Saving Alberta's Resource Revenues:
Role of Intergenerational and Liquidity Funds

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Abstract

We use a welfare-based intertemporal stochastic optimization model and historical data to estimate the size of the optimal intergenerational and liquidity funds and the corresponding resource dividend available to the government of the Canadian province Alberta. To first-order of approximation, this dividend should be a constant fraction of total above- and below-ground wealth, complemented by additional precautionary savings at initial times to build up a small liquidity fund to cope with oil price volatility. The ongoing dividend equals approximately 30 per cent of government revenue and requires building assets of approximately 40 per cent of GDP in 2030, 100 per cent of GDP in 2050 and 165 per cent in 2100. Finally, the effect of the recent plunge in oil prices on our estimates is examined. Our recommendations are in stark contrast with historical and current government policy.

Keywords: oil price volatility, precautionary saving, resource wealth, fiscal policy

JEL codes: E21, E22, D91, Q32

Revised September 2016

1. Introduction

The mission of the Alberta Heritage Savings Trust Fund is “to provide prudent stewardship of the savings from Alberta’s non-renewable resources by providing the greatest financial returns on those savings for the current and future generations of Albertans.” The fund was created in 1976 when 30 per cent of government resource revenue was transferred to the fund. With the economic crises of the early 1980s, this percentage was halved and eventually cut to zero in 1987. Once the Alberta government had eliminated its accumulated debt in 2005 and showed budget surpluses, revenue was again transferred to the fund. Since its inception, \$¹33 billion has been withdrawn from the Alberta Heritage Fund to support spending in health care, education, infrastructure, debt reduction and social programs. The value of this fund stood at \$15.1 billion, or 4.7 per cent of Alberta’s GDP in March 2014 (\$14.9 billion or 4.8 per cent of GDP in March 2013).² In addition to this fund, a second, much smaller fund, the Contingency Account, with a value of \$4.7 billion or 1.5 per cent of Alberta’s GDP in March 2014 (\$2.7 billion or 0.9 per cent of GDP in March 2013) is used to smooth revenue arising from volatilities in oil and gas prices.³ These two funds are examples of what are known in the literature as, respectively, an intergenerational fund and a liquidity fund. We will call the combined total of these two funds simply “the fund.”⁴

With fossil fuel extraction rates remaining high for years to come, but the decline in crude oil prices toward the end of 2014 illustrating their inherent uncertainty, the time is ripe to take a

¹ All dollar values (\$) reported are Canadian dollars, unless indicated otherwise.

² We use the book values reported in the annual budget documents by Alberta Finance. Using the slightly higher current fair market value would only marginally affect our calculations and leave our qualitative policy recommendations unaltered.

³ Given the objective of fiscal stabilization, the contingency account is much more invested in short-term, fixed-income securities than the Heritage Savings Fund.

⁴ Both figures come from Alberta’s 2014 provincial budget (<http://finance.alberta.ca/publications/budget/budget2014/fiscal-plan-savings-plan.pdf>).

The Alberta Government has a number of smaller funds, which include the Medical Research Endowment Fund, the Science and Engineering Endowment Fund and the Scholarship Fund. Their total value is \$3.4 billion or 1.1 per cent of Alberta’s GDP as of March 2014 (\$3.5 billion or 1.1 per cent of Alberta’s GDP as of March 2013). We do not include these smaller funds, since they are domestic investment funds. The merit of these funds should be decided on the basis of their social returns. If these returns are satisfactory, Alberta can make use of international capital markets to finance these and not the Heritage Fund.

more structural approach to managing Alberta's fund. We argue that it is useful to distinguish between an intergenerational fund to distribute the temporary proceeds from resource wealth over many generations and a liquidity or precautionary savings fund to cushion the adverse impact on government income of a drop in the world price of oil. We use intertemporal stochastic welfare optimization to derive the optimal savings policy. This distinguishes our paper from Landon and Smith (2015), who use Monte-Carlo techniques to quantitatively compare welfare of several ad-hoc saving rules. Our approach is similar to that of Bems and de Carvalho Filho (2013), who examine the effect of precautionary saving on the current account on a number of countries, and van den Bremer and van der Ploeg (2013), who examine Norway, Iraq and Ghana. Specifically, our focus here is on the implication for government fiscal policy for the Albertan government.

In addition to the recent work by Landon and Smith (2015), Bems and de Carvalho Filho (2013) and van den Bremer and van der Ploeg (2013) discussed, many authors have studied different aspects of the important question which share of volatile and temporary resource revenues to save, invest and spend and even more have examined its operational policy implications. For example, Barnett and Ossowski (2003) have examined how volatile government resource revenues can lead to the unproductive use of government funds. Based on historical experience, Fasano (2000), Bacon and Tordo (2002) and Kumar et al (2009) have argued for clear and transparent fiscal rules for payment into and out of a fund. Arrau and Claessens (1992), Engel and Valdes (2009) and Bartsch (2006) among others have used Monte Carlo simulations to assess the performance of stability funds. What sets our paper apart from this applied policy literature is that we have set out to expose the fundamental economic channel to optimal policy. Ultimately, this relies on the permanent income hypothesis modified for uncertain income to reveal the effect of prudence and precautionary saving (Kimball 1990).

We use historical data on extraction costs, prices and tax revenues and official projections of extraction rates for Alberta to calculate the size and development of the *optimal*

intergenerational and liquidity funds and the corresponding resource dividends, the amount taken annually from the fund and from the resource revenues to be used for general budget purposes. In doing so, we distinguish oil, natural gas and bitumen revenues. How much of the dividend is allocated to public spending, tax cuts or handouts depends on political preferences.⁵ The Mintz Commission recommended a target of \$100 billion in net financial assets by 2030 and saving a fixed percentage of Alberta's total revenues each year as part of the budget (Alberta Financial Investment and Planning Advisory Commission, 2007). Once this target is achieved, the commission foresaw a permanent annual income of \$4.5 billion to fund public services and/or maintain low taxes in the future.

Although we focus on oil and gas price volatility, long-term risk is also based on future, unknown changes in technologies, resource discoveries and transportation investments (e.g., approval of the extended Keystone Pipeline System) and uncertainties about future carbon-emission constraints and other policies that impact Alberta's ability to maintain or expand resource production. Our estimates of optimal precautionary saving which only take into account resource price volatility thus provide a lower bound.

This paper is laid out as follows. Our principles of managing the intergenerational and liquidity funds are derived and outlined in sections 2 and 3, respectively. Our estimates of the optimal sizes of these funds for Alberta, based on the data discussed in section 4, are presented in section 5. Crucially, section 6 discusses the sensitivity of our results. Finally, section 7 concludes.

2. How to Build an Intergenerational Fund

⁵ To strengthen the supply side, one could use the dividend for investment, infrastructure and tax cuts. The Mintz Commission (Alberta Financial Investment and Planning Advisory Commission, 2007) dismissed Alaska-style dividend payments as they are lump-sum in nature and have little benefit for the economy. We abstract from the specific allocation of the resource dividend herein, but focus on its optimal size.

Revenue from fossil fuel⁶ extraction is temporary, as revenues end when resources are exhausted or too costly to extract, and volatile due to volatile prices. For these reasons, the revenues provide a rationale for an *intergenerational* fund to smooth consumption per capita across generations and a *liquidity* fund to cushion the impact of volatility of the world oil price. We discuss the former first, abstracting from oil price volatility, and discuss the latter in section 3. We assume a deterministic return on foreign assets r and a fixed marginal cost of extracting one unit of oil. Utility increases at a decreasing rate in the resource dividend D . The government maximizes utilitarian welfare:

$$(1) \quad J(t, F, P, Y) = \max_D E_t \left[\int_t^\infty U(C(\tau) / L(\tau)) L(\tau) e^{\rho(\tau-t)} d\tau \right],$$

where $\rho > 0$ is the social discount rate and L the population size. We explicitly define the resource dividend as the difference between total consumption and non-oil production in the rest of the economy: $D \equiv C - Y$. Non-oil production Y is assumed to be an exogenous process that grows exponentially at a rate of $n + g$, with n denoting population and g productivity growth. Equation (1) must be solved subject to the budget constraint:

$$(2) \quad \dot{F} = rF + \Omega - D, \quad F(0) = F_0,$$

where F denotes the fund size and Ω the oil rents. Equations (1-2) give the Keynes-Ramsey rule for consumption growth:

$$(3) \quad \frac{dC}{dt} = [n + \theta(r - \rho)]C,$$

where $\theta > 0$ is the elasticity of intertemporal substitution, having assumed a utility function of the form $U(C) = C^{1-1/\theta} / (1-1/\theta)$, and n is the rate of population growth. The coefficient of relative intergenerational inequality aversion is $1/\theta$. If we further assume the ratio of consumption and non-oil production is constant in the absence of oil revenues

⁶ Throughout, we refer to ‘oil’ as a general term to include conventional oil, natural gas and bitumen.

