Non-Plastic Silty Sand Liquefaction, Screening, and Remediation

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ABSTRACT

Assessing liquefaction potential, in situ screening using cone penetration resistance, and liquefaction-remediation of non-plastic silty soils are difficult problems. Presence of silt particles among the sand grains in silty soils alter the moduli, shear strength, and flow characteristics of silty soils compared to clean host sand at the same global void ratio. Cyclic resistance (CRR) and normalized cone penetration resistance (q_{c1N}) are each affected by silt content in a different way. Therefore, a unique correlation between cyclic resistance and cone resistance is not possible for sands and silty sands. Likewise, the response of silty soils subjected to traditional deep dynamic compaction (DC) and vibro-stone column (SC) densification techniques is influenced by the presence of silt particles, compared to the response in sand. Silty soils require drainage-modifications to make them amenable for dynamic densification techniques. The first part of this paper addresses the effects of silt content on cyclic resistance CRR, hydraulic conductivity k , and coefficient of consolidation C_v of silty soils compared to clean sand. The second part of the paper assesses the effectiveness of equivalent intergranular void ratio ($e_{c,eq}$) concept to approximately account for the effects of silt content on CRR. The third part of the paper explores the combined effects of silt content (viz effects of ($e_{c,eq}$, k , and C_v) on q_{c1N} using laboratory model cone tests and preliminary numerical simulation experiments. A possible inter-relationship between q_{c1N} , CRR, accommodating the different degrees of influence of ($e_{c,eq}$, k , and C_v on q_{c1N} and CRR, is discussed. The fourth part of the paper focuses on the detrimental effects of silt content on the effectiveness of DC and SC techniques to densify silty soils for liquefaction-mitigation. Finally, the effectiveness of supplemental wick drains to aid drainage and facilitate densification and liquefaction mitigation of silty sands using DC and SC techniques is discussed.

1. Introduction

1.1. Soil Liquefaction and Screening

Current liquefaction screening techniques rely on knowledge from extensive laboratory research conducted on liquefaction resistance of clean sands and field performance data during past earthquakes. Field observations have been documented in the form of normalized penetration resistance (SPT $(N_1)_{60}$, CPT q_{c1N}) (Seed et al. 1983, Youd et al. 2001, Robertson and Wride 1998, Idriss and Boulanger 2008), and shear wave velocity (v_{s1}) (Andrus and Stokoe, 2000) versus cyclic stress ratio ($CSR = \tau / \sigma'_{vo}$) induced by the earthquakes, corrected for magnitude, for many sites. Cyclic resistance ratio (CRR), applicable for a standard earthquake magnitude of 7.5, of a soil deposit with a known value of q_{c1N} is obtained from a demarcation line drawn between the field-observation-based data points which correspond to liquefied sites and those that did not liquefy as shown in Fig.1. The CRR determined in this manner depends on fines content of the soil for a given q_{c1N} .

This has sparked numerous research on the effects of fines on cyclic liquefaction

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resistance of silty sands (e.g. Chang et al. 1982, Kuerbis et al. 1988, Vaid 1994, Koester 1994, Zlatovic and Ishihara 1997, Polito and Martin 2001). Results show that silt content affects liquefaction resistance of silty soils compared to sand at the same void ratio. Studies also show that silt content also significantly affects permeability, compressibility, and consolidation characteristics of silty sands compared to sand (Thevanayagam et al. 2001). The latter characteristics could influence cone penetration resistance as well. Two soils with the same stress-strain characteristics and liquefaction resistance but with different silt contents may have different permeability, compressibility, and coefficients of consolidation. Their cone resistance could be different due to different degrees of partial drainage, which may occur around the cone during penetration in each soil. A unique correlation between cyclic liquefaction resistance and penetration resistance may not be possible without considering the effects of fines, viz. coefficient of consolidation, on penetration resistance (Thevanayagam and Martin 2002, Thevanayagam et al. 2003, Thevanayagam and Ecmis 2008). A correlation between cyclic resistance, cone resistance, compressibility and permeability characteristics may be possible.

Figure 1. Liquefaction Screening Charts – CPT

1.2. Liquefaction Mitigation by Densification

Densification techniques such as dynamic compaction (DC) and vibro-stone column (SC) are among the most field proven and commonly used techniques for liquefaction mitigation in sands (Figs. 2a and 2c). The DC technique involves high-energy impacts to the ground surface by systematically dropping heavy weights of 5 to 35 Mg from heights ranging from 10 to 40 m to compact the underlying ground using heavy crawler cranes (Lukas 1995). Vibro-stone column installation (FHWA 2001) process involves insertion of a vibratory probe with rotating eccentric mass and power rating in the vicinity of 120kW. The probe plunges into the ground due to its self-weight and vibratory energy, which facilitates penetration of the probe. Once the specified depth (depth of stone column) is reached, the probe is withdrawn in steps (lifts) of about 1m. During withdrawal of the probe, the hole is backfilled with gravel. During each lift the probe is then reinserted expanding the stone column diameter. This process is repeated several times until a limiting condition is achieved.

Densification of silty sand deposits containing high silt contents appears to be feasible only when these techniques are supplemented with wick drains (Figs. 2 b and d) (Dise et al. 1994, Han 1998, Luehring et al. 2001). Traditionally, field design of these approaches rely on site specific field pilot trials and/or past experience based on case histories (Lukas 1995, Baez 1995). In the case of silty soils case histories are scarce. More recently advances have been made that enable detailed analyses of site response and changes in soil densities during DC

and SC installations with due consideration for the influence of soil conditions including effects of silt content and soil permeability (Shenthan 2005, Nashed 2005). These advances allow a study of the effects of wick drains, spacing between wick drains, soil permeability, impact grid pattern and impact energy in the case of DC and diameter and spacing of stone columns in the case of SC on the degree of soil densification improvement achievable in the field, and select optimum field operation parameters for DC and SC for a site.

