Spatial Variation of Magnitude Scaling Factors During the 2010 Darfield and 2011 Christchurch, New Zealand, Earthquakes

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Abstract: The combination of well-documented liquefaction response during the Darfield and Christchurch, New Zealand, earthquakes, densely-recorded ground motions for the events, and detailed subsurface characterization provides an unprecedented opportunity to investigate the significance of the spatial variation of magnitude scaling factors (MSF) on liquefaction triggering. Towards this end, MSF were computed at 15 SMS sites across Christchurch and its surroundings using two established approaches. Trends in the spatial variation of the MSF computed using number of equivalent cycles (n_{eq}) from both approaches were similar, with the spatial variation being more significant for the Christchurch earthquake than the Darfield earthquake. However, there was no consistent trend for regions with lower computed MSF having experienced more severe or widespread liquefaction. Additionally, there is a general correlation between MSF and a_{max} , but because a_{max} ranges more widely than MSF it has a greater influence on the resulting seismic demand imposed on the soil than MSF does. Nevertheless, the spatial variation of the MSF is deemed significant enough that it warrants being considered for incorporation into future variants of simplified liquefaction evaluation procedures.

Keywords: liquefaction; magnitude scaling factors; Christchurch; Canterbury earthquakes

1. Introduction

The objective of the study presented herein is to assess the significance of the spatial variation of the magnitude scaling factors across Christchurch, New Zealand, and its environs on the assessment of liquefaction triggering during the 2010 Darfield and 2011 Christchurch earthquakes. Liquefaction is a phenomenon that generally occurs in loose to medium dense saturated cohesionless soil subjected to cyclic loading. It results from the contractive tendency of the soil skeleton during shaking and the consequential transfer of the overburden stress from the soil skeleton to the pore fluid. The “simplified” liquefaction triggering evaluation procedure is a total stress framework that was originally developed by Whitman (1971) and Seed and Idriss (1971) and has been continually revised as a result of the increase in the number of field case histories, etc. (e.g., Youd et al., 2001; Cetin et al., 2004; Idriss and Boulanger, 2008). In this procedure, the durational effects of the ground motions are accounted for by magnitude scaling factors (MSF), which traditionally have only been correlated to earthquake magnitude (e.g., Youd et al., 2001; Cetin et al., 2004; Idriss and Boulanger, 2008). As a result, within the context of the simplified procedure, the spatial variation in the seismic demand imposed on the soil traditionally has been assumed to be solely a function of the spatial variation of the peak amplitude of the ground motions and the characteristics of the soil profile. However, these traditional assumptions fail to appreciate the inverse correlation between the peak amplitude of ground motions and strong ground motion duration. The extreme cases of this are motions with forward versus reverse directivity effects, where the former tend to have higher amplitudes and shorter durations and the latter tend to have the opposite trends (Archuleta and Hartzell, 1984; Somerville et al., 1997). Additionally, it is known that ground motion duration varies with site-to-source distance (e.g., Bommer and Martinez-Pereira, 1999; Kempton and Stewart, 2006; Stafford et al., 2009; Lee and Green, 2014), and thus, it would be expected that MSF will also vary with distance from the source.

Christchurch, New Zealand, and its surroundings experienced widespread liquefaction as a result of the 2010-2011 Canterbury Earthquake Sequence (CES) (e.g., Cubrinovski and Green, 2010; Cubrinovski et

al., 2011; Green et al., 2014). However, of the 10 events in the CES that are known to have caused liquefaction (Quigley et al., 2013), the 4 September 2010, M_w 7.1 Darfield and 22 February 2011, M_w 6.2 Christchurch earthquakes were the most significant (Figure 1). The combination of well-documented liquefaction response during the Darfield and Christchurch earthquakes, densely-recorded ground motions for the events, and detailed subsurface characterization provides an unprecedented opportunity to investigate the significance of the spatial variation of MSF on liquefaction triggering. Of particular relevance to this study is that directivity effects were prevalent in the ground motions recorded during the Darfield earthquake, and to a much lesser extent, during the Christchurch earthquake (e.g., Bradley and Cubrinovski, 2011; Bradley, 2012a,b,c; Shahi and Baker, 2012). The strike slip rupture mechanism of the Darfield earthquake and the orientation of the causative fault relative to Christchurch resulted in the manifestation of forward directivity effects throughout much of the city. In contrast, the predominantly reverse rupture mechanism of the Christchurch earthquake and the causative fault orientation only resulted in forward directivity effects in areas south of the city along the Port Hills. This allows a comparison to be made between the spatial variation of MSF across the same region for motions where directivity effects were and were not prominent.

In the following, background information on the geology and geomorphology of the Canterbury region is presented. This is followed by a brief overview of the CES. Next, background information is given on the approaches used to compute MSF. These approaches are then used to compute MSF across the Christchurch region, with the trends in the results discussed.