$(\theta(r - \rho) - g = 0)$, an assumption discussed further below, we obtain for the resource dividend:

$$dD/dt = [n + \theta(r - \rho)]D.$$

By substituting (3) into the present-value budget constraint and solving, we find that the optimal resource dividend is a constant fraction of total financial and subsoil oil wealth:

$$(4) \quad D(t) = [r - \theta(r - \rho) - n][F(t) + V(t)], \quad V(t) \equiv \int_t^\infty e^{-r(\tau-t)} \Omega(\tau) d\tau,$$

where oil wealth V is the present value of oil rents. Lower oil extraction costs and larger reserves imply larger oil wealth.

2.1. Policy implications

We choose the social discount rate so that the resource dividend and thus the total of financial and oil wealth grow at the same rate as the rest of the economy.⁷ Having denoted the per-capita growth rate of non-oil GDP by $g > 0$, non-oil GDP, the resource dividend and total wealth all grow at the rate $g + n$, if we set the social discount rate to $\rho = r - g/\theta < r$. The social discount rate must thus be lower in a growing economy to ensure that more saving occurs and the per-capita resource dividend grows over time. If it is easier to substitute present for future consumption (high θ), this correction term is smaller. From (4) the propensity to consume out of total wealth is $r - \theta(r - \rho) - n = r - g - n$. Both the resource dividend and total wealth per capita then grow at the rate of productivity growth g . As fractions of GDP they are fully smoothed across different generations.

The *permanent* component of oil revenue is the annuity value of current and future oil revenues, which is the growth-corrected interest on oil wealth $(r - n - g) V$. The *temporary* component of oil revenue is current minus permanent revenue. If oil revenue is expected to increase (decrease)

⁷ Since $\dot{V} = rV - \Omega$ and $\dot{F} = rF + \Omega - D$, with dots denoting time derivatives, we obtain

$(\dot{F} + \dot{V})/(F + V) = r - D/(F + V) = \theta(r - \rho) + n$ (from (4)).

over time, temporary revenue is negative (positive). The deterministic permanent income hypothesis thus offers the following guidelines for managing resource wealth:

- (i) The resource dividend that is available to fund the government budget is a constant proportion of total above- and below-ground wealth. It grows at the rate of GDP growth even if oil revenues decline over time, and remains a constant proportion of each generation's non-oil income.
- (ii) The decline in below-ground oil wealth is exactly compensated by an increase in above-ground financial wealth so total wealth remains a constant fraction of total GDP (Hartwick, 1977).
- (iii) The faster the rate of oil depletion and decline in oil revenues, the larger the proportion of revenue that is saved in the intergenerational fund in order that future generations benefit from the current boom in oil revenue. The savings rate out of oil revenues thus varies over time.

2.2. Other choices of discount rates

Our pragmatic choice for the social discount rate $\rho = r - g/\theta < r$ has its merits, but two alternatives should be kept in mind. First, $\rho = r$ ensures that per-capita consumption is constant and reduces or reverses the rationale for an intergenerational fund if productivity growth is positive. With the prospect of even small productivity growth over an infinite horizon, an incentive arises to borrow heavily to start consuming the permanent value of non-resource GDP now, which goes against the motive to save in the face of declining oil revenues. In the absence of present oil revenue, this borrowing can often not be realized, as it requires borrowing with future growth as collateral. Crucially, the uncertain nature of future GDP growth would need to be taken into account, significantly depressing its expected present value and the corresponding consumption increment. Secondly, if incumbent politicians try to secure re-election and become impatient, we might have $\rho > r$ so the propensity to consume out of current wealth is higher and the economy saves less and gets poorer with the passage of time. This effect is less pronounced

if politicians have a high willingness to substitute present for future consumption, i.e., a low elasticity of intergenerational inequality aversion (high θ). Although aware of its implications, we proceed under the assumption $\rho = r - g/\theta$, as it allows us to assess the incremental effect of the temporary oil revenues on optimal savings, which would be zero in their absence.

3. Oil Price Volatility and the Case for a Liquidity Fund

To derive the optimal size of the liquidity fund, we extend section 2 to allow for oil price uncertainty, where the oil price⁸ is assumed to follow an autoregressive process with high persistence (see appendix B for details of the calibration). The problem is thus to maximize (1) subject to:

$$(2') \quad \frac{dF}{dt} = rF + \Omega - D, \quad \Omega(t) = \sum_{i=\text{bitumen, crude oil, natural gas}} (P_i(t) - \lambda_i) O_i(t),$$

where P_i is the price in \$/barrel of oil equivalent (b.o.e.), λ_i the constant unit extraction cost in \$/b.o.e. and O_i the extraction rate in b.o.e./year. The Keynes-Ramsey rule then becomes:

$$(3') \quad \frac{1}{dt} E_t [dD] = [\theta(r - \rho) + n] D + \frac{1}{2} CRP \left(\frac{D}{D+Y} \right)^2 \sigma_D^2 D,$$

where CRP denote the coefficient of relative prudence and σ_D the volatility of the dividend (see appendix A). Prudent policy-making is built on a greater desire to avoid negative outcomes than to seek positive outcomes. We have from (3') with our choice of the discount rate that the dividend as fraction of GDP grows at the rate:

$$(5) \quad \frac{1}{D} \frac{1}{dt} E_t [dD] - n - g = \frac{1}{2} CRP \left(\frac{D}{D+Y} \right)^2 \sigma_D^2 > 0,$$

where σ_D is not a constant (see appendix A).

⁸ We adopt three separate correlated price processes for conventional oil, natural gas and bitumen.

Hence, the greater the coefficient of relative prudence and the greater the volatility of the dividend, the greater the optimal precautionary buffers that are needed to act as insurance against future drops in oil prices.⁹ Furthermore, volatility and the buffers are higher if oil price shocks are less transient, as a greater part of the revenue resulting from shocks is consumed in terms of the resource dividend if these shocks are more permanent thus resulting in larger values for the partial derivatives in (6) (see appendix A for details). If the stochastic shocks are permanent (cf., random walk) and all future oil prices change by the same amount as the initial shock, the required precautionary buffers are large. If shocks are transient and do not impact future oil prices, very little precautionary saving is required. With mean reversion in price shocks, the precautionary buffers are smaller.¹⁰ Finally, there is less need for buffers if productivity growth g makes future generations richer and hence better able to deal with future income shocks, as reflected by the ratio of the dividend D and total consumption $C = D+Y$ in equation (5).

4. Data and Assumptions for Alberta¹¹

To calculate the optimal intergenerational and liquidity funds and resulting dividends for Alberta, we distinguish between rents from bitumen, conventional oil and natural gas. Although we follow official projections until 2022, we examine two scenarios for the bitumen-extraction paths after that date, where the second scenario is considered in the sensitivity analysis. This section introduces the parameter choice for the base case presented in this paper. A sensitivity analysis is undertaken in section 6.

4.1. Extraction rates and reserve estimates

⁹ Here, precautionary savings are channeled into a fund, but they can also appear as current account surpluses in a small open economy (e.g, Bems and de Carvalho Filho, 2013).

¹⁰ In contrast, temporary revenue requires more saving in the *intergenerational* fund.

¹¹ Further details can be found in appendix B.

For the extraction rates of bitumen, conventional oil, and natural gas, we use official projections available until 2022. In these official projections production of bitumen rises from 0.72 to 1.4 billion barrels per year during the period 2012–2022. Production of conventional oil and natural gas are set to decline from 0.20 and 0.58 to 0.17 and 0.44 barrels of oil equivalent,¹² respectively, over the same period. Allowing for some new discoveries, we set initial reserves to 168 billion barrels of bitumen, 4.7 billion barrels of conventional oil and 15.4 billion barrels of oil equivalent of natural gas. Based on these numbers, figure 1 presents two scenarios for the period after 2022. Scenario 1 is the base case scenario. In this scenario, extraction of bitumen continues to increase linearly after 2022 until reaching a value of 2.0 billion barrels per year, remaining constant afterwards until exhaustion.