Figure 2. Dynamic compaction and Vibro-stone columns with and without wick drains

This paper presents a summary of (a) the recent advances on understanding of the influences of non-plastic fines on undrained cyclic resistance (CRR), permeability, coefficient of consolidation C_v , and cone penetration resistance q_{c1N} of silty soils, (b) possible relationships between CRR, q_{c1N} , and C_v , and (c) effectiveness of dynamic compaction and stone columns supplemented with pre-installed wick drains for liquefaction mitigation of silty sands. Simplified design charts for liquefaction mitigation using vibro-stone columns and dynamic compaction are also presented.

2. Effects of Silt Content on Soil Properties

2.1 Cyclic Resistance (CRR)

Effect of non-plastic silt content on cyclic resistance has been the subject of research and much controversy in the early 80's until recently. A large data base (Veluchamy 2012) has been recently compiled based on available data in the literature on the effects of non-plastic fines content on undrained cyclic resistance of silty soils. The results indicate that the effect of non-plastic silt content on cyclic resistance can be appropriately accounted for using the equivalent intergranular void ratio concept (Thevanayagam 1998, Thevanayagam et al. 2002, Thevanayagam 2007). This is illustrated below, using a few example data sets.

Fig.3a shows the number of cycles (N_L) required to reach liquefaction versus void ratio for Ottawa sand/silt mix obtained from undrained cyclic triaxial tests conducted at a constant stress ratio (CSR) of 0.2 and initial confining stress of 100 kPa. The specimens were prepared by mixing Ottawa sand ($D_{50}=0.25\text{mm}$) with a non-plastic silt ($d_{50}=0.01\text{mm}$) at different silt contents (0 to 100% by weight). OS-15 in this figure refers to sand-silt mix at 15% silt content. At the same void ratio, liquefaction resistance of silty sand decreases with an increase in silt content. Beyond a transition silt content of about 20 to 30%, the trend reverses and liquefaction resistance increases with further increase in silt content. Similar observations have been widely reported in the literature (e.g. Zlatovic and Ishihara 1997, Koester 1994, Vaid 2004, Polito and Martin 2002).

(a) e versus N_L

(b) $(e_c)_{eq}$ versus N_L

(c) $(e_f)_{eq}$ versus N_L

(d) $(CRR)_{lx}$ at 15 cycles versus $(D_{rc})_{eq}$

Figure 3. Effect of silt content on liquefaction resistance CRR

The equivalent intergranular void ratio concept (Thevanayagam et al. 1998, 2002, 2003) proposes that mechanical properties such as liquefaction resistance of soils are dependent on the intergrain contact density of a soil, among other factors. Silty sand and sand at the same global void ratio are not expected to have the same intergrain contact density. Therefore, it is not appropriate to compare the liquefaction resistance of silty soil with that of clean sand using global void ratio as depicted in Fig.3a. Sand-silt mixes and host sand are expected to show similar mechanical behavior if compared at a same contact density index. A soil classification system based on contact density was developed (Thevanayagam et al. 2002) and two contact density equivalent void-ratio indices, $(e_c)_{eq}$ and $(e_f)_{eq}$, respectively, were introduced for soils at silt content (FC) less than a threshold silt content FC_{th} and more than FC_{th} , respectively. $(e_c)_{eq}$ and $(e_f)_{eq}$ have been defined as

$$(e_c)_{eq} = [e + (1-b)fc] / [1 - (1-b)fc] \quad (1a)$$

$$(e_f)_{eq} = [e / (fc + (1-fc)/R_d^m)] \quad (1b)$$

Where fc = fines content by weight, R_d = ratio of the d_{50} 's of the host sand and silt in the soil mix; b and m are soil parameters (Thevanayagam et al. 2003, Kanagalingam and Thevanayagam 2006) depending on gradation and grain size characteristics of the soil such as uniformity coefficient of coarse grain soil (C_{us}) and fine-grained soil (C_{uf}) in the soil mix (Thevanayagam 2007a, b).

The data shown in Fig3a has been regrouped into two groups, one for fines content (FC) less than a threshold value (FC_{th}) and the other for fines content exceeding the threshold value, and replotted against $(e_c)_{eq}$ and $(e_f)_{eq}$, instead of global void ratio e in the x-axis, in Figs.3b and 3c, respectively. The cyclic strength data falls in a narrow band in each case, indicating the unifying influence of $(e_c)_{eq}$ and $(e_f)_{eq}$, respectively. These relationships have not been tested at medium to high confining stresses beyond 400 kPa (Thevanayagam., et al, 2003). Further analyses show that host sand and silty sand have similar undrained shear strength and stress-strain characteristics if compared at the same contact density indices $(e_c)_{eq}$ or $(e_f)_{eq}$. This holds for many other non-plastic silty soils as recently shown in Veluchamy (2012) based on a large data base collected from the literature, including those reported by Thevanayagam et al. (2002), Thevanayagam and Martin (2002), Thevanayagam (2007), Carraro et al. (2003), Cubrinovski and Rees (2008), and numerous others, as shown in Fig.3d, as an example. While the intergranular concept does unify and narrow-down the scatter otherwise observed in the CRR data plotted against global void ratio for sands and silty sands, it does not completely eliminate the scatter, due to influence of other grain size/gradation characteristics which are not completely captured by the parameter b .