2. Background Information

2.1 Geology and Geomorphology of the Christchurch Area

The Canterbury Plains are ~160 km long and up to 60 km wide. The plains are the result of overlapping alluvial fans produced by glacier-fed rivers from the Southern Alps, the main mountain range of the South Island (Forsyth et al., 2008). Uplift of the Southern Alps resulted in rapid deposition during the late

Quaternary and inundation of the Canterbury Plains by alluvial and fluvial sediments. The Canterbury Plains are underlain by a deep and complex network of overlapping fingers of fluvial gravels interbedded with swamp, estuarine, lagoon, and beach deposits of sand, silt, clays, and peat as a result of the glacial and post-glacial movements of the coastline (Brown et al., 1995). Figure 2 shows an idealized cross section of the interbedded sedimentary deposits beneath the region. Also shown in this figure is the approximate depth to the uppermost gravel formation (i.e., Riccarton Gravel Formation) for several of the strong motion recording stations (SMSs) located in the Central Business District (CBD) and eastern suburbs of Christchurch (note that the simplified model of the depth to the Riccarton Gravel Formation shown in this figure is in accord with the 3D model of the formation developed by Lee et al. (2015)). The characterization of these SMS sites and several others are discussed later in the paper.

In the recent geologic past, most of Christchurch was low-lying floodplains and swamps behind a series of barrier dunes (composed of fine-grained beach/dune sand), estuaries, and lagoons (underlain by fine-grained deposits) of Pegasus Bay. The Waimakariri River regularly flooded Christchurch before levee construction and river realignment, shortly after the city was established in 1850. The original city center was constructed on slightly higher ground compared to areas to the north and east. Of particular relevance to liquefaction susceptibility of the region are the locations of abandoned paleo-channels of the Waimakariri, Heathcote, and Avon Rivers, and former swamps. These areas are underlain by, and filled with, young loose sandy sediments, with shallow groundwater levels (from 1 to 5 m below ground surface), which are highly susceptible to liquefaction (e.g., Wotherspoon et al., 2012).

2.2 Canterbury Earthquake Sequence (CES)

The 2010-2011 CES started at 4:35 am on 4 September 2010 NZ Standard Time (16:35 3 September 2010 UTC), when a previously unmapped fault west of Christchurch ruptured, producing the M_w 7.1 Darfield earthquake (Bradley et al., 2014). Although the earthquake caused major damage to the built environment and induced widespread liquefaction, there were no fatalities or major injuries. The CES included twelve

other events having $M_w \geq 5.0$ with epicentral locations within 20 km of Christchurch (GeoNet, 2012), and up to ten of these larger events are known to have induced liquefaction (Quigley et al., 2013). However, the $M_w 6.2$, 22 February 2011 Christchurch earthquake was the most damaging event, due to the close proximity of its rupture plane to Christchurch, resulting in 185 fatalities and causing widespread liquefaction (e.g., Cubrinovski et al., 2011; Green et al., 2011; Orense et al., 2011; Cubrinovski et al., 2012; Robinson et al., 2013; Maurer et al., 2014).

The motions from both the Darfield and Christchurch earthquakes were recorded by a dense network of strong ground motion stations (e.g., Cousins and McVerry, 2010; Bradley and Cubrinovski, 2011; Bradley, 2012a,b,c; Bradley et al., 2014). The recordings included a significant number of near-fault motions, many of which show clear evidence of the forward directivity phenomenon (Bradley, 2012c; Bradley and Cubrinovski, 2011; Joshi, 2013). In an effort to better understand site effects in the ground motions recorded across Christchurch, Wotherspoon et al. (2014, 2015a,b) performed a detailed characterization of the shallow (~ 40 m) subsoil conditions at the 15 SMSs shown in Figure 3. The profiles obtained by Wotherspoon et al. form the basis of the site response analyses performed herein.

2.3 Magnitude Scaling Factors (MSF)

As stated previously, magnitude scaling factors account for the influence of the durational effects of strong ground motion on liquefaction triggering. For historical reasons, MSF are normalized to $M_w 7.5$ events and traditionally have been computed from number of equivalent cycle correlations using the following relationship (e.g., Green, 2001; Boulanger and Idriss, 2015):

$$MSF = \left(\frac{n_{eq M7.5}}{n_{eq M}} \right)^b \quad (1)$$

where $n_{eqM7.5}$ and n_{eqM} are the number of equivalent cycles for $M_w7.5$ and M_w events, and b is the negative slope of the $\log(CSR)$ versus $\log(N_{liq})$ curve for the soil of interest; CSR is cyclic stress ratio (i.e., cyclic shear stress (τ) divided by the initial vertical effective stress(σ'_{vo})), and N_{liq} is the number of cycles required to trigger liquefaction in a soil specimen subjected to sinusoidal loading having an amplitude of CSR , typically determined using cyclic triaxial or cyclic simple shear tests. Alternatively, MSF have been developed directly from the statistical analysis of liquefaction case histories (e.g., Cetin et al., 2004). However, this approach is limited by the relatively narrow variable ranges of the cases in the liquefaction case history databases.