INSERT FIGURE 1

4.2. Government resource rents

In order to calculate government resource rents, we must first calculate resource rents $\Omega(t) = \sum_i (P_i(t) - \lambda_i) O_i(t)$. We use extraction costs of \$15 per barrel of oil equivalent for both conventional oil and natural gas. To reflect the large costs associated with bitumen production, we use an extraction cost of \$32 per barrel. We assume conventional oil is sold at the WTI price and natural gas at the Henry Hub NYMEX natural gas price, but use the much lower (also below Western Canadian Select) average field gate price to estimate the actual price of a barrel of bitumen. For all three resource prices, we adopt AR(1) price processes, reflecting the significant reversion to the mean observed in resource prices. We use the calibration in van den Bremer and van der Ploeg (2013) for the conventional oil and natural gas price with a mean price of \$110 per barrel, a mean reversion of six per cent per year, and a volatility of 26 per cent

¹² 1,000 bbl of natural gas corresponds to 1.000 bbl of oil equivalent (Norwegian Petroleum Directorate, “Facts: The Norwegian Petroleum Sector” (Oslo: Ministry of Petroleum and Energy, 2011), <http://www.npd.no/en/Publications/Facts/Facts-2011>), which corresponds approximately to equivalent energy content. Under this definition, the per barrel of oil equivalent price of natural gas is significantly lower than the price of oil per barrel, which reflects imperfect substitution and, to a lesser extent, transportation costs.

for conventional oil. For natural gas, we take a mean price of \$32 per barrel of oil equivalent, a mean reversion of six per cent per year, and a volatility of 20 per cent. For bitumen, we adopt the same mean reversion and volatility, but a substantially lower mean price of \$80 per barrel.

We assume these prices are perfectly correlated. Initial prices at the start of 2013 are \$96 per barrel natural of oil, \$64 per barrel of bitumen and \$11 per barrel of oil equivalent of natural gas. Extraction of natural gas will initially not be profitable, but becomes profitable when the price reverts back to the mean. If extraction cost exceeds the price of natural gas, gas rents are zero. To reflect the very significant effect the choice of initial (and mean) prices has on our estimates, illustrated, once again, by the drop in prices towards the end of 2014, we consider the effect of such a drop in section 6.4.

As our focus lies on optimal fiscal policy for the government of Alberta, we assume a constant share of 34 per cent of resource rents accrues to the government through different taxes and levies, as supported by the data (the 2002–2012 average), thus abstracting from any non-linearity in the tax regime. Finally, we assume that the share of the non-oil part of government revenue as a share of non-oil GDP is constant at 14 per cent (corresponding to the 2002–2012 average). We report the optimal resource dividend: the increase in government spending that is made possible by the resource revenues.

4.3. Return on the fund and general economic trends

The initial size of the fund is \$17.6 billion (both the Contingency Account and the Heritage Savings Trust Fund in March 2013) and is almost 6.0 per cent of total GDP. We set the real return on the fund to $r = 6.1$ per cent per year (the average annual real return on the Alberta Heritage Savings Trust Fund from 2002 to 2012). We will also present, to verify robustness, our estimates for a lower real return on 4.5 per cent per year in section 6.2. Trend population growth n is set at 1.3 per cent per year, the long-term projected growth rate for 2014–41.¹³ The trend

¹³ Taken from Alberta Finance, *Population Projection: Alberta 2014-2041* (2014), <http://finance.alberta.ca/aboutalberta/population-projections/2014-2041-alberta-population->

productivity growth rate g is set at 2.0 per cent per year, so trend growth of non-resource GDP is 3.3 per cent per year. We take an elasticity of intertemporal substitution of $\theta = 0.5$ and thus set the rate of discount to $\rho = r - g/0.5 = 2.1$ per cent per year.

5. Optimal Intergenerational and Liquidity Funds for Alberta

5.1. Benchmark estimates and the effects of prudence

Figure 2 reports the optimal dividend and size of the fund for extraction scenario 1 for various degrees of prudence. The continuous (red) line in figure 2a corresponds to the optimal resource dividend, expressed as a percentage of government revenue, to build up an intergenerational fund. The continuous (red) line in figure 2b shows the optimal size of the intergenerational fund, which corresponds to the case without volatility or without prudence. The intergenerational fund grows gradually from 5.7 per cent of GDP in 2013 to 159 per cent in 2100. This sustains an annual dividend between 25 and 31 per cent of government revenue.¹⁴ The dashed (purple) and dotted (light blue) lines in figure 2 correspond to a moderate (benchmark) and high prudence.

INSERT FIGURE 2

The optimal initial dividend drops from 28 ($CRP = 0$) to 26 and 21 per cent for degrees of relative prudence of 3 and 10, respectively. The additional initial precautionary saving leads to the buildup of a larger fund with a final fund size in 2110 of 6.5 and 21 percentage points larger for degrees of relative prudence of 3 and 10, respectively. For the benchmark case of $CRP = 3$, the liquidity fund, given by the difference between the $CRP = 0$ and $CRP = 3$ lines, is thus small compared with the intergenerational fund: it grows gradually to a mere 6.5 per cent of GDP in 2100. However, with a much larger relative prudence of 10, the dotted (light blue) lines

[projections.pdf](#). In the past, Alberta has seen high rates of population growth with a 10-year average of 2.2 per cent and 20-year average of 2.0 per cent population growth (Statistics Canada, 2013).

¹⁴ In fact, the optimal dividend as a share of non-oil GDP is constant in the absence of uncertainty. Variations here merely reflect normalization by total GDP (non-oil + oil GDP), which does not grow at a constant rate unlike non-oil GDP due to changes in the rates of resource extraction.

indicate that the accumulated liquidity fund is much larger, as reflected by a smaller initial dividend and larger expected resource dividends in the long run.

Table 1 reports the optimal fund sizes as percentages of GDP and the resource dividends as percentages of government revenue if $CRP = 3$. We also report, in brackets, our estimates for the optimal fund sizes and resource dividends in thousands of 2013 dollars per capita, corrected for productivity growth (the per capita fund sizes grow at the rate of 2.0 per cent per year) and, finally, uncorrected for this growth.

INSERT TABLE 1

The total fund starts at about \$4,500 per capita in 2013 (5.7 per cent of GDP) and grows to \$32,600 per capita in 2030 (39 per cent of GDP) and then to \$76,900 per capita in 2050 (101 per cent of GDP) and \$117,000 per capita in 2100 (165 per cent of GDP) — all figures in 2013 constant dollars, corrected for growth. This sustains an annual dividend of \$2,800 in 2013 (26 per cent of government revenue) and \$3,200 per capita from 2050 onwards (approximately 30 per cent of public revenue).¹⁵

This dividend in per capita terms is corrected for productivity growth too, so grows with the rest of the economy at 2.0 per cent per year. This means that the per capita dividend and per capita GDP grow by a factor of 2.1 ($\approx \exp(0.02 \times (2050 - 2013))$) between 2013 and 2050. In real terms, the uncorrected per capita dividend grows from \$2,800 in 2013 to \$6,600 in 2050.

It is instructive to compare our results for Alberta with those for Norway, Iraq and Ghana (van den Bremer and van der Ploeg, 2013). The dividend of \$2,800 per capita is much larger than that for Ghana (U.S.\$37 per capita), larger than that for Iraq (U.S.\$1,528 per capita), but roughly a factor three smaller than that for Norway (U.S.\$8,537 per capita). The optimal final size of the intergenerational and liquidity fund for Alberta reached in 2100 (159 per cent and 6.5

¹⁵ Since government revenue as percentage of non-resource GDP is constant and resource rents decline, government revenue as a percentage of total GDP rises slightly.

per cent of non-resource GDP, respectively) are rather less than the final fund sizes for Norway (677 per cent and three per cent of non-resource GDP) and very much smaller than those for Iraq (172 and 12 times non-resource GDP), but larger than those for Ghana (115 per cent and 0.2 per cent of GDP). Norway is perhaps the most natural comparison for Alberta. Natural resource revenues last longer in Alberta and thus there is less need to smooth resource dividends across generations and a smaller intergenerational fund is needed. Comparing to Iraq, it is evident that both windfalls may last for an equally long time, but that they make up a much smaller share of total GDP in the case of Alberta, thus considerably reducing the precautionary motive.