Figure 4. Parameter b and Soil Grain Characteristics

It has been further shown that the above relationships hold well for many sands and silty sands if plotted against equivalent intergrain relative density $(D_{rc})_{eq}$ [calculated using the above contact density indices instead of global void ratio]:

$$(D_{rc})_{eq} = ((e_{max})_{HS} - (e_c)_{eq}) / ((e_{max})_{HS} - (e_{min})_{HS}) \quad (2)$$

where HS=host sand. It has also been shown that other measures of liquefaction resistance such as energy E_L (per unit volume of soil) required to cause liquefaction, shear modulus, and shear wave velocity also correlate with $(e_c)_{eq}$ or $(e_f)_{eq}$ and $(D_{rc})_{eq}$ (Thevanayagam 1999, Thevanayagam and Martin 2002). As an example, Fig.3d shows a relationship between CRR (at 15 cycles to liquefaction) versus $(D_{rc})_{eq}$, for many different sands and sand-silt mixes based on the data collected from the literature. The data points for all soil mixes fall in a narrow band, further illustrating that the effects of silt content on CRR can be characterized using the inter-grain contact density $(e_c)_{eq}$ or $(D_{rc})_{eq}$ of the soil. Further a preliminary analysis of the dependence of b -parameter in Eq.1a on grain characteristics indicate an approximate relationship between b and $(R_d, C_{us}$ and $C_{uf})$ as shown in Fig.4, based on a large number of data for more than 30 soils obtained from the literature. The data sources for these soils are summarized in Veluchamy (2012).

The literature contains other correlations for b with soil gradation, including fines content (e.g. Ni et al. 2004, Kanagaingam and Thevanayagam 2006, Rahman et al. 2008, Rahman et al. 2014). Particles reorient as soil is sheared and it would be simplistic to expect a constant b for the entire range of strains. The scatter seen in Fig.4 could be partly due to the different levels of strains involved in large strain monotonic shear tests compared to relatively small strain cyclic tests up to initiation of liquefaction. Nevertheless, the intergranular void ratio concept appears to simply capture the essence of fines' participation in the shear strength of soils for practical purposes.

2.2. Hydraulic Conductivity k and Coefficient of Consolidation C_v

For the same Ottawa sand-silt soil mixes discussed above in Fig.3a, laboratory tests were also conducted to measure pre- and post-liquefaction compressibility m_v , hydraulic conductivity k , and coefficient of consolidation c_v . These data were collected using stress and volume measurements during consolidation of soil specimens prepared for monotonic and/or cyclic triaxial tests and pore pressure dissipation and consolidation following liquefaction and permeability measurements (Shenthan 2001, Thevanayagam et al. 2001). Fig.5a shows the variation of k against silt content. Hydraulic conductivity k decreases steadily with an increase in silt content due to reduction in pore size due to addition of silt up to a threshold value of FC. Beyond that, since the pore size characteristics between silt particles themselves primarily control the flow behavior at high fines contents, k did vary somewhat and was found to primarily depend on the inter-fine void ratio. Fig.5b shows the measured c_v versus $(e)_{eq}$ [$(e_c)_{eq}$ or $(e_f)_{eq}$] values, corresponding to effective confining stress of 100 kPa. Fig.5b is plotted against $(e)_{eq}$, not expecting a relationship for c_v vs $(e)_{eq}$, but to illustrate that there cannot be a narrow-banded relationship between c_v and $(e)_{eq}$ for sands and silty soils, for reasons discussed below, and to contrast it with the existence of narrow-band relationship for cyclic resistance vs $(e)_{eq}$. Fig.5c shows the normalized (c_{v0}/c_v) values at nearly the same $(e)_{eq}$ versus silt content, where c_v and c_{v0} are the coefficient of consolidation of Ottawa sand-silt mix and clean Ottawa sand, respectively.

Figure 5. Effect of silt content on k and c_v for virgin sand and sand-silt mixes

Unlike the narrow-banded liquefaction resistance versus $(e)_{eq}$ relationship, there is no apparent correlation for c_v with $(e)_{eq}$ in Fig.5b, except that c_v decreases with increasing silt content. (c_{v0}/c_v) increases steadily with an increase in silt content up to a threshold value of silt content (FC_{th}) and it is little affected with further increase in silt content. Fig.6 shows the effect of silt particles on compressibility of sands and silty sands. The effect of fines on compressibility is much less compared to its significant effects on k and, therefore has a

significant effect on c_v compared at the same $(e)_{eq}$. Unlike cyclic resistance, a correlation of c_v with $(e)_{eq}$ is not likely.

In general, an increase in silt content significantly affects the pore size and hydraulic conductivity, and hence reduces c_v . With further increase in silt content beyond FC_{th} , the degree of change in pore size is small as compared to the change before reaching the threshold silt content value. Although the absolute magnitude variation pattern for c_{v0}/c_v with silt content and the threshold silt content are dependent on the characteristics of the coarse and silt grains themselves, the general pattern as shown in Fig.5c holds well for many sand-silt mix soils.

Figure 6. Effect of fines on compressibility of virgin sand and sand-silt mixes

Notation: os00-752 = OS-00 at $e=0.752$

2.3. Summary

The above combined observations from 2.1 and 2.2 indicate that a sand and sand-silt mix show similar liquefaction resistance as the host sand or silt, when compared at the same contact density indices $(e_c)_{eq}$ or $(e_f)_{eq}$, or $(D_{rc})_{eq}$ respectively. But there is some difference in m_v and major difference in c_v ($=k/(m_v \gamma_w)$) between sand and sand-silt mixes even if compared at the same contact density indices ($(e_c)_{eq}$ or $(e_f)_{eq}$, or $(D_{rc})_{eq}$). The latter has significant implication on consolidation behavior of a silty sand compared to that of a host sand at the same $(e_c)_{eq}$, although the silty sand and sand at the same $(e_c)_{eq}$ have similar liquefaction resistance. This difference in consolidation behavior has a paramount influence on any potential correlation between cyclic resistance (which is primarily influenced by intergranular contact density) and cone resistance (which is influenced by intergranular contact density and c_v as well), as illustrated next.

