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5 **Dating buried glacier ice using cosmogenic ^3He in surface clasts: theory and**
6 **application to Mullins Glacier, Antarctica.**
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ABSTRACT

We develop a modeling framework to describe the accumulation of terrestrial cosmogenic ^3He in Antarctic debris-covered glaciers. The framework helps quantify the expected range in cosmogenic-nuclide inventories for measured clasts at the surface of supraglacial debris. We first delineate the physical factors that impact clast movement within, and on top of, debris-covered glaciers, including the effects of (1) ice ablation, (2) erosion at the debris surface, and (3) stochastic geomorphic processes that impact clast movement within and on top of supraglacial debris; we then explicitly calculate the impact of each process in altering the total inventory of cosmogenic nuclides in surface clasts. Assuming basic elements of ice-dynamics and debris entrainment are known, the model results provide an estimate for the total accumulation of cosmogenic nuclides, as well as the expected range in nuclide inventories, for any clast at the surface of debris-covered glaciers. Because the values are quantified, the approach can be applied to help evaluate the robustness of existing and future cosmogenic datasets applied to these systems. As a test, we applied our model framework towards Mullins Glacier, a cold-based debris-covered alpine glacier in the Dry Valleys of Antarctica. Our simulated values for cosmogenic-nuclide inventories compare well with those previously measured from fifteen surface cobbles along Mullins Glacier (^3He), both in terms of expected ranges and absolute values, and suggest that our model framework adequately incorporates most of the complicating factors that impact cosmogenic datasets for cold-based, debris-covered glaciers. Relating these cosmogenic-nuclide inventories to ice ages, the results show that ice within Mullins Glacier increases non-linearly, ranging from 12 ka to ~ 220 ka in areas of active flow, to $\gg 1.6$ Ma in areas of slow-moving-to-stagnant ice.

1. INTRODUCTION

Debris-covered glaciers are common in high-elevation, high-latitude glacial and periglacial environments. Due to their protective, ablation-limiting cover of debris (Brock et al.; Clark et al., 1994; Konrad and Humphrey, 2000; Kowalewski et al., 2012; Kowalewski et al., 2006; Kowalewski et al., 2011; Mihalcea et al., 2008a; Mihalcea et al., 2006), buried glaciers may be extremely long-lived – especially those that are slow-moving-to-stagnant (Clark et al., 1996; Konrad et al., 1999; Mackay et al., 2014; Steig et al., 1998; Sugden et al., 1995; Yau et al., 2015). Unfortunately, dating buried glacier ice is complicated by several factors. First, the paucity of organic material in many high-elevation and high-latitude glacial systems limits attempts to derive ages from both radiocarbon dating (Balco, 2011) and lichenometry (e.g., Konrad and Clark, 1998). Second, recent attempts to date buried glacier ice through geochemical analysis of englacial gasses (e.g., Bender et al., 2008; Yau et al., 2015) have proved difficult because englacial gasses, at least in the uppermost ~30 m, are typically contaminated with recent air introduced via thermal-contraction cracks and brittle deformation (though pristine atmosphere may be present in ice at greater depths) (Yau et al., 2015).

As an alternative, we explore dating buried glacier ice by assessing the inventory of cosmogenic nuclides in clasts on top of the protective supraglacial debris. Although this approach is not new, the problem is perhaps more complex than previously envisaged because the inventory of cosmogenic nuclides varies greatly due to differences in clast-transport pathways and from stochastic geomorphic processes that result in complex burial-and-erosion dynamics near the ground surface (e.g., Balco, 2011; Dunai, 2010; Lal, 1991).

Assuming that a given clast on the surface of a buried glacier has not yet reached secular equilibrium (see Section 2.3.3), the local *ice age* directly beneath the clast can theoretically be estimated from the cosmogenic nuclide inventory within the sample if (1) the environmental forcing

conditions, (2) the exposure history of the sample, and (3) the sample's dynamic relationship to the glacier ice in question are known. Our approach is to first define a range of potential transport pathways for clasts within debris-covered glaciers. We then model the specific differences in transport pathways that arise from changes in (1) ice ablation (e.g., the apparent movement of englacial debris toward the surface of down-wasting ice); (2) surface erosion; and, (3) stochastic clast movement (complex burial and exposure) from a combination of site-specific surficial geomorphic processes. Next, we examine the impact of these varied clast-transport pathways on altering the inventory of cosmogenic nuclides. Through understanding the potential spread in cosmogenic nuclide inventories that result from different exposure histories and transport pathways, we develop a framework to estimate the potential range in ice ages directly beneath a measured surface sample. Our modeling efforts include analytical approximations, which are best for simplified systems and long time frames, and numerical forward models. At the end of the paper, we apply our modeling framework toward understanding the age of Mullins Glacier, a well-studied debris covered alpine glacier in Antarctica that is thought to be several million years old in its distal, stagnant regions (Kowalewski et al., 2011; Levy et al., 2006; Mackay et al., 2014; Shean and Marchant, 2010; Yau et al., 2015).

2. MODELING FRAMEWORK

2.1. Model assumptions and application

Our model applies to high-latitude, cold-based debris-covered glaciers. By targeting *cold-based* glaciers, we remove the possibility of significant basal entrainment, and thus reduce potential problems associated with multiple source areas and multiple transport pathways for supra- and englacial debris (Atkins, 2013; Cuffey et al., 2000; Denton et al., 1993). Further, recent studies have shown that, for cold-based debris-covered glaciers in the Transantarctic Mountains, source areas for

englacial and supraglacial debris are restricted to rockfall from steep bedrock cliffs at valley heads (Kowalewski et al., 2011; Mackay et al., 2014; Marchant et al., 2013; Swanger et al., 2010). Accordingly, distal rockfall that lands onto ice-ablation zones lacks a component of englacial flow and is transported solely as supraglacial debris. In contrast, debris that comes to rest in ice accumulation zones is buried by snow and ice and subsequently is transported as englacial debris. Eventually, this englacial debris returns to the ice surface as overlying ice sublimates in ice ablation zones. Together, these processes create a layer of supraglacial debris that thickens with increasing distance down glacier (Fig. 1). In addition to the above framework for clast entrainment in our model framework, we make the following approximations: first, we assume that the time it takes for rockfall on ice accumulation zones to transition into englacial debris (via burial beneath snow and ice) is negligible compared to the ultimate age of the ice (Mackay et al., 2014); second, we approximate that the age of the ice is uniform with depth, especially when compared to ice ages as a function of distance down glacier (Fig. 1b).

2.2. Model set up

Following previous investigators (e.g., Marchant et al., 2002; Morgan et al., 2010a; Morgan et al., 2010b; Ng et al., 2005; Schaefer et al., 2000), we model clast movement within the ablating portions of debris-covered glaciers as one-dimensional columns of debris-laden ice undergoing slow surface lowering (Fig. 1b-c). The additional effects of longitudinal ice flow are addressed in the Supplementary Information. Within this one-dimensional model, two factors dominate: first, the thickness of the supraglacial debris will increase at a rate that is proportional to the sublimation rate and the concentration of englacial debris (with supraglacial debris thickness increasing at a faster rate with higher concentrations of englacial debris and higher sublimation rates); second, the supraglacial debris will decrease in thickness via erosion:

$$\frac{dH(t)}{dt} = \frac{cS}{1 - \varphi} - \varepsilon_d = Sc^* - \varepsilon_d, \quad (1)$$

where $H(t)$ is the thickness of the supraglacial debris layer (cm), c is the englacial debris concentration (by volume) ($\text{cm}^3 \text{cm}^{-3}$), S is the sublimation rate (cm a^{-1}), ε_d is the bulk erosion rate of the supraglacial debris (cm a^{-1}), φ is the average bulk porosity of the resulting supraglacial debris ($\text{cm}^3 \text{cm}^{-3}$) and c^* is a shorthand used throughout the paper to denote $c/(1 - \varphi)$. Other than the average bulk porosity of the supraglacial debris, φ , which likely varies by relatively small amounts, none of the parameters in Equation 1 are constant. Over long time periods, ε_d will vary with climate forcing, c may vary by location within the dirty ice column, and S will vary as both a function of H and climate forcing. However, we initially assume that the debris concentration is broadly constant throughout the ice column (we explore the effect of variable englacial debris concentration in Section 2.3.5.4) and assume that, over the time period of interest, climate forcing remains roughly consistent (see Supplementary Information for a discussion of the impact of variable climate forcing). This allows us to approximate ε_d as a constant in Equation 1 and address the dependency of S on $H(t)$.

Several field, experimental, and modeling studies have investigated the impact of supraglacial debris thickness on the ablation rate of underlying glacier ice (e.g., Benn and Evans, 1998; Bozhinskiy et al., 1986; Brock et al., 2010; Brock et al., 2007; Hagg et al., 2008; Humlum, 1996; Juen et al., 2014; Kirkbride and Deline, 2013; Kirkbride and Dugmore, 2003; Konrad and Humphrey, 2000; Kowalewski et al., 2011; Mattson et al., 1993; Mihalcea et al., 2008b; Mihalcea et al., 2006; Nakawo and Rana, 1999; Nakawo and Young, 1982; Nicholson and Benn, 2006; Östrem, 1959; Reznichenko et al., 2010; Richardson and Brook, 2010; Schomacker, 2008; Yang et al., 2010). Although the physical drivers of ablation along debris-covered glaciers are complex and commonly site-specific, several investigators have observed that the general form of the relationship between

ablation rate and supraglacial debris thickness can be well approximated with a simple power-law (e.g., Juen et al., 2014; Richardson and Brook, 2010) or exponential function (e.g., Brock et al., 2010; Clark et al., 1994; Hagg et al., 2008; Humlum, 1996; Konrad and Humphrey, 2000; Schomacker, 2008). Following this simplified approach, we parametrize the general dependency of S on H as

$$S(H) = ae^{-bH}, \quad (2)$$

where a (cm a^{-1}) and b (cm^{-1}) are experimentally derived constants specific to the local ablation environment (See Supplementary Fig. S4). We note that Equation 2 does not adequately capture the ablation behavior for exposed ice or for ice beneath very thin debris ($H < \sim 2 \text{ cm}$) (See Supplementary Information). However, our focus is on glaciers with relatively thick surface debris, $H > \sim 2 \text{ cm}$, and consequently the resulting slow ice loss is well represented by Equation 2.

Substituting Equation 2 into Equation 1 and solving the differential equation yields an expression for the debris thickness over time:

$$H(t) = \frac{\ln[ac^* + e^{-b\varepsilon_d t}(\varepsilon_d e^{bH_r} - ac^*)] - \ln(\varepsilon_d)}{b}, \quad (3)$$

where H_r (cm) is the initial debris thickness at time zero. Physically, H_r represents the cumulative thickness of debris that could potentially have accumulated during a phase of ice loss prior to the development of a thick debris cover; this could arise at an early stage when H is $< \sim 2 \text{ cm}$, or from any rockfall debris that lands directly onto ice-ablation zones and bypasses englacial transport (e.g. Mackay et al., 2014). We note that in the limit as t approaches infinity, $H(t)$ asymptotically approaches

$$H_{eq} = \frac{\ln\left(\frac{ac^*}{\varepsilon_d}\right)}{b}, \quad (4)$$

where H_{eq} (cm) represents the dynamic equilibrium thickness at which the growth rate of the supraglacial debris exactly equals the rate of material removed by surface erosion (Fig. 1d-e and Fig. 2). Under these steady-state conditions, sublimation is constant and linearly related only to the surface erosion rate and englacial debris concentration as $S(H_{eq}) = \varepsilon_d/c^*$. In order to estimate how long it takes for the supraglacial debris to reach its final equilibrium thickness, we set Equation 3 equal to Equation 4 multiplied by a small tolerance value and solve for t . This results in:

$$T_{eq}(\gamma) = \frac{\ln\left((ac^* - \varepsilon_d e^{bH_r}) - \ln\left[ac^* - \varepsilon_d \left(\frac{ac^*}{\varepsilon_d}\right)^\gamma\right]\right)}{\varepsilon_d b}, \quad (5)$$

where $T_{eq}(\gamma)$, the equilibrium time, is the time at which the supraglacial debris thickness reaches γ percent of H_{eq} and γ is an arbitrarily chosen tolerance value equal to ~ 0.95 (Fig. 2). Substituting Equation 3 into Equation 2 gives an expression for the sublimation rate as a function time, $S(t)$,

$$S(t) = \frac{a\varepsilon_d}{ac^* + e^{-b\varepsilon_d t}(\varepsilon_d e^{bH_r} - ac^*)}, \quad (6)$$

and integrating Equation 6 with respect to time yields an expression for the total thickness of ice lost to ablation over time, $I(t)$ (cm):

$$I(t) = \frac{b\varepsilon_d t + \ln(-\varepsilon_d e^{-bH_r}) - \ln[e^{b\varepsilon_d t}(ac^* - \varepsilon_d e^{-bH_r}) - ac^*]}{bc^*}. \quad (7)$$

As expected, as t approaches infinity, the form of Equation 7 asymptotically approaches a line with the slope ε_d/c^* , reaching a nearly linear state at time T_{eq} .

2.2.1. Clast transport through the system

With the top of the supraglacial debris as a frame of reference, we can define clast movement during a period of pre-accretion dynamics and post-accretion dynamics. In the pre-accretion stage, clasts approach the ground surface at the rate (termed the *apparent emergence velocity*)

$$v_i(t) = S(t) - S(t)c^* + \varepsilon_d(t), \quad (8)$$

where v_i is the apparent emergence velocity of englacial clasts (cm a^{-1}) (Fig. 1d-e). The first term on the right hand side represents the overlying ice lost to sublimation, the second term represents the debris added to the base of the supraglacial debris, and the third term is the material lost at the surface of the supraglacial debris due to erosion. Post-accretion, clasts approach the ground surface at the rate

$$v_d(t) = \varepsilon_d(t) , \quad (9)$$

where v_d is the apparent emergence velocity of clasts within the supraglacial debris (cm a^{-1}) (Fig. 1d-e). The above assumes negligible clast transport arising from active ice flow, e.g., ice advection along shear planes or other types of ice deformation. Finally, if we include processes affecting clasts once they emerge *at the ground surface*, any mineral within the clast approaches the surface of the clast (e.g., exposure) at the rate

$$v_c(t) = \varepsilon_c(t) , \quad (10)$$

where v_c is the apparent emergence velocity of a particle within the clast (cm a^{-1}) and ε_c is the erosion rate of a single clast (cm a^{-1}).