5.2. Comparison with the spend-all and bird-in-hand rules

Figure 3 compares the benchmark with $CRP = 3$ and the intergenerational fund outcomes corresponding to $CRP = 0$ with a spend-all policy. The dash-dotted (blue) line denoted by “Spend all” shows government resource rents as percentage of total government revenue and thus corresponds to spending all resource rents directly without saving. This spend-all policy is suboptimal for three reasons. Firstly, with excessive spending in the first two decades and a much too rapid decline thereafter not leaving a dividend for future generations, benefits are clearly not smoothed optimally across generations. Secondly, precautionary buffers are not built up to protect against a future drop in oil prices. Finally, with a significant degree of mean reversion in the oil prices, the resource dividend with a spend-all policy leaves the government budget exposed to extreme volatility.

The dotted (orange) lines in figures 3a and 3b illustrate a Norwegian style bird-in-hand (BIH) rule, which does not allow the use of reserves as collateral, puts all resource revenue in the fund and withdraws a fixed 4.0 per cent per year from the fund for general purposes (Bjerkholt, 2002; Barnett and Ossowski, 2003). We observe that under this rule, wealth is accumulated much more quickly than under the optimal rule, even allowing for the effects of prudence and precautionary savings (i.e., contrasting with the continuous (red) and dashed (purple) lines).

INSERT FIGURE 3

Finally, figure 4 shows that, compared with the optimal policy, dividends under the bird-in-hand rule are much too low in the initial periods of the windfall and too high once the windfall has faded away. The optimal policy thus spends a much larger percentage of the fund in the early years and a much lower percentage in later years compared to the bird-in-hand rule. Hence, given substantial amount of below-ground natural resource wealth, it is sub-optimal to set the resource dividend (as Norway does) to a fixed percentage of just above-ground financial wealth.

INSERT FIGURE 4

6. Sensitivity Analysis

This section discusses the sensitivity of the results presented in the previous section to changes in the production scenario, the real return on assets in the fund, the correlation between oil and gas prices and the initial price level.

6.1. Alternative production scenarios

As discussed in section 2, the timing of the windfall has important implications for optimal savings behavior. In the benchmark extraction scenario 1 rents reach a peak of approximately 40 per cent of government revenue in 2030. Such an increase reduces the need for intergenerational saving. In the second scenario, the increase in production of bitumen only continues until reaching a value of 1.4 billion barrels per year (compared to 2.0 billion barrels per year in scenario 1), followed by a similar plateau until exhaustion at a later date, as illustrated in figures 1a and 1b. Extraction paths for conventional oil and natural gas, which are set to run out much sooner, are not varied across the scenarios.

The dashed- and solid (green) lines denoted by $CRP = 3$ in figure 5a show that the initial optimal spending increment initially drops from 26 per cent in scenario 1 to 23 per cent of government revenue in scenario 2 (with $CRP = 3$). Since, in the alternative scenario 2, the

windfall is more spread out over time, a smaller fund has to be built up in the long run. However, more funds have to be accumulated in the short run as production reaches a plateau earlier. The dashed- and solid (green) lines in figure 5b illustrate the effects on the total fund ($CRP = 3$) for the two scenarios.

INSERT FIGURE 5

6.2. Effects of a lower real return on assets

Figure 6 compares the case of a real return on assets of 6.1 per cent (base case) based on realized returns by the Alberta Heritage Fund over 2002-2012 to a perhaps more realistic long-term return of 4.5 per cent. Lowering the rate of return, depresses the dividend in the long run, from 33 per cent to 19 per cent of government revenue. It also leads to a greater accumulation of assets and thus to a larger fund (figure 6b), as the below-ground wealth that is being converted into above-ground wealth is simply worth more when discounted at a lower rate. The fund size in 2100 is now 215 per cent instead of 165 per cent of GDP.

INSERT FIGURE 6

6.3. Correlation between gas and oil prices

Short-term instability of revenue in Alberta can be driven as much by fluctuations in gas prices as by fluctuations in bitumen prices. This is why it is important to stabilize revenue through resource diversification. Empirically, there has been a high degree of negative correlation between oil and gas prices. Although we can allow for such a negative correlation, we find that this does not matter much as rents for natural gas only make up a small part of total resource rents. For example, if the correlation coefficient between gas and oil prices is taken to be -0.5 instead of 1.0, the resource dividend as fraction of public revenue and the fund size as a percentage of GDP are hardly affected, simply reflecting the fact that most revenues are derived from conventional oil and bitumen and not from natural gas.

6.4. The plunge in oil price

To illustrate the potential effect of a sudden plunge in oil prices, such as the one observed since the end of 2014, figure 7a shows the initial resource dividend as a function of the initial (and mean) oil price with figure 7b illustrating the final fund size. Although only the initial conventional oil price is shown on the horizontal axis, we vary the initial prices of bitumen, conventional oil and natural gas by applying the same scale factor to each. We perform two experiments. In the first experiment, we adjust both initial and mean prices reflecting a price jump that is permanent (the two steepest (blue) lines denoted by “Initial and mean adjusted”). In the second experiment, we only adjust the initial prices reflecting a price jump that is temporary and prices reverting to the original mean values (the two shallowest (red) lines denoted by “Initial adjusted”). In doing so, we intend to capture the arbitrary nature of any initial price assumption of a process with strong random walk characteristics and the lack of robust estimates of the mean price despite evidence of mean reversion and the relative stability of estimates of the rate of mean reversion.

INSERT FIGURE 7

It is evident then from figure 7 that a temporary drop in conventional oil prices to 60 dollars per barrel, reduces the initial resource dividend as percentage of government revenue from 26 to 18 percent of government revenue, with resource revenues dropping from 29 to 7.9 per cent, and cuts the size of the sovereign wealth fund in 2100 from 165 to 114 percent of total GDP. If we also modify the mean prices proportionally and thus consider a permanent plunge, the resource dividend and the size of the fund in 2100 drop even further, to 9.0 and 78 per cent, respectively. Since oil price shocks are very persistent, the size of the resource dividend and the fund that is accumulated varies strongly with the initial oil price that pertains after a truly permanent shock. Compared to our base case, the 2014 plunge implies 30 per cent drop in current resource dividend and final fund size, whereas the drop is a staggering 65 per cent for the dividend and 53 per cent for the final fund size, when the effect is permanent and mean prices also adapt.

Finally, to obtain a sense of the sensitivity to the degree of mean reversion, figure 8 compares the base case with and without mean reversion. As discussed in appendix A (and B.5), both a pure random walk and a mean-reverting process for the oil price are difficult to reject on statistical grounds. The (red) lines denoted by AR(1) correspond to the base case discussed in section 5 with rates of mean reversion of 6.0 per cent for the three price processes, initial prices of \$64, \$96 and \$11 per barrel of oil equivalent for bitumen, conventional oil and natural gas, reverting to mean prices of \$80, \$100 and \$32 per barrel of oil equivalent, respectively.

INSERT FIGURE 8

Setting the degree of mean reversion to zero and thus adopting random walk processes for the prices, the (blue) lines denoted by RW show the corresponding resource dividends and fund buildup. From these lines it is evident that the absence of a reversion to a higher mean, reduces the final size of the intergenerational fund from 165 per cent of GDP to 136 per cent in 2100. Accordingly, the initial dividend is lower: 22 per cent versus 28 per cent with mean reversion. More importantly, the persistence of shocks now necessitates much greater precautionary savings. At the initial time, the resource dividend drops from 26 to 15 per cent of government revenue and the liquidity fund now constitutes 18 per cent instead of a mere 6.5 per cent with mean reversion at $t = 2100$.

7. Conclusions and Policy Implications

Following Ossowski (2002), Kneebone (2006) and the Mintz Commission (Alberta Financial Investment Planning Advisory Commission, 2007), our welfare-theoretic analysis examines the optimal savings path of resource revenues in an intergenerational fund to spread the resource wealth across generations and in a liquidity or buffer fund to deal with oil price volatility. We focus our attention on the three main non-renewable resources, bitumen, conventional oil and natural gas, and do not consider renewable resources such as forestry. Crucially, we have chosen the social discount rate such that the optimal resource dividend is a constant fraction of GDP. The per-capita resource dividend thus grows in line with the rest of the economy. Our

results suggest that policy in Alberta can be improved in two ways. Firstly, the amount taken from either the fund or resource revenues for general budget purposes — the resource dividend — should neither be a fixed percentage of financial wealth, as done in Norway, nor should a fixed percentage of annual resource revenues be saved, as recommended by the Mintz Commission¹⁶. Instead, to first-order of approximation, the resource dividend should be a fixed percentage of the total of above-ground financial and below-ground resource wealth. In the presence of uncertainty, this result is modified slightly, as a small amount of precautionary saving is needed to cope with volatile oil and gas prices. The percentage that the resource dividend makes up out of total wealth is then slightly lower in the short term and higher in the long term reflecting the precautionary motive. Our optimal policies differ from Norway's bird-in-hand rule, which requires that all resource revenues are deposited in the fund and an annual dividend of 4.0 per cent of the fund is withdrawn (e.g., Bjerkholt, 2002; Barnett and Ossowski, 2003). As the fund grows, the amount withdrawn from it each year increases. Yet, this bird-in-hand rule violates the permanent-income hypothesis and is therefore suboptimal.