Well-established fatigue theories have been proposed for computing n_{eq} for materials having varying phenomenological behavior; reviews of different approaches for computing n_{eq} are provided in Kaechele (1963), Stallymeyer and Walker (1968), Green and Terri (2005), and Hancock and Bommer (2005), among others. Developed specifically for use in evaluating liquefaction potential, the approaches proposed by Seed et al. (1975) and by Green and Terri (2005) are used herein. These approaches are variants of the Palmgren-Miner (P-M) fatigue theory, with the approach by Green and Terri (2005) better accounting for the non-linear behavior of the soil than the Seed et al. (1975) variant.

Assuming that ground surface accelerations (a) and corresponding shear stresses (τ) in the upper ~6 m (20 ft) of a soil profile are proportional, the Seed et al. (1975) variant of the P-M fatigue theory can be expressed as:

$$n_{eq} = \sum_i 0.5 \cdot \left(\frac{1}{0.65} \cdot \frac{|a_i|}{a_{max}} \right)^{\frac{1}{b}} \quad (2)$$

where $|a_i|$ is the absolute value of the amplitude of the i^{th} peak (or pulse) in a horizontal acceleration time history recorded at the ground surface, and a_{max} is the peak ground acceleration for the same acceleration time history; b is as defined for Eq. 1. Inherent to the Seed et al. (1975) procedure, the ratio of acceleration and peak ground acceleration (a/a_{max}) is assumed equal to τ/τ_{max} , where τ_{max} is the maximum shear stress induced in the soil profile at the depth of interest.

There are several different “peak counting” methods (ASTM, 2011) that are used in fatigue analyses. Seed et al. (1975) used a version of the mean crossing peak counting (also known as zero-crossing) method wherein the amplitude of one cycle is taken as the maximum value of the response history between successive zero crossings. Also, Seed et al. (1975) excluded peaks having amplitudes less than approximately $0.3 \cdot a/a_{\text{max}}$ in computing n_{eq} because their contribution was considered to be negligible. To calculate the equivalent number of cycles from two mutually perpendicular horizontal components of motion, Seed et al. suggested two ways without giving a clear preference for one or the other: (1) Normalize each component of motion by its own a_{max} ; and (2) Normalize both components of motion by the larger a_{max} of the two motions. Although Seed et al. do not state a preference for one approach over the other, approach (1) has been widely used in subsequent studies. Using this approach, Seed et al. (1975) computed a mean value of $n_{\text{eqM7.5}} = 15$ from a suite of ~60 motions from earthquakes in the western US and South America.

In the Green and Terri (2005) approach, dissipated energy is explicitly used as the damage metric. n_{eq} is determined by equating the energy dissipated in a soil element subjected to an earthquake motion to the energy dissipated in the same soil element subjected to a sinusoidal motion of a given amplitude and a duration of n_{eq} . Dissipated energy was selected as the damage metric because it has been shown to correlate with excess pore pressure generation in saturated cohesionless soil samples subjected to undrained cyclic loading (e.g., Green et al., 2000; Polito et al., 2008). Furthermore, from a microscopic

perspective, energy is thought to be predominantly dissipated by the friction between sand grains as they move relative to each other as the soil skeleton breaks down, which is requisite for liquefaction triggering.

Conceptually, the Green and Terri (2005) approach for computing n_{eq} is shown in Figure 4. Stress and strain time histories at various depths in the soil profile are obtained from a numerical site response analysis. By integrating the variation of shear stress over shear strain, the cumulative dissipated energy per unit volume of soil can be computed (i.e., the cumulative area bounded by the stress-strain hysteresis loops). n_{eq} is then determined by dividing the cumulative dissipated energy for the entire earthquake motion by the energy dissipated in one equivalent cycle. For historical reasons, the shear stress amplitude of the equivalent cycle (τ_{avg}) is taken as 0.65 time the maximum induced shear stress (τ_{max}) at the depth of interest, and the dissipated energy associated with the equivalent cycle is determined from the constitutive model used in the site response analysis. Additionally, the b value that is needed to relate n_{eq} to MSF per Eq. 1 can also be determined from the constitutive model used in the site response analysis, by assuming that the CSR vs. N_{liq} curve is a contour of constant dissipated energy (Lasley et al., 2016). Finally, from equivalent linear site response analyses performed on 50 soil profiles compiled by Cetin (2000) that are representative of cases in the liquefaction case history database, using 195 pairs of horizontal rock motions recorded during 47 earthquakes as input motions, Lasley et al. (2016) determined the median $n_{eqM7.5}$ to be equal to 8.2.

