Role of response time of a Babcock-Leighton solar dynamo model in meridional flow-speed reconstruction by EnKF data assimilation

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Abstract

Ensemble Kalman Filter in the framework of Data Assimilation Research Testbed (DART) has been successfully implemented into a 2D kinematic flux-transport dynamo model by Dikpati and colleagues in order to do a parameter estimation, the parameter being the meridional flow-speed as function of time. They performed several 'Observing System Simulation Experiments' (OSSEs), and showed that an optimal reconstruction of time-series of meridional flow-speed can be obtained by using 16 ensemble members and only one surface magnetic observation with 30% observational error. Error in reconstruction can be reduced by increasing the ensemble size and number of observations. However, this parameter reconstruction has been found to be sensitive to locations from where observational data are taken. While assimilation of low-latitudes' surface poloidal magnetic field data can produce good reconstruction, medium-frequency oscillations appear in time-series of reconstructed flow-speed if tachocline toroidal field data are assimilated. These oscillations occur primarily because tachocline toroidal fields change very little during an assimilation interval taken to be 15 days, due to changes in meridional flow. A Babcock-Leighton dynamo model's response time to changes in meridional flow-speed is a few months. We show here that rms error in reconstruction can be significantly reduced if model's response time is taken into consideration in assimilation of tachocline toroidal field data.

1 INTRODUCTION

Babcock-Leighton type flux-transport solar dynamo models in both 2D (Wang & Sheeley , 1991; Choudhuri, Schüssler & Dikpati , 1995; Dikpati & Charbonneau , 1999; Guerrero, Dikpati & de Gouveia Dal Pino , 2009; Nandy, Muñoz-Jaramillo & Martens , 2011; Belucz, Dikpati & Forgács-Dajka , 2015) and 3D (Yeates & Muñoz-Jaramillo , 2013; Miesch & Dikpati , 2014) have been successful in explaining many large-scale solar cycle features, including a particularly difficult one – the correct phase relationship between the equatorward migrating sunspot belt and the poleward drifting large-scale, diffuse fields. This class of models have four major dynamo ingredients, namely the differential rotation, meridional circulation, Babcock-Leighton type surface poloidal source and magnetic diffusivity. These models can be best calibrated for the Sun (see, e.g. Dikpati et al (2004)) if they operate with a primarily single-celled meridional circulation pattern in each hemisphere.

The meridional circulation, along with magnetic diffusivity, play crucial roles in determining (i) the dynamo cycle period and (ii) governing the memory of the model’s past magnetic fields. However, the profile and spatio-temporal pattern of this circulation is known at/near the surface down to a shallow layer, not in the deeper convection zone and tachocline. Even in the upper half of the convection zone, different observations infer different profiles. Using time-distance helioseismology the most recent observations from SDO/HMI data infer meridional circulation with two cells in depth (Zhao et al. , 2013). Ring-diagram analysis from GONG data gives poleward surface flow up to about $\sim 60^\circ$ latitude (Komm et al. , 2013), whereas Doppler measurements from MWO data, which can measure this flow at higher latitudes, show a high-latitude, reverse flow associated with the primary poleward surface flow (Ulrich , 2010). Using a very long-term GONG database and applying time-distance technique Kholikov, Serebryanskiy & Jackiewicz (2014) have found signatures of equatorward return-flow in the lower half of the convection zone, indicating a long deep one cell flow-pattern. A p-mode perturbation analysis by Schad, Timmer & Roth (2013) yields four cells in latitude, each going down to about $0.8R$. Thus observations do not yet give us a unique answer about the Sun’s meridional circulation pattern.

Models produce more complex flow patterns that vary from model to model. For example, mean-field models produce a long, counterclockwise primary cell often associated with a weak, reverse cell at high latitudes, both extending down to the bottom of convection zone (Dikpati , 2014), whereas full 3D

convection simulations produce multiple cells in latitude and depth (Guerrero et al , 2013; Featherstone & Miesch , 2015).