Using historical data, we apply our results to the Alberta natural resource windfalls consisting of bitumen, conventional oil and natural gas with 2013 as the start date of our analysis and a corresponding initial oil price of \$96 per barrel¹⁷. Our base case estimates suggest that the dividend that can be used to finance government spending or tax cuts is approximately \$2,800 per capita per year in 2013, subsequently growing at 2.0 per cent per year in real terms or, equivalently, at about 30 per cent of total government revenue at all times. Most of the corresponding saving is needed to smooth the dividend as a fraction of GDP. This necessitates a

¹⁶ More recently, Landon and Smith (2013) advocate a rule that would deposit half of revenues in the fund and set resource dividends at 25 per cent of the fund. Norway deposits all revenues in its fund and withdraws 4 per cent of the fund each year. Although very useful from a policy perspective, such arbitrary rules are suboptimal from a welfare-optimizing perspective across the whole time horizon, must be re-optimized periodically and are never sustainable in the long run.

¹⁷ Our results assume parity between the U.S. dollar, in which oil prices are typically denoted, and the Canadian dollar, which we use to present our results, based on the situation in 2013. Since 2013 the Canadian dollar has depreciated in value by approximately 20 per cent. Although we have not modelled any such trends nor possible additional volatility due to exchange rates, the depreciation of the Canadian dollar with resource fixed in U.S. dollars would act to increase the value of resource rents and corresponding dividends as expressed in Canadian dollars.

growth in the fund from 5.7 per cent of GDP in 2013 to 39 per cent in 2030, 101 per cent in 2050, and 165 per cent in 2100. In monetary terms, this corresponds to a size of net financial assets of \$46,000 per capita in 2030 and \$161,000 per capita in 2050 (both in 2013 dollars, not corrected for growth). This is equivalent to having a target fund in the aggregate of at least \$200 billion by 2030 and \$1 trillion by 2050, compared to the \$17.6 billion that the fund held as of March 2013. The amount that is needed to cushion against oil price volatility — the liquidity fund or Contingency Account — only plays a leading role in the early years, unless policy makers are very prudent.

Although we have abstracted from the stochastic nature of above-ground investments, consideration must be given to the type of investment. The portfolio of assets should be fully diversified, both internationally and across different types of asset groups to minimize risk. The large amount of below-ground resource wealth necessitates that the optimal holdings of risky assets are leveraged up with a factor equal to the ratio of oil wealth to fund wealth, if necessary by going short and taking a negative position in the safe asset (Gintschel and Scherer, 2008; van den Bremer et al., 2016). The leveraging up of risky assets in the fund's portfolio will be gradually undone as subsoil wealth is depleted. From a financial portfolio management perspective, it is important to have two different funds. The intergenerational fund has to smooth welfare across generations and is thus larger the more transitory the windfall. The liquidity fund, in contrast, has to collect precautionary buffers in the face of stochastic volatility which are larger when shocks are more permanent. In practise, the types of asset invested in and the maturity of the assets will also be very different for the two funds.

Since Alberta has good access to international capital markets, as illustrated by the very low rates the Canadian government pays on international borrowing (see appendix B.1), there is no need to spend any part of the fund on public investment projects or to have a separate Alberta Heritage Capital Fund (e.g., Collier et al, 2010; van der Ploeg and Venables, 2012). The decision to invest in domestic capital should be solely based on a cost-benefit analysis,

independent of the availability of windfall proceeds, as access to international capital markets guarantees the availability of funds if needed. Moreover, such funds carry the danger of improper calculation of costs and benefits and of political manipulation.

As with all welfare-theoretic analysis stretching across many decades, the figures reported here depend strongly on our assumptions, crucially here on the choice of social discount rate, the return on the fund and the initial oil price. Firstly, our results assume that the resource dividend is indexed to wages and productivity, as is typical for welfare benefits. However, if it is desirable to have a dividend that is constant in per capita terms, the case for building a large fund is much weakened or even reversed. It is implemented by setting the rate of time preference to the market rate of interest minus the product of the growth rate and intergenerational inequality aversion, so the rate of time preference is lower than the market rate of interest. It implies that, in a political sense, the resource dividend (associated wages, profits and benefits) is tilted towards future generations, as all benefit from productivity growth. This has been politically acceptable in Norway for many years and in many other countries too. However, this may be a much harder sell, if the country has not managed to build a fund when oil and gas exports and prices were high with substantial terms-of-trade improvements at that time. Unfortunately, this seems to be the case for Alberta, where only a very small fund has been built up. Policy makers might be more impatient politically and thus prefer to hand out hydrocarbon wealth much more quickly than our calculations suggest, evidently at the expense of future generations.

Our sensitivity analysis confirms the order of magnitude of our estimates, but shows considerable variation in the actual numbers, largely reflecting the enormous and close to permanent nature of the windfall. Nevertheless, our estimates for precautionary saving provide a lower bound; the size of the reserves, future productivity of the non-resource part of the economy, extraction and transportation costs and the long-term cost of carbon emission provide considerable additional sources of uncertainty. Modelling the resource prices as random walk

processes, an hypothesis which cannot be rejected statistically, indeed significantly increases optimal precautionary savings, as shown. Our analysis is partial equilibrium in nature, thus takes macroeconomic outcomes and asset returns as exogenous and excludes human capital, or in fact any other types of wealth other than resource wealth, and future pension liabilities. Secondly, if fund managers only achieve a real return of 4.5 per cent per year instead of our benchmark of 6.1 per cent per year, the optimal resource dividend drops from 30 to 20 per cent of government revenue. Yet, the required fund size by the end of this century increases from 165 to 215 per cent of GDP. Finally, if the current plunge in oil prices turns out to be permanent, then our recommendation is to build up a fund of \$75 billion by 2030, much more in line with the \$100 billion advocated by the Mintz Commission. In practise, today all revenues from the existing Heritage Fund are consumed (with the exception of a small amount for inflation proofing). Further, with the Contingency Account set to disappear in the next few years and the new government (2015) to start borrowing for the first time in decades in light of sustained low oil prices,¹⁸ the game-changing nature of oil price volatility is once more emphasized.

Finally, an important proviso must be made relating to climate policy, stranded hydrocarbon assets and endogenous extraction paths. McGlade and Ekins (2015) have calculated that, if policy makers throughout the world commit to their announced target of keeping global warming limited to 2 degrees Celsius, 80% of global coal reserves, half of global gas reserves and a third of global oil reserves should stay in the ground and never be burnt. More interestingly, these authors show that in view of the relatively high extraction costs and the large associated emissions, the Canadian oil sand reserves should not be burnt altogether (and the same applies to all hydrocarbon reserves in the Arctic). As carbon is gradually being priced higher and higher and this price is shifted to producers, especially if supply does not react much to prices and demand does, Canadian producers extracting oil from the oil sands will be hit

¹⁸ From Alberta's 2016 provincial budget
(<http://www.finance.alberta.ca/publications/budget/budget2016/index.html>).

more and more. As time passes, there comes a moment that the price fetched for a barrel of oil from the oil sands on international markets falls below the sum of extraction costs and the carbon tax. This will happen most quickly for the most expensive fields and those fields will be taken out of production first. Hence, one way or another, Canada and Alberta in particular face substantial risks of stranded hydrocarbon assets. This makes it even more important for Alberta to save a larger fraction of hydrocarbon revenues, as its resource boom is likely to last a shorter time when global warming is taken more seriously and carbon policy uncertainty reduces the value of the reserves.