3. Computation of MSF for the Darfield and Christchurch Earthquakes

MSF were computed for both the Darfield and Christchurch earthquakes at the 15 SMS sites shown in Figure 3 and listed in Table 1. These sites were selected because of the detailed site characterization performed at them (Wotherspoon et al., 2014, 2015a,b). As described below, equivalent linear site response analyses were performed to compute n_{eq} as a function of depth at each SMS site per the Green and Terri (2005) procedure and from the computed surface acceleration time histories per the Seed et al. (1975) procedure. Bedrock or the Riccarton Gravel Formation was assumed as the elastic half space in the

equivalent linear site response analyses for these sites. For the cases where the Riccarton Gravel Formation was assumed to be the elastic halfspace, this assumption is based on the sharp impedance contrast between the Riccarton Gravel Formation and the looser overlying sediments (Brown and Weeber, 1992). For all the analyses, the input motions were input as rock-outcrop motions at the depth of the elastic halfspace. Additional information on the SMS site profiles and the selection of the input motions for the site response analyses are given in the following sections.

3.1 SMS Profiles

As stated above, detailed site characterization was performed at the 15 SMS sites listed in Table 1. Methods used to characterize the site included active and passive-source surface wave testing, standard penetration testing (SPT), cone penetration testing (CPT), and horizontal to vertical spectral ratio (H/V) calculations. The shear wave velocity profiles for the 15 SMS sites used in the site response analyses were developed from the combination of the active and passive-source surface wave techniques, and layering constrained using subsurface investigation data. The idealized soil stratigraphy was inferred for each profile using the collection of CPT soil behavior type indices, and SPT boreholes performed at each site. Because the depth to bedrock at some of the SMS sites is ~500 m, detailed characterization down to bedrock was only feasible for HVSC, where the depth to bedrock was only 19 m. Twelve of the other sites were characterized down to the Riccarton Gravel Formation (i.e., CACS, CBGS, CCCC, CHHC, CMHS, KPOC, PPHS, PRPC, REHS, RHSC, SHLC, and SMTC); for all of these sites, the Riccarton Gravel Formation is at a depth less than 30 m. For the remaining two sites (i.e., HPSC and NNBS), the depth to the Riccarton Gravel is greater than 30 m and the sites were only characterized down to ~30 m. For these latter two sites, the small strain shear wave velocity (V_s) profiles were extrapolated to the estimated depth of the Riccarton Gravel Formation using the relationship (Menq, 2003):

$$V_s = A_s \cdot \left(\frac{\sigma'_0}{P_a} \right)^n \quad (3)$$

where A_s is the small strain shear wave velocity corresponding to an initial effective mean stress equal to 1 atm (P_a), n is a calibrated exponent of normalized effective mean stress, and σ'_0 is the mean effective stress at the depth of the Riccarton Gravel Formation. The values of A_s and n used to model dense gravel were 312 m/s and 0.331, respectively (Menq, 2003). Values of V_s for the Riccarton Gravel Formation at HPSC and NNBS SMS sites were calculated to be 400 and 415 m/s, respectively, which fall in the range of 400-450 m/s measured at the twelve sites noted above and listed in Table 1.

Figure 5 shows the profile for the North New Brighton School (NNBS) SMS site used in the equivalent linear site response analyses, performed using ShakeVT2 (Lasley et al., 2014). The profiles for all 15 SMS sites are shown in Part A of the Electronic Supplement. The non-linear behavior of the soil was assumed to follow the Darendeli and Stokoe (2001) shear modulus and damping degradation curves.

3.2 Input Ground Motions

An ensemble of site- and event-specific motion sets were selected for use in the site response analyses. Towards this end, 15 sets of horizontal motions were selected for each SMS-earthquake scenario using the generalized conditional intensity measure (GCIM) approach (Bradley 2010a, 2012d). The GCIM-based ground motion selection process for a given scenario can be broken down into two main steps. The first step entails the selection of motions that fall within a range of target causal parameters (e.g., M_w , R_{rup} , V_{s30} , source mechanism, etc.). For this study, the following causal parameters and ranges were used: $M_w \pm 0.75$ magnitude units (i.e., M_w : 6.35-7.85 and 5.45-6.95 for the Darfield and Christchurch earthquakes, respectively), $R_{rup} < 75$ km, and $V_{s30} > 150$ m/s. Additionally, to ensure the selected motions were from shallow crustal events in active tectonic regions, analogous to the tectonic settings of the Darfield and Christchurch events, motions were only selected from the NGA West ground motion

database (Chiou et al., 2008), supplemented with “pulse-like” motions from the NGA West2 database (Seyhan et al., 2014) to ensure an adequate number of candidate motions with directivity effects.