Recently Dikpati, Anderson & Mitra (2014) have demonstrated that Ensemble Kalman Filter data assimilation method in the DART framework has the potential for successfully reconstructing the time-variation of the Sun’s meridional circulation. By building a Dynamo-DART system, Dikpati et al. (2014) performed Observing System Simulation Experiments (OSSEs) using a 2D Babcock-Leighton flux-transport dynamo model operating with a time-varying meridional flow-speed, which they assumed. Note that in OSSEs, the observational data are ”synthetic” data, which are obtained by adding noise to model-output. Dikpati et al. (2014) found that their Dynamo-DART system can best reconstruct the time variation in meridional flow-speed if 160 ensemble members are used in assimilation of 180 magnetic field observations with 1% observational error in each of them, although a fairly reasonable reconstruction of meridional flow-speed was possible by just using only 16 ensemble members and one observation with 30% error.

By performing several OSSEs, these authors showed recently (Dikpati, Anderson & Mitra , 2016) that the reconstruction is sensitive to locations from where observational data is taken. For example, a very good reconstruction of the time series of meridional flow-speed is obtained when surface poloidal magnetic field observations from low latitudes are assimilated, but the reconstructed meridional flow-speed shows some oscillations when tachocline toroidal field data are assimilated.

This medium-frequency oscillation occurs due to the fact that the assimilation interval of 15 days is too short a time for the tachocline toroidal fields to respond to the changes in meridional circulation. A Babcock-Leighton solar dynamo model’s overall response time to changes in meridional circulation is a few months (see, e.g. Dikpati & Anderson 2012). Here we show that such oscillations in the time series of meridional flow-speed can be reduced if the information of the model’s response time is given to the Dynamo-DART system.

In the next section we describe the methodology for including the dynamo model’s response time in the assimilation. We present our results in §3 and close the paper with concluding remarks.

2 DESCRIPTION OF METHODOLOGY

The Data Assimilation Research Testbed (DART) is a flexible sequential data assimilation framework that allows experimenting with different filter

algorithms, such as Ensemble Kalman Filter (EnKF), Ensemble Adjustment Kalman Filter (EAKF) conveniently. More details on the method can be found in Anderson (2001) and Anderson & Collins (2007), and the software is available in IMAGE department of NCAR/CISL (Computer Information System Laboratory) from the url <http://www.image.ucar.edu/DARes/DART>. The EnKF-DART scheme is based on statistical multidimensional regression analysis using a Bayesian approach – a well-developed method that has been applied in atmospheric and oceanic studies, such as in the Earth’s upper atmosphere models (see Matsuo et al. 2013), and recently in a 2D Babcock-Leighton solar dynamo model (Dikpati et al. 2014).

A different EnKF scheme was applied in the past (Kitiashvili & Kosovichev, 2008) in a one-dimensional solar dynamo model for predicting solar cycle 24 amplitude. For real-life predictions of polar field patterns, ADAPT scheme based on EnKF data assimilation has been implemented by Arge et al (2010) and Hickmann, Godinez, Henney & Arge (2015). Fournier, Nerger & Aubert (2013) have implemented the EnKF data assimilation scheme in a geodynamo model.

Fig. 1. Schematic representation of the EnKF method as applied in present problem. Three members of the ensemble of flow-speeds at time t , namely V_1^t , V_2^t and V_3^t are evolved using Equation (1) to generate corresponding prior state at time $t + \delta$, shown as evolution from (a) to (b). In the “Filter” box, observations O_1 , O_2 and O_3 are generated by applying forward operator (FTD) on the prior estimates. Then following a linear regression analysis on these observations with corresponding flow-speeds, the “Filter” estimates posterior flow-speeds using the synthetic observation with errors. In the process it also generates the posterior observation for evaluation of innovation at time $t + \delta$. Time advances from t to $t + \delta$ (denoted by solid line at the top) during the evolution from (a) to (b), while in the “Filter” procedure, time remains frozen at step $t + \delta$ (denoted by dotted line from (b) to (c)) (adopted from Dikpati, Anderson & Mitra (2014)).

Following Dikpati, Anderson & Mitra (2014), we will again implement here the EnKF data assimilation in a Babcock-Leighton flux-transport (BLFT) dynamo model of Dikpati et al (2010). Note that the physical model (i.e. BLFT dynamo model) is the forward operator in data assimilation terminology, and we call it FTD. Our goal is to investigate the role of the dynamo model’s response time to changes in meridional flow in reconstructing the time variation in meridional flow-speed. The time-varying meridional flow-speed at a given time is the “model state” or “true state”, to be estimated by the EnKF system. For convenience of comparison between the true state of the time-varying meridional flow-speed and the flow-speed reconstructed from ensemble mean, we present the true state in Figure 2, which is adopted from figure 2(a) of Dikpati, Anderson & Mitra (2014).

