models

2.1 Slip length in the cavitation region

Fig. 1(a) shows a typical cylindrical journal bearing. The inner part is a rotor with a radius R and a rotational speed Ω (surface velocity $U=R\Omega$), and the outer part is a bearing with inner radius $R+h_0$. Within the converging film region, the hydrodynamic pressure rises to a peak, decreasing to ambient

pressure at the end sides. In zones where the film thickness locally increases, the fluid pressure may drop to ambient, or below ambient to its vapor pressure causing lubricant cavitation.

The problem to be considered is that of a laminar flow slip effect in the cavitation zone of a journal bearing, as shown in Fig. 1(b). A cavity layer separates oil lubricant from a journal bearing wall in the cavitation zone by Mistry et al. [19] and Groper et al. [20]. Under this condition, the large cavity adheres to the bearing wall in a state of laminar flow, leading to a smooth interface between gas and liquid phase. As an ideal case, a slip length L_s due to the pure air layer of thickness δ can be represented by Choi et al. [21]

$$\begin{cases} L_s = \delta \left(\frac{\mu_l}{\mu_a} - 1 \right) \\ \delta = (1 - \alpha)h \end{cases} \quad (1)$$

where μ_l and μ_a are the viscosity of liquid and air, respectively, α is local air volume fraction ($\alpha=1$ in oil film zone, and $0 < \alpha < 1$ in the cavitation zone), h is local bearing clearance. A effective slippage is expected due to the sizable viscosity difference between liquid and air, larger with a thicker air layer. Thus, the cavitation boundaries should be modified to make it suitable for the slip flow in the journal bearing.

2.2 Derivation of Reynold equation

The methodology used in deriving the Reynolds equation is quite similar to that employed to obtain the conventional Reynolds equation. Using the usual assumptions of lubrication theory and considering a laminar flow with the inertial effects neglected in the film, the Navier-Stokes equations can be reduced to

$$\begin{cases} \frac{\partial p}{\partial x} = \mu_l \frac{\partial^2 v_x}{\partial y^2} \\ \frac{\partial p}{\partial z} = \mu_l \frac{\partial^2 v_z}{\partial y^2} \\ \frac{\partial p}{\partial y} = 0 \end{cases} \quad (2)$$

where p is local oil film pressure, x is circumferential coordinate, y is radial coordinate, z is axial coordinate, v_x and v_z are x component of fluid velocity and z component of fluid velocity, respectively.

In order to integrate Eq. (2), both the no-slip boundary conditions for the oil film zone and the slip boundary conditions for cavitation zone must be used. Considering that the critical shear stress at the air-liquid interface is equal to zero and that the slip flow can easily appear at the gas-liquid interface within the cavitation zone, the rotor boundary conditions are

$$\begin{cases} v_x \big|_{y=0} = U \\ v_z \big|_{y=0} = 0 \end{cases} \quad (3)$$

and the gas-liquid interface boundary conditions are [22]

$$\begin{cases} v_x|_{y=\alpha h} = -L_s \frac{\partial v_x}{\partial y}|_{y=\alpha h} \\ v_z|_{y=\alpha h} = -L_s \frac{\partial v_z}{\partial y}|_{y=\alpha h} \end{cases} \quad (4)$$

The solution of the fluid velocities (v_x and v_z), subject to the boundary conditions Eq. (3) and Eq. (4), is

$$\begin{cases} v_x = \frac{y}{\alpha h} \cdot v_x|_{y=\alpha h} + U \frac{\alpha h - y}{\alpha h} - \frac{1}{2\mu_l} \frac{\partial p}{\partial x} (y\alpha h - y^2) \\ v_z = \frac{y}{\alpha h} \cdot v_z|_{y=\alpha h} - \frac{1}{2\mu_l} \frac{\partial p}{\partial z} (y\alpha h - y^2) \end{cases} \quad (5)$$

The partial derivative of v_x in Eq. (5), with respect to y , is further obtained as follows

$$\frac{\partial v_x}{\partial y} \Big|_{y=\alpha h} = \frac{1}{\alpha h} \cdot v_x|_{y=\alpha h} - \frac{U}{\alpha h} + \frac{\alpha h}{2\mu_l} \frac{\partial p}{\partial x} \quad (6)$$