The second step in the GCIM-based ground motion selection process entails the use of target distributions of intensity measures generated from ground motion predictive equations (GMPEs). To capture ground motion severity for a specific SMS-earthquake scenario, a multivariate distribution of intensity measures (e.g., spectral acceleration, cumulative absolute velocity, Arias intensity, significant duration, etc.) computed from GMPEs is used as the target. Each of the chosen intensity measures can be weighted according to their deemed significance, and to optimize the fit of candidate motions to the target distributions, scale factors can be applied to the amplitude of the motions.

For this study, spectral acceleration (SA), cumulative absolute velocity (CAV), Arias intensity (I_a), and 5-75% and 5-95% significant durations (D_{s75} and D_{s95} , respectively) were chosen as the target intensity measures. These intensity measures were chosen because each is considered to influence liquefaction triggering (e.g., Kramer and Mitchell, 2006). Listings of the GMPEs used to compute the respective target distributions of intensity measures and the weights assigned to each intensity measure are given in Table 2 (Tarbali and Bradley, 2014). The target distributions of the intensity measures were computed for each SMS site using the small strain shear wave velocity for the assumed elastic half space, listed in Table 1. The target SA for a damping ratio of 5% was defined at 21 oscillator periods: $T = 0.05, 0.075, 0.1, 0.15, 0.2, 0.25, 0.3, 0.4, 0.5, 0.75, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 7.5, \text{ and } 10.0$ s. For the scenarios where forward directivity was considered (Joshi, 2013), the spectral accelerations were amplified at the pulse period (T_p) determined from the actual recorded motions at the site using the Shahi and Baker (2011) narrowband model. Scale factors ranging from 0.5 to 2 were applied to the amplitude of the motions to optimize the fit of the motions to the target distributions. In total, 15 sets of horizontal motions were selected for each SMS-earthquake pair, resulting in a total of 900 motions selected (2 motions per horizontal set $\times$ 15 sets of motions per SMS-earthquake scenario $\times$ 15 SMS $\times$ 2 earthquakes = 900

motions). The sets of ground motions selected for the NNBS SMS site for both the Darfield and Christchurch earthquakes using the GCIM-based process are discussed in the following.

Figure 6 shows the M_w vs. R_{rup} distributions of selected ground motions for the NNBS SMS site for both the Darfield and Christchurch earthquakes. As expected, based on the adopted causal parameter bounds, it can be seen that the selected ground motions have magnitude and distance values that are relatively consistent with the ‘target’ values for the considered events (albeit the source-to-site distances for the Christchurch earthquake scenario are larger than the target due to the paucity of as-recorded motions for <5km distances). Figure 7 shows the median and the 16th and 84th percentiles of the 5% damped target response spectrum. For comparison, the response spectra for the selected motions are also shown, along with the median and the 16th and 84th percentiles for the selected motions. As may be observed, the distributions of SA fit the target distributions reasonably well for both earthquakes, albeit more so for the Christchurch earthquake which does not include the pronounced long-period forward directivity spectral peak. Figure 8 shows the target distributions for the other intensity measures chosen, along with those for the selected motions. For these other intensity measures, the selected motions appropriately represent the target distribution.

4. Results and Discussion

Using the Seed et al. (1975) and Green and Terri (2005) approaches for computing n_{eqM} in conjunction with the results from the site response analyses performed as described above, MSF were computed per Eq. 1 for each of the 15 SMS sites for both the Darfield and Christchurch earthquakes. In computing n_{eqM} per the Seed et al. (1975) procedure (i.e., per Eq. 2) and in computing the MSF for using the n_{eqM} from both the Seed et al. (1975) and Green and Terri (2005) (i.e., per Eq. 1), b was assumed to equal 0.34. As shown in Lasley et al. (2016), this value is consistent with the Darendeli and Stokoe (2001) shear modulus and damping degradation curves for a range of densities and effective confining stresses, based on the assumption that a CSR vs. N_{liq} curve is a contour of constant dissipated energy. This assumption

has been shown to be reasonably in accord with laboratory data (e.g., Kokusho and Kaneko, 2014; Polito et al., 2013). Additionally, $b = 0.34$ is consistent with laboratory curves developed from high-quality undisturbed samples obtained by freezing (Yoshimi et al., 1984).