2.2.2. Complications and stochastic processes

In the sections above, we provide analytical approximations for the basic dynamics of supraglacial debris layer formation, underlying ice wastage, and clast-transport pathways. However, this framework does not capture several additional processes that may impact clast movement and transport. Some of these processes are modelled specifically, though others are introduced only to provide a qualitative assessment of stochastic material transport.

2.2.2.1. Clast intrusion into the base of the supraglacial debris

Consider the top of the supraglacial debris layer as the frame of reference. As noted in Section 2.2.1, clasts approach the ground surface at a rate that is dependent on their location within the

surrounding substrate – either ice or surface debris. We approximate that as clasts breach the surface of the ice and enter the supraglacial debris, they transition immediately from a pre-accretion emergence velocity (Equation 8) to a post-accretion emergence velocity (Equation 9) (Fig. 1d-e). This is broadly correct for small particle sizes. However, for larger clasts sizes this approximation breaks down. Large clasts remain fixed in the ice for a period of time after their uppermost surface has entered the supraglacial debris (Fig. 3a). As more of the clast is freed from ice via sublimation, the preexisting supraglacial debris layer is modified (draped) around clasts that still remain partly entombed in ice. During this transition period, various surface processes, including the settling of fines around the larger emerging clast and the subtle redistribution of grains within the overlying debris matrix will all modify the vertical transport of material directly above and adjacent to the emerging clast. The clast will essentially penetrate into the supraglacial debris as opposed to simply accreting vertically to its base (Lewis, 2005) (Fig. 4a). Thus, instead of transitioning immediately to the post accretion emergence velocity (Equation 8), the clast will acquire an apparent transitional emergence velocity (v_t) (cm a^{-1}) of

$$v_t(t) = S(t) + \varepsilon_d(t) \quad (11)$$

over the time period required for the base of the clast to fully exit the ice, at which point the clast fully transitions to a post-accretion emergence velocity (Equation 9).

2.2.2.2. *Rapid exhumation at polygon margins*

In the discussion above, we have ignored changes in supraglacial debris imparted by the development and maturation of sublimation polygons that dominate the meter-scale topography of debris-covered glaciers in hyper-arid, cold-polar deserts (Levy et al., 2006; Marchant et al., 2002). However, as described in Marchant et al. (2002) and Kowalewski et al. (2011), heightened sublimation at polygon margins (relative to that at polygon centers) gives rise to deep (1-3 m)

troughs that fringe elevated polygon centers. The troughs develop slopes that exceed the angle of repose for supraglacial debris and give rise to sporadic slumping and translational slides. Through this process, material at depth within supraglacial debris along steep slopes at polygon margins is periodically exhumed, while debris at the center of polygon troughs is incrementally buried (debris at polygon centers remains unaffected by this process). The effect of rapid exhumation and exposure of once-buried clasts and debris at polygon margins is equivalent to having clasts located at some depth in supraglacial debris (z_d) instantaneously transition to the surface such that $z_d \approx 0$ (Fig. 5a). We refer to the depth at which this occurs as the exhumation depth (z_e) (cm).

To determine the value for z_e directly would require a coupled model for polygon development and trough-slope stability, which is beyond the scope of the present work. However, we can approximate z_e by assuming a linear relationship between z_e and supraglacial debris layer thickness, H . This simplification is justified by field observations that show a strong depth dependence between H and mass-wasting events at the margin of sublimation polygons: thin supraglacial debris (H), which is typically associated with immature polygons and shallow bounding troughs, appears relatively stable and does not appear to undergo slumping at polygon margins (Levy et al., 2006; Lorrey, 2002). Thick supraglacial debris, on the other hand, is typically found in association with mature polygons and deep polygon troughs; the depressed scars of former slumps and exposed surfaces on which translation slides have propagated are readily apparent on the flanks of such polygons and appear noticeably less weathered than adjacent supraglacial debris (Marchant et al., 2002). Using these two extreme cases as end points, we fit a simple linear relationship between z_e and H of the form

$$z_e = \begin{cases} mH(t) - k & \text{for } H(t) \geq H_{min} \\ 0 & \text{for } H(t) < H_{min} \end{cases}, \quad (12)$$

where m (cm cm^{-1}) and k (cm) are empirically derived constants and H_{min} (cm) is the minimum thickness required before slumping can occur.

2.2.2.3. *Episodic erosion of surface clasts by fracture and spalling*

Spalling of rock fragments caused by thermal stress-induced fracture may produce fresh exposures on surface clasts (e.g., Bao et al., 2008; Hall, 1999; Hall and André, 2001; Lamp, 2016; Marchant et al., 2013; McFadden et al., 2005; Swanger et al., 2011). Although the frequency, magnitude, and geographic distribution of thermal fracture is poorly characterized, the low temperatures and rapid thermal contrasts induced by abrupt changes in solar exposure, cloud cover, and winds, along with abundant field evidence for fractured surface clasts (e.g., “puzzle rocks” of Marchant and Head (2007)) and exfoliated slabs (Fig. 3b-d) suggest that thermal fracture and spalling are important in the weathering of supraglacial debris in hyper-arid, cold-polar deserts.

To model the rock spalling process we impose simplified fracture geometry where all fractures occur in the horizontal plane (Fig. 6a-b). We model multiple spalling events by introducing an average estimated spalling recurrence frequency, f , and specifying a model for fracture depth. Swanger et al. (2011) suggested a model of fracture depth where the depth of each successive fracture is equal to half of the vertical dimension of the initial block before the fracture occurs (see Swanger et al. [2011] Fig. 7). However, in agreement with field observations from the Dry Valleys (Fig. 3b-d), we introduce a second and preferred model of fracture depth that calls for a constant average recurring fracture depth that is independent of the original clast size pre-fracture (Fig. 6b).

2.2.2.4. *Heterogeneous englacial debris concentration*

In section 2.2.1 we made the assumption that the concentration of englacial debris, c , was constant. However, c may vary with depth in the ablating ice column (e.g., Mackay et al., 2014). If

the variation departs significantly from the mean value, then Equations 3-7 are invalid and the dynamics of supraglacial debris formation and ice loss must be modelled numerically.

2.2.2.5. *History of clasts prior to entering the debris-covered glacier system*

A detailed model showing the precise source area and detachment process for rockfall events that supply debris to buried glacial systems is beyond the scope of this work. However, in any rockfall deposit, some clasts will inevitably contain evidence for early episodes of in-situ weathering acquired on cliff faces prior to detachment and rockfall deposition. These weathered faces will almost surely be oriented in different fashion upon deposition (e.g., weathered surfaces may be buried face down). It is also likely that due to kinetic effects and grain-to-grain impacts during transport, most of the clasts will likely have lost evidence for prior exposure and show only fresh faces without weathering rinds (Kowalewski et al., 2011).

2.2.2.6. *Reorientation of clasts across material boundaries*

The majority of clasts exposed at the surface of supraglacial debris have been reoriented since initial deposition. For the simple case where rockfall (1) lands on ice-ablation zones, (2) remains at the ground surface throughout subsequent transport, and (3) is eventually sampled at polygon centers (away from bounding troughs), the effect of clast reorientation is most likely at its minimum value (in the absence of liquid water, active layer cryoturbation is nearly non-existent in high-elevation regions of the Transantarctic Mountains (Marchant and Head, 2007)). However, for complex cases where clasts are deposited onto ice accumulation zones, reorientation will likely occur through englacial transport and across material boundaries. Near the surface, reorientation may occur when a clast transitions from the surface of buried ice into the base of supraglacial debris and again when the clast reaches the ground surface. This process is not modeled explicitly, but as seen in Fig. 7 the significance of this effect likely scales with clast size and shape.

2.2.3. Summary of clast exposure models and model variants

To simplify the discussion that follows, we have classified the various transport pathways and stochastic processes noted above into a suite of eight models (Fig. 7).

(1) clast erosion-only (EO) model. The clast erosion-only model (EO) is appropriate for those clasts that have been exposed continuously at the surface of supraglacial debris and have subsequently been modified by constant erosion.

(2) single fracture model (variant of the *clast EO model*). The single fracture / spalling model includes the same dynamics as the *clast erosion-only* model but with the additional specification that clasts have undergone a single fracture and spalling event.

(3) recurrent fracture model (variant of the *clast EO model*). The recurrent fracture / spalling model includes the same dynamics as the *clast erosion-only* model but with the additional condition that clasts have undergone multiple recurrent fracture and spalling events.

(4) erosion-sublimation (ES) model. The erosion-sublimation model (ES) is appropriate for those clasts that (a) were initially buried by snow/ice/debris in the accumulation zone, and (b) have just recently reached the top of the supraglacial debris layer via sublimation and/or erosion. This model assumes that clasts transition instantaneously into the supraglacial debris when they reach the surface of the ice.

(5) clast intrusion model (variant of the *ES model*). The clast intrusion model includes the same dynamics as the *erosion-sublimation model* but with the additional stipulation that as clasts reach the surface of the ice they do not transition immediately into the base of the supraglacial debris layer. Instead, they remained locked within the ice for a period of time and penetrate into the base of the supraglacial debris.

(6) *polygon exhumation model* (variant of the *ES model*). The polygon exhumation model includes the same dynamics as the *ES model* but includes the additional condition that the clasts have been rapidly exposed via slumping at polygon margins.

Finally, we include two additional reference models that do not have physical meaning for our system but are used for illustrative and comparison purposes:

(7) *debris erosion only (EO) model*. The debris erosion only model is appropriate for samples that are not sourced from sublimating ice but have recently been exposed via erosion of overlying debris.

(8) *apparent model*. The apparent model is similar to the *clast EO* model in that it considers clasts that have been exposed continuously at the surface of supraglacial debris. However, this model assumes the clasts have not been modified by erosion or any other post-depositional disturbance.

2.3. Cosmogenic nuclide accumulation in a debris-covered ice system

With the preceding discussion on transport pathways as a framework for defining potential exposure histories for clasts on debris-covered glaciers, we now develop analytical and numerical models to quantify the build-up of cosmogenic nuclides in such clasts. Once these accumulation rates are established, we use this framework to estimate a process-based (model-framework) exposure age range.

2.3.1. Cosmogenic nuclide accumulation in erosion only (EO) models

As high energy cosmic rays impact the earth, collisions with the nuclei of surface materials periodically cause the loss of protons and produce cosmogenic nuclides. This process, termed spallation, is the dominant production pathway for most cosmogenic nuclides at shallow depths (<~1.5 m) and the only significant producer of cosmogenic ^3He in most minerals (Dunai, 2010). If

we limit our discussion to spallation reactions, the production of cosmogenic nuclides at any vertical depth below the surface (or below any homogeneous shielding layer) can generally be described by

$$P(z, t) = P(0, t)e^{-(\rho z/\Lambda)}, \quad (13)$$

where $P(0, t)$ is the time dependent surface production rate of the nuclide of interest ($\text{atom g}^{-1} \text{a}^{-1}$), z is the depth beneath the surface (cm), ρ is the density of the overlying material (g cm^{-3}), and Λ is the absorption mean free path (g cm^{-2}). We initially assume that the production rate at the surface has been constant over the time period of interest (the case of a non-constant production rate is addressed in the Supplementary Information, Section 5) and adopt the notation $P(0, t) = P_0$. If we allow depth to vary as a function of time, the change in nuclide inventory is:

$$\frac{dN(z, t)}{dt} = -N(z, t)\lambda + P_0 e^{-(\rho z(t)/\Lambda)}, \quad (14)$$

where $N(z, t)$ is the accumulated concentration of nuclides (atom g^{-1}) in a sample at depth z and time t , and λ is the decay constant for the nuclide of interest (a^{-1}) (if measuring radionuclides). Assuming zero erosion and excluding other dynamics that may cause vertical clast movement relative to the surface (the *apparent* model), $z(t)$ is a constant, and Equation 14 is solved for t to yield the *apparent age* (Dunai, 2010), T_{app} , of a sample:

$$T_{app} = -\frac{1}{\lambda} \ln \left(1 - \frac{\lambda N(z, t)}{P_0 e^{-(\rho z/\Lambda)}} \right). \quad (15)$$

For an eroding system with a constant erosion rate (ε) Equation 14 is solved to give an expression for the cosmogenic nuclide inventory of a sample as a function of time and depth (Lal, 1991):

$$N(z, t) = N(z, 0)e^{-\lambda t} + \frac{P_0}{(\lambda + \rho\varepsilon/\Lambda)} e^{-(\rho z/\Lambda)} \left[1 - e^{-(\lambda + \rho\varepsilon/\Lambda)t} \right], \quad (16)$$

where $N(z, 0)$ is any pre-existing inventory at time zero for a clast that is now at depth z . Rearranged for t and assuming zero inheritance ($N(z, 0)=0$), Equation 16 becomes

$$t = T_{\varepsilon} = -\frac{1}{(\lambda + \rho_{\varepsilon}/\Lambda)} \ln \left[1 - \frac{(\lambda + \rho_{\varepsilon}/\Lambda)N(z, t)}{P_0 e^{-(\rho_z/\Lambda)}} \right]. \quad (17)$$

Equation 17 can be used to solve for exposure age using nuclide inventories assuming two transport pathways: the *debris erosion* only model or the *clast erosion-only* model (and their variants), where T_{ε} is an *erosion-only age*. However, expressions 16 and 17 do not adequately account for the additional dynamics imparted by the continual transport of clasts to the base of supraglacial debris, nor for the accumulation of cosmogenic nuclides in englacial clasts while they are still in the ice before reaching the supraglacial debris layer, e.g., *erosion-sublimation* models (Morgan et al., 2010a; Ng et al., 2005; Schaefer et al., 2000).