EnKF sequential data assimilation is a two-step procedure, a forecast followed by an analysis each time an observation is available. In the first step, from initial state vectors at time t , a forecast at time $t+\delta$ is made using an assimilation model (the BLFT dynamo model here); thus the “prior states” of the state vectors are generated. In the second step of the two-step process, their “posterior states” are estimated by employing multidimensional regression among the observation vectors and the prior state vectors. Figure 1 schematically describes these steps (see Dikpati, Anderson & Mitra (2014) for details).

In Figure 1, v_1^t, v_2^t, v_3^t near the label (a) denote three different realizations of initial flow-speeds. The EnKF requires a prediction model, given by Equation (1), that generates a forecast of the meridional flow-speed at a later time given the value at the current time (i.e., v_1^t, v_2^t, v_3^t):

$$v_1^{t+\delta^{\text{prior}}} = v_1^t + \Gamma_0 \times \gamma, \quad (1),$$

in which, γ is a function for generating normalized Gaussian random numbers with unit amplitude and unit standard deviation, and the amplitude of the prediction model noise is governed by Γ_0 . “Prior states”, denoted by $v_1^{t+\delta^{\text{prior}}}, v_2^{t+\delta^{\text{prior}}}, v_3^{t+\delta^{\text{prior}}}$ near the label (b)) are generated by using Equation (1) to advance the estimates of the flow-speed from the initial time (t) to the time ($t+\delta$) at which the first observations of magnetic field are available.

In the second step, posterior flow-speeds ($v_1^{t+\delta^{\text{posterior}}}, v_2^{t+\delta^{\text{posterior}}}, v_3^{t+\delta^{\text{posterior}}}$) are estimated by employing regression among the model-outputs of magnetic fields, “synthetic” observations of magnetic fields and prior flow-speeds. flow-speeds. The forward operator (BLFT dynamo in this case) is denoted by FTD along the solid black arrows, which uses the prior estimates of flow speed to produce a prior estimates of ensemble of magnetic field observations. Three realizations of magnetic fields from the forward operator are denoted by O_1, O_2, O_3 in Figure 1.

Medium frequency oscillations in the reconstruction occur because changes in

tachocline magnetic fields during 15 days of assimilation interval are generally small due to changes in flow speed. The flow-speed is much smaller at/near the base of the convection zone than at the surface because of the density stratification. So a plausible remedy can be to run the model longer than the assimilation interval of 15 days, and obtain significant changes in the tachocline magnetic fields. So we again consider an assimilation interval of 15 days, but another longer time span (say 30 days) during which the flow speed continues to vary with time so that significant changes in tachocline magnetic fields occur. So we start integration of the forward operator at time t and continue the integration up to $t + 30$ days; we obtain the model-outputs at the time $t + 30$ days, and consider the “synthetic” observations also at $t + 30$ days, and regress them with prior states of flow-speeds. The posterior states of velocities obtained from this regression we implement as prior states at the time of $t + 15$ days instead of at the time $t + 30$ days. In this way, we keep our assimilation interval as 15 days, but apply an operation time (> 15 days) to the forward operator for using its response time.

3 RESULTS

We consider 10 toroidal magnetic field observations, which are taken from the center of the tachocline, particularly from the depth of $0.704R$, and from the low latitudes ranging from 23.4° to 31.5° with an interval of 0.9° magnetic field observations, and perform assimilation runs with 192 ensemble members with an observational error of $\sim 30\%$, which means an error of $\sim 30\%$ about the ideal magnetic field generated using the true meridional flow speed. To estimate the prior states of flow-speed, we use Equation (1), in which we set $\Gamma_0 = 0.5\text{ms}^{-1}$. If the meridional flow-speed varies up to $\pm 20\%$ (i.e. $\sim \pm 2.8\text{ms}^{-1}$ for a mean flow-speed of 14ms^{-1}) during six months, the variation in 15 days can be $\sim 0.23\text{ms}^{-1}$. Thus we chose $\Gamma_0 = 0.5\text{ms}^{-1}$ so that it is large enough to capture the variation in flow-speed within our selected updating time-step of 15 days and also large enough to avoid ensemble collapse, but not so large as to produce unusual departures from cyclic behavior in a flux-transport dynamo.