The MSFs for the NNBS SMS site are shown in Figure 9 and those for all 15 SMS sites are shown in Part B of the Electronic Supplement. A few trends may be observed from this figure. First, the MSF for the Darfield earthquake are generally less than those for the Christchurch earthquake, which is expected because the magnitude for the Darfield earthquake is larger than that for the Christchurch earthquake (i.e., $M_w 7.1$ vs. $M_w 6.2$). Second, the MSF computed using the n_{eqM} from the Green and Terri (2005) approach is relatively constant with depth, although the standard deviation decreases slightly with depth; this trend was also identified by Lasley et al. (2016) (Note that the MSF computed using the n_{eqM} from the Seed et al. (1975) approach are computed using surface ground motions; hence, the corresponding MSFs and standard deviations are assumed constant with depth). Third, the MSFs computed using the n_{eqM} from the Seed et al. (1975) approach are larger than those computed using the n_{eqM} computed using the Green and Terri (2005) approach. This actually has more to do with the values used for $n_{eqM7.5}$ (i.e., 15 for the Seed et al. approach versus 8.2 for the Green and Terri approach) rather than with the computed values for n_{eqM} , which were very similar for both approaches. However, the focus of this study is on the spatial variation of MSF during the Darfield and Christchurch earthquakes, not on the absolute values of the MSF at any given site. Similarly, it should be noted that the MSF computed herein (either those computed using the n_{eqM} from the Seed et al. (1975) or the Green and Terri (2005) approaches) should not be used in conjunction with any of the existing simplified liquefaction evaluation procedures. Rather, the MSF recommended by a given procedure should be used with that procedure.

Contour plots of MSF computed using the n_{eqM} from the Green and Terri (2005) approach for both the Darfield and Christchurch earthquakes are shown in Figure 10; similar plots for MSF computed using n_{eqM} from the Seed et al. (1975) approach are shown in Part C of the Electronic Supplement. As may be

observed, similar trends are shown in contour plots of the MSF computed using the Green and Terri (2005) and the Seed et al. (1975) approaches. These contours are superimposed on maps of the Christchurch region, with areas that experienced liquefaction also shown. Because the MSF computed using the n_{eqM} from the Green and Terri (2005) approach vary to some extent with depth, the contour plots are developed from the MSF computed at the “critical” depths for each of the SMSs, where critical depth is assumed herein to be that to the layer most susceptible to liquefaction (i.e., depth to the layer having the lowest computed factor of safety against liquefaction). For SMSs sites where no layers were deemed susceptible to liquefaction, the critical depth was taken as ~4.75 m which corresponds to the median critical depth of compiled liquefaction case histories (e.g., Cetin et al., 2004; Boulanger and Idriss, 2014). Both the depths to the layer having the lowest factor of safety against liquefaction and to an initial vertical effective stress of 1 atm are identified in Figure 9 for the NNBS SMS site and are tabulated in Table 3 for all sites.

As may be observed from Figure 10, the MSF for the Darfield and Christchurch earthquakes range from approximately 0.9 to 1.2 and from approximately 0.95 to 1.46, respectively, with a general trend of MSF being higher in the southern part of the Christchurch and increasing in a northerly direction. However, despite these ranges in MSF being relatively significant, particularly for the Christchurch earthquake, there does not appear to be a consistent correlation between lower computed MSF and observed liquefaction. For example, widespread liquefaction was observed in Kaiapoi during the Darfield earthquake, which is an area having a relatively low MSF (~0.9). However, Kaiapoi did not experience as severe or as widespread liquefaction during the Christchurch earthquake, but the MSF for this area for this event are also relatively low (~0.95). More significantly, portions of the CBD and eastern Christchurch experienced severe and widespread liquefaction during the Christchurch earthquake, yet relatively high MSF are computed for these regions for this event. However, it should be realized that the seismic demand imposed on the soil is both a function of the amplitude and the duration of the motions, where the former is proportional to a_{max} . Additionally, the capacity of the soil to resist liquefaction varies

spatially, which inherently influenced the liquefaction response observed during the Darfield and Christchurch earthquakes.

In comparing contour maps of the median $a_{\max}$ values from the site response analyses (Figure 11) with the contour maps for MSFs (Figure 10), it is observed that there is a general correlation between MSF and $a_{\max}$ (i.e., regions having higher $a_{\max}$ values also have higher MSFs). This results in opposing effects on the seismic demand imposed in the soil (i.e., higher $a_{\max}$ induces higher seismic demand in the soil, while a higher MSF results in a lower seismic demand). However, of the two parameters, $a_{\max}$ ranges more widely than MSF and thus has a greater influence on the resulting seismic demand imposed on the soil. Nevertheless, the spatial variation of the MSF is significant enough that it could potentially contribute to discrepancies between predicted versus observed liquefaction.