3. Effects of Silt Content on Cone Penetration Resistance

3.1 Partial drainage around CPT tip

Consider penetration of a CPT probe into a saturated soil. The penetration causes high stresses and shear strains in the soil around the probe. The spatial distribution of shear strain and the excess pore pressures around the probe are highly non-uniform. Therefore, depending on the rate of penetration, geometry of the penetrating object, stress-strain characteristics, and consolidation characteristics of the soil a different degree of partial dissipation of excess pore pressures and consolidation is expected to occur around the probe depending on silt content. Therefore, the penetration resistance is expected to differ. It may be argued that if the stress-strain characteristics of sand and silty sand are the same, the penetration resistance would be

identical in both soils if the penetration rate is either fast enough for partial drainage around the probe to be negligible or penetration is slow enough for fully drained conditions to develop around the probe. It means that for the same penetration resistance obtained from such ideal tests, the liquefaction resistance of both soils must be the same as well.

In reality, however, even if the stress-strain characteristics and the liquefaction resistance of two soils are the same, the degree of drainages around the probe are not the same for the same rate of penetration due to major differences in coefficients of consolidation between the two soils. Therefore, the penetration resistances would be different.

3.2. Model Cone Tests

Recently, an exploratory model cone test was performed at University at Buffalo on clean Ottawa sand and silty sand mix at 25% silt content, the same soils reported in Figs.3-6. Cone penetration tests were done on each soil mix while they were dry and again while they were fully saturated. The test hypothesis was that, if cone resistance is affected by primarily intergranular contact density alone, the cone resistances must be very similar for both soils at the same $(D_{rc})_{eq}$. If k and c_v do affect the cone resistance, then the cone resistances for sand and silty sand should be very similar for dry soils prepared at the same $(D_{rc})_{eq}$, but the resistances should differ for the two fully saturated soils, prepared at the same $(D_{rc})_{eq}$.

Figure 7a. Overview of CPT Chamber
(Unit: mm in parenthesis)

Figure 7b. CPT Chamber system

The tests were conducted using a 1.27 cm diameter (d) scaled-model CPT probe in a cylindrical test chamber (internally 50.8 cm dia, 49 cm high, Fig.7). It involved dry pulviation of the soil, and back-pressure saturation for fully-saturated tests. Soil dry weight, initial moisture content (typically ranging in 0 to 0.3%), volume of the container when the soil is filled, soil volume changes during saturation and consolidation, the volume of water introduced to the specimen, and compliance of the chamber were recorded. They were used to estimate the final void ratio of the specimen prior to cone penetration. Two different methods were used to estimate the final void ratio: (a) based on test chamber volume after correcting for settlement in soil height during consolidation, and small chamber expansion due to pressure, and (b) based on dry weight of sand and volume of water used to saturate the specimen, which does account for volume changes during saturation. Where applicable, this is shown by a band in Fig.8a, introduced later. The effective vertical stress in the soil prior to

cone penetration was typically about 50 kPa. Sidewall soil friction in the chamber was addressed using a lubricated thin film placed between the soil and the side wall of the chamber. A needle-tip piezometer was planted at a vertical distance δ below the cone tip (Fig.7a) to monitor how pore pressure develops as cone is penetrated. Porewater pressure at the piezometer needle-tip and the distance δ were continuously monitored during cone penetration until the cone tip reached the needle-tip, viz. until $\delta=0$. The measured cone resistance data was normalized for 100 kPa of confining stress to obtain q_{c1N} .

Fig.8a shows measured q_{c1N} versus $(D_{rc})_{eq}$ data from the above tests. The following observations can be made.

- When the sand (open triangle) and silty sand (solid triangle) were dry, the q_{c1N} versus $(D_{rc})_{eq}$ data for sand and silty sand follow a similar trend, not indicating any significant effect of silt content on q_{c1N} . This is because the cone resistance in dry soils is influenced by the intergranular void ratio, and not (air) permeability or (air) coefficient of consolidation. The tests were fully drained for both soils.
- When the q_{c1N} versus $(D_{rc})_{eq}$ data for dry sand (open triangle) is compared with saturated sand (open square), again, no difference is observed in the q_{c1N} versus $(D_{rc})_{eq}$. Moreover no excess pore pressures were observed for saturated sand as shown in Fig.8b. This is because, for all practical purposes, the cone penetration is almost fully drained response in saturated sand, and hence does not noticeably differ from q_{c1N} for dry sand at the same $(D_{rc})_{eq}$.
- When saturated soils are compared, saturated silty sand (solid square) shows significantly low q_{c1N} compared to q_{c1N} for saturated sand (open square) at the same $(D_{rc})_{eq}$. This is because, the presence of silt reduces the k and c_v in saturated soils, and therefore makes the penetration in saturated silty sand partially or fully undrained response, whereas the penetration in sand is fully or nearly drained response. Therefore they have different q_{c1N} response at the same $(D_{rc})_{eq}$.
- Likewise, saturated silty sand (solid squares) exhibits a low q_{c1N} compared to dry silty sand (solid triangle), at the same $(D_{rc})_{eq}$, for the same reasons as above.

Fig.8b shows the pore pressure at a normalized distance δ/d below the cone tip as the cone was continuously pushed into the soil in saturated silty sand (25% silt) compared to saturated sand. A decreasing δ/d means the cone tip was approaching the piezometer location. $\delta/d = 0$ means that the cone tip has reached the piezometer location and the test was terminated at that point. There was no noticeable change in pore water pressure for saturated sand for the entire penetration process, indicating mostly fully drained cone penetration. In silty sand, no pore pressures were felt at the piezometer location far below the cone tip until the cone tip reached a normalized distance of about 4. From then on, there was noticeable change in porewater pressure increasing as the cone tip approached the piezometer location. This observation indicates that pore pressures are generated around the cone in a silty sand and it is not fully dissipated immediately, unlike in sand. Only a partial drainage occurs around the cone tip in silty sands. These tests were performed using a small diameter cone and penetrated at a slow rate of 0.4 cm/s. A standard cone is 4.37 cm penetrated at 2 cm/s. It would be expected that the porewater dissipation rates around the large standard cone would be much slower by about an order of magnitude compared to the laboratory cone used in the above experiments, and therefore larger porewater pressures would remain around such a standard cone in silty sands and influence the q_{c1N} .