Substitute Eq. (6) and Eq. (1) into Eq. (4), the slip velocity v_x at $y=\alpha h$ can be rewritten as

$$v_x|_{y=\alpha h} = K_s \cdot \left(U - \frac{(\alpha h)^2}{2\mu_l} \frac{\partial p}{\partial x} \right) \quad (7)$$

where, K_s is slip coefficient at gas-liquid interface

$$K_s = \frac{(1-\alpha) \cdot \left(\frac{\mu_l}{\mu_a} - 1 \right)}{(1-\alpha) \cdot \left(\frac{\mu_l}{\mu_a} - 1 \right) + \alpha}$$

Similarly, the slip velocity v_z at $y=\alpha h$ can be rewritten as

$$v_z|_{y=\alpha h} = K_s \cdot \left(-\frac{(\alpha h)^2}{2\mu_l} \frac{\partial p}{\partial z} \right) \quad (8)$$

Considering that the fluid density is assumed to be constant, the volume flows in x , y and z direction can be expressed as

$$\begin{cases} M_x = \int_0^{\alpha h} v_x dy \\ M_y = \int_0^{\alpha h} v_y dy \\ M_z = \int_0^{\alpha h} v_z dy \end{cases} \quad (9)$$

Substitute the Eq. (5), (7) and (8) into the Eq. (9), M_x and M_z are rewritten as:

$$\begin{cases} M_x = \frac{\alpha h}{2} (K_s + 1) U - \frac{(\alpha h)^3}{12\mu_l} \frac{\partial p}{\partial x} \\ M_z = -\frac{(\alpha h)^3}{12\mu_l} \frac{\partial p}{\partial z} \end{cases} \quad (10)$$

Continuity equation is expressed as

$$\frac{\partial(M_x)}{\partial x} + \frac{\partial(M_y)}{\partial y} + \frac{\partial(M_z)}{\partial z} = 0 \quad (11)$$

Substitute equations (10) into continuity equation (11), it was rewritten as

$$\frac{\partial}{\partial x} \left[\frac{(\alpha h)^3}{\mu_l} \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial z} \left[\frac{(\alpha h)^3}{\mu_l} \frac{\partial p}{\partial z} \right] = \frac{\partial}{\partial x} (6\alpha h (K_s + 1) U) + 12 \cdot \frac{\partial(\alpha h)}{\partial t} \quad (12)$$

Boundary conditions are as follows:

- ① $p=101325\text{Pa}$ on the boundary $z = \pm L/2$ or $x = 0^\circ$ (because of the atmospheric pressure)
- ② $p > 0, \alpha = 1, K_s = 0$ in full oil film region
- ③ $p = 0, 0 < \alpha < 1, 0 < K_s < 1$ in cavitation region (because the saturated vapor pressure of water is close to zero)
- ④ $p = 0$ on the internal boundary between these two regions

2.3 Static and dynamic characteristics

Fig.2 shows a stationary operating state of the bearing with a journal displacement. The bearing unit load W can be calculated by the following equations [23]

$$W = \sqrt{(W_x)^2 + (W_y)^2} \quad (13)$$

$$\begin{cases} W_y = -\int_{-L/2}^{L/2} \int_0^{2\pi} p \cos(\varphi + \theta) d\varphi dx \\ W_x = -\int_{-L/2}^{L/2} \int_0^{2\pi} p \sin(\varphi + \theta) d\varphi dx \end{cases}$$

where, φ is circumferential coordinate ($\varphi = x/R$), θ is the attitude angle, L is axial length of journal bearing, W_x and W_y are the horizontal and vertical components of the load carrying capacity, respectively.

By the finite perturbation technique [24], the oil film stiffness and damping coefficients can be determined in matrix form as

$$\begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix} = \begin{bmatrix} \frac{\partial W_x}{\partial X} & \frac{\partial W_x}{\partial Y} \\ \frac{\partial W_y}{\partial X} & \frac{\partial W_y}{\partial Y} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{W}_x}{\partial \dot{X}} & \frac{\partial \dot{W}_x}{\partial \dot{Y}} \\ \frac{\partial \dot{W}_y}{\partial \dot{X}} & \frac{\partial \dot{W}_y}{\partial \dot{Y}} \end{bmatrix} \quad (14)$$

where, X and Y are the coordinates of journal center, $\dot{X}$ and $\dot{Y}$ are the velocity perturbation of journal center.