5. Summary and Conclusions

MSF account for the durational effects of strong ground shaking on liquefaction triggering within the simplified liquefaction evaluation procedure, and it has traditionally been assumed that they are only a function of earthquake magnitude. However, this assumption fails to appreciate the inverse correlation between the peak amplitude of ground motions and strong ground motion duration, and thus would seemingly vary spatially (e.g., distance from source). The combination of well-documented liquefaction response during the Darfield and Christchurch, New Zealand, earthquakes, densely-recorded ground motions for the events, and detailed subsurface characterization provides an unprecedented opportunity to investigate the significance of the spatial variation of MSF on liquefaction triggering. Towards this end, MSF were computed at 15 SMS sites across Christchurch and its environs using n_{eqM} computed using the approaches proposed by Seed et al. (1975) and by Green and Terri (2005). Both of these approaches are inherently “total stress” approaches (i.e., no consideration is given to influence of excess pore water pressure on soil stiffness), consistent with the total stress-based simplified liquefaction evaluation procedure framework.

Trends in the spatial variation of the MSF computed using n_{eqM} from both the Seed et al. (1975) and Green and Terri (2005) approaches were similar, and the MSF computed using the n_{eqM} from the Green and Terri (2005) approach ranged from 0.9 to 1.2 and from 0.95 to 1.46 across Christchurch for the Darfield and Christchurch earthquakes, respectively. Note that the impact of the spatially varying MSF on the computed factor of safety against liquefaction would more directly indicate the significance of the results presented herein. However, the absence of a cyclic resistance ratio (CRR) curve developed from liquefaction field case histories analyzed using a spatially varying MSF precludes this at present.

From examining the spatial variation of the MSF across Christchurch, there was no consistent trend for regions with lower computed MSF having experienced more severe or widespread liquefaction. However, it was observed that there is a general correlation between MSF and a_{max} (i.e., regions having higher a_{max} values also have higher MSFs), but a_{max} ranges more widely than MSF and thus has a greater influence on the resulting seismic demand imposed on the soil. Nevertheless, the spatial variation of the MSF is deemed significant enough that it could result in discrepancies between predicted versus observed liquefaction and thus warrants consideration for incorporation in future variants of simplified liquefaction evaluation procedures.

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Table 1. Strong motion station details.

Station Name	Code	Network	Latitude	Longitude	V_s^* (m/s)	R_{rup} (km)	
						Dar eqk	Chch eqk
Canterbury Aero Club	CACS	NSMN	-43.48316539	172.5300139	600	11.9	12.9
Christchurch Botanic Gardens	CBGS	NSMN	-43.52933938	172.6198776	400	14.4	4.8
Christchurch Cathedral College	CCCC	CanNet	-43.5380850	172.6474270	400	16.3	2.9
Christchurch Hospital	CHHC	CanNet	-43.53592591	172.6275195	400	14.8	3.9
Cashmere High School	CMHS	NSMN	-43.56561744	172.6241694	400	14.0	1.5
Hulverstone Dr Pumping Station	HPSC	CanNet	-43.50157144	172.7021909	400	21.7	4.1
Heathcote Valley School	HVSC	CanNet	-43.57977835	172.7094230	760	20.8	3.9
Kaipoi North School	KPOC	CanNet	-43.37646016	172.6637603	450	27.7	17.5
North New Brighton School	NNBS	NSMN	-43.49541878	172.7179969	415	23.2	4.0
Papanui High School	PPHS	NSMN	-43.49284238	172.6069135	400	15.4	8.8
Pages Rd Pumping Station	PRPC	CanNet	-43.52580347	172.6827633	400	19.4	2.6
Christchurch Resthaven	REHS	NSMN	-43.52194513	172.6351501	400	15.9	4.9
Riccarton High School	RHSC	CanNet	-43.5361720	172.5644040	450	10.0	6.6
Shirley Library	SHLC	CanNet	-43.50533475	172.6633938	400	18.7	5.3
Styx Mill Transfer Station	SMTc	CanNet	-43.46752930	172.6138611	400	17.6	10.9

*The listed V_s are for the elastic halfspace assumed in the equivalent linear site response analyses, which corresponded to the Riccarton Gravel Formation of all sites except HVSC, where the listed value is for the Banks Peninsula volcanic rock.

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Table 2. Summary of intensity measures and weighting scheme used in GCIM ground motion selection

Intensity Measure	GMPE	Weight
Spectral Acceleration (SA)	Bradley (2013b)	60% *
Cumulative Absolute Velocity (CAV)	Campbell and Bozorgnia (2010)	10%
Arias Intensity (I_a)	Campbell and Bozorgnia (2012)	10%
5-75% Significant Duration (D_{s75})	Bommer et al. (2009)	10%
5-95% Significant Duration (D_{s95})	Bommer et al. (2009)	10%

*Evenly distributed to 21 SA ordinates (i.e. 60%/21 $\approx$ 2.86% per SA ordinate)

Table 3. a_{max} and MSF computed for the SMS sites.