2.3.2. Cosmogenic nuclide accumulation in the erosion-sublimation (ES) models

In order to incorporate the effect of ice sublimation on cosmogenic-nuclide inventories we first represent the accumulation of cosmogenic nuclides at any point beneath three vertically stacked shielding materials (ice, debris, and whole rock) as

$$\frac{dN(z, t)}{dt} = -N(z, t)\lambda + P_0 \exp \left[-\frac{1}{\Lambda} (\rho_i z_i(t) + \rho_d z_d(t) + \rho_c z_c(t)) \right], \quad (18)$$

where ρ_i is the bulk density of the dirty ice (g cm^{-3}), z_i is the thickness of the overlying ice (cm), ρ_d is the mean bulk density of the debris (g cm^{-3}), z_d is the depth of the overlying supraglacial debris layer (cm), ρ_c is the density of the rock (g cm^{-3}), z_c is the thickness of the overlying rock layer (cm) and $z = z_i + z_d + z_c$. The bulk density of the dirty ice is calculated by scaling it with the englacial debris concentration such that $\rho_i = (1 - c)\rho_{io} + c\rho_c$ where ρ_{io} is the density of clean ice (g cm^{-3}). When considering the cosmogenic nuclide inventory of only the uppermost portion of individual clasts, the third term in the exponential can be ignored and Equation 18 reduces to

$$\frac{dN(z, t)}{dt} = -N(z, t)\lambda + P_0 \exp \left[-\frac{1}{\Lambda} (\rho_i z_i(t) + \rho_d z_d(t)) \right] \quad (19)$$

In order to derive an exposure age from the cosmogenic nuclide inventory of clasts that have traveled through an ablating debris-covered glacier, we require a solution to Equation 19 for subsurface clasts or surface clasts that have recently emerged and have only undergone minimal erosion, or a solution to Equation 18 for the case where surface clasts have been significantly eroded.

Solving Equations 18 or 19 requires that we track the location of clasts moving within the ice layer, the supraglacial debris layer, and on the surface, and then determine the amount of time spent in each location/layer (e.g., Morgan et al., 2010a; Ng et al., 2005). In the pre-accretion stage, a clast approaches the surface with an emergence velocity given by Equation 8 and is shielded from cosmic rays by both the mass of overlying ice, $\rho_i z_i$ (which is diminishing at the rate $S(t)$) and the mass of the supraglacial debris layer, $\rho_d z_d = \rho_d H(t)$. Post-accretion, the clast approaches the surface with an emergence velocity given by Equation 9 and is shielded only by the mass of the overlying eroding supraglacial debris layer, $\rho_d z_d$. Finally, we include the dynamics governing the clast at all times after its uppermost portion has been exposed to the surface of the debris layer as a separate surface erosion state. Under the surface erosion state, sample locations at any depth z_c within an individual rock approach the surface with an emergence velocity according to Equation 10 and are shielded only by the mass of the overlying eroding whole rock, $\rho_c z_c$.

Morgan et al. (2010a) provide an analytical solution to Equation 19 that includes both the effects of sublimation and erosion, but does not include the change in ice loss rate caused by the feedback between debris thickness and sublimation rate. Ng et al. (2005) provides a solution to Equation 19 that allows for a non-constant sublimation rate, but does not include the effects of erosion or the coupling between supraglacial debris thickness and sublimation rate. We have developed coupled expressions for ice loss and supraglacial debris thickness change over time (Section 2.2.1). However, inserting these relationships into a representation of Equation 18 or 19 does not result in an

expression that can be solved analytically in closed form and must be solved numerically (see Section 2.3.4).

It should be noted that some authors express the depth of shielding material and the erosion rates in terms of the shielding mass (units of g cm^{-2}) which is the depth times the density of the material. This is an equivalent representation, but we have chosen to represent material depths and densities separately in order to simplify geomorphological interpretation.

2.3.3. Cosmogenic nuclide accumulation in old systems approaching secular equilibrium

After a critical period of exposure, a steadily eroding system will reach secular equilibrium where the production of new cosmogenic nuclides is exactly balanced by the removal of existing cosmogenic nuclides due to erosion at surface (and radioactive decay for radionuclides). In a similar way, a debris-covered glacier system will also eventually reach a form of secular equilibrium where the cosmogenic nuclide inventories within the ice and supraglacial debris layer stabilize in response to the dynamics of sublimation and erosion. We denote the critical time at which this occurs as T_{s_crit} . In the Supplementary Information, we derive an expression for T_{s_crit} as well as expressions for the cosmogenic nuclide concentrations within englacial clasts, $N_i(z_i)$, and the supraglacial debris $N_d(z_d)$ at time points leading up to and reaching T_{s_crit} . Here we use the subscripts i and d to denote ice and supraglacial debris respectively. If a debris-covered ice system has reached (or is assumed to have reached) secular equilibrium, we can estimate the average erosion rate of the supraglacial debris over the time period of $\sim T_{s_crit}$ as

$$\varepsilon_d = \frac{\Lambda P_0}{N_d(0)} \left[e^{-\left(\frac{\rho_d H}{\Lambda}\right)} \left(\frac{c^*}{\rho_i} - \frac{1}{\rho_d} \right) + \frac{1}{\rho_d} \right], \quad (20)$$

where H is the thickness of supraglacial debris underneath a surface sample that has a measured nuclide inventory of $N_d(0)$ (See Supplementary Information, Section 2).

2.3.4. Numerical modeling of cosmogenic nuclide accumulation

The analytical framework presented above provides closed-form approximations for the basic *erosion-only* models, as well as the dynamics of sublimation till formation, underlying ice wastage, and cosmogenic nuclide production within englacial and supraglacial debris for the *erosion-sublimation* model. However, this treatment explicitly assumes that ε , c , and the parameterized sublimation rate as a function of debris thickness (characterized by an exponential function of a and b in Equation 2) remain roughly constant through time. Furthermore, the cosmogenic nuclide profiles within the ice and supraglacial debris layer are only predicted analytically for the limited cases of $t > T_{eq}$ (for $N_i(z_i)$) and $t > T_{s_crit}$ (for $N_d(z_d)$) (See Supplementary Information, Section 2); the transient stages of cosmogenic nuclide accumulation in the system before these time points are not adequately represented analytically.

To address these limitations and to process the full suite of *erosion-only* and *erosion-sublimation* model variants (Section 2.2.3), we developed a numerical forward modeling scheme in which we directly track the location and cosmogenic exposure history of individual tracer clasts within a vertical column of ablating dirty ice. The forward modeling framework allows for variable environmental forcing as well as non-uniform debris concentrations with depth. For the complete model description see the Supporting Information.

2.3.4.1. Test case scenario

We investigate the general behavior of the *erosion* only model (Equation 16) and the *erosion-sublimation* model (Section 2.3.2) using a primary test case scenario – designated ‘SC1’ – in which all parameter values represent mean measured or modeled values similar to estimated values for debris-covered glaciers in Antarctica. For the SC1 scenario, $\rho_c = 3.0 \text{ g cm}^{-3}$, $\rho_i = 0.9 \text{ g cm}^{-3}$ (Cuffey and Paterson, 2010), $\varphi = 0.3$ (Kowalewski et al., 2011), $\varepsilon_d = \varepsilon_c = 40 \text{ cm Ma}^{-1}$ (Morgan et al., 2010a),

$c = 0.008$ (Mackay et al., 2014), $H_r = 5$ cm, $\rho_d = 150$ g cm⁻² (Gosse and Phillips, 2001), $P_0 = 560$ atom g⁻¹ a⁻¹ (consistent with a latitude of -77 S), an elevation of 1420 m, and a sea level high-latitude production rate of 121 ± 6 ³He atoms g⁻¹ (Dunai and Wijbrans, 2000) adjusted with the scaling function of Stone (2000)), and $\rho_d = (1 - \varphi) \rho_c = 2.1$ g cm⁻³. In the SC1 scenario, we use a parameterized sublimation rate curve (Equation 2) M defined by $a = 0.08932$ cm a⁻¹ and $b = 0.6114$ cm⁻¹ (Supplementary Fig. S4). Although the general ES model is applicable to several cosmogenic nuclides produced by spallation reactions, we limit this study to the analysis of cosmogenic ³He and therefore do not include the dynamics of radioactive decay (i.e. $\lambda = 0$ a⁻¹). To simplify the discussion that follows we use the nomenclature N_{surf} to refer to a modeled cosmogenic nuclide inventory at the surface ($N(z_d = 0, t)$) and the specific notation N_{surf_DEO} , N_{surf_CEO} , and N_{surf_ES} for the modeled N_{surf} predicted by the *debris erosion-only* model, *clast erosion-only* model, and *erosion-sublimation* model respectively.

2.3.4.2. Test case results: time evolution of the erosion-sublimation and erosion-only models

Fig. 8 and Fig. 9 show ³He cosmogenic nuclide depth profiles for both ice (N_i) and supraglacial debris (N_d) predicted by the *ES model* for the SC1 scenario. Also displayed, for comparison, are the cosmogenic nuclide concentrations predicted by the *debris EO model* –using the same set of applicable SC1 input parameters. Note that N_d and N_i refer to the nuclide inventory of only the uppermost portion (first ~1-3 cm) of hypothetical clasts located along the profile.

Within the ice, the modeled evolution of N_i through time reflects the increasing shielding of the thickening supraglacial debris layer up until $t = T_{eq}$. Beyond T_{eq} , N_i increases predictably in response to a constant shielding of overlying debris (see Supplementary Equation S5) and eventually reaches its final maximum configuration at time $t = T_{i_crit}$ (Fig. 8c and Fig. 9, see also Supplementary

Information Section 2). N_i displays an exponential decrease in cosmogenic inventories with depth at all times. Within the supraglacial debris layer, the ES modeled time evolution of N_d (Fig. 8a and Fig. 9) is strongly controlled by the continual supply of debris to its base that is depleted in cosmogenic nuclides. Due to this nuclide inventory deficit at the base of the supraglacial debris, the sublimation profile develops a number of distinguishing characteristics that differentiate it from the erosion-only profile. First, only the upper-most portion ($z_d \leq H_r$) of the sublimation profile is identical to the erosion-only profile and only over the short duration of time required to erode the initial thickness of supraglacial debris, i.e. over the time period $t < \varepsilon/H_r$. At all other depths and times, the sublimation profile shows a significant deficit in nuclide concentrations (N) compared to the N for the erosion-only profile. The difference in N between the *ES model* and the *EO model*, initially most pronounced at the base of the profile, increases with time until it reaches a constant deficit at all depths at the time both profiles reach secular equilibrium (Fig. 8d). At the surface of the supraglacial debris, the *ES model* dynamics result in a N_{surf} that is initially nearly identical to the erosion-only model but eventually departs significantly and attains a maximum inventory much earlier than the EO case (Fig. 9). Second, the curvature, $\partial^2 N / \partial z^2$ of the ES sublimation profile is positive in the early stages of the sublimation till development ($t < T_{eq}$) (Fig. 8g-h). This is in direct contrast to the EO case that maintains a negative curvature consistent with an exponential profile at all times. Third, the gradient, $\partial N / \partial z$, of the sublimation profile is significantly more negative than the erosion-only case for all times $t < T_{s_crit}$ (Fig. 8e-f).

2.3.4.3. Test case results: sensitivity analysis of the erosion-sublimation model

Increasing the englacial debris content (c) in the ES model while keeping all other parameters consistent with the SC1 test scenario yields an increase in the final equilibrium supraglacial debris thickness (H_{eq}) and a reduction in the rate of ice loss (Fig. 10a). The value of c , however, has a minor

impact on T_{eq} , the amount of time required to reach H_{eq} . As c increases, the minimum cosmogenic nuclide concentration of surface clasts (N_{surf}) in the erosion-sublimation model approaches the value predicted by the debris erosion-only model that disregards sublimation. As c decreases, N_{surf} departs from the erosion-only profile earlier, reaches T_{s_crit} earlier, and achieves a lower final maximum cosmogenic inventory in casts that have just reached the surface (N_{dmax}) (Fig. 10a).

Increasing the erosion rate (ϵ_d) in the ES model causes a decrease in H_{eq} , a reduction in T_{eq} , and an increase in the rate of ice loss (Fig. 10b). Higher ϵ_d also results in overall lower values of N_{surf} at all times and a reduction in both T_{s_crit} and N_{dmax} .

Decreasing the sublimation rate (S) sensitivity to supraglacial debris thickness (and thus increasing S overall) causes an increase in H_{eq} , but T_{eq} remains largely unaffected (Fig. 10c). Increasing the sublimation rate increases the total amount of ice loss, but the final equilibrium rate of ice loss $S(H_{eq})$ is not affected. Increasing the sublimation rate results in higher values of N_{surf} overall and causes an increase in both T_{s_crit} and N_{dmax} .

Multifactor sensitivity analysis for c and ϵ_d are shown in Supplementary Fig. S3.

2.3.5. Impact of additional stochastic processes on cosmogenic-nuclide accumulation

Above we have outlined the cosmogenic nuclide accumulation dynamics of the *erosion-only* model and the *erosion-sublimation* model. In the following sections we develop variations of these two primary models to address how each of the additional stochastic physical processes introduced in Section 2.2.2 impact the cosmogenic nuclide inventories of surface clasts.