These model cone tests are preliminary and the interpretations reported herein are subject

to further research study, and analysis, with several series of new data sets to be generated on this soil mix and other soil mixes, including tests involving larger size cones and faster penetration rates before definitive conclusions can be made.

However, when compared at the same $(D_{rc})_{eq}$ (or $(e_c)_{eq}$), it is noted that the presence of silt, which influences c_v , q_{c1N} of saturated silty sand is lower than that of clean sand (Fig.8a), whereas the cyclic resistances are nearly the same (Fig.3b) for sand and silty sand. This points to a three-way CRR- q_{c1N} - c_v relationship that can lead to use of in situ q_{c1N} and c_v to determine CRR to perform liquefaction screening in sands and non-plastic silty soils.

Figure 8. Effect of fines (k and c_v) on q_{c1N} and porewater pressure (Δu) for sand and silty sand (25% silt)

The above experimental observations in Fig.8 could be explained in a critical state soil mechanics framework. Recent work has shown that the equivalent intergranular void ratio concept unifies the behavior of sand and silty sand within the critical state soil mechanics framework (Thevanayagam and Mohan 2000, Rahman, Lo, and Dafalias 2014). The critical state soil mechanics principles and conceptual soil models developed for sand in terms of global void ratio and state parameter can be applied equally well to silty sands using intergranular void ratio and intergranular state parameter instead of global void ratio and state parameter. Fig. 9 shows a schematic diagram of the soil behavior of a contractive sand and silty sand around a cone tip in a $(e_c)_{eq}$ vs $\log(p')$ plane.

For host sand, the shear response around a cone tip is nearly fully drained due to high permeability (and high c_v) and therefore the shear path would resemble that of AB_1 . For a silty sand, at the same initial state with the same $(e_c)_{eq}$ as sand, the permeability and c_v are much lower than that of sand. Therefore, the shear path is partially drained, and follows a path (schematically AB_2 or AB_3) located between AB_1 and AC . AC is a fully undrained path, which may be relevant only when the fines content is very high leading to very low permeability and c_v . This in turn reduces the cone resistance of the silty sand compared to the resistance of sand at the same $(e_c)_{eq}$ or $(D_{rc})_{eq}$ and initial confining stress.

Figure 9. Effect of fines and drainage on shear behavior of a normally consolidated soil

4. Numerical Simulation Experiments of Cone Penetration

4.1. Cone Penetration Simulation

The influence of k and c_v and the associated possible differences in partial drainage conditions that may prevail around a cone tip during cone penetration and their influence on cone penetration resistance, q_{cIN} of sand and silty sand were studied using finite element numerical simulation experiments using finite element software, ABAQUS (2000). As an exploratory study, the soil was simulated using Drucker-Prager model. An axisymmetric soil model was set up as shown in Fig.10. The cone tip was placed at a depth below the top surface of the finite element mesh. On the bottom and two vertical sides, the normal component of displacement and fluid flow were fixed at zero. No pore fluid flow was permitted across cone body. A vertical effective stress of 100 kPa was imposed at the top surface of the mesh. The soil was fully saturated. The diameter d of the cone was 4.37cm.

Two independent sets of numerical experiments (referred to as analysis 1 and analysis 2 in Fig.11 later) were done by two different students. In the first case, the cone was placed at 35 cm below the top surface of the mesh, and the mesh extended to a distance of 54cm (about 15d) below the cone tip and 40 cm away horizontally (diameter ratio of about 18d) from the cone axis (Ecemis 2008). In the second case, the cone was placed at 50cm below the top surface, and the mesh extended to a distance of 1.3m below the cone tip and 80cm away horizontally (diameter ratio of 36d) from the cone axis (Huang 2014). Each case utilized a different mesh technique.

Several numerical cone penetration simulations were done for a select number of soil densities, $(D_{rc})_{eq}$. Material properties relevant for each $(D_{rc})_{eq}$ (Huang, 2014), including dilation angles, required for these were obtained from several sets of undrained triaxial test data on Ottawa sand and sand-silt mixes for a wide range of relative densities characterized by $(D_{rc})_{eq}$ (Thevanayagam et al. 2003). The mechanical material parameters required to model stress-strain characteristics, except for permeability, are the same for a silty sand and sand at the same $(D_{rc})_{eq}$ (Thevanayagam and Mohan 2000). For each density $(D_{rc})_{eq}$, the simulated triaxial stress-strain was compared with the experimental triaxial data to verify that the major features of experimental stress-strain data were captured by the triaxial test simulations. Each case (analyses 1 and 2) had a slightly different set of material parameters for the same soils, as a matter of different human interpretations of the same source triaxial test data. Following this, the material parameters were utilized for cone penetration simulations. For each simulation at a given $(D_{rc})_{eq}$, the permeability was changed to account for the effects of fines on permeability k . The soil was modelled as isotropic material and

both horizontal and vertical permeabilities were assumed to be the same. Pore pressure responses and cone penetration resistances were monitored while the cone was penetrated at a constant rate $v = 2 \text{ cm/s}$. The normalized cone resistance q_{c1N} for each sand and silty sand simulated was plotted against a normalized parameter T :

$$T = vd/c_v \quad (3)$$

where v = penetration velocity, d =diameter of cone, c_v =coefficient of consolidation ($=k/m_v\gamma_w$). Figs.11a-b show the results for the cases of analysis 1 and 2.

Figure 10. Finite element mesh – CPT model

4.2. Effect of k and c_v on q_{c1N}

Apart from the differences in the absolute magnitudes of q_{c1N} due to the differences in mesh techniques, and slight different input soil parameters between the two cases of numerical analyses, a clear trend emerges from Figs.11a-b. For each $(D_{rc})_{eq}$, the q_{c1N} depends on consolidation characteristics parameter T . The q_{c1N} at the cone tip steadily decrease with an increase in T (decrease in c_v). It is noted that such a relationship trend between cone resistance and the parameter T has also been reported for clay soils (Schneider et al. 2007).