With an increasing risk of stranded assets, it is also important to examine the impact on the optimal extraction path of an individual country.¹⁹ If markets perceive a risk, however small, that global leaders will finally undertake serious action to limit global warming to 2 degrees Celsius by curbing cumulative emissions to at most a few hundreds Giga tons of carbon, then the optimal rational response of each individual oil- or gas-producing country is to extract its hydrocarbon reserves as quickly as possible, before other countries sell their reserves and effectively exhaust the global carbon budget. Failure to cooperate can thus induce a race to burn the last ton of carbon with all the inefficiencies that result. If the risk of stranded assets speeds up oil and gas extraction, the expected net present value of future oil revenue increases due to reduced discounting of less distant rents, assuming the same amount of reserves is still extracted in total. The resource dividend increases because of the increase in net present value of the reserves. More is also saved in the intergenerational fund, as the windfall becomes more temporary and a greater initial build-up of the fund results. Crucially, more uncertainty results and the motive for precautionary saving becomes apparent once again.

¹⁹ In our exercises we have kept our optimal extraction paths exogenous as given by various government projections. That is not unreasonable given that once fields are open extraction rates are pinned down by geological considerations such as Darcy's law. However, the opening of fields itself is endogenous and is governed by Hotelling-type considerations (Anderson, et al., 2015). The dynamics may be different for bitumen produced from oil sands.

Acknowledgements

An earlier version appeared as the working paper *Digging deep for the heritage fund: why the right type of fund pays dividend long after oil is gone* (van den Bremer & van der Ploeg, Research Paper 7-32, The School of Public Policy, University of Calgary), for which we acknowledge financial support from the School of Public Policy, University of Calgary. We are grateful to Beverly Dahlby and Jennifer Winter of the School of Public Policy, University of Calgary, Matthew Foss of the Alberta Department of Energy, and Mark Parsons of Alberta Finance for their advice and help in obtaining relevant data for Alberta. The recommendations and any errors are our own.

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Appendix A: Volatility of the Resource Price and the dividend

Empirical evidence (e.g., Hamilton, 2009) suggests that it is hard to reject the hypothesis that the crude oil price follows a random walk (a Brownian Motion process in continuous time). However, it is also not possible to reject a high degree of persistence with mean reversion (van den Bremer and van der Ploeg, 2013). To avoid heteroskedasticity of the standard errors, we thus assume an AR(1) stochastic processes for the logarithm of the price of bitumen, conventional oil or natural gas (Schwartz, 1997):

$$(A1) \quad dP_i(t) = \left\{ \eta_i \left[\mu_i + \nu_i t - \log(P_i(t)) \right] P_i(t) + \nu_i \right\} dt + \sigma_i P_i(t) W(t),$$

where μ_i is the mean, σ_i the volatility, ν_i the drift, η_i the rate of mean reversion and $W(t)$ a Wiener process. Equation (A1) can be written as an AR(1) stochastic process:

$$(A2) \quad d \log(P_i(t)) = \left\{ \eta_i \left[\mu_i^* + \nu_i t - \log(P_i(t)) \right] P_i(t) + \nu_i \right\} dt + \sigma_i dW(t),$$

where $\mu_i^* \equiv \mu_i - 0.5\sigma_i^2 / \eta_i$. Details of the calibration can be found in appendix B.2. Using three correlated stochastic processes for bitumen, conventional oil and natural gas, we can write the volatility of the dividend as:

$$(A3) \quad D^2 \sigma_D^2 = \left(\frac{\partial D}{\partial P_B} \right)^2 P_B^2 \sigma_B^2 + \left(\frac{\partial D}{\partial P_O} \right)^2 P_O^2 \sigma_O^2 + \left(\frac{\partial D}{\partial P_G} \right)^2 P_G^2 \sigma_G^2 + 2 \frac{\partial D}{\partial P_B} \frac{\partial D}{\partial P_O} P_B P_O \rho_{BO} \sigma_B \sigma_O + 2 \frac{\partial D}{\partial P_B} \frac{\partial D}{\partial P_G} P_B P_G \rho_{BG} \sigma_B \sigma_G + 2 \frac{\partial D}{\partial P_O} \frac{\partial D}{\partial P_G} P_O P_G \rho_{OG} \sigma_O \sigma_G,$$

where σ_B , σ_O and σ_G are the volatilities of the prices of bitumen, conventional oil and natural gas and ρ_{BO} , ρ_{BG} and ρ_{OG} are the correlations between the respective price processes.

Instead of solving the system of partial differential equations numerically, we use the solutions to the deterministic solution to solve the problem approximately and obtain insight into the role of uncertainty. Formally, this approach would correspond to taking the first-order term in a Taylor-series expansion with the volatility σ_i as the small parameter, where the zeroth-order

term would correspond to the deterministic solution (see also van den Bremer & van der Ploeg, 2013). We thus have from (4) that the effect of a shock at time t on the resource dividend at that same time is the net present value of all future effects of this shock:

$$(A4) \quad \frac{\partial D(t)}{\partial P_i(t)} = [r - \theta(r - \rho) - n] \int_t^{\infty} \frac{\partial E_t[P(\tau)]}{\partial P(t)} O(\tau) e^{-r(\tau-t)} d\tau,$$

where the price process in (A3) gives:

$$(A5) \quad \frac{\partial E_t[P_i(\tau)]}{\partial P_i(t)} = \begin{cases} 1 & \text{random walk,} \\ e^{\mu_i[1 - \exp(-\eta_i(\tau-t))] + \log(P_i(t))[\exp(-\eta_i(\tau-t))]} & \text{AR(1) (A1-A2),} \\ P(t) e^{-\eta_i(\tau-t)} < 1 \end{cases}$$

It is evident then from (A4-A5) that mean reversion acts to decrease the effect of a price shock of the corresponding increment in the resource dividend and thus reduces the need for precautionary savings.

Appendix B: Detailed Description of Data

B.1. Real interest rates

Over the 2002–2012 period, the average real annual rate of return on the Alberta Heritage Savings Trust Fund was 6.1 per cent with a nominal rate of return of 8.1 per cent²⁰ and average inflation in that period of 2.0 per cent,²¹ compared to 3.7 per cent for the Norwegian Pension Fund Global over the same period.²²

The average real annual interest rates on Canadian and U.S. government bonds with maturities of 1 year, 5 years and 10 years over the same period were 0.4 per cent for Canadian and -0.6 per cent for U.S.; 1.1 per cent for Canadian and 0.4 per cent for U.S.; and 1.7 per cent for Canadian

²⁰ Alberta Treasury Board and Finance, Heritage Fund Annual Report 2012-2013, www.finance.alberta.ca/business/ahstf/publications.html.

²¹ Statistics Canada, Cansim Online Statistics Database (2013), <http://www5.statcan.gc.ca/cansim/home-accueil?lang=eng&p2=49&MM>.

²² Norges Bank Investment Management, Annual Report 2012, <http://www.nbim.no/Global/Reports/2012/Annual%20report/Annual%20report%202012.pdf>.

and 1.2 per cent for U.S..²³ For the 1992–2012 period, the same rates were 1.9 per cent for Canadian (0.8 per cent U.S.), 2.7 per cent (1.7 per cent) and 3.2 per cent (2.7 per cent). The average real annual interest rates paid on Alberta provincial debt over the period 2005–2012 were 0.1 per cent, 1.3 per cent and 2.2 per cent on bonds with maturities of one year, 5 years and 10 years respectively.²⁴ We set $r = 6.1$ per cent per year and abstract from the risky nature of the returns.

B.2. Estimates of reserve stocks

We use “remaining established reserves” as defined by the Alberta Energy Regulator (“recoverable quantities known to be left”)²⁵. Remaining established reserves correspond approximately to proven reserves. We then allow for discoveries based on historical data. At the end of 2012, remaining established reserves are:²⁶

- Bitumen (or oil sands): 168 billion barrels.
- Conventional oil (light and heavy crude): 1.7 billion barrels.
- Natural gas: 5.8 billion barrels of oil equivalent (916 billion SM3).

We obtain the following R/P (reserves to production ratios) for 2012:

- *Bitumen*: at 2012 production rates of 0.72 billion bbl/year, we obtain a R/P ratio of 230 years. We do not allow for future discoveries.
- *Conventional oil* (encompassing light, heavy, and crude oil): at 2012 production rates of 0.20 billion bbl/year we obtain an R/P ratio of 8.5 years. Although for conventional oil there are significant new discoveries over many decades, production has only marginally exceeded new discoveries in the last 10 years with 10-year (20-year) averages of 0.19 (0.25) and 0.20 (0.20) billion bbl/year, respectively. To reflect this, we

²³ Bank of Canada, Private Communication (2013); Statistics Canada, Cansim Online Statistics Database (2013); U.S. Federal Reserve, “Historical data on selected interest rates,” (Washington, D.C.: 2013), <http://www.federalreserve.gov/releases/h15/data.htm>.