2.3.5.1. Clast intrusion model (erosion-sublimation model variant)

In order to investigate the impact of transient clast intrusion into the base of supraglacial debris, we modify the numerical *ES model* framework to assign the transitional emergence velocity

(Equation 11) to clasts when they enter the supraglacial debris layer. We define the vertical dimension of a clast as it transitions from the ice and enters the supraglacial debris layer as the *effective emergence dimension* denoted by D_{eff} (cm). Clasts maintain this emergence velocity until they have transited the vertical distance of D_{eff} through the supraglacial debris, after which time they approach the surface at the post-accretion emergence velocity (Equation 9). Numerical results show a dramatic reduction in N_{surf} for clasts with large D_{eff} (Fig. 4c). We use the notation $N_{surf_ES}(D_{eff})$ to denote this modified N_{surf} calculated by the ES model when including the intrusion dynamics of realistic clast sizes. As expected, early in the system development when the supraglacial debris is thin relative to clast size, the normalized difference $[N_{surf_ES} - N_{surf_ES}(D_{eff})]/N_{surf_ES}$ is near its maximum (Fig. 4d) and then stabilizes as the supraglacial debris thickness approaches H_{eq} . Analytical approximations of these quantities are developed in the Supporting Information.

For surface debris that is sourced entirely from the sublimation of debris-covered ice, this analysis suggests the testable hypothesis that, ignoring all other competing processes, a significant correlation should exist between clast size and cosmogenic nuclide inventory: larger clasts in the supraglacial debris will contain lower nuclide inventories than co-located smaller clasts.

2.3.5.2. *Polygon exhumation model (erosion-sublimation model variant)*

To estimate the impact of rapid exhumation at polygon margins on the cosmogenic nuclide inventory (Fig. 5), we use the linear relationship of Equation 12 and determine the exhumation depth (z_e) as a function of time to extract the cosmogenic nuclide inventory of a clast at that depth ($N_d(z_e, t)$) from the numerical erosion-sublimation model. We use the notation N_{surf_poly} to represent $N_d(z_e, t)$. Thus defined, N_{surf_poly} represents the estimated minimum bound on the cosmogenic nuclide inventory of surface clasts impacted by polygon margin formation processes. Fig. 5c shows the behavior of N_{surf_poly} for the SC1 test scenario with $\varepsilon_d = 30 \text{ cm a}^{-1}$. The differences between the basic

EO model and the modified version that accounts for polygon processes, can reach a very large magnitude. For example, for a model time of $t = 1$ Ma, $(N_{surf_ES} - N_{surf_poly}) \cong 190 \times 10^6$ atom g^{-1} , or ~43% of the expected N_{surf} (Fig. 5c). For older samples, this potential difference in N increases.

2.3.5.3. *Fracture and spalling model (clast erosion-only model variant)*

Clast surfaces that have been recently exposed by in-situ fracture and spalling will always contain lower nuclide concentrations than would be predicted by a steady-erosion model. Swanger et al. (2011) conjectured on qualitative estimates that recurrent spalling events of surface clasts in the MDV could result in apparent age scatter equal to 200 and 1000 ka for deposit ages of 500 and 2000 ka respectively. Here, we quantify the potential impact of rock fracture and spalling on cosmogenic nuclide inventories using the simplified physical models of rock fracture and spalling geometry /frequency introduced in section 2.2.2.3. In the simplest case of a single spalling event, the N contained in a fresh surface shortly after being exposed by the removal of a single spalled block, designated N_{surf_frac} , is modeled with Equation 16 solved for $N(\chi, t)$ where χ is the estimated thickness of the removed capping block.

Under a recurrent fracture and spalling scenario, adopting the fracture depth geometry model of Swanger et al. (2011) (Fig. 6b) and ignoring sources of erosion other than rock fracture and spalling, we can quantify the cosmogenic nuclide inventory of each freshly exposed layer as

$$N_{layer}(n, t) = N(n, 0) + \frac{P_0}{f} \sum_{j=1}^n \exp \left[\frac{\rho_c D_0}{\Lambda} (2^{-j} - 1) \right] + P_0 \left(t - \frac{n}{f} \right) \quad \text{for } n \leq ft, \quad (21)$$

where $N_{layer}(n, t)$ is the cosmogenic nuclide inventory of the n th layer through time after it has been exposed, $N(n, 0)$ is the inherited nuclide inventory of layer n before the fracture / spalling process begins (assumed zero), f is the average spalling recurrence frequency (a^{-1}), n is the layer number,

and l_0 is the initial vertical dimension of the clast at time zero (cm), and f is the average spalling frequency (a^{-1}). Within our preferred model for fracture depth (Fig. 6a-b), ignoring all other sources of erosion, the expected cosmogenic nuclide inventory at the surface of each freshly exposed layer can be written

$$N_{layer}(n, t) = N(n, 0) + \frac{P_0}{f} \sum_{j=1}^n e^{-\left(\frac{\rho_c \chi j}{\Lambda}\right)} + P_0 \left(t - \frac{n}{f}\right) \quad \text{for } n \leq ft, \quad (22)$$

where χ is the average thickness of spalled fragments (cm). The second term in Equations 21 and 22 specifies the initial N at the surface of layer n directly following a spalling event. This second term is significant to our analysis because it can be used to approximate the minimum allowable N at any time for a clast suspected of undergoing recurrent spalling. We quantify this approximation by interpolating between the minimum N values of bounding fracture events resulting in

$$N_{surf_frac_r} = \frac{P_0}{f} \left[\sum_{j=1}^{n^*} e^{-\left(\frac{\rho_c \chi j}{\Lambda}\right)} + \left(\sum_{j=1}^{n^*+1} e^{-\left(\frac{\rho_c \chi j}{\Lambda}\right)} - \sum_{j=1}^{n^*} e^{-\left(\frac{\rho_c \chi j}{\Lambda}\right)} \right) (t - n^*) \right], \quad (23)$$

where $N_{surf_frac_r}$ is the approximate minimum N possible on the surface of a clast undergoing recurrent fracture and $n^* = (t/f)$. As time and the number of spalling events increase, the second term in Equation 22 asymptotically approaches the constant

$$N_{frac_lim} = \frac{P_0}{f} \sum_{j=1}^{\infty} e^{-\left(\frac{\rho_c \chi j}{\Lambda}\right)} = \frac{P_0}{f \left(e^{\left(\frac{\rho_c \chi}{\Lambda}\right)} - 1 \right)}, \quad (24)$$

where N_{frac_lim} defines a dynamic equilibrium bound on the maximum N within the fresh faces of clasts undergoing recurrent spalling.

To include the additional effect of constant low magnitude steady-state surface erosion, we model the scenario numerically using a discretization of Equation 14 in which the full vertical cosmogenic nuclide depth profile is computed at each time step within the clast and surface material

is removed via both constant erosion and the instantaneous removal of material to a depth χ at time intervals indicated by f . We use this numerical approach to model N_{layer} , N_{frac_lim} , and $N_{surf_frac_r}$ for clasts undergoing recurrent spalling in conjunction with steady surface erosion (Fig. 6c). Even for low estimates of χ and f , the result is a very large potential deficit in N_{surf} for clasts affected by recurrent fracture and spalling. The magnitude of this deficit increases for older clasts and for increased spalling frequency and/or spall thickness.

The number of possible recurrent spalling events is ultimately limited to $l_0/(\chi + \varepsilon_c/f)$. Beyond this number of spalling events, the clast will essentially have deteriorated to a size that is typically too small for sample collection. Therefore, by assuming the clast has undergone recurrent spalling since its emplacement, we can limit the estimated maximum amount of time the clast has spent within the system (T_{frac_max}) to

$$T_{frac_max} = \frac{l_0}{f \left(\chi + \frac{\varepsilon_c}{f} \right)}. \quad (25)$$

We note that the use of an *effective* clast erosion rate, defined as $\varepsilon_{c_eff} = (\varepsilon_c + \chi f)$, within Equation 16 can provide an approximation of $N_{surf_frac_r}$ (i.e. in lieu of Equation 22 for $\varepsilon_c = 0$ or a numerical model for $\varepsilon_c \neq 0$). However, this analytical approach, although simpler, neglects the impact of rapid changes in shielding mass and ultimately overestimates the true value for $N_{surf_frac_r}$ by a substantial margin (Fig. 6c).

2.3.5.4. *Heterogeneous concentration of englacial debris (erosion-sublimation model variant)*

Fig. 11 shows the behavior of the erosion-sublimation forward model executed over a set of dirty ice profiles that contain variable concentrations of englacial debris. Based on the concentration of englacial debris observed in cold-based, debris-covered glaciers in Antarctica (Mackay et al., 2014),

we model ‘background’ englacial debris at ~ 0.01 and introduce narrow bands of higher debris concentrations at various depths in the ice column. Consistent with Equation 3, the thickness of the supraglacial debris and associated rate of ice-loss approach equilibrium values that reflect a dynamic balance between the amount of material being introduced via sublimation of debris-laden ice and the amount of supraglacial debris being removed via surface erosion. These dynamics result in a significant thickening in the supraglacial debris layer that begins when a layer of debris-rich ice (10% debris by volume) intersects the buried ice surface and continues until ice in the englacial debris-rich layer has completely ablated (Fig. 11). For example, for the parameters of the SC1 scenario, ablating ice in a 1-2 m thick englacial debris layer ($c = \sim 0.1-0.2$) located initially $> \sim 50$ m below ice surface, causes the supraglacial debris to rapidly increase $\sim 20-40$ cm above the thickness expected for the conditions of homogeneous englacial debris concentration. Following the complete ablation of ice in the englacial debris layer, the supraglacial debris thickness eventually reduces back down to the H_{eq} that is consistent with a lower englacial debris concentration (Fig. 11). If the debris band is located higher in the initial ice column (Fig. 11d), the material is assimilated earlier into the supraglacial debris layer and produces a rapid increase in the supraglacial debris thickness, but does not generate a local maximum in H in excess of the value of H_{eq} expected for the background englacial debris concentration.

The dynamic fluctuations in supraglacial debris thickness, sublimation rate, and associated accretion rate of new debris to the base of the supraglacial debris caused by spikes in englacial debris concentration directly impact the shape of the cosmogenic nuclide inventory depth profile through time ($N_d(z_d, t)$) (Fig. 11a). As ice in the englacial debris layer ablates under supraglacial debris, the rate at which fresh clasts accrete to the base of the supraglacial debris is elevated. The relatively rapid accumulation of this low N debris to the base of the supraglacial debris coupled with increased

shielding by the rapidly thickening supraglacial debris layer results in a steepening in the local gradient of the cosmogenic profile with depth, $\partial N/\partial z$. Originating at the base of the supraglacial debris layer, this discontinuity in the cosmogenic nuclide depth profile will persist until erosion has removed a thickness of debris equivalent to the maximum depth of the supraglacial debris achieved at the time the N -depleted clasts were initially accreted. At the surface, $N_{surf}(t)$ also reflects this hysteresis in the system; the local maximum in N_{surf} will occur after the spike in supraglacial debris thickness has subsided (and the debris band has completely ablated away) (Fig. 11). If multiple debris bands are present in the ablating ice column, the situation is compounded and a series of several local maximums in the time series of both supraglacial debris thickness and N_{surf} are present. For englacial debris-rich layers located near the surface of the ice column (that have ablated earlier in the system evolution), N_{surf} may never exceed N_{dmax} to produce a local maximum, but instead will increase at an accelerated rate (Fig. 11d). The exact magnitude and timing of these N_{surf} anomalies are controlled by the location, depth, and debris concentration of englacial debris bands within the ice column.

From the above analysis, we infer that relatively minor variations in englacial debris concentration can have a significant impact on the accumulation of cosmogenic nuclides in supraglacial debris (as also suggested by Ng et al. (2005)). Moreover, the presence of englacial debris-rich layers may introduce local maxima into the $H(t)$ and $N_{surf}(t)$ profiles (Fig. 11).

2.3.5.5. *Inheritance from prior exposure*

Clasts that have undergone exposure prior to entering the debris-covered glacier system will contain a cosmogenic nuclide inventory that is greater than any of our modeled values. The theoretical maximum magnitude of this potential inherited nuclide inventory is limited only by the constraints of secular equilibrium. Detection of prior inheritance is difficult using a single nuclide

approach, but its presence may be expressed in measured data as individual outliers that contain a significantly higher-than-expected N when compared to nearby samples. For debris-covered glaciers with measurable ice flow, we can make the first-order prediction that the cosmogenic nuclide inventory of samples should generally increase with increasing distance from the ice accumulation source. If a sample violates this general trend to a significant degree when compared to several additional samples, we may assume that it has been compromised by prior exposure. Identifying the presence of inherited nuclides in older samples becomes increasingly difficult.

2.3.5.6. *Reorientation of clasts across material boundaries*

Any rotation or reorientation of clasts following initial deposition (Section 2.2.2.6) (Fig. 7) will have a significant impact on the distribution of cosmogenic nuclides within the clast. The assumption that the maximum nuclide concentration will be found on exposed, horizontal surfaces may be invalid, and the simple models developed above are inadequate to quantify the expected N_{surf} . Assuming the extreme scenario where all reorientation occurs at the ice-to-supraglacial debris layer transition and that the clast is sufficiently large that the base of the clast, while in the ice, has acquired only a negligible inventory of cosmogenic nuclides, the nuclide inventory acquired while in the ice, $N_i(0,t)$, can be ignored and the eventual N_{surf} may simply be equal to the amount of cosmogenic nuclides acquired while the clast is within the supraglacial debris layer. If all reorientation occurs as the clast reaches the surface of the supraglacial debris layer, then the resulting N_{surf} will reflect a more complicated history of partial shielding. In all reorientation scenarios, the number of unknowns preclude an attempt to quantify the resulting N_{surf} . However, this qualitative analysis does suggest that for clasts affected by positional reorientation (which may be the majority of sampled surface clasts): (1) N_{surf} will always be less than that predicted by the standard *clast*

erosion-only model or *erosion-sublimation model* and (2) N_{surf} will likely vary as a function of micro-location on current surface-parallel faces (e.g., Mackey and Lamb, 2013) (Fig. 7c).