2.4 Stability analysis

The bearing stability is further analyzed by evaluating the critical mass (M_{cr}), which represents the upper limit for stability. M_{cr} is function of dynamic characteristic coefficients [25]:

$$M_{cr} = \frac{K_{st}}{(\gamma_{st})^2} \quad (15)$$

where K_{st} is the effective bearing stiffness and γ_{st} is the instability whirl frequency:

$$K_{st} = \frac{K_{xx}C_{yy} + K_{yy}C_{xx} - K_{xy}C_{yx} - K_{yx}C_{xy}}{C_{xx} + C_{yy}}$$

$$(\gamma_{st})^2 = \frac{(K_{xx} - K_{yy})(K_{yy} - K_{xx}) - K_{xy}K_{yx}}{C_{xx}C_{yy} - C_{xy}C_{yx}}$$

If the mass of the system is smaller than the critical mass M_{cr} , the bearing system is stable. The bearing system is unstable for a mass M greater than M_{cr} .

2.5 Extension to dynamical problems

The film-thickness function $h(x, z, t)$ results from the dynamical interaction of the bearing with an applied load w . To demonstrate such a problem, a journal bearing centered at the origin O_J is considered. $(X(t), Y(t))$ denote the position of the shaft's center O_R at time t . With the geometry of the problem as defined in Fig. 2, the film-thickness function over the domain which is the rectangle $((0, 2\pi) \times (-L/2, +L/2))$ is given by Ausas et al. [26]

$$h = h_0 - X(\theta) \sin(\theta) - Y(\theta) \cos(\theta) \quad (16)$$

and the boundary conditions of the pressure field at the time t

$$p(x, z = -0.5L, t) = 101325 \text{ Pa} \quad \text{and} \quad p(x, z = +0.5L, t) = 101325 \text{ Pa}$$

The dynamical equations for $X(t)$ and $Y(t)$ are as follows [26]

$$\begin{cases} M \frac{d^2(X(t))}{dt^2} = W_X(t) + w_X(t) \\ M \frac{d^2(Y(t))}{dt^2} = W_Y(t) + w_Y(t) \end{cases} \quad (17)$$

where, M is the mass of the shaft, $W_X(t)$ and $W_Y(t)$ are X and Y components of the load carrying capacity W , $w_X(t)$ and $w_Y(t)$ are X and Y components of the applied load w .

To solve strong coupling equations (17), the adapted method [26] is based on the following Newmark scheme

$$\begin{cases} X^n = X^{n-1} + \Delta t \left(\frac{dX}{dt} \right)^{n-1} + \frac{\Delta t^2}{2M} \left((W_X)^n + (w_X)^n \right) \\ Y^n = Y^{n-1} + \Delta t \left(\frac{dY}{dt} \right)^{n-1} + \frac{\Delta t^2}{2M} \left((W_Y)^n + (w_Y)^n \right) \end{cases} \quad (18)$$

and

$$\begin{cases} \left(\frac{dX}{dt} \right)^n = \left(\frac{dX}{dt} \right)^{n-1} + \frac{\Delta t}{M} \left((W_X)^n + (w_X)^n \right) \\ \left(\frac{dY}{dt} \right)^n = \left(\frac{dY}{dt} \right)^{n-1} + \frac{\Delta t}{M} \left((W_Y)^n + (w_Y)^n \right) \end{cases} \quad (19)$$

where, Δt is time step, n is the number of time step, $(dX/dt)^n$ and $(dY/dt)^n$ approximate $X'(t_n)$ and $Y'(t_n)$, respectively.

3 Numerical formulation

After introducing dimensionless parameters into the Eq. (12), the dimensionless modified Reynolds equation is obtained.

$$\frac{\partial}{\partial \varphi} \left[(\bar{h})^3 \frac{\partial \bar{p}}{\partial \varphi} \right] + \left(\frac{D}{L} \right)^2 \frac{\partial}{\partial \bar{z}} \left[(\bar{h})^3 \frac{\partial \bar{p}}{\partial \bar{z}} \right] = \frac{\partial}{\partial \varphi} [\alpha \bar{h} \cdot (K_s + 1)] + \frac{1}{\pi} \frac{\partial (\alpha \bar{h})}{\partial \bar{t}} \quad (20)$$

where the dimensionless variables and parameters are defined by

$$\varphi = \frac{x}{R}, \quad \bar{y} = \frac{y}{h}, \quad \bar{z} = \frac{2z}{L}, \quad \bar{h} = \frac{h}{h_0}, \quad \bar{p} = \frac{h_0^2}{6\Omega\mu R^2} p, \quad T = \frac{U}{2\pi R} t$$