We present the results in three panels of Figure 2; panel (a) shows the true state of the time-varying flow-speed to be reconstructed by EnKF assimilation in our DART-Dynamo system. The reconstructed flow-speed, obtained by assimilating 10 synthetic observations of low-latitude tachocline magnetic fields in an assimilation interval of 15 days, has been plotted in panel (b).

Fig. 2. Panel (a): true state (i.e. time-varying meridional flow speed) that varies by $\pm 70\%$ about mean flow-speed, 14 ms^{-1} ; Panel (b): red curve denotes ensemble-mean of reconstructed meridional flow-speeds as a function of time; superimposed green curves denote all 192 ensemble members; true state is plotted in blue for comparison. Panel (c): rms error in reconstruction as function of assimilation time interval.

Defining the rms error in reconstruction as, $\sqrt{\frac{\sum_{i=1}^{n_{\max}} (v_T^i - v_R^i)^2}{n_{\max}}}$, in which i denotes the index for every assimilation interval, v_T^i the true state at the i th assimilation step and v_R^i the reconstructed state, and $n_{\max}$ the total number of assimilation steps during the entire time span of the assimilation runs, we plot the rms error as function of assimilation interval and present in panel (c). Panel (b) of Figure 2 reveals that the reconstructed flow-speed starts deviating from the truth after 10 years after which medium frequency oscillations in the reconstructed flow speed occur, when the tachocline magnetic fields are used. Apart from oscillations, we find that the reconstruction is showing major deviations with opposite trends with respect to the true state late in the assimilation runs, namely beyond $t = 27$ years.

Variation in tachocline toroidal magnetic fields is much smaller than that in surface magnetic fields, because (i) the flow-speed is smaller at the bottom of the convection zone and in the tachocline, (ii) the magnetic diffusivity is smaller there than in the upper half of the convection zone and at the surface. So the flux-freezing inertia felt by the tachocline toroidal magnetic fields is large, and these fields are not prone to significant changes in 15 days by flow-speed variation. As a consequence the reconstructed flow-speed deviates from the true state. If we just assimilate only one observation of tachocline toroidal field, the reconstruction continues to deviate without approaching towards the truth. However the assimilation of more than one observations of tachocline toroidal fields produces such deviations at different times, and the net result is a back and forth oscillation in reconstruction about the truth.

The obvious question arises whether the reconstructions can be improved if the assimilation interval is increased. To answer this question, we run the assimilations using different assimilation time intervals, and plot in Figure 2(c) the rms error in reconstruction as function of assimilation time interval. Figure 2(c) reveals that increasing the assimilation interval is not the answer, because the reconstruction deteriorates with longer updating intervals. The optimum assimilation interval is about 15 days. **Reconstruction starts deteriorating again when assimilation interval is decreased from 15 days to a smaller value. By performing extensive assimilation runs, Dikpati, Anderson & Mitra (2016) have recently shown in their figure 14 that the RMS error in reconstruction is indeed minimum for 15 days assimilation interval.**

(a)

(b)

Fig. 3. (a) RMS errors in reconstruction (in units of ms^{-1}) as functions of response time for assimilation intervals of 15, 30 and 45 days, respectively plotted in solid triangles, squares and circles implemented in the assimilation runs; (b) reconstructed flow-speed as function of time, obtained by implementing 75 days response time.

Speculating that allowing a response time of a few months, or in other words, allowing the model to feel the change of meridional flow speed for a few months, can improve the reconstruction, we perform the assimilation runs using different response times, using three different assimilation intervals – 15, 30 and 45 days. We present our results in Figure 3; the upper panel shows the rms error in reconstruction as function of response time, for assimilation intervals of 15, 30 and 45 days, respectively plotted in solid triangles, squares and circles. The lower panel shows the reconstructed flow-speed for the case with 75 days response time. We find in Figure 3(a) that the, for each assimilation interval chosen, the rms error in reconstruction decreases with the increase in response time up to a certain value (75, 90 and 90 days respectively), and then starts increasing again. Note that the response time can be only the integral multiple of assimilation interval in these experiments. Examining the three curves in Figure 3(a) we can clearly find that the minimum RMS error in reconstruction can be obtained when 15 days assimilation interval is used. For a Babcock-Leighton dynamo model, updating the Dynamo-DART system in every 15 days is optimal (see figure 14 of Dikpati, Anderson & Mitra (2016); see also Sanchez, Fournier & Aubert (2014), for discussion on optimal updating interval in a Babcock-Leighton dynamo model). Therefore, a corresponding curve of RMS error as function of response time for 5 days assimilation interval will shift upward of solid triangles.