2.3.5.7. *Impact of stochastic processes on cosmogenic-nuclide concentrations: summary*

We have identified several dynamic processes active within debris-covered ice systems that contain stochastic elements (Table 1). These processes will result in a range of measured cosmogenic nuclide inventories for surface samples, and the direction and magnitude of these values will vary significantly (Fig. 12). Of the phenomena considered, prior nuclide inheritance and recurrent fracture and spalling will likely result in the largest range of potential scatter in N_{surf} . However, neither of these phenomena are typically well constrained and each will result in modeled inventories that diverge in opposite directions (Table 1). In a flowing debris-covered glacier, scatter in N_{surf} as a result of prior inheritance will be greatest nearest the accumulation area and decrease as the system ages down-glacier. The magnitude of the expected range in scatter in N_{surf} produced by each of the remaining identified processes will increase with increasing distance from the accumulation area and/or the age of the system.

As an additional complication, any individual clast may be subjected to any combination of these processes throughout the course of its exposure history. Through this compounding mechanism, it is possible for a clast to exhibit a surface-nuclide inventory that exceeds the range predicted by the above semi-quantitative analysis. As an example, consider an initially englacial clast that penetrated partially into the base of supraglacial debris, was rapidly exhumed near a polygon margin, and then underwent a series of fracture and spalling events. The measured N_{surf} for this clast would fall short of the minimum possible N_{surf} calculated based on any of these processes individually.

2.4. Presentation of the modeling framework to estimate the age of debris-covered ice

For debris-covered ice of a *known* age, the suite of models presented above predicts the range in cosmogenic nuclide scatter that should be expected amongst adjacent surface clasts. To address the opposite problem of an *unknown* ice age using measured cosmogenic nuclide abundances, we simply invert this modeling approach and extract the model time at which each of the potential exposure pathways results in a modeled N_{surf} that agrees with measured values (Fig. 12). This process results in an ensemble of modeled ages – one for each of the possible exposure pathways – for ice underneath a measured clast. Together, this group of modeled ages comprises the estimated range for the true age of the underlying ice (Fig. 12). We note that this approach is similar to the hybrid statistical / modelling efforts of Applegate et al. (2012) and Applegate et al. (2010), but purposely deviates from purely statistical analyses (e.g., Balco and Schaefer, 2006; Barrows et al., 2002).

Although we do not attempt to determine, for any individual clast, which of the possible exposure pathways it underwent before being sampled, we nevertheless assign a qualitative *confidence level* to each of the potential exposure pathways and resultant age predictions. In general, we assume that the more complex the proposed exposure pathway, the less likely it is that it actually occurred for a particular clast. Under this scheme, and ignoring the potential issue of prior exposure, results from the clast erosion-only model and the erosion-sublimation model are given the highest confidence and the recurrent fracture and multiple disturbance models the lowest confidence (Fig. 12). We note that, although outside of the scope of this current work, future efforts may directly utilize predictions of expected N_{surf} from each exposure model in conjunction with a large sample size to more directly determine the likelihood of each model's occurrence for a specific setting.

3. APPLICATION TO MULLINS GLACIER

We now apply our modeling framework to the Mullins Valley debris-covered glacier (hereafter, Mullins Glacier). Mullins Glacier represents an end-member class of cold-based, debris-covered glaciers in which supra- and englacial debris is derived solely from rockfall off the valley headwall (Kowalewski et al., 2011). Further, Mullins Glacier is an ideal test case for our model framework because ice ablation is relatively straightforward, dominated by sublimation, and ice-flow velocities/dynamics are reasonably well constrained (Rignot et al., 2002; Mackay et al., 2014). We pursue two related applications of our model framework. The first application takes advantage of existing, (non-cosmogenic) chronological control for Mullins Glacier and applies our model framework to compute a range of potential ages for hypothetical, “tracer” clasts located at various positions along the glacier. The goal of this approach is to determine the likely *range of exposure ages* that could be possible for clasts at any point (“known age”) along the glacier; the wide results for each clast are presented as *model-framework age ranges*. The second approach is to compare these model-framework age ranges with actual data from ^3He cosmogenic nuclide analyses on 15 clasts collected on the surface of Mullins Glacier (Marchant et al., 2007). Ultimately, comparison of our model-framework age range with measured ^3He data will help refine ice ages, and – to a first order – help determine past transport pathways and stochastic processes that have likely impacted the measured clasts.

3.1. Physical Setting for Mullins Glacier

Mullins Glacier (77.89°S, 160.58°W, 1557msl) originates from snow and ice accumulation directly beneath bedrock cliffs at the headwall of Mullins Valley, ~2100 m elevation. The glacier flows for ~ 3.5 km between Vestal and Rector ridges, and then bends eastward into upper Beacon Valley (Fig. 13 and Supplementary Fig. S5), where it travels an additional ~ 5 km. Peak horizontal ice-surface velocities occur near the glacier headwall, $\sim 40\text{mm a}^{-1}$, but thereafter decelerate to near 0

mm a⁻¹ (within measurement error of $\sim \pm 2$ mm a⁻¹) at ~ 4.8 km (Rignot et al., 2002). The ice thins from a maximum of ~ 125 m near its headwall toward a local minimum of ~ 22 m at 3.4 km; thereafter, ice thickness increases in central Beacon Valley to $> \sim 50$ m (Mackay et al., 2014; Shean et al., 2007; Shean and Marchant, 2010).

With an accumulation area ratio of only ~ 0.14 (Marchant et al., 2013) the majority of Mullins Glacier lies within a slow moving-to-stagnant ablation zone (Regions 2, 3, and 4 in Fig. 13). As much as 93% of this ablation zone is mantled with a continuous sheet of supraglacial debris that increases in thickness from ~ 5 cm at its onset to a maximum of ~ 75 cm several km down valley (Supplementary Fig. S6). Ice loss occurs through down-wasting only, and ablation is accommodated almost entirely by sublimation. The modern ablation rate is extremely low and ranges from ~ 5 cm a⁻¹ for exposed glacier ice in the uppermost portion of the ablation area to well below 6.6×10^{-7} cm a⁻¹ under ~ 50 cm of supraglacial debris > 5 km down-valley (Supplementary Fig. S4) (Kowalewski et al., 2011; Mackay et al., 2014). The sole source for debris on and within Mullins Glacier is rockfall from cliffs at the valley headwall (Kowalewski et al., 2011; Mackay et al., 2014). Most rockfall debris ($\sim 70\%$) comes to rest in the ice accumulation area (Mackay et al., 2014), where it undergoes burial by subsequent snow and ice. Overall, the concentration of englacial debris, c , changes down glacier, increasing in step fashion across Region boundaries (Fig. 13). c ranges from as low as < 0.01 in Region 1 to as much as 0.25 in Region 4.

A network of sublimation-type polygons (Marchant et al., 2002) dominates the meter-scale topography of the surface debris (Levy et al., 2006) (Fig. 3e). At finer scales, clast erosion and rock-surface topography is dominated by wind abrasion, salt weathering, and thermal fracture (Marchant and Head, 2007; Marchant et al., 2013; Swanger et al., 2011).

3.2. Existing Chronological Control for Mullins Glacier

The age of Mullins Glacier is not yet defined precisely, but preliminary age control comes from (1) integrating the rate of modern horizontal ice flow back in time (Mackay et al., 2014; Rignot et al., 2002), (2) geochemical analysis of trapped air within stagnant portions of Mullins Glacier (Yau et al., 2015), and (3) ^3He cosmogenic-nuclide analyses of 15 surface clasts along Mullins Glacier (Marchant et al., 2007); in addition to improving chronological control, the latter permit the testing and application of our overall modeling framework toward understanding the evolution of debris-covered glaciers.

3.2.1. Integrating modern horizontal ice velocity of Mullins Glacier

Chronological control for areas of active ice flow within Mullins Glacier can be inferred from integrating measured horizontal ice-surface flow velocities (InSAR) back in time (Rignot et al., 2002) (Fig. 14a). This can be done by (1) dividing the measured horizontal distance by the *mean* velocity over the region of interest or (2) integrating the measured surface ice-flow velocities at specific points along the glacier (smoothed with a moving average filter of 50 m). Both approaches are vulnerable to the assumption that modern flow velocity is a reasonable indicator of paleo-flow velocities (i.e. that accumulation rate and environmental conditions have remained roughly constant) and that ice flow has been continuous over the time period of interest. Results from this exercise suggest that Mullins Glacier ice is ~200-300 ka near the distal portion of conspicuous active flow (down to the end of Region 2 at ~4.0 km down glacier; Fig. 13 and Fig. 15).

3.2.2. Preliminary chronology from trapped gas in Mullins Glacier

Preliminary chronological control for slow-moving-to-stagnant regions of Mullins Glacier come from geochemical analyses of trapped gasses recovered in shallow ice cores. Yau et al. (2015)

measured the $^{40}\text{Ar}/^{36}\text{Ar}$ and $^{38}\text{Ar}/^{36}\text{Ar}$ ratios of gas in bubbles trapped within ice cores recovered ~4.9 km from the valley headwall (Region 3, Fig. 15) and used the approach of Bender et al. (2008) to determine the estimated age. In the first attempt at using this method, measured gasses in shallow ice cores (< 35 m deep) reveal significant modification by gas loss, thermal and gravitational fractionation, and contamination via the introduction of young air through thermal-contraction cracks. Even so, the measured gas ages of up to 1.6 Ma (Yau et al., 2015) provide robust estimates for *minimum ice ages* at this location (Fig. 15).

3.3. Establishing model-framework age ranges for surface clasts along Mullins Glacier.

In this section, we use the existing chronological control for Mullins Glacier to assign proxy ages for hypothetical surface clasts (tracer clasts) at any point along glacier surface. One desired outcome is to assess better the potential transport pathways and stochastic processes that might have impacted measured clasts in cosmogenic-nuclide datasets (see also section 3.4 below). As a first step, we set our model times equal to the estimated ice age at a given location on Mullins Glacier (Section 3.2) and compute N_{surf} for each exposure pathway model (erosion-only and sublimation-erosion models, along with model variants). The resulting spread in modeled values (model-framework age range) equates to the expected variation in cosmogenic ^3He that could be measured in clasts at each location; ages within the *high confidence* interval are based on results assuming either the clast erosion-only model or the erosion-sublimation model including clast intrusion (Fig 12).

Tables 2 and 3 list all parameters used to model the accumulation of cosmogenic nuclides in Mullins Glacier. The model input values for c , ρ_c , and P_0 are specific to each sample; all other parameters are uniform across all model runs. Values for c are based on observed concentrations of englacial debris at the ice surface and in shallow ice cores (Mackay et al., 2014) (to account for possible variability in englacial debris concentrations, we also include model results that utilize a

small range of c deviating from the mapped averages). For the region of Mullins Glacier first under study (Region 2), the effect of down-valley flow and ice-surface lowering are negligible ($\ll 1\%$ offset in age (however, see also the treatment of this parameter in the Supplementary Information).

Fig. 14b displays the model-framework range in cosmogenic ^3He inventories for surface clasts as a function of distance down-valley. Although limited to the uppermost ~ 4.5 km of Mullins Glacier, and predicated on the approximate age model for Mullins Glacier provided by integrating modern flow velocities back in time, these results show that the potential range in N_{surf} above debris-covered ice at any point along Mullins glacier increases rapidly down glacier.

3.4. Comparison of our model-framework age ranges with ^3He cosmogenic-nuclide data

In a prior study, Marchant et al. (2007) measured the concentration of cosmogenic ^3He within fifteen clasts of Ferrar Dolerite collected along the surface of Mullins Glacier (Fig. 13). In Region 2, the measured samples were collected on or near major arcuate surface ridges (Mackay et al., 2014) and at the center of sublimation polygons (to avoid complications with slumping at modern polygon boundaries). The ~ 8 km longitudinal transect included a cluster of three closely spaced (~ 90 m separation) samples at ~ 4.0 km, near the termination of Region 2 (Fig. 13), as well as a small set of three samples collected along a transverse line at ~ 5.2 km (~ 500 m down from the ice cores measured for $^{40}\text{Ar}/^{36}\text{Ar}$ and $^{38}\text{Ar}/^{36}\text{Ar}$, Yau et al., 2015). Measured values of cosmogenic ^3He , reported in Table 2, range from 8×10^6 to 1540×10^6 atom g^{-1} . Apart from some notable exceptions, the measured ^3He concentrations generally increase down glacier.