Using a finite difference method [27], the dimensionless modified Reynolds equation (20) is solved numerically. The computational domain is assumed to be the rectangle $(0, 2\pi) \times (-L/2, L/2)$, and a uniform cartesian mesh of $n_1 \times n_2$ nodes, with spacing $\Delta\varphi = 2\pi/n_1$ and $\Delta\bar{z} = L/n_2$ is defined. Subscripts j and k at a variable refer to its value at the discrete location $\varphi_j = j\Delta\varphi$ and $\bar{z}_k = k\Delta\bar{z}$. The time variable t is discretized into uniform time steps of length Δt , and a superscript n on a variable refers to its value at time $t_n = n\Delta t$. The first term in Eq. (20) in finite increment format is expressed as

$$\begin{aligned} \frac{\partial}{\partial \varphi} \left[(\bar{h})^3 \frac{\partial \bar{p}}{\partial \varphi} \right]_{j,k}^n &= \frac{(\bar{h}_{j+1}^3 + \bar{h}_j^3)^n (\bar{p}_{j+1,k} - \bar{p}_{j,k})^n - (\bar{h}_j^3 + \bar{h}_{j-1}^3)^n (\bar{p}_{j,k} - \bar{p}_{j-1,k})^n}{2\Delta\varphi^2} \\ &= \frac{(\bar{h}_{j+1}^3 + \bar{h}_j^3)^n \bar{p}_{j+1,k}^n + (\bar{h}_j^3 + \bar{h}_{j-1}^3)^n \bar{p}_{j-1,k}^n - [\bar{h}_{j+1}^3 + 2\bar{h}_j^3 + \bar{h}_{j-1}^3]^n \bar{p}_{j,k}^n}{2\Delta\varphi^2} \end{aligned} \quad (21)$$

Similarly, the second term on the left side of Eq. (20) is given as

$$\left(\frac{D}{L} \right)^2 \frac{\partial}{\partial \bar{z}} \left[(\bar{h})^3 \frac{\partial \bar{p}}{\partial \bar{z}} \right]_{j,k}^n = \left(\frac{D}{L} \right)^2 (\bar{h}_j^3)^n \frac{\bar{p}_{j,k+1}^n + \bar{p}_{j,k-1}^n - 2\bar{p}_{j,k}^n}{\Delta\bar{z}^2} \quad (22)$$

The two terms on the right side of Eq. (20) are given as

$$\frac{\partial}{\partial \varphi} [\alpha \bar{h} (K_s + 1)]_{j,k}^n = \frac{(K_s + 1)_{j,k}^n \alpha_{j,k}^n \bar{h}_j^n - (K_s + 1)_{j-1,k}^n \alpha_{j-1,k}^n \bar{h}_{j-1}^n}{\Delta\varphi} \quad (23)$$

$$\left[\frac{\partial (\alpha \bar{h})}{\partial T} \right]_{j,k}^n = \frac{\alpha_{j,k}^n \bar{h}_{j,k}^n - \alpha_{j,k}^{n-1} \bar{h}_{j,k}^{n-1}}{\Delta T} \quad (24)$$

Substituting the equations (21), (22), (23), (24) into equation (20) and rearranging the equation, one obtains

$$\begin{aligned} \bar{p}_{j,k}^n &= \left[\frac{(\bar{h}_{j+1}^3 + \bar{h}_j^3)^n \bar{p}_{j+1,k}^n + (\bar{h}_j^3 + \bar{h}_{j-1}^3)^n \bar{p}_{j-1,k}^n}{2\Delta\varphi^2} + \left(\frac{D}{L} \right)^2 (\bar{h}_j^3)^n \frac{\bar{p}_{j,k+1}^n + \bar{p}_{j,k-1}^n}{\Delta\bar{z}^2} \right. \\ &\quad \left. - \frac{(K_s + 1)_{j,k}^n \alpha_{j,k}^n \bar{h}_j^n - (K_s + 1)_{j-1,k}^n \alpha_{j-1,k}^n \bar{h}_{j-1}^n}{\Delta\varphi} - \frac{1}{\pi} \frac{\alpha_{j,k}^n \bar{h}_{j,k}^n - \alpha_{j,k}^{n-1} \bar{h}_{j,k}^{n-1}}{\Delta T} \right] \\ &\quad \left[\frac{[\bar{h}_{j+1}^3 + 2\bar{h}_j^3 + \bar{h}_{j-1}^3]^n}{2\Delta\varphi^2} + \left(\frac{D}{L} \right)^2 (\bar{h}_j^3)^n \frac{2}{\Delta\bar{z}^2} \right]^{-1} \end{aligned} \quad (25)$$