An intriguing question arises from the observations in Fig.11, for high density soils. The cone resistance for an undrained penetration (at high T values) appears to be lower than the resistance for a drained penetration (at very low T values) for high density soils as well. An undrained triaxial test at low confining stresses such as 100 kPa on a dense specimen would be expected to provide a high undrained shear strength, due to dilation, compared to the drained shear strength of a specimen subjected to drained triaxial test at the same density. This trend is not observed in Fig.11, and this appears to be a paradox.

It is surmised as follows. Apart from possible contributions from numerical accumulation of errors during step-wise penetration simulation, the reason for the above paradox is likely due to differences on the size and geometry of the ‘deformed soil failure block’ around the cone between drained penetration and undrained penetration in the same soil at the same density. The two triaxial tests referenced above reflect simply the strength of the soil subjected to no excess pore pressures (drained test), or spatially equal excess pore pressures throughout the specimen (undrained test) caused by dilation for dense specimens (or contraction for loose specimens). Cone penetration is not a single-soil-element test. In contrast, cone resistance is a function of the strength of the soil elements around the cone and

the geometry of deformation shape of the soil around the cone and the spatially varying pore pressures, if they are induced. Due to development of high and spatially varying excess pore pressures around the cone tip and the consequential spatial differences in the distribution of effective stresses, the deformed size and geometry of the soil around the cone for an undrained penetration would be different from the case for a drained penetration, where there is no excess pore pressure. Therefore, the cone resistance for the respective two cases referenced above would not be directly proportional to the respective triaxial strengths at the same density, but other geometric factors induced by the penetration. This reasoning applies to loose specimens as well. However, such a hypothesis has not been thoroughly investigated. Furthermore, in the laboratory cone tests on dense silty sand specimens did not show higher q_{c1N} than sand at the same density.

In Fig.11a, for a given value of $(D_{rc})_{eq}$, at values of T less than about 0.01 to 0.05 (high c_v) the q_{c1N} is high and is little affected by further decrease in T , indicating a nearly fully drained penetration. For values of T higher than about 5 to 10 (low c_v), the q_{c1N} reaches a low value and remains little affected by further increase in T , indicating nearly undrained penetration. For intermediate values of T of about 0.01 to 5, q_{c1N} steadily decreases with an increase in T (decrease in c_v), indicating the effect of a partially drained condition around the probe on q_{c1N} . Similar trend is observed in Fig.11b, with slight differences in magnitudes of q_{c1N} and T compared to Fig.11a.

Figure 11. q_{c1N} versus T at constant $(D_{rc})_{eq}$

4.3. Summary

The observations from Fig.11a-b imply that, q_{c1N} for a low permeable silty sand would be smaller than that of highly permeable clean sand at the same $(D_{rc})_{eq}$. This difference is attributable to the presence of fines, which causes low k and c_v and undrained or partially drained conditions during penetration in silty sands leading to a decrease in tip resistance compared to highly permeable sand. The above observations indicate that the fines-content-dependent liquefaction screening chart shown in Fig.1 could be further refined to develop a more rational T -dependent q_{c1N} -CRR relationship (Thevanayagam et al. 2016).

5. Effects of Silt on Liquefaction Remediation of Silty Soils

5.1. Soil Response During DC and SC

Soil response during dynamic compaction (DC) and stone column (SC) installation involves complex processes. Modeling of these processes and developing analytical tools to assess the increase in soil density, resistance to liquefaction, and cone resistance due to DC and SC are even more complex. The following sections present a simplified approach to model these processes and the results from such analyses.

Excess pore pressure generation and liquefaction in saturated granular soils is a process involving energy dissipation due to friction along grain contacts during cyclic loading, leading to contact slips and concurrent increase in excess pore pressures. The energy required to reach a certain level of pore pressure or cause liquefaction depends primarily on the density of packing and effective confining stress. The magnitude of induced excess pore pressure depends on the cumulative energy dissipated per unit volume of soil, soil density, and confining stress (Berrill and Davis 1985, Figueroa et al. 1994, Thevanayagam et al. 2000, Kayen and Mitchell 1997, Green and Mitchell 2004). If the energy dissipated in a saturated loose granular soil due to vibratory tamping, such as DC, or vibratory stone column installation approaches or exceeds the energy required to cause liquefaction, pore pressure in localized zones around the impact area increases. Soil density increases during dissipation of excess pore pressures (Figs. 12a-b).

During the vibratory process, the energy delivered at the vibratory source generates body waves and Rayleigh waves. As these waves radiate and spread through the soil deposit causing vibrations of soil grains, the intensity of energy decays due to geometric radiation due to spreading and loss due to material damping. The energy loss due to material damping causes rise in excess pore pressures. The induced pore pressures are high near the impact zone and decreases with distance from the impact zone. In the case of sands, the permeability of the soil may be large enough for rapid dissipation of the excess pressures. In the case of silty sands supplemented with wick drain, these wick drains facilitate dissipation of excess pore pressures. In both cases, due to repeated vibratory applications, pore pressures increase and dissipate cyclically, and the soil density and the lateral confining stresses around the impact zones increase, resulting in an increase in resistance to liquefaction as well as cone resistance.