To facilitate comparisons among our model-framework age ranges with direct measurements of cosmogenic ^3He in surface clasts, we extract the model time at which the calculated N_{surf} matches the measured ^3He abundance for a given sample. The resulting spread in model ages (derived from the range of transport pathways and stochastic processes) represents the estimated quantifiable age range

805 for ice at a given location. Two key components of our model include (1) the selection of appropriate
806 rates for surface erosion and (2) the assessment of confounding down-valley ice flow and surface
807 lowering. We derive estimates for average erosion rates by first assuming that our sample with the
808 highest measured nuclide inventory (DXP-06-05) (Table 2) has reached secular equilibrium. If this
809 sample has remained on the surface since being emplaced in the ablation area (e.g., assuming the
810 clast erosion-only model), then the maximum possible clasts erosion rate, ε_c , can be computed by
811 assuming an infinite exposure period in Equation 16 (e.g., Dunai, 2010; Lal, 1991). This yields a
812 maximum ε_c of $\sim 16 \text{ cm Ma}^{-1}$. Considering that DXP-06-05 may not have yet reached equilibrium, we
813 choose the low range of $\varepsilon_c = 5 - 15 \text{ cm Ma}^{-1}$ as our preferred range for calculations of age. We take a
814 similar approach to estimating the erosion rate for the bulk supraglacial debris layer, ε_d . In this case,
815 assuming that DXP-06-05 is at secular equilibrium but has reached the surface after transiting
816 sequentially from buried ice via sublimation and then through the supraglacial debris layer via
817 erosion (e.g., assuming the erosion-sublimation model), the maximum allowable ε_d , as computed
818 from Equation 20 is $\sim 24 \text{ cm Ma}^{-1}$. This value is within the range of published regolith erosion rates
819 for nearby Arena Valley ($\sim 19 \text{ cm Ma}^{-1}$) (Morgan et al., 2010b), but lower than the regolith erosion
820 rates computed for lower Beacon Valley ($\sim 45\text{-}120 \text{ cm Ma}^{-1}$) (Morgan et al., 2010a). Considering that
821 the debris thickness (H in Equation 20) at the location of DXP-06-05 (0.35 m) is anomalously low
822 for the region (e.g., Region 4) (Supplementary Fig. S6), and that the erosion rates of Morgan et al.
823 (2010a) were calculated for a raised and convex moraine surface where erosion rates are likely
824 higher than average for the valley floor, we use the low range of $\varepsilon_d = 15\text{-}25 \text{ cm Ma}^{-1}$ in our
825 calculations.

To incorporate the potential effects of down-valley ice flow and ice-surface lowering over time, we include a small adjustment factor to the P_0 value computed for each sample location (see Supplementary Information, Section 5).

Model-framework ages using our ensemble approach are displayed in Fig. 15 and Table 4. A small subset of the potential exposure pathways evaluated over a broad range of c and erosion rates are additionally displayed in Supplementary Fig. S7, Supplementary Fig. S8, and Supplementary Table 1.

3.5. Applying our model framework age ranges to refine the age of Mullins Glacier

An analysis of ages derived from our model-framework for clasts on Mullins glacier (Table 4) reveal several inconsistencies that can be exploited to help refine the exposure history of surface clasts. First, if we assume that all processes impacting the inventories of cosmogenic ^3He in surface clasts on Mullins Glacier have been captured by our ensemble modeling approach (transport pathways and stochastic processes), then the minimum age measured for a given sample should not exceed the maximum, model-framework age range for samples located further down glacier. If a sample fails this quantitative test, then we can assume that the clast has been impacted by physical processes beyond our limited set of exposure models. This might suggest one or more of the following confounding factors: inherited cosmogenic nuclides from prior exposure, complex and heterogeneous concentrations of englacial debris, and/or the impact of multiple and varied stochastic disturbances. Examination of Fig. 15 shows that the samples that fall into this category on Mullins Glacier include DXP-99-2 and DXP-99-4.

Likewise, samples with model-framework age ranges that fall below the age range of samples in both the up glacier and down glacier directions are likely to have been impacted by disturbances not resolvable in our current model. Samples in this category include DXP-99-31 and DXP-99-07 (Fig.

15). We note that in the relatively simple case of one or two clasts lying outside a well-defined, linear age progression (as for DXP-99-2 and DXP-99-4) the identification of outliers is relatively straightforward; however, for complex datasets in which ages are seemingly chaotic, or lack a well-defined age progression, this technique can be used to identify outliers in a robust and quantifiable manner.

Finally, in the case where multiple samples from a single location show overlapping age ranges (e.g., DXP-99-22, DXP-99-23, and DXP-99-24), the simplest approach in reducing age uncertainty is to restrict the plausible age range to include only those overlapping ages that lie within the *high-confidence* age intervals (e.g., pink horizontal bar in Fig. 15).

With the above assumptions and caveats in mind, our best age assessment for Mullins Glacier is as follows: at the onset of continuous debris cover (~1.7 km from the headwall), Mullins Glacier is most likely ~12-14 ka (assuming these clasts have not been impacted by single and/or recurrent spalling). Thereafter, Mullins Glacier increases in age. At ~4.0 km down valley near the limit of Region 2, a collection of three, closely-spaced samples < 90 m apart (DXP-99-22, DXP-99-23, DXP-99-24) show a tight cluster of overlapping ages. Restricting the age overlap to high-confidence intervals yields an ice age of ~210-225 ka. Samples located further up glacier at 2.6 km and 3.4 km, DXP-99-5 and DXP-99-7, show model-framework age ranges that fall within expected values assuming ice at ~4.0 km is 210-225 ka (Fig. 15 and Table 4).

The situation for clasts collected between ~4.5 km and ~8.3 km on Mullins Glacier (e.g., in areas of slow-moving-to-stagnant ice) is not nearly as straightforward. First, due to the strong possibility of recurrent spalling in these clasts, an upper age bound for most of the clasts cannot be defined (Fig. 15). Second, samples DXP-99-31 and DXP-06-07 are identified as outliers, because they are too young for their positions on the glacier surface and they show no overlap in model-framework ages

with samples on either up glacier or down glacier locations. The cosmogenic data are, however, consistent with preliminary gas analyses (Yau et al., 2015), which show that beyond ~ 5 km the ice in Mullins Glacier is older than 1.6 Ma. Although the cosmogenic dataset presented in Fig. 15 does not permit further improvement in age assessment, it does clearly indicate that the examined clasts have been modified by stochastic processes, most likely (at a minimum) some combination of recurrent spalling and polygon exhumation.

4. SUMMARY AND CONCLUSIONS

We developed a modeling framework to describe the accumulation of terrestrial cosmogenic ^3He in Antarctic debris-covered glaciers. The framework helps quantify the expected range in cosmogenic-nuclide inventories for measured clasts at the surface of supraglacial debris. The framework includes analytical approximations appropriate for old systems approaching secular equilibrium and numerical models applicable to young systems and complex clast transport pathways. The framework includes dynamic feedbacks among ice ablation and the vertical movement of clastic material in ice, the development of supraglacial debris, and several stochastic processes that impact clast position at and near the ground surface.

Results indicate that the expected range of cosmogenic-nuclide inventories within clasts above debris-covered ice can be broadly quantified. The wide range in cosmogenic inventories arises from stochastic processes that impact clast movement within and on debris-covered glaciers, including: (1) uncertainty in the initial location of rockfall deposition onto glacier ice (accumulation area vs ablation area) (Mackay et al., 2014); (2) episodic slumping and exhumation of supraglacial debris at the margin of contraction-crack polygons; (3) episodic rock fracture via recurrent spalling; (4) variable concentrations of englacial debris; (5) rotation, disturbance, and differential penetration of

clasts at transitions across the ice-to-debris and debris-to-ground surface interfaces, and (6) prior inheritance of cosmogenic nuclides.

We applied this modeling framework to assess an existing β -chronology for the Mullins debris-covered glacier, southern Victoria Land, Antarctica. Assuming the whole-clast erosion rate = 10 cm Ma^{-1} and the debris-layer erosion rate = 20 cm Ma^{-1} , the age of Mullins Glacier increases non-linearly from ~12 ka near the onset of continuous debris cover to > 1.6 Ma about half-way down the glacier. An upper bound cannot be prescribed due to the potential for some sampled clasts to have reached secular equilibrium assuming certain exposure pathways (Marchant et al., 2002).

In this study, we focused on evaluating cosmogenic datasets for individual clasts at the surface of debris-covered glaciers. In the future, work should incorporate sampling strategies that include multiple samples at specific locations, including the collection of samples from depth profiles (e.g., Marchant et al., 2002; Morgan et al., 2010a; Ng et al., 2005; Schaefer et al., 2000), and /or samples of varying size (sand, gravel, cobble, boulder) in order to (1) more precisely date underlying glacier ice, (2) better constrain erosion and sublimation dynamics, and (3) refine the dominant surface processes operating at a given location on the glacier surface.

Acknowledgements

Funding for this research was provided by NSF Graduate Research Fellowship awarded to S.L. Mackay and NSF Polar Programs grants NSF ANT-0944702, NSF ANT-0739700, and NSF ANT-1043706 awarded to D.R. Marchant. Logistical support for this project in Antarctica was provided by the U.S. National Science Foundation through the U.S. Antarctic Program. Measurements of cosmogenic nuclides reported in this study were completed by Joerg Schaefer at the cosmogenic-nuclide lab at the Lamont Doherty Earth Observatory and by William Phillips and Gary Landis at the USGS noble gas laboratory, Denver Federal Center.

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Figure 1 [1.5 column]: Basic glacial dynamics incorporated in our model framework. (a) Cross section of idealized cold-based, debris covered glacier with exposed bedrock cliffs at the valley headwall. Ice ages increase nonlinearly with distance from the headwall (source area). (b) Simplified form of (a) showing additional approximations used in our model. We assume that (1) all clasts originate from rockfall at the valley headwall; (2) clasts that initially land in the ice accumulation zone are buried by snow/ice and travel englacially (pathways simplified as white lines); (3) englacial clasts eventually reach the surface as overlying ice ablates; (4) ice age is uniform with depth. (c) Model of an ablating dirty ice column through time (location shown as a gray vertical bar in (b)). Time steps increase from $t=1$ to $t=4$. Colored dashed lines track the location of individual clasts (labeled A-F) as they move toward the ice surface and add to the base of a thickening layer of supraglacial debris. The debris not only thickens from input at its base, but thins due to erosion at its surface; we define a bulk debris-erosion rate ε_d and an erosion rate for individual clasts, ε_c ; supraglacial clast “A” at time step $t=1$ is reduced in size at $t=4$ (see text). (d) Sketch of particle trajectory (gray lines) for englacial debris/clasts over time, first through the ice and then through the supraglacial debris layer (shaded portion); the surface of the debris layer is the frame of reference. Clasts approach the ice surface, with an *apparent emergence velocity* ($v_i(t)$) and then approach the surface of the debris layer with an apparent emergence velocity of $v_d(t)$. Initially, ice loss and ($v_i(t)$) is rapid under a relatively thin debris layer. As the debris layer thickens a negative feedback reduces ice ablation and the rate at which supraglacial debris thickens slows. Eventually, the system reaches a dynamic equilibrium in which the thickening of the debris layer via ablation of dirty ice is exactly equal to the loss of material via surface erosion, e.g. when $H = H_{eq}$. The apparent emergence velocity of clasts within the debris layer, $v_d(t)$, is a function of surface erosion. (e) The same as (d) except the frame of reference is now the surface of the ice. Both reference frames are used throughout the text. See text for explanation of additional notation. We emphasize that some confusion exists when considering the age of supraglacial debris. The age could be defined as (1) the time that has elapsed since the debris has been on the present surface or (2) the time since debris now at the surface initially entered the ice at the accumulation zone. In both cases, however, the values do not directly equate to the age of the underlying ice; this is due to differing emplacement locations of the debris particles and divergent travel paths to the surface.

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956 **Figure 2** [*single column*]: Modelled ice loss (black line) and supraglacial debris thickness (gray line)
 957 as a function of time using porosity of the supraglacial debris, $\phi = 0.3$; debris erosion rate, $\varepsilon_d = 40$
 958 cm Ma^{-1} ; concentration of englacial debris, $c = 0.008$; and with the initial thickness of the debris
 959 layer at time=0, $H_r = 0.05$ m. The debris layer thickens rapidly in the transient period as rapid ice
 960 sublimation delivers a high volume of debris to the buried ice surface, and then asymptotically
 961 approaches the dynamic steady-state thickness (H_{eq}) at time $\sim T_{eq}$. Ice loss is fastest when the
 962 supraglacial debris layer is thin and then slows until it reaches a nearly constant depletion rate as the
 963 debris layer thickness reaches $\sim H_{eq}$.

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Figure 3 [*single column*]: (a) Excavation through the supraglacial debris layer in Region 3 of Mullins Glacier exposing the buried ice surface, an emerging englacial clasts transitioning into the supraglacial debris layer, and surface clasts modified by wind erosion (wind-polished facets). (b-d) Examples of fracture and spalling observed in surface clasts located in Regions 2, 3, and 4 respectively. (e) Off nadir aerial photograph of the surface topography in Region 3. The high-centered polygonal pattern is caused by preferential sublimation along polygon margins (see text). Material freshly exposed at polygon margins is visible as areas of less surface staining and grayer in color.

Figure 4 [1.5 column]: Impact of clast sizes on clast emergence dynamics and exposure history. (a) Exaggerated time-sequence sketch of size-dependent clast movement near the ice-debris boundary. Reference frame is the valley bed. The top surface of each clast enters the debris layer at the same time. By t=3, the top surfaces have separated a distance of D_{eff} . (b) Sketch of expected cosmogenic nuclide inventories (along depth profiles) for the surface of an idealized (sand-sized) clast and a larger clast at an arbitrary system age. The gray hatched region indicates the range in inventory that should be expected due to in the differing clast sizes. (c) Modeled N_{surf} as a function of time for various values of effective emergence dimension (D_{eff}). (d) The modelled ratio $(N_{surf_ES} - N_{surf_ES}(D_{eff})) / N_{surf_ES}$ as a function of time. According to this model, larger clasts in the supraglacial debris will contain lower nuclide inventories than co-located smaller clasts.

Figure 5 [1.5 column: Impact of rapid exhumation at polygon margins on clast exposure history. (a) middle panel: cross section of idealized sublimation polygon, showing the location of two clasts, A and B, at the polygon center and trough wall, respectively. Both clasts initially developed under the same set of dynamics and have gained the same cosmogenic nuclide inventory (accepting minor local shielding by polygon trough in the case of B). Clast B has recently emerged at the surface due to slumping of overburden at the polygon margin, which removed z_e thickness of overlying debris. Top panel in (a): the cosmogenic nuclide depth profiles for both A (left) and B (right). Lower panel: spatial context for the middle panel illustrating the potential impact of ‘relict’ polygon troughs (Marchant et al., 2002). (b) Modeled debris thickness and excavation depth as a function of time for the SC1 scenario with $\varepsilon_d=30$ cm Ma^{-1} . Excavation depth is determined from Equation 12 using $m = 0.6$ cm, $k = 9$ cm, and $H_{min} = 15$ cm. (c) Modelled N_{surf_ES} and N_{surf_poly} for SC1 scenario and the excavation depths in (b).