and

$$\alpha_{j,k}^n = \frac{\left[\frac{(\bar{h}_{j+1}^3 + \bar{h}_j^3)^n \bar{p}_{j+1,k}^n + (\bar{h}_j^3 + \bar{h}_{j-1}^3)^n \bar{p}_{j-1,k}^n - [\bar{h}_{j+1}^3 + 2\bar{h}_j^3 + \bar{h}_{j-1}^3]^n \bar{p}_{j,k}^n}{2\Delta\varphi^2} + \left(\frac{D}{L}\right)^2 (\bar{h}_j^3)^n \frac{\bar{p}_{j,k+1}^n + \bar{p}_{j,k-1}^n - 2\bar{p}_{j,k}^n}{\Delta z^2} + \frac{\alpha_{j-1,k}^n (K_s + 1)^n \bar{h}_{j-1}^n}{\Delta\varphi} + \frac{1}{\pi} \frac{\alpha_{j,k}^{n-1} \bar{h}_j^{n-1}}{\Delta T} \right]}{\bar{h}_j^n \left[\frac{(K_s + 1)^n}{\Delta\varphi} + \frac{1}{\pi\Delta T} \right]} \quad (26)$$

The mesh has 120 intervals in the φ direction, and 100 intervals across the bearing width z . The tolerance of the convergence criterion is 0.001. By applying the boundary conditions ①, ②, ③ and ④ for the lubricant in the film region, the film pressure p and local air volume fraction β ($\beta=1-\alpha$) can be obtained.

Convergence is assumed that, the calculated sums of fractional changes as well as pressure changes in the film content between two successive SOR iterations fall below the user-defined tolerance value ($\xi=0.001$). That is,

$$\sum_{j=0}^j \sum_{k=0}^k \left| \frac{\alpha_{j,k}^{new} - \alpha_{j,k}^{old}}{\alpha_{j,k}^{new}} \right| + \sum_{j=0}^j \sum_{k=0}^k \left| \frac{p_{j,k}^{new} - p_{j,k}^{old}}{p_{j,k}^{new}} \right| < \xi \quad (27)$$

4 Results and discussion

To verify the theoretical model proposed in this paper, a journal bearing published by [28] is chosen as an example. The detail parameters of the journal bearing are listed in Table 1.

According to the study of Arumugam et al. [28], load on the test bearing is 255 N, and the oil film thickness is 29 μm for 1000 rpm, 35 μm for 2000 rpm and 38 μm for 2500 rpm. So the corresponding eccentricity is 0.61, 0.53 and 0.49, respectively. In the present work, the variable parameters, including the rotating speed Ω and the eccentricity ε , are listed in Table 2. The static and dynamic characteristics are analyzed by the proposed model and compared with the test ones. The proposed methodology is then extended to the fully dynamical case of a journal bearing under time-dependent load.

4.1 Static load capacity

Fig.3 illustrates circumferential distribution of oil film pressure under the slip and no-slip conditions. The results show that, the slip effects within cavitation region have almost no influences when the rotating speed is higher than 1500 rpm, and increase the peak pressure when the rotating speed is less than 1500 rpm, which can reflect the load capacity of the oil film in the journal bearing.