²⁴ Alberta Treasury Board and Finance, Private Communication (2013).

²⁵ Energy Resources Conservation Board (ERCB), “ST98-2013 Alberta’s Energy Reserves 2012 and Supply/Demand Outlook 2013–2022” (2013), <http://www.aer.ca/data-and-publications/statistical-reports>.

²⁶ *ibid.*

assume discoveries decline linearly from 0.20 billion bbl/year to zero in 30 years and increase the current reserves by 3.0 billion barrels accordingly.

- *Natural gas*: at 2012 production rates of 0.59 billion b.o.e./year, we obtain an R/P ratio of 10 years. We note significant new discoveries. Production has marginally exceeded new discoveries during the last 10 years with 10-year averages (20-year) of 0.78 (0.82) and 0.64 (0.60) billion b.o.e./year, respectively. We thus assume discoveries decline linearly from 0.64 billion b.o.e./year to zero in 30 years and increase current reserves by 9.6 billion b.o.e.

We exclude gas from oil wells (circa 10 per cent) and other natural resources such as coal and sulphur. Including our estimates for new discoveries, we use the following reserve estimates:

- Bitumen: 168 billion barrels.
- Conventional oil (light and heavy crude): $1.7 + 3.0 = 4.7$ billion barrels.
- Natural gas: $5.8 + 9.6 = 15.4$ billion b.o.e.

B.3. Official projections of extraction rates

Official projections are available until 2022.²⁷ In these official projections:

- *Bitumen* (or oil sands): production rates almost double and reach 1.4 billion bbl/year in 2021 from 0.72 billion bbl/year in 2012.
- *Conventional oil*: production rates decline marginally from 0.20 billion bbl/year in 2012 to 0.17 billion bbl/year in 2022.
- *Natural gas*: production rates decline from 0.58 billion b.o.e./year in 2012 to 0.44 billion b.o.e./year in 2022.

We use these official projections until 2022 and from then on we assume:

- *Bitumen* (or oil sands): in scenario 1 a continued linear increase of the production rate until 2.0 billion bbl/year in 2030 followed by a plateau at this rate of production until

²⁷ *ibid.*

exhaustion in 2100; in scenario 2 production reaches a plateau in 2022 and continues at the constant rate of 1.44 bbl/year until exhaustion at a later time.

- *Conventional oil*: continued flat rate of production of 0.17 billion bbl/year until exhaustion in 2038.
- *Natural gas*: continued flat rate of production of 0.44 billion b.o.e./year until exhaustion in 2044.

B.4. Extraction costs

Van den Bremer and van der Ploeg²⁸ estimate that, apart from in the initial years 1970–75, when extraction costs were still very high as the very first exploratory and extraction activity took place, average extraction costs for Norway were U.S.\$9/b.o.e. in the period 1990–2000, U.S.\$6/b.o.e. for 2000–2005 and U.S.\$14/b.o.e. for 2005–2010 (2013 prices). In the absence of data for extraction costs for conventional oil in Alberta, we thus set extraction costs to \$15/bbl for conventional oil.

Extraction costs are significantly higher for bitumen. The Canadian Energy Research Institute²⁹ provides estimates of the extraction costs (calculated from subtracting its estimates of royalties and income taxes from its estimates of total supply costs; see its figure E.1) for four different types of plants: 23, 36, 79 and 51 2011-WTI equivalent U.S.\$/bbl. (i.e. the price at which extraction would just become profitable ignoring taxes and royalties). The Canadian Energy Research Institute³⁰ assumes a constant price differential of 15 U.S.\$/bbl between WTI and WCS, the price at which bitumen is sold following dilution for pipeline transportation. Ignoring diluent costs, we thus estimate extraction costs to be the average of these estimates minus the WTI-WCS price differential: $47 - 15 = \$32/\text{bbl}$.

²⁸ van den Bremer and van der Ploeg (2013).

²⁹ Canadian Energy Research Institute, “Canadian oil sands supply costs and development projects (2012–2046)” (2013), http://www.ceri.ca/images/stories/2013-06-10_CERI_Study_133_-_Oil_Sands_Update_2012-2046.pdf.

³⁰ *ibid*.

Furthermore, we note that there has been a significant increase in extraction costs in recent years. Comparing estimates of supply costs from 2012³¹ of 72 U.S.\$/bbl WTI equivalent averaging across different extraction methods to comparable estimates from 2005³² of U.S.\$35/bbl WTI equivalent reveals a twofold increase in costs in seven years (all prices are 2013 prices). To reflect this increase, we assume a linear increase from \$20/bbl in 2006 to \$32/bbl to calculate historical rents in appendix B.7.

Due to the shale gas revolution in the U.S. and the resulting sharp decline in North American natural gas prices, many of the reserves in Western Canada are in fact not economical to extract at current natural gas prices. For natural gas, the Canadian Energy Research Institute³³ estimates extraction costs for vertical and horizontal extraction to be \$7.60/mcf and \$2.60/mcf or \$43 and \$20 /b.o.e.³⁴ The 2012 price of natural gas is below this at \$11/b.o.e. (see figure B.2). Despite the large variation of extraction estimates across different extraction methods and across different Canadian provinces (\$2/mcf–\$10.20/mcf or \$11/b.o.e.–\$57/b.o.e.) provided by the Canadian Energy Research Institute,³⁵ we use \$15/b.o.e., corresponding to extraction costs of conventional oil.

We thus use the following extraction costs to calculate future resource rents:

- *Oil sands*: \$32 \$ 15/bbl.
- *Conventional oil* (light and heavy crude): \$15/b.o.e..
- *Natural gas*: \$15/b.o.e..

For natural gas, extraction costs may exceed prices, in which case we set resource rents to zero.

Since we assume that Alberta gas is sold at the Henry Hub price and Alberta conventional oil is

³¹ ERCB, “ST98-2013 Alberta’s.”

³² ERCB, “ST98-2013 Alberta’s.”

³³ Canadian Energy Research Institute, “Conventional natural gas supply costs in western Canada” (2013), http://www.ceri.ca/images/stories/ceri_study_136_-_conventional_natural_gas_supply_cost_-_final_june_2013.pdf.

³⁴ *ibid.*, figures 3.2 and 3.3.

³⁵ *ibid.*

sold at the WTI price, we effectively abstract from transportation costs. Transportation costs only account for a small reduction in resource rents of the order of 5 per cent.

B.5. Price processes

Figure B.1 shows historical records of real oil and gas prices (discounted using Canadian CPI)³⁶ in Canadian dollars.³⁷

INSERT FIGURE B.1

We assume conventional oil is sold at the WTI price and natural gas at the Henry Hub NYMEX natural gas price.³⁸ Also shown are the longer historical records: the world crude oil price³⁹ and the U.S. natural gas wellhead price.⁴⁰ To calculate the value of a barrel of bitumen, the costs of diluting heavy crude to make it transportable via pipelines have to be taken into account. Western Canadian Select therefore only provides an upper bound to the actual bitumen price. The average field gate price for bitumen⁴¹ provides our estimate of the actual price of bitumen.

We use the values of the mean-reversion and volatility for the oil and gas price as estimated in van den Bremer and van der Ploeg (2013) (there is approximate parity of U.S. and Canadian dollars in 2013):

- *Conventional oil*: a mean price of \$110/bbl, a mean-reversion coefficient of 6.0 per cent per year, and a volatility of 26 per cent.

³⁶ Statistics Canada, Cansim Online Statistics Database (2013), <http://www5.statcan.gc.ca/cansim/home-accueil?lang=eng&p2=49&MM>.

³⁷ Exchange rates from Statistics Canada, Cansim Online Statistics Database (2013).

³⁸ Government of Alberta, Department of Energy, Private Communication (2013).

³⁹ BP, Statistical Review (London: BP, 2013), <http://www.bp.com/en/global/corporate/about-bp/statistical-review-of-world-energy-2013.html>.

⁴⁰ USEIA, "Independent Statistics and Analysis" (Washington, D.C.: U.S. Energy Information Administration, 2013), www.eia.gov/dnav/ng/hist/n9190us3a.htm.