6. Numerical Simulation of DC and SC

6.1. Numerical Simulation

Based on energy principles governing pore pressure generation, simple models for energy dissipation in soils during DC and SC, and consolidation theory a set of numerical simulation models have been developed to simulate the performance of soil deposits and determine density changes, and increase in liquefaction resistance and cone resistance due to DC and SC. The theoretical and numerical aspects of these simulations are presented in detail in Shenthan (2005), Nashed (2005), Thevanayagam et al. (2006), and Thevanayagam et al. (2009). It involved use of attenuation relationships for propagation of ground vibrations (Richart et al. 1970) to estimate the spatial distribution of vibration amplitude, and energy dissipated in the soil due to vibration caused during DC and SC. Experimental relationship between magnitude of normalized pore pressure generated versus cumulative energy dissipated per unit volume of soil due to cyclic loading (Figueroa et al. 1994, Kanagalingam and Thevanayagam 2006) was used to estimate the spatial distribution of field excess pore

pressures generated by vibrations caused by DC and SC based on energy dissipated in the soil. Coupled consolidation equations were used to simulate excess pore pressure dissipation and soil consolidation to determine post-improvement soil density profiles. The influence of non-plastic fines content was taken into account in this simulation model by considering their effects on liquefaction resistance using equivalent void ratio concept as well as the effects of silt content on c_v and k .

Figure 12. Soil response during dynamic compaction and stone column installation, supplemented with wick drains

6.2. Dynamic Compaction

Several parametric studies were conducted, using the above simulation approach, to study the effects of k , wick drain spacing S_w and diameter d_w , impact grid spacing S , impact energy

WH, number of impacts per grid point, and number of passes on the effective depth of influence $d_{\max}$ feasible by dynamic compaction as well as to determine post-improvement density profile. The cumulative energy applied at the simulation sites ranged from 100 to 300 $\text{Mg}\cdot\text{m}/\text{m}^2$. In each case, the soil profile at the site was considered to be uniform loose soil with initial equivalent normalized SPT blow count of $(N_1)_{60\text{cs}}$ of 7.5 (Nashed 2005, Thevanayagam et al. 2006). The blow counts were converted to equivalent clean sand relative densities using Tokimatsu and Seed (1984) (Nashed 2005, Thevanayagam et al. 2006, 2009). The groundwater was assumed to be at 2 m below the ground surface. The impact grid pattern for silty sand sites was assumed as shown in Fig. 13, with $S=15.2$ m. Each grid point received a total of 12 impacts. The time cycle to between any two consecutive impacts was selected as 2 minutes. k was varied in the range of 10^{-7} to 10^{-8} m/s to represent the effect of different amounts of silt content. S_w was varied from 1 to 2 m. The equivalent diameter of the wick drains was 5 cm. Although most of the studies for silty soil site included presence of pre-installed wick drains, for comparative analyses purposes a few simulations were also conducted assuming presence of no pre-installed drains using an impact grid pattern shown in Figure 14. Effective depth of influence of dynamic compaction $d_{\max}$ was determined at a location midway between primary and secondary impact locations.

Figure 13. Typical impact grid pattern in silty sand site with wick drains

Figure 14. Typical impact grid pattern in sand site without wick drains

The studies for clean sand sites were done without pre-installed wick drains using the grid pattern shown in Fig. 14, with $S = 6$ m. Each grid point received a total of 8 impacts. k was set in the range of 10^{-5} to 10^{-6} m/s, representing fine sand. The d_{max} was determined at the center of the square impact grid pattern.

Effect of k on d_{max} – DC in clean sand without Wick Drains: Fig. 15 shows the results for d_{max} versus WH for clean sand, using grid pattern shown in Fig. 14, without pre-installed wick drains. The empirical relationship for d_{max} ($=n(WH)^{0.5}$ for sands with $n=0.5$, Lukas 1995) is also shown in this figure. The numerical simulations are in close agreement with the empirical relationship. The numerical simulations for silty soil sites, without pre-installed wick drains, with k less than 10^{-6} m/s showed little or no improvement using grid pattern in Figure 14. Therefore, further simulations were not carried out.

Effect of k on d_{max} – DC in silty sand with Wick Drains: Fig. 16 shows the relationship between d_{max} and WH for silty sands with wick drain spaced at 1.5 m for two different values of k . The empirical relationship for d_{max} ($=0.5(WH)^{0.5}$) for highly permeable sand sites is also superimposed in this figure. The results show that, when closely spaced wick drains are present, d_{max} increases with WH. d_{max} increases with an increase in k and approaches that of sands. The results show that silty soils with k values as low as 10^{-7} m/s to 10^{-8} m/s may be improved by dynamic compaction provided that pre-installed wick drains are present.

Figure 15. Clean sand site without wick drain
(N_1)_{60cs} = 7.5

Figure 16. Silty sand site with wick drain
(N_1)_{60cs} = 7.5, S_w = 1.5 m

Effect of Drain Spacing on d_{max} : Figs. 17 and 18 show the effect of wick drain spacing on d_{max} for a silty sand deposit with $k = 10^{-7}$ m/s and pre-improvement (N_1)_{60cs} of 7.5. As the drain spacing gets closer, the tributary area covered by the drains become smaller and the drains become more effective in dissipating the excess pressures during DC installation. As a result, the depth of influence of dynamic compaction increases with a decrease in drain spacing.

Pre- and Post-DC (N_1)_{60cs} Profile: Several additional numerical simulations were conducted to obtain a relationship between pre- and post-dynamic compaction densities for various uniform silty soil sites, pre-installed with wick drains. For all simulations, the impact grid pattern was assumed to be as shown in Fig.13. Each simulation included three phases of impact, primary, secondary, and tertiary, respectively, at the grid locations shown in Fig.13. The energy per impact (WH), impact grid spacing S , total number of impacts per grid point

during each phase (N_I), wick drain spacing S_w , wick drain size d_w , and time cycle between impacts t_0 were varied for each simulation. Groundwater level was assumed to be at 2.0 m depth from impact surface. After each simulation, the density profiles were converted to $(N_I)_{60cs}$ (Tokimatsu and Seed 1984, Nashed 2005, Thevanayagam et al. 2006, 2009).