Figure 3(b) shows that the reconstructed flow-speed is largely improved compared to that in Figure 2(b); the oscillations in reconstructions are much reduced, but they are not completely gone. The opposite trends in the time window of 27 to 35 years are rectified. However, this reconstruction is still poorer than the best reconstruction obtained by assimilating surface magnetic observations, as was seen in figure 5 of Dikpati, Anderson & Mitra (2014).

Dikpati, Anderson & Mitra (2014) used 180 observations from the surface and near surface region. We perform here an experiment to examine how much improvement we can obtain if we include one surface poloidal field observation and nine tachocline toroidal field observations. Nine tachocline toroidal field observations are taken from the tachocline center and from the latitude range of $23.4^\circ - 30.6^\circ$, and the surface poloidal field observation is taken from $0.98R$ and 4.5° latitude. We present the result in Figure 4. We can see that the reconstruction is greatly improved when we include surface magnetic observations along with tachocline magnetic field observations.

The reasons for this significant improvement in reconstruction by including surface poloidal field observation in the assimilation runs is that the flow speed at the surface is about 10 to 15 times larger than that at the bottom of the convection zone; therefore the variation in flow speed is also larger there. Furthermore, the magnetic diffusivity being larger near the surface,

Fig. 4. Same as in Figure 2(b) but with incorporation of response time in the assimilation runs using nine tachocline toroidal magnetic field and one surface poloidal field observations.

the flux-freezing inertia is much less for the surface poloidal fields. Thus the changes in the poloidal fields in 15 days, due to variation in flow speed, is significant, but also not too large to be uncorrelated to the poloidal fields of the previous assimilation, i.e. the poloidal fields at that location 15 days ago. So the resulting reconstruction is largely improved.

4 COMMENTS

It was previously demonstrated through OSSEs that EnKF data assimilation into a DART-Dynamo system can be a powerful tool for successfully reconstructing time-variation in meridional circulation speed for more than a solar cycle from observations of the magnetic field. However, the reconstruction is sensitive to the locations from where observations are taken. Assimilation of tachocline magnetic field observations produce oscillations affecting the faithful reconstruction.

We demonstrated here that these oscillations can be reduced and reconstruction can be improved up to a certain extent if the physical model's response time is taken into consideration and OSSEs are performed with incorpora-

tion of such response time. However, in order to obtain the most faithful reconstruction of time variation in meridional flow-speed it is necessary to include surface observations of weak poloidal magnetic fields along with strong tachocline magnetic field observations.

One of the major purposes of data assimilation techniques is to estimate parameters which are difficult to obtain from models and/or observations. Application of data assimilation technique is fairly new in solar physics, but the implementation of both the EnKF techniques and variational methods in solar models have been demonstrated to be successful in the reconstructions of different types of parameters, such as α -effect (see, Jouve, Brun & Talagrand (2011)), time series of meridional circulation (Dikpati, Anderson & Mitra, 2014). These observing system simulation experiments are very important for eventually assimilating instruments' data in the model for real time predictions.

Acknowledgments

We thank Hanli Liu for reviewing the entire manuscript. We extend our thanks to two anonymous reviewers for their helpful comments, which have helped improve the paper significantly. Our thanks go to Nancy Collins and Tim Hoar for their invaluable help with assimilation tools and software. The DART/Dynamo assimilation runs have been performed on the Yellowstone Supercomputer of NWSC/NCAR under project number P22104000, and two NCAR Strategic Capability project with award numbers NHAO0009 and NHAO0010. All assimilation tools used in this work are available to the public from [http : //www.image.ucar.edu/DARes/DART](http://www.image.ucar.edu/DARes/DART). National Center for Atmospheric Research is sponsored by National Science Foundation.

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