Fig.4 shows the load capacity versus rotating speed of rotor under the slip and no-slip conditions. The results show that, the load capacity of journal bearing increases slowly as the rotating speed increases, and the maximal load is gradually close to 295 N as the rotating speed is more than 2500 rpm, which is different from a constant load 255 N [28]. One possible reason is that, in present

theoretical model, circumferential grooves in the upper half of journal bearing are ignored and only smooth surfaces are used, resulting in a difference in load capacity of journal bearing. The results also show that, the slip effect within the cavitation region will increase the load capacity as the rotating speed is less than 2000 rpm, and the increment of increasing load gets smaller with the increase of the shift speed. As is well known that, the cavitation zone in the journal bearing is near the inlet of the supply of lubricant [29, 30], the slip effects at the inlet has the enhanced hydrodynamic properties, reported by Ma et al. [31]. We think that, the slip direction is the same as that of the fluid flow at the cavitation zone of the journal bearing, while no-slip condition at oil film zone as well as the outlet or two sides of the journal bearing will retard the fluid flowing out the gap, leading to a increase of load support. However, it is interesting that, this growth rate is close to zero when the rotating speed is higher than 2000 rpm in Fig.4. In order to indicate the reason for the formation of this phenomenon. The slip coefficient and bubble concentration distribution are further obtained in the following work.

4.2 Slip coefficient and cavitation zone

Fig.5 presents slip coefficient distribution for rotating speed 1000 rpm, 2000rpm and 3000rpm. It can be observed that, the value of slip coefficient K_s can quickly reach from 0 to 1 in divergent region along the circumferential direction, and K_s is equal to 1 in most of cavitation region. Fig. 6 presents bubble concentration distribution for rotating speed 1000 rpm, 2000 rpm and 3000 rpm. The results show that, the bubble concentration can reach 100% in region of $K_s = 1$, and the shape of cavitation region is changing from the wedge to the rectangle with the increase of rotating speed. From Fig.6, it can be also seen that, although the eccentricity (0.65) at 1000 rpm is higher than that (0.45) at 3000 rpm, the cavitation area for 1000 rpm is clearly lower than that of 3000 rpm. Thus, the rotating speed of rotor has more serious impact on the cavitation area of the oil film, and the cavitation area increases with the increase of the rotating speed. On the other hand, through the results from Fig.5, the high slip coefficient at cavitation zone can lead to the high slip velocity at corresponding location by Eq. (4), which could increase the load support for a high speed condition (>2000 rpm). But it could not do this, as shown in Fig.4. We think that, this phenomenon is possibly caused by a high bubble concentration at the cavitation zone. As is well known that, the slip length model [32, 33] states that the “slip length” (L_s , described in Eq. (1)), the fictive distance below the solid surface where the velocity extrapolates linearly to zero, is the proportional constant. However, the local bubble concentration at the cavitation zone is reaching to $\beta=100\%$ in the study, so the flow lubricant at those zone is not oil film but only cavitation bubble, lead to the failure of the slip length model. Furthermore, with the increase of the rotating speed, the journal bearing enlarges the cavitation area of $\beta=100\%$. Consequently, the slip effect in the cavitation zone gradually has no affect on the static load capacity of the journal bearing.

4.3 Dynamic characteristics

Fig.7 illustrates the stiffness coefficients versus the rotating speed. It can be seen from Fig.7 (a) that, for the slip and no slip condition, greater error of K_{YY} (0.65 N/ μm) and K_{XX} (0.42 N/ μm) occurs at the speed of 1500 rpm, which shows slip effect within the cavitation region can raise the direct stiffness (K_{YY} and K_{XX}) of journal bearing. In contrast, at the speed of 1000 rpm, slip effect within the cavitation region can reduce the direct stiffness (K_{XX}) of journal bearing, and raise the direct

stiffness (K_{YY}). However, the error of K_{YY} and the error of K_{XX} are gradually reduced to zero with the increase of the shift speed. On the other hand, in Fig.7 (b), the cross-coupled stiffness coefficients (K_{XY} as well as K_{YX}) increases with the increase of the shift speed. Different to the results of direct stiffness coefficients, the greater error for K_{XY} (1.3 N/ μ m) occurs at the speed of 1500 rpm, and the greater error for K_{YX} (0.87 N/ μ m) occurs at the speed of 1000 rpm. Fig.8 shows the damping coefficients versus the rotating speed. It can be seen from Fig.8 (a) that, at the speed of 1000 rpm, slip effects within the cavitation region can reduce the direct damping coefficients (C_{YY} and C_{XX}) of journal bearing, and the greater errors are 3.93 kNs/m and 19.1 kNs/m, respectively. However, these errors are reduced to zero as the shift speed is higher than 1500 rpm. On the other hand, in Fig.8 (b), the cross-coupled stiffness coefficients decrease with the increase of the shift speed. Different to the results of direct damping coefficients, the errors of cross-coupled stiffness coefficients (C_{XY} and C_{YX}) are very close to zero for all rotating speed (from 1000 rpm to 3000 rpm), and C_{XY} and C_{YX} overlap to each other as the shift speed exceeds than 1500 rpm.