Station	a _{max} (g) (Surface)		σ' _{vo} = 1 atm Depth (m)	MSF (Reference Depth)				Critical Depth (m)	MSF (Critical Depth)			
	Darfield Eqk	Chch Eqk		Darfield Eqk		Chch Eqk			Darfield Eqk		Chch Eqk	
				Sea75 ¹	GT05 ²	Sea75	GT05		Sea75	GT05	Sea75	GT05
CACS	0.31	0.30	4.5	1.11	0.85	1.07	0.86	4.5	1.11	0.85	1.07	0.86
CBGS	0.19	0.26	6.5	1.28	1.08	1.48	1.26	13.5	1.28	1.14	1.48	1.35
CCCC	0.15	0.33	9.4	1.35	1.13	1.56	1.39	5.8	1.35	1.13	1.56	1.39
CHHC	0.18	0.35	8.5	1.24	1.06	1.53	1.34	6.5	1.24	1.08	1.53	1.35
CMHS	0.24	0.55	7.4	1.43	1.20	1.56	1.47	8.2	1.43	1.22	1.56	1.47
HPSC	0.23	0.48	9.5	1.33	1.05	1.53	1.29	2.4	1.33	1.05	1.53	1.28
HVSC	0.29	0.71	9.5	1.02	0.86	1.57	1.20	4.5	1.02	0.85	1.57	1.33
KPOC	0.23	0.27	8.9	1.11	0.92	1.13	0.90	4.3	1.11	0.90	1.13	0.95
NNBS	0.25	0.44	8.9	1.35	1.10	1.58	1.22	9.8	1.35	1.10	1.58	1.22
PPHS	0.22	0.18	9.2	1.37	1.23	1.43	1.25	3.3	1.37	1.16	1.43	1.25
PRPC	0.16	0.34	8.5	1.37	1.17	1.55	1.41	4.5	1.37	1.13	1.55	1.41
REHS	0.14	0.37	10.5	1.41	1.07	1.66	1.38	2.6	1.41	1.17	1.66	1.45
RHSC	0.46	0.49	6.3	1.19	1.05	1.30	1.21	4.8	1.19	1.04	1.30	1.18
SHLC	0.24	0.49	7.5	1.15	0.91	1.60	1.32	4.5	1.15	0.91	1.60	1.34
SMTC	0.16	0.18	9.8	1.19	0.95	1.40	1.20	11.5	1.19	0.97	1.40	1.22

¹ Sea75 are MSF based on n_{eqM} computed using the Seed et al. (1975) approach

² GT05 are MSF based on n_{eqM} computed using the Green and Terri (2005) approach

Figure 1. Ariel extents of liquefaction observed in Christchurch, NZ following the two largest events in the CES. Areas delineated in black represent liquefaction from the Sept. 4, 2010 M_w 7.1 Darfield earthquake. Areas highlighted in dark gray represent liquefaction from the Feb. 22, 2011 M_w 6.2 Christchurch earthquake. The finite fault model boundaries shown are simplified from Holden et al. (2011) and Beaven et al. (2011) to highlight the rupture portions believed to have contributed the majority of the released energy.

Figure 2. Geologic cross section of Christchurch showing the complexity of the interbedded sedimentary deposits and depth to the Riccarton gravel formation for several SMSs (adopted from Forsyth et al., 2008 and Wotherspoon et al., 2015)

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606 Figure 3. Overview of the fifteen SMS sites characterized by Wotherspoon et al. (2014, 2015a,b) and used
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612 Figure 4. Illustration of the Green and Terri (2005) procedure to compute n_{eq} (Green, 2001).

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Figure 5. North New Brighton (NNBS) SMS soil profile along with profiles of: (a) CPT soil behavior type index, and (b) shear wave velocity obtained by Wotherspoon et al..

Figure 6. Distributions of causal parameters, M_w and R_{rup} , for the selected motion sets for both the: (a) Darfield earthquake, and (b) Christchurch earthquake scenarios at the NNBS SMS site.

Figure 7. 5% damped response spectra of the target motions and selected motion for the NNBS SMS site for the: (a) Darfield earthquake, and (b) Christchurch earthquake.

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Figure 8. Cumulative distributions of the target and selected motions for the NNBS SMS site for both the Darfield and Christchurch earthquakes: (a) CAV; (b) I_a ; (c) D_{s75} ; and (d) D_{s95} 5-95%.

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Figure 9. MSF computed for NNBS SMS site using the Green and Terri (2005) and Seed et al. (1975) approaches for computing number of equivalent cycles for: (a) Darfield earthquake, and (b) Christchurch earthquake.

(a)

(b)

Figure 10. Contour plots of MSF computed using n_{eqM} from the Green and Terri (2005) approach for the:
(a) Darfield earthquake, and (b) Christchurch earthquake.

(a)

(b)

Figure 11. Contour plots of median a_{max} computed at the ground surface from the site response analyses for the: (a) Darfield earthquake, and (b) Christchurch earthquake.