⁴¹ Government of Alberta, Department of Energy (2013).

- *Bitumen*: the same mean–reversion coefficient and volatility as conventional oil, but a substantially lower mean price of \$80/bbl,⁴² which assumes a constant price differential of \$15/bbl between WTI and WCS).
- *Natural gas*: a mean price of \$32/b.o.e., a mean–reversion coefficient of 6.0 per cent per year, and a volatility of 20 per cent.

We set the correlation coefficients between the different prices to one. The time horizon of our analysis starts Jan. 1, 2013. We use the 2012 prices as the initial prices:

- *Conventional oil*: $P_o(t = 2013) = 96 \$/\text{bbl}$.
- *Bitumen*: $P_b(t = 2013) = \$64/\text{bbl}$.
- *Natural gas*: $P_g(t = 2013) = \$11/\text{b.o.e.}$.

Extraction of natural gas will not be profitable in the initial years with extraction costs of 15\$/b.o.e., but will eventually become profitable as a result of reversion to the mean. All prices are in 2013 Canadian dollars unless otherwise indicated.

B.6. Economic and population growth

Alberta has seen relatively high rates of population growth, with a 10-year average of 2.2 per cent growth and 20-year average of 2.0 per cent growth,⁴³ but growth is forecasted to decline to 1.3 per cent in the next three decades. In part due to volatile oil prices, Alberta’s GDP has been very volatile, with 10-, 20- and 30-year average real per capita growth rates of 3.8 per cent, 3.8 per cent and 1.4 per cent, respectively. In part these growth rates reflect the expansion of the resource-extraction sector. To calculate growth in non-resource GDP, we calculate resource rents as described above. Figure B.2 shows the share of resource revenues and resource rents of total Alberta GDP with averages of 24 per cent and 12 per cent for the range for which data is available. Subtracting resource rents gives per capita non-resource GDP growth rates of 3.2 per

⁴² cf. CERI, “Canadian oil sands.”

⁴³ Statistics Canada, Cansim.

cent for the last 10 years and 3.0 per cent for the period for which we have calculated resource rents (17 years). Excluding not just resource rents, but total resource revenues we obtain 3.8 per cent and 3.2 per cent, respectively.

We set trend population growth to $n = 1.3$ per cent with population size at end of 2012 equal to 3,873,745⁴⁴ and set trend growth of non-resource GDP to $n + g = 3.3$ per cent per year, which implies a trend productivity growth of $g = 2.0$ per cent per year.

INSERT FIGURE B.2

B.7. Historical series of government resource rents

Alberta government income derived from the extraction of oil and gas and bitumen consists of a variety of fees and royalties of which the conventional oil royalty, the oil sands royalty and the natural gas and by-product royalty are the major components. In addition, the Alberta government receives a share of the corporate income tax paid by the resource-extracting sector. Figure B.3 shows the sum of these rents as received by the government in absolute values⁴⁵ and as a share of total resource rents.

On average, resource revenues constitute approximately one-third of total Alberta government revenues (part of the income from corporate income taxation is received at a national level by the federal government). The government take (the share of total resource rents that is ultimately received by the Alberta government) is 34 per cent for the period 2002–2012. To get at optimal savings for the Alberta government, we take optimal savings for the economy as a whole and multiply it by 0.34. The government can only save that part of resource rents that it receives as royalty or tax income in the first place. We also suppose in our calculations that the size of the Alberta government relative to the total Alberta economy stays constant at 14 per cent based on an historical average.

INSERT FIGURE B.3

⁴⁴ *ibid.*

⁴⁵ Government of Alberta, Treasury Board and Finance, Private Communication (2013).

B.8. Initial size of the fund

We include the Contingency Account (\$2.7 billion) and the Heritage Savings Trust Fund (\$14.9 billion) to give a total initial fund size of \$17.6 billion (5.7 per cent of total GDP in 2012) at the end of March 2013.⁴⁶ We do not include the much smaller funds, such as the Medical Research Endowment Fund and the Scholarship Fund, since these have not been funded by oil and gas revenues and reflect savings as part of the non-resource part of the economy.

⁴⁶ Government of Alberta, Budget 2013, <http://budget2013.alberta.ca/>.

Figure 1: Historical data and projections for extraction rates and reserves

a. Bitumen reserves

b. Bitumen extraction

c. Conventional oil and gas reserves

d. Conventional oil and gas extraction

Figure 2: Dividend and fund size with different degrees of prudence
(extraction scenario 1)

a. Resource dividend

b. SWF buildup

Figure 3: Spend all, permanent-income hypothesis and bird-in-hand
(extraction scenario 1)

a. Resource dividend

b. SWF buildup

Figure 4: Resource dividends as percentage of the fund

Figure 5: Optimal spending and build-up of fund for different extraction scenarios (CRP=3)

a. Resource dividend

b. SWF buildup

Figure 6: Effects of a lower real return on fund assets

a. Resource dividend

b. SWF buildup

Figure 7: Initial resource dividend and final fund size ($t = 2100$) as function of initial and mean prices

a. Initial resource dividend

b. Final fund size ($t = 2100$)

Note: In the base case, initial prices are \$64, \$96 and \$11 per barrel of oil equivalent for bitumen, conventional oil and natural gas, reverting to mean prices of \$80, \$100 and \$32 per barrel of oil equivalent, respectively. Although only the conventional oil price is shown on the x -axis for reference, an equivalent scale factor ranging between 0.4 and 1.6 is applied to all three initial prices. For the two steepest lines (blue) the scale factor is applied to both initial and mean prices, whereas for the two less steep lines (red) the scale factor is only applied to the initial prices.

Figure 8: Effect of price process (mean reversion vs. random walk)

a. Resource dividend

b. SWF buildup

Figure B.1: Real oil and natural gas prices (2013 prices)

a. Historical oil prices

b. Historical natural gas prices

Figure B.2: Resource revenues and resource rents as a share of total GDP

Figure B.3: Alberta government resource revenues

a. Total resource rents

b. Share of government revenue

Table 1: Estimates of the optimal fund sizes, resource dividends and savings for the Alberta government (*CRP* = 3 and extraction scenario 1)

	Intergenerational fund (per cent of GDP)	Liquidity fund (per cent of GDP)	Total fund (per cent of GDP)	Dividend (per cent of government revenue)	Saving (per cent of government revenue)
2013	4.8% (\$3,800 pp \$3,800 pp)	0.9% (\$700 pp \$700 pp)	5.7% (\$4,500 pp \$4,500 pp)	26% (\$2,800 pp \$2,800 pp)	3.5%, 2.1% (\$400 pp \$400 pp)
2020	12% (\$9,900 pp \$11,000 pp)	1.9% (\$1,500 pp \$1,800 pp)	14.0% (\$11,000 pp \$13,000 pp)	25% (\$2,900 pp \$3,300 pp)	12%, 1.5% (\$1,300 pp \$1,500 pp)
2030	35% (\$29,000 pp \$41,000 pp)	4.0% (\$3,400 pp \$4,700 pp)	39% (\$33,000 pp \$46,000 pp)	26% (\$3,000 pp \$4,200 pp)	16%, 0.4% (\$1,900 pp \$2,700 pp)
2050	95% (\$72,000 pp \$151,000 pp)	6.4% (\$4,900 pp \$10,000 pp)	101% (\$77,000 pp \$161,000 pp)	30% (\$3,200 pp \$6,600 pp)	-6.7%, -1.0% (-\$700 pp -\$1,500 pp)
2100	159% (\$112,000 pp \$639,000 pp)	6.5% (\$26,000 pp \$148,000 pp)	165% (\$117,000 pp \$665,000 pp)	33% (\$3,200 pp \$18,400 pp)	-28%, 1.6% (-\$2,700 pp -\$16,000 pp)

Note: The size of the fund in 2013 is \$17.6 billion. The size of resource wealth in 2013 is \$1.24 trillion in 2013 or \$320,000 per capita or 400 per cent of GDP. For comparison with the figures in the table, we must multiply this by 0.34, the share of resource rents that accrues to the government, to give \$420 billion, \$109,000 per capita, or 137% of GDP. In each cell, the first figure in brackets is in dollars per person. They are corrected for productivity growth and thus grow at 2.0% per year. The second figure in brackets is uncorrected for productivity growth. The figures in the last column report total and precautionary saving as percentage of government revenue; figures in brackets are total saving, growth-corrected and uncorrected.