In the Fig.7 and the Fig.8, the stiffness hardening and the damping softing effects are due to the fluid slip effects at the cavitation zone. In fact, the load capacity and the dynamic stiffness can be improved because of a critical shear stress of zero at the interfaces between the liquid and the solid bearing surface, which has been reported by [34]. This prediction is important because, just like in gas-liquid lubrication condition, the mixture (bubbles and oil) can be tailored to produce either increased damping or increased stiffness [35]. Nevertheless, with the increase of the rotating speed, the stiffness hardening and the damping softing are gradually not affected due to the high bubble concentration ($\beta=100\%$) at the most cavitation zone, as mentioned in section 4.2.

Fig. 9 shows the critical mass of journal bearing as function of rotational speed. It can be seen that, a greater error of the critical mass (84 Kg) occurs at the rotating speed of 1000 rpm (the critical mass obtained in slip condition is 215 Kg, which is much higher than 131 Kg in no slip condition), and the errors decrease with the increase of the rotating speed, and are close to zero when the rotating speed is equal or greater than 2000 rpm. Thus, the slip effect can also further affect the whirl instability of the rotor-bearing system. From Fig. 9, when the rotating speed is less than 3500 rpm, the critical mass is greater than the mass ($M=W/g=26$ kg) of the system related to one bearing and the bearing should run stably whether slip condition or no slip condition. Around a rotational speed of 4000 rpm, the critical mass is very close to M . For rotational speeds greater than 4000 rpm, the critical mass is smaller than M . Consequently, it may be concluded that the system is unstable for rotational speeds greater than 4000 rpm according to [25]. Actually, an analysis from Yang [36] has accounted for the effects of the eccentricity, rotor speed, and restriction parameter on the whirl instability of the rotor-bearing system. In the present work, we think, the slip effect at cavitation zone affects the stability more than no-slip effect for lower rotor speed. So the stability threshold decreases as the slip area at the inlet of journal bearing increases. On the other hand, for higher rotor speed, the slip effect at cavitation zone doesn't dominate the stability more than no-slip effect.

4.4 Dynamic evolution under transient loads

The study is next extended to dynamic load conditions. An essential test for a dynamical slip effect is that of transient loads of car engines, which essentially consist of two sharp impulses with a time interval of a quarter of a period. For this example, the nondimensional period of the load is set to $T=1$ ($T=tU/(2\pi R)$) and an analytical load given by (for $0 \leq T \leq 1$)

$$w_Y = 0.01 \cdot \left[\exp\left(-400(T - 0.25)^2\right) + 0.95534 \exp\left(-400(T - 0.5)^2\right) \right] \cdot \frac{6\mu UR^2 (L/2)}{h_0^2} \quad (28)$$

$$w_X = 0.0029552 \left[\exp\left(-400(T - 0.5)^2\right) \right] \cdot \frac{6\mu UR^2 (L/2)}{h_0^2} \quad (29)$$

is chosen. The mass of the shaft is set to 2 Kg, and the initial conditions under two shift speed are set to the following values, respectively

a) $\Omega=2500$ rpm, $\varepsilon=0.49$

$$X|_{t=0}=-32.5, Y|_{t=0}=-18, dX/dt|_{t=0}=0, dY/dt|_{t=0}=0 \quad (30)$$

b) $\Omega=1000$ rpm, $\varepsilon=0.61$

$$X|_{t=0}=-38.5, Y|_{t=0}=-25.5, dX/dt|_{t=0}=0, dY/dt|_{t=0}=0 \quad (31)$$

Under the two conditions, the obtained evolution of the journal's center with nondimensional time is as shown in Fig. 10, where the components of applied loads w_Y , w_X , and load capacity W_Y and W_X are plotted as well.