Figure 6 [1.5 column: Model of recurrent spalling and its impact on cosmogenic-nuclide inventories in surface clasts. (a) The geometry of our simplified fracture model. Each panel shows the idealized configuration following a spalling event and the surface of each new exposed layer; the surface showing the highest expected nuclide inventory is labeled 0. (b) Three models of recurrent spalling. Top: recurrent spalling at a constant depth and constant recurrence frequency and where recurrent spalling is the only source of erosion. Middle: erosion occurs via recurrent, episodic spalling and constant surface erosion. Lower: model of (Swanger et al., 2011), where occurs via spalling only and the depth of each successive spall is equal to one half the current thickness of the clast. The black dashed line in the top two panels represents the “effective erosion rate.” (c) Modelled cosmogenic nuclide inventories for various spalled surfaces over time. Labeled gray solid lines are modelled N_{surf} for each successively exposed surface; increasing numbers correspond to the sequence represented in (a). The shaded areas in top panels represent N of ‘surfaces’ within the host rock before spalling; the rapid, vertical drops show the reduction in N after each spalling event. The dashed orange line represents the minimum N to expect through time for a clast undergoing recurrent spalling. The blue line is the inventory at a freshly exposed surface if only a single spalling event has occurred prior to collecting; the dashed red line shows the maximum N_{surf} that will be achieved by a clast that is undergoing recurrent spalling. The insets (top panels only) show the isolated single fracture N_{surf} (blue line), the recurrent fracture N_{surf} (dashed orange line), as well as an approximate N_{surf} calculated using a simple effective erosion rate method (see text) (black line). Model details: upper left panel: recurrent spalling-only model for the SC1 scenario with $\varepsilon_d = 0 \text{ cm Ma}^{-1}$, $\rho_c = 3 \text{ g cm}^{-3}$, $\chi = 15 \text{ cm}$, and $f = 2.0 \times 10^{-6} \text{ Ma}^{-1}$; upper right panel: recurrent spalling plus steady erosion model for the SC1 scenario with $\varepsilon_d = 30 \text{ cm Ma}^{-1}$, $\rho_c = 3 \text{ g cm}^{-3}$, $\chi = 15 \text{ cm}$, and $f = 2.0 \times 10^{-6} \text{ Ma}^{-1}$; lower left panel: recurrent spalling-only model for the SC1 scenario with $\varepsilon_d = 0 \text{ cm Ma}^{-1}$, $\rho_c = 3 \text{ g cm}^{-3}$, $\chi = 15 \text{ cm}$, and $f = 5.0 \times 10^{-6} \text{ Ma}^{-1}$; lower right panel: recurrent spalling plus steady erosion model for the SC1 scenario with $\varepsilon_d = 30 \text{ cm Ma}^{-1}$, $\rho_c = 3 \text{ g cm}^{-3}$, $\chi = 15 \text{ cm}$, and $f = 5.0 \times 10^{-6} \text{ Ma}^{-1}$.

Figure 7 [two column]: Summary of potential clast transport pathways and complications. All diagrams use the debris surface as the frame of reference. (a) Scenarios used to calculate model-framework age ranges for hypothetical clasts following defined pathways. (b) Reference exposure models. (c) Exposure complications discussed in text but not explicitly modeled

Figure 8 [1.5 column]: Erosion sublimation (ES) and erosion-only (EO) exposure model output for the SC1 scenario. (a, b) cosmogenic ^3He inventories (N) as a function of depth (including clasts in supraglacial debris and those in ice) for the ES model (solid lines) and EO model (dashed lines) (panel a and b are similar but show different time periods). The break in slope in ES profiles, detailed in the inset of (a), marks the change in N that occurs at ice-debris interface, the depth of which changes with time (see Fig. 1d); this break does not exist in the EO profile which ignores the dynamics of ice sublimation. (c) Detail of the englacial cosmogenic ^3He inventory depth profile with the ice surface as the reference frame. (d) The difference between the englacial cosmogenic ^3He inventory as a function of depth for the EO and ES models through time. (e, f) The gradient of the cosmogenic ^3He inventories for the ES (solid lines) and the EO (dashed lines) models covering different time periods. (e, f) The curvature of the cosmogenic ^3He inventories for the ES (solid lines) and the EO (dashed lines) models covering different time periods.

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1054 **Figure 9** [one column]: Modelled cosmogenic ^3He inventories at the supraglacial debris surface
 1055 (black lines, solid and dashed) and ice surface underneath the supraglacial debris (gray line) as a
 1056 function of model and time for the SC1 scenario. The modelled cosmogenic nuclide inventories of
 1057 surface clasts in the erosion-sublimation scenario (N_{surf_ES}) are always lower than the cosmogenic
 1058 nuclide inventories of surface clasts in the erosion-only scenario (N_{surf_DEO}). The modelled maximum
 1059 inventory values, when the system reaches secular equilibrium at the ice surface (N_{imax} ;
 1060 Supplementary Equation S7) and debris surface (N_{dmax} ; Supplementary Equation S10), are marked
 1061 with horizontal hatched lines. The vertical hatched lines show that the cosmogenic nuclide inventory
 1062 of clasts at the ice surface ($N_i(0,t)$) reaches a maximum at an earlier time (T_{i_crit}) than clasts at the
 1063 supraglacial debris surface (T_{s_crit} ; Supplementary Equation S1). See text for additional explanation
 1064 of notation.
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Figure 10 [1.5 column]: Single parameter sensitivity tests based on the SC1 scenario. (a) Impacts of various englacial debris concentrations on N_{surf} (left panel); supraglacial debris layer thickness (middle panel); and ice loss (right panel) over time. (b) Impacts of changes in erosion rate on N_{surf} (left panel); supraglacial debris layer thickness (middle panel); and ice loss (right panel) over time. (c) Impacts of sublimation rate parameterizations on N_{surf} (left panel); supraglacial debris layer thickness (middle panel); and ice loss (right panel) as a function of time. See Supplementary Fig. S4 for details of sublimation rate parameterization cases.

Figure 11 [1.5 column]: Changes in supraglacial debris thicknesses, ice loss rates, and cosmogenic nuclide inventories as predicted by the sublimation-erosion (SE) model executed for several concentrations of englacial debris, c , including concentrated bands of englacial debris. All model runs are for the SC1 scenario, with only the effects of a modified englacial debris concentration profile plotted. (a) englacial debris content is 1% everywhere except for a 2 m thick layer of 10% debris content at 50 m depth: upper left panel, englacial debris profile; upper middle panel, ice loss and supraglacial debris thickness; lower middle panel, cosmogenic nuclide inventory at the surface for the test case, as well as the EO and homogeneous englacial debris content ES reference models; right panel, cosmogenic nuclide depth profiles through time. (b-d) Left panels show concentration of englacial debris as a function of depth for several different englacial debris concentration profiles; middle panels plot ice losses and supraglacial debris thicknesses over time; right panels plot cosmogenic nuclide inventory for clast at the ground surface.

Figure 12 [one column]: Summary schematic to show predicted N_{surf} as a function of time according to several of the exposure models discussed in Section 2.2.3. Results shown are exaggerated to show general trends. On the right side of the plot, the full range of potential N_{surf} produced as a result of various stochastic physical processes is shown for an arbitrary time t^* . Shading indicates the qualitative relative probability of the indicated process occurring; we choose an approach described by Occam's Razor in that the most complex exposure histories are those ranking in lowest confidence. In general, the spread in predicted nuclide inventories (N) increases with model time, which for most glaciers is comparable to distance down valley. Shading along the x-axis illustrates the relative confidence level in the predicted ice age at an arbitrary measured nuclide inventory, N_{surf}^* , as a function of exposure history; ages derived from simple exposure histories are assigned high confidence values. Shading here corresponds to similar shading in Fig. 15.

Figure 13 [1.5 column]: Mullins Valley (which is occupied by Mullins Glacier) and upper Beacon Valley. Left image, hill-shaded relief map from airborne LiDAR (2m resolution (Schenk et al., 2004)); draped on top of multispectral satellite imagery (GeoEye01, 1.5 m resolution, January 2004) in headwall region (lower right corner). Locations of samples collected for cosmogenic nuclide analysis are shown as white dots and numbered at far left. Colored regions depict divide Mullins Glacier into distinct regions based on surface morphology, ice-surface velocity, and englacial debris concentration (Mackay et al., 2014). Middle panel plots elevation of the central flow-line of Mullins Glacier, with elevations of cosmogenic-nuclide samples indicated. Right panel shows field photographs of selected cosmogenic-nuclide samples.

Figure 14 [one column]: Inferred age of Mullins glacier based on integrating modern horizontal ice-flow velocities back in time. (a) Gray data points show the measured centerline ice-flow velocities derived from InSAR (Rignot et al., 2002); gray line shows data with a 100 m running mean smoothing function. Black lines show continuous age estimates derived from mean InSAR velocity (dashed line) and from integrating the InSAR velocity at each measured point over time (solid line). The velocity integration model was tuned to match the model age of DXP-99-1 (~13 ka) at ~1500 m. (b) Modelled N_{surf} (colored circles) as a function of distance down-valley for upper Mullins Glacier (see text, Section 3.3). Values assume an ice age according to the velocity integration model (panel (a)). All model inputs follow values in Table 2 and Table 3. P_0 is computed according to the current location of the modelled sample (no adjustment is made for elevation loss over the exposure history).

Figure 15 [1.5 column]: Ice chronology for Mullins Glacier inferred from cosmogenic ^3He in surface clasts. Upper panel: Full estimated age range for each sample location based on the sample's measured N , density, elevation, and the local englacial debris concentration evaluated over a suite of potential exposure models and a selection of probable values for ε_d , and ε_c . EO models and model variants are evaluated using erosion rates of 10 cm Ma^{-1} (central symbol), 5 Ma^{-1} (lower error bar) and 15 Ma^{-1} (upper error bar). ES models and model variants are evaluated using erosion rates of 20 cm Ma^{-1} (central symbol), 15 Ma^{-1} (lower error bar) and 25 Ma^{-1} (upper error bar). Gray shading in each age range indicates regions of high (dark gray), medium (medium gray), and low confidence (light gray) (see Figure 12 and Section 2.4 for details). Arrows at the upper extent of some age ranges indicate that no upper age bound can be determined as the sample has reached secular equilibrium for at least one of the process models considered. Also shown are gas ages (black stars) determined from $^{40}\text{Ar}/^{36}\text{Ar}$ analysis of trapped gas in Mullins Glacier (Yau et al., 2015). Lower panel: Potential age for Mullins Glacier inferred from our model-framework ages as a function of distance down valley. Gray shading indicates relative confidence in the age estimate as per the shading scheme of the upper panel. Sample outliers, defined as such in Section 3.5 are grayed out. Light blue shaded area indicates our estimated age range for Mullins glacier. Also shown are the continuous age estimates derived from integrating the mean (blue line) and point (red line) InSAR surface flow velocities (e.g. Figure 14a).

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TABLES

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1153 **Table 1: Stochastic Processes and Error Inducing Dynamics**

Process	Details	Probability ^a	impact on age estimate ^b	Scatter / Error increased for	Modelling approach	Poorly constrained parameters ^c
<u>Stochastic / Scatter inducing:</u>						
Unknown clast emplacement location	Rockfall will deposit clasts in both (1) the accumulation area where they spend part of their exposure history englacially and (2) the upper ablation area where they remain supraglacial for their entire exposure history	high	underestimate	older samples		- rockfall runout location - initial emplacement location of modern surface samples
Clast intrusion into debris layer	Emerging englacial clasts remain embedded in the ice and will penetrate some distance into the base of supraglacial debris layer before becoming embedded	high	underestimate	larger samples older samples	<i>clast intrusion model</i> ; model assumes clast penetration to a specified distance	- effective emergence dimension of clasts
Clast exhumation at polygon margins	Clasts at polygon margins may undergo rapid exhumation (and/or burial) due to slumping of destabilized material	low	underestimate	increase in slope angle and prevalence of polygons	<i>polygon exhumation model</i> ; model assumes an instantaneous excavation depth	- polygon formation frequency - slope stability dynamics
Variable englacial debris concentration	High debris content englacial debris bands observed within otherwise uniform areas of low debris concentration	med	overestimate; not single valued	older samples; samples associated with englacial debris bands	<i>No explicit model</i> . Age range constrained by the EO and ES models	- englacial debris concentration of ice that has already ablated
Spalling	Surface clasts may undergo periodic thermal-stress induced fracture and spalling	med	underestimate	older samples; larger samples	<i>spalling model</i> ; model assumes average fracture depth and spalling frequency	- fracture frequency/probability - avg. fracture depth and orientation
Prior inheritance	Some clasts may contain cosmogenic nuclide inventories inherited from exposure in the rockfall source areas before falling onto the glacier	low	overestimate	all samples	<i>No explicit model</i> . Potential inheritance identified by context with other samples	- percentage of clasts with prior inheritance - average magnitude of inheritance
Reorientation and multiple disturbances	Some clasts may change orientation when transitioning across the ice – debris layer and the debris layer – surface boundary	med	underestimate	older samples	<i>No explicit model</i>	- Initial clast orientation within ice
<u>Non-stochastics errors/ complications:</u>						
Non-constant surface production rates	Down valley glacial flow and surface lowering will decrease a sample's elevation, and therefore, P_0 over time.	all	overestimate	samples furthest from accumulation area	Apply general elevation function to approximate sample lowering	- Paleo surface flow velocities - surface lowering rates
Flow deformation of ice column	Lateral surface parallel strain in the flowing portion of the glacier will disrupt the initial configuration of the ice column	all	underestimate	older samples sourced from deepest portion of original ice body	<i>No explicit model</i> .	- current and paleo strain rates

1154 ^a The probability that the process will occur. This is a subjective qualifier based on field observations (e.g., Kowalewski et al., 2011; Mackay et al., 2014; Marchant et al., 2013; Swanger et al., 2011) and
1155 roughly follows a structure in which the more complex the proposed process, the less likely it is that it actually occurred for a particular clast.