Figure 17. Effect of wick drains spacing on d_{max}
($k=10^{-7}$ m/s, $(N_I)_{60cs}=7.5$, $S_w=1.5$ m)
(silty sand site, WH=500 Mg.m)

Figure 18. Effect of k on d_{max}
($k=10^{-7}$ m/s, $(N_I)_{60cs}=7.5$)

a) $k=10^{-7}$ m/s, $pre-(N_I)_{60cs}=7.5$ b) $k=10^{-7}$ m/s, $pre-(N_I)_{60cs}=16$ c) $k=10^{-8}$ m/s, $pre-(N_I)_{60cs}=7.5$ d) $k=10^{-8}$ m/s, $pre-(N_I)_{60cs}=16$

Figure 19. Pre- and Post-improvement $(N_I)_{60cs}$ for $S = 15$ m (Post 750: WH = 750 Mg. m)

a) $k=10^{-7}$ m/s, $pre-(N_I)_{60cs}=7.5$ b) $k=10^{-7}$ m/s, $pre-(N_I)_{60cs}=16$ c) $k=10^{-8}$ m/s, $pre-(N_I)_{60cs}=7.5$ d) $k=10^{-8}$ m/s, $pre-(N_I)_{60cs}=16$

Figure 20. Pre- and Post-improvement $(N_I)_{60cs}$ for $S = 12$ m (Post 500: WH = 500 Mg. m)

Two sets of results are presented in Figs. 19 and 20. For these examples, $d_w = 5$ cm and $S_w = 1.5$ m. Wick drains were pre-installed in a rectangular pattern. The number of impacts per grid location and the time cycle between impacts were set at $N_I = 12$ and $t_0 = 2$ min., respectively. Fig. 19 shows the pre- and post-improvement $(N_I)_{60cs}$ profiles for two uniform

soil deposits with pre-improvement $(N_1)_{60cs}=7.5$ and 16, respectively and impact grid spacing of $S=15$ m. Figs.19a-b are for $k=10^{-7}$ m/s, and Figs. 19c-d are for $k=10^{-8}$ m/s. Each curve in these figures show the pre-improvement profile and post improvement profiles, respectively, for a different energy per impact WH of 100, 250, 500, and 750 Mg.m, respectively. Fig. 20 is for impact grid spacing of $S = 12$ m and energy per impact WH of 100, 250, and 500 Mg.m, respectively. The improved soil profiles follow a pattern similar to those observed in field case histories. Comparisons of simulation results for specific case histories are presented elsewhere (Nashed et al. 2009a-b).

6.3. Vibro-stone Columns

In the case of vibro-stone columns, numerical simulations were conducted to obtain the relationship between pre- and post-improvement densities for various uniform soil sites containing clean sands to non-plastic silty soils supplemented with or without wick drains. The diameter of stone columns was set at 0.9 m installed in a triangular pattern. Three different area replacement ratios were considered ($A_r = 5.6, 10$, and 22.5%). Area replacement is defined as the cross sectional area divided by the tributary area of the soil surrounding each stone column. For all simulations, the power rating of the vibratory probe was set at 120 kW. In cases where supplementary wick drains were considered, the diameter of wick drains was assumed to be 5 cm, pre-installed at midpoints between stone columns. The results for post-improvement density profiles were converted to equivalent normalized clean sand SPT blow counts $(N_1)_{60cs}$. Fig.21 shows these results, expressed in terms of post-improvement $(N_1)_{60cs}$ for soils with different hydraulic conductivities k , for a set of pre-improvement values of $(N_1)_{60cs}$ and A_r . The three figures in the first row (Fig.21a) represent soils with pre-improvement $(N_1)_{60cs}$ of 7, 11 and 16, respectively, improved using $A_r=5.6\%$. The second and third rows (Figs.21b-c) are for soils improved using $A_r=10\%$ and 22.5% , respectively. Each figure has two curves, one for improvement with stone columns only, and the other for improvement by stone columns supplemented with pre-installed wick drains.

Figure 21. Vibro-stone columns – Post-Improvement design charts

(SC + Wicks = vibro-stone column with wicks; SC = vibro-stone column without wicks)

Results indicate the following. Stone columns without pre-installed wick drains are effective in improving sands with k values larger than 10^{-5} m/s. Effectiveness of SC diminishes with a decrease in k (or with an increase in silt content). At A_r approaching 22.5% or higher, stone columns may be effective in improving silty soils with k values as low as 10^{-7} m/s, provided that wick drains are pre-installed. The degree of improvement that can be achieved diminishes with a decrease in k (or silt content).

7. Conclusions

Non-plastic silt content in silty sands affects liquefaction resistance, permeability, coefficient of consolidation, cone penetration resistance and soil response during ground improvement using dynamic compaction and stone columns in different ways. Silt content affects the intergrain contact density of the soil compared to that of a sand at the same void ratio. When this is approximately taken into account, sand and silty sand show similar liquefaction resistance at the same equivalent void ratio $(e_c)_{eq}$. Silt content also significantly affects the hydraulic conductivity k and coefficient of consolidation c_v in silty soils compared to sand. Cone resistance is sensitive to $(e_c)_{eq}$ as well as k and c_v . Cone resistance of a silty sand is smaller than that of a sand at the same equivalent void ratio $(e_c)_{eq}$. This difference is apparently caused by different drainage conditions that prevail around a cone tip in silty sand compared to that of sand, due to differences in c_v . It appears that normalized cone resistance q_{c1N} is dependent on a parameter $T (=vd/c_v)$ that represents c_v , cone diameter d and penetration speed v . There is likely a correlation possible between liquefaction resistance CRR, q_{c1N} and T .

Low hydraulic conductivity and c_v for silty soils appear to adversely affect soil densification process during dynamic compaction and stone column installation. Pre-installation of closely spaced wick drains appears to expedite dissipation of excess pore pressures during DC and SC installation and enhance soil densification. In both cases, silty soils with k values as low as 10^{-7} m/s may be effectively improved using DC and SC, with preinstalled wick drains, for liquefaction mitigation. Additional field test data are needed to further verify and refine these findings.

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