As is shown in Fig.10, the whole system becomes periodic. In a cycle, the two impulses (at $T=0.25$ and $T=0.5$) of w_Y in Fig.10 (a) successively brings the shaft, which was **revolving** freely to shake up and down for two times, as is shown in Fig.10 (c). Consequently, two increasing impulses of W_Y in Fig. 10 (a) occur correspondingly so as to balance Y component of the applied load in a cycle. In this process, a spike in the load W_X (**in Fig. 10 (b)**) as well occurs at time 0.4 due to the inertia of the shaft. Furthermore, it is found from **Fig. 10 (c)** that, the slip effects can increase the Y, X component of the journal's center in contrast to the results with no-slip effects. For example, at 2500 rpm, the maximum value of increasing amplitude is 0.5 μm for Y component at $T=3.45$, and only 0.2 μm for X component at $T=3.16$, as shown in local amplified figures.

In order to study the effects of the slippage on the dynamic evolution of journal bearing under transient loads, the whirl orbits of the rotor system in the two rotating speeds (1000 rpm and 2500 rpm) is further illustrated in Fig.11. It can be seen clearly that, at 2500 rpm, the journal center orbits with dynamic slip effects are not coincidence with that with no-slip effects. **The possible reason is that, in the dynamic load conditions, the bubble concentration at the cavitation zone is in the state of transient change ($\beta < 100\%$), forming an effective slip effect all the time at the cavitation zone. As a result, the slip effect in the cavitation zone still affects on the dynamic evolution of the journal's center at a higher rotor speed.** Fig.11 also shows that, the whirling center of the journal bearing moves from the location O_1 (-38.5 μm , -25.5 μm) under 1000 rpm to the location O_2 (-33 μm , -20 μm) under 2500 rpm, and the vibration amplitude of the journal center increases from 4 μm around the point O_1 to 6 μm around the point O_2 . **Thus, at the two rotating speed, the whirling center always belongs to the indicated static equilibrium position, meanwhile, with the increase of the rotating speed, the whirling center moves closer to the bearing center to satisfy overall dynamic equilibrium. The above is a better validation for the stability of rotor-bearing system, as mentioned in section 4.3.**

5 Conclusions

- 1) A velocity-slip model for analysis of the fluid film in a journal bearing, including slip length within the cavitation region and slip boundary conditions at the gas-liquid interfaces, has been developed, and well predicts the load capacity by the available experiment results.
- 2) At a lower rotating speed of rotor, the slip effects within the cavitation region can increase the

static load capacity as well as the direct stiffness coefficient of journal bearing, but decrease the damping coefficients and the critical mass of the journal bearing.

- 3) High slip coefficient ($K_s=1$) and bubble concentration ($\beta=100\%$) can form in divergent region of the journal bearing; while for a higher rotating speed, the slip effects within the cavitation region have no influence on the static and dynamic characteristics of the journal bearing as well as the critical mass of the journal bearing.

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Table 1 Calculated parameters[28]

axial length L (mm)	journal diameter D (mm)	radial clearance h_o (μm)	working temperature T_0 ($^{\circ}\text{C}$)
21	28	75	25
liquid viscosity μ_l (Pa.s)	gas viscosity μ_g (Pa.s)	load on the bearing W (N)	slip coefficient K_s
0.132	0.00002	255	0-1

Table 2 Variable parameters[28]

the rotating speed (rpm)	1000	1500	2000	2500	3000
the eccentricity ε	0.61	0.57	0.53	0.49	0.45

Fig. 1 Schematic of (a) a journal bearing and (b) a slip velocity.

Fig. 2 Force equilibrium of the bearing under a static load W

Fig.3 Oil film pressure versus circumferential angle under the slip and no-slip conditions ($1000 \text{ rpm} < \Omega < 3000 \text{ rpm}$, Load on the test bearing is 255 N).

Fig.4 The load capacity versus rotating speed of rotor under the slip and no-slip conditions

Fig.5 Slip coefficient distribution for different rotating speed. (a)1000 rpm; (b)2000rpm; (c) 3000rpm.

Fig.6 Bubble concentration distribution for different rotating speed. (a)1000 rpm; (b)2000rpm; (c) 3000rpm.

Fig.7 (a) Direct stiffness coefficients vs. rotating speed; (b) Cross-coupled stiffness coefficients vs. rotating speed.

Fig.8 (a) Direct damping coefficients vs. rotating speed; (b) Cross-coupled damping coefficients vs. rotating speed.

Fig. 9 Predicted critical mass of journal bearing as function of rotational speed.

Fig.10 Evolution with time of (a) Y component of the load, (b) X component of the load, and (c) X, Y component of the journal's center.

Fig.11 Evolution with time of the journal's center, applied loads w_Y , w_X , and load capacity W_Y and W_X