1156 ^b The direction of the age estimate error (compared to the true age of the ice) if the process is not correctly taken into account.

1157 ^c Parameters that would be required to fully quantify the process

Table 2. Sample Parameters and Analytical Results^a

Sample ID	Latitude (°)	Longitude (°)	MSL elev. (m)	Dist. from datum (m) ^b	Region ^c	Supraglacial debris thick. (cm)	Sample Density. (g cm ⁻³); ρ_c	Englacial debris conc.; c^d	base P_0 (atom g ⁻¹ a) ^e	³ He/ ⁴ He (x 10 ⁻⁶) ^f	⁴ He (10 ⁻¹² atom g ⁻¹) ^f	³ He (10 ⁻⁶ atom g ⁻¹) ^f
DXP-99-1	-77.9040	160.5975	1611.8	1500	2	6	2.85	0.008	652	0.0239 (0.0004)	344.92 (1.97)	8.2 (0.1)
DXP-99-2	-77.9035	160.5954	1614.6	1580	2	8	2.76	0.008	654	0.6936 (0.0002)	262.47 (1.50)	182.0 (1.0)
DXP-99-4	-77.8986	160.5873	1577.7	2160	2	10	2.76	0.008	635	0.8440 (0.0002)	335.79 (1.92)	283.4 (1.7)
DXP-99-5	-77.8949	160.5797	1548.8	2632	2	11	3.03	0.008	621	0.6534 (0.0003)	128.61 (0.73)	84.0 (0.5)
DXP-99-7	-77.8886	160.5642	1497.3	3405	2	15	2.78	0.008	596	0.6012 (0.0007)	170.92 (0.98)	102.8 (0.6)
DXP 99-22	-77.8855	160.5455	1425.5	3960	2	18	2.91	0.008	563	1.1751 (0.0004)	107.86 (0.02)	126.7 (0.2)
DXP 99-23	-77.8852	160.5411	1413.7	3990	2	18	2.72	0.008	558	0.8509 (0.0009)	123.76 (0.02)	105.3 (0.2)
DXP 99-24	-77.8849	160.5463	1422.6	4002	2	15	2.90	0.008	562	0.3567 (0.0053)	325.39 (0.05)	116.1 (1.7)
DXP-99-30	-77.8819	160.5354	1384.0	4426	3	35	2.92	0.015	544	2.4695 (0.0001)	139.98 (0.80)	345.7 (2.0)
DXP-99-31	-77.8797	160.5305	1367.3	4710	3	38	2.78	0.015	537	1.0927 (0.0007)	92.67 (0.53)	101.3 (1.0)
DXP 06-07	-77.8744	160.5377	1335.0	5286	4	55	3.0 ^h	0.20	523			124.4
DXP 06-08	-77.8737	160.5284	1329.4	5298 ^g	4	50	3.0 ^h	0.20	521			643.7
DXP 06-05	-77.8749	160.5479	1333.5	5307 ^g	4	40	3.0 ^h	0.20	523			1540. 2
DXP-99-34	-77.8735	160.547	1326.8	5414	4	50	2.48	0.20	520	1.9736 (0.001)	191.06 (1.09)	377.1 (2.2)
DME-3	-77.8498	160.5940	1269.9	8320	4	55	3.0 ^h	0.20	496			892 (22) ^g

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^aEach sample varied between 10 and 20 cm in thickness. Noble gas analyses for DXP-99-1, DXP-99-2, DXP-99-4, DXP-99-5, DXP-99-7, DXP-99-22, DXP-99-23, DXP-99-24, DXP-99-30, DXP-99-31, and DXP-99-34 were conducted at the USGS noble gas laboratory, Denver Federal Center, on a MAP 215-50 mass spectrometer (William Phillips). Samples DXP-06-05, DXP-06-07, DXP-06-08, and DME-3 were analyzed at the Cosmogenic-nuclide lab at Lamont Doherty Earth Observatory (Joerg Schaefer). 1 σ analytical errors are quoted in parenthesis; preliminary results for this same dataset were reported in Marchant et al. (2007)

^bDistance computed along the central flow-line.

^cRegions as defined by (Mackay et al., 2014).

^dDebris concentration at the general location of the sample as mapped by Mackay et al. (2014).

^eDerived using the conservative sea level-high latitude surface production rate of 121 $\pm$ 6 ³He atoms g⁻¹ (Dunai and Wijbrans, 2000) adjusted to the current elevation of each sample using the atmospheric scaling of Stone (2000). This scaling accounts for the pressure anomalies of Antarctica. Referred to as the ‘base’ P_0 because it does not reflect adjustments for dynamic changes in the sample’s elevation.

^f1 σ analytical errors are quoted in parenthesis

^gSample not located along central flow-line. Distance measured to the intersection of the central flowline and an orthogonal line drawn through the sample location.

^hMeasured density not available. Value is an estimate only.

Table 3. Model input parameters for Mullins Glacier

parameter		units	value	source/notes
	absorption mean free path	g cm^{-2}	150	Gosse and Phillips (2001)
a	ablation parameterization constant 1	cm a^{-1}	0.08932	parameterized from Kowalewski et al. (2011); see Supplementary Figure S4 ^a
b	ablation parameterization constant 2	cm^{-1}	0.06114	parameterized from Kowalewski et al. (2011); see Supplementary Fig. S4 ^a
D_{eff}	effective emergence dimension	cm	20	Mackay et al. (2014) and this study ^b
f	spalling frequency	a^{-1}	$5 \times 10^{-6}, 10^{-5}$	Swanger et al. (2011) and this study ^c
H_{min}	min. depth for slumping on polygon margins	cm	15	this study, see Supplementary Fig. S6
H_r	initial debris thickness	cm	5	Mackay et al. (2014) ^d
k	exhumation depth parameterization constant	cm	9	this study, see Supplementary Fig. S6
m	slope of exhumation depth parameterization	-	0.6	this study, see Supplementary Fig. S6
χ	average clast fracture depth	cm	15	this study ^e
ϵ_c	whole clast erosion rate	cm Ma^{-1}	5-15	this study
ϵ_d	bulk debris erosion rate	cm Ma^{-1}	15-25	this study
λ	decay constant	$\text{atom g}^{-1} \text{a}^{-1}$	0	³ He is stable
ρ_i	density of ice	g cm^{-3}	0.9	Cuffey and Paterson (2010)
ρ_d	density of bulk debris	g cm^{-3}	2.1	assuming $\rho_d = (1 - \varphi)\rho_c$ with avg. $\rho_c = 3.0$
φ	porosity of bulk debris	-	0.3	Kowalewski et al. (2011)

^a The use of a uniform sublimation rate curve parameterization (constant a and b) for all samples is justified by the minimal spatial variation in measured environmental conditions along the length of Mullins Glacier (Kowalewski et al., 2011; Mackay et al., 2014).

^b This value equals the minimum c-axis dimension of the smallest sample. This value is poorly constrained but is likely conservative because it does not account for the potentially inclined a-axis positions of clasts while they are englacial in the pre-accretion stage (Mackay et al., 2014; Shean and Marchant, 2010).

^c The average fracture frequency of Mullins supraglacial clasts is poorly constrained but likely lies within $10^{-4} - 10^{-6} \text{ a}^{-1}$.

^d Estimated based on the rock-fall runout distance modelling results of Mackay et al. (2014). This value is considered a maximum value and is likely lower. As H_r increases, the modelled exposure age according to the erosion-sublimation models (and variants) decreases. Therefore, we consider this to be a conservative estimate.

^e This value is derived from measurements of in-situ “puzzle rock” (e.g., Marchant et al., 2013) fragments and observed average minimum dimension of detached fresh appearing slabs left in situ near their detachment points from parent clasts (Fig. 3b-d).

Table 4. Model Ages Including Stochastic Processes and Non-constant Surface Production Rates^a

	T_{app} ^b	ES model ^c			Clast intrusion ^d = 20 cm			D_{eff}	Clast intrusion ^e = 30 cm			D_{eff}	Polygon exhumation ^f				EO model ^g			Single spall ^h χ = 15cm				Recurrent spalling ⁱ χ =15 (10), f = 1×10 ⁻⁵				Recurrent spalling ^j χ =15 (10), f = 5×10 ⁻⁶				
		ϵ_d =			ϵ_d =				ϵ_d =				ϵ_d =				ϵ_e =				ϵ_e =				ϵ_e =				ϵ_e =			
		15	20	25	15	20	25		15	20	25		0	15	20	25	5	10	25	0	5	10	15	0	5	10	15	0	5	10	15	
DXP-99-1	12.5	13	13	13	14	14	14		14	14	14		NA	NA	NA	NA	13	13	13	17	17	17	17	17	17	17	17	17	17	17	17	
DXP-99-2	278	288	290	296	334	344	356		396	415	438		NA	NA	NA	NA	282	286	290	376	370	380	387	690	690	740	850	680	690	740	850	
DXP-99-4	446	470	482	493	592	621	653		674	729	674		NA	NA	NA	NA	455	465	476	585	603	618	640	900	0	0	0	449	938	1022	1132	
DXP-99-5	135	137	138	138	161	176	183		227	236	245		NA	NA	NA	NA	135	136	137	184	184	186	189	217	219	221	224	184	184	186	189	
DXP-99-7	172	175	176	177	194	217	224		253	282	292		NA	NA	NA	NA	173	173	175	223	225	228	233	281	283	288	297	228	233	237	242	
DXP 99-22	225	228	230	232	269	276	285		326	340	357		228	233	236	240	224	227	229	296	301	305	311	478	483	513	547	330	334	341	348	
DXP 99-23	189	190	191	192	222	235	242		286	298	310		195	198	201	206	187	189	191	241	246	248	251	325	328	335	343	258	258	262	268	
DXP 99-24	207	208	210	212	249	256	264		308	319	334		212	218	220	224	206	208	210	271	276	279	286	402	406	416	437	294	297	305	313	
DXP-99-30	636	672	693	708	706	736	775		750	792	850		744	805	829	858	645	666	690	834	870	911	953	(4000)	(3810)	(3636)	(3478)	(8000)	(7270)	(6660)	(6150)	
DXP-99-31	189	189	190	191	226	235	249		285	298	310		213	226	232	241	187	188	190	242	247	250	252	329	333	336	348	259	263	270	277	
DXP 06-07	238	238	240	242	281	290	299		340	356	375		333	336	344	351	236	239	242	313	318	323	329	578	585	622	676	355	360	365	376	
DXP 06-08	1236	1408	1494	1583	1405	1493	1602		1412	1504	1611		1628	2003	2194	2497	1291	1390	1512	1630	1843	1985	2262	(4000)	(3810)	(3636)	(3478)	(8000)	(7270)	(6660)	(6150)	
DXP 06-05	2945	4507	6486	15974	4576	7880	-		4659	15857	-		4013	88290	16542	20751	3414	4352	7014	3891	4958	7752	-	(4000)	(3810)	(3636)	(3478)	(8000)	(7270)	(6660)	(6150)	
DXP-99-34	725	770	794	821	856	918	1005		955	1070	1253		988	1121	1169	1233	732	756	782	959	1008	1067	1135	(4000)	(3810)	(3636)	(3478)	(8000)	(7270)	(6660)	(6150)	
DME-3	1798	2199	2444	2791	2212	2455	2835		2221	2494	2891		2367	3293	4044	9636	1932	2172	2519	2364	2709	3235	4238	(4000)	(3810)	(3636)	(3478)	(8000)	(7270)	(6660)	(6150)	

^a All values listed are ages in units of ka. All ages are adjusted (0-3%) for elevation loss – and thus surface production rate decrease – according to the procedure outlined in the Supplementary Information, Section 5. Polygon exhumation models are not applicable for samples where the supraglacial debris thickness is too thin for slumping to occur; these values are indicated as ‘NA.’ Model runs for which secular equilibrium is reached at a value that is lower than the measured nuclide inventory (and hence and age estimate is undefined) are indicated as ‘-’. Ages in parenthesis are estimated maximum ages for clasts that have reached secular equilibrium according to the recurrent spalling model. These ages indicated not cosmogenic exposure ages, but are estimated based on the physical limitations of the recurrent spalling assumption (Equation 25) using $l_0 = 400$ cm as an estimated initial clast size.

^b Apparent age. No adjustment has been made for any post-depositional disturbance or for elevation loss.

^c Ages differ from those listed in Table 2 due to adjustments for elevation loss.

^d Ages computed assuming 20 cm of effective clast intrusion into the base of the debris layer. Model as outlined in Section 2.3.5.1.

^e Ages computed assuming 30 cm of effective clast intrusion into the base of the debris layer. Model as outlined in Section 2.3.5.1.

^f Ages computed assuming the clast has been abruptly exposed at a polygon margin at some point in its exposure history. Model as outlined in Section 2.3.5.2.

^g Ages differ slightly from those listed in Table 2 due to adjustments for elevation loss.

^h Ages computed assuming a single spall to a depth of ~15 cm has occurred before sampling. Model as outlined in Section 2.3.5.3.

ⁱ Ages computed assuming a recurrent spalling at a frequency of $f = 1 \times 10^{-5}$ have occurred before sampling. Model as outlined in Section 2.3.5.3.

^j Ages computed assuming a recurrent spalling at a frequency of $f = 5 \times 10^{-6}$ have occurred before sampling. Model as outlined in Section 2.3.5.3.

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