A novel interpretation of fatigue delamination growth behavior in CFRP multidirectional laminates

Libin Zhao¹, Yu Gong¹, Jianyu Zhang^{2, *}, Yana Wang¹, Zizi Lu³, Lei Peng³, Ning Hu²

1 School of Astronautics, Beihang University, Beijing 100191, China

2 College of Aerospace Engineering, Chongqing University, Chongqing 400044, China

3 Beijing Aeronautical Science & Technology Research Institute, Beijing 102211, China

Abstract: Two Paris-type models are proposed to characterize the fatigue delamination growth (FDG) behavior in CFRP laminates. The crack-driving forces denoted by two definitions of the strain energy release rate (SERR) range against the fatigue delamination resistance of composite materials are introduced to control the FDG. The first definition is the square of the difference between the square root of maximum SERR and that of minimum SERR. The second one is the arithmetic difference between the maximum and minimum SERR. A series of fatigue delamination tests of CFRP multidirectional laminates under various stress ratios and mode-mixity ratios were carried out. Among different Paris relations, the FDG rate corrected by the first model exhibits a linear relationship with R -ratio dependency, and the second model further removes the R -ratio dependence, under different mode-mixity ratios, on a log-log scale. The experimental results indicate that the presented models can provide more generic descriptions and a proper interpretation of the FDG behavior in CFRP multidirectional laminates.

Keywords: B. Delamination; B. Fatigue; C. Strain energy release rate; Fatigue delamination growth

* Corresponding author: Jianyu Zhang, E-mail: jy Zhang@cqu.edu.cn

1 Introduction

Composite laminates consisting of continuous carbon-fiber-reinforced plies are now widely employed in engineering structures due to their improved performance over conventional metallic materials. However, the lack of through-thickness reinforcement means that delamination between two adjacent layers is one of the most serious damaging behaviors in laminated structures [1-5]. When subjected to fatigue loading, delamination usually originates from manufacturing flaws or interlaminar stress raises and propagates after initiation, which causes the degradation of the structural behavior and even the catastrophic failure of the laminated structures [6]. To increase the fatigue life and reliability of composite structures, numerous efforts have been made to understand the fatigue delamination growth (FDG) behavior in unidirectional laminates [7]. However, regarding widely applied multidirectional laminates, the studies are still limited due to the complication that both material properties and mechanical factors affect the FDG behavior and some different phenomena, such as fiber bridging, crack migration and matrix cracking. Furthermore, plies with different orientations in the multidirectional laminates usually make the delamination grow in mixed modes [8].

Fracture mechanics approaches have been very useful in providing phenomenological descriptions of different failure processes in materials. Analogous to metal crack growth in terms of the stress intensity factor K , the FDG rate in composite laminates is known to be correlated to different formulations of the strain energy release rate (SERR) G , which usually exhibits a linear relationship on a log-log scale. The use of G is preferred over the use of K when characterizing the steady-state FDG in composite laminates due to the relative ease on the calculation of G during tests and the avoidance of oscillatory singularities of complex K solutions for inhomogeneous layered materials. Analogous to

the Paris relation for metal crack growth, a consensus view stating that the FDG rate possessing the phenomenal and functional form given below has been reached, because the FDG rate demonstrates a linear relationship on a log-log scale with different formulations of the SERR from experimental observations:

$$\frac{da}{dN} = Cf(G)^r \quad (1)$$

where C and the exponent r are material constants, which must be determined experimentally.

So far, the formulation of $f(G)$ for quantifying the delamination growth has still been the bone of contention in the research community. For a cyclic loading that causes a variation of the SERR from $G_{\min}$ to $G_{\max}$, the stress state ahead of the crack tip is expected to be described by $G_{\max}$, the stress ratio R and the mode-mixity ratio ϕ , because these parameters describe both the intensity and variation of the stress field and exhibit the experimentally observed effect on the delamination growth behavior in composite laminates [1]. Up to now, the FDG behavior under cyclic loadings have been extensively investigated, and various models have been proposed by numerous researchers [9]. Kenane and Benzeggagh [10] as well as Tumino and Cappello [11] used $G_{\max}$ to characterize the FDG behavior under mixed mode loadings. The prevalent use of $G_{\max}$ is due to its importance in assessing static failure limits. However, it neglects the information of the minimum load [12], which results in R -ratio dependency. Hojo et al. [13] studied the near-threshold growth of mode I fatigue delamination in unidirectional laminates made of T300/914 and Toray P305 prepregs. Their results showed that the FDG rates were related to $G_{\max}$, $\Delta G = G_{\max} - G_{\min}$ and ΔK . It was found that the R -ratio effect was negligible in the stable growth stage for T300/914 laminates when the FDG rates were corrected with ΔG , while a small dependency existed for

Toray P305 laminates. Hence, a complex controlling parameter $\Delta K_{eq} = \Delta K(1-R)^{\gamma}$ was proposed to merge various stress ratio curves into a single one. However, Rans et al. [12] thought that the characterization of fatigue crack/delamination growth based on K rather than G should not be fundamentally different, because K and G are equivalently parameters in linear elastic fracture mechanics. They highlighted a potential misinterpretation of the FDG behavior when ΔG was chosen as the arithmetic difference between $G_{\max}$ and $G_{\min}$, which is analogous to the use of ΔK . Therefore, a new definition of the G range, $G_{\Delta P} = \left(\sqrt{G_{\max}} - \sqrt{G_{\min}} \right)^2$, which was defined using a linear superposition approach and in accordance with the principle of similitude for characterizing metal crack growth, was presented to remove the R -ratio dependency when adopting $\Delta G = G_{\max} - G_{\min}$. This expression was also used in other studies [14-16]. Wang et al. [17] firstly introduced the material resistance in the expression of delamination growth rate. They suggested that G should be normalized by the fracture toughness (G_c) rather than being entered into the Paris relation directly. The quantity G/G_c presented the real crack-driving force relative to the material resistance. This approach was subsequently adopted by many other researchers [18-22]. In this approach, the fracture toughness is assumed to be a constant value. However, an obvious R -curve exhibiting the relationship between the fracture toughness and delamination length shows that the fracture toughness was actually variable [4, 23]. Shivakumar et al. [24], Chen et al. [25] and Murri [26] proposed that G should be normalized by $G_c(a)$ in the Paris relation. They supposed that the quasi-static delamination curve presented the changing resistance against delamination growth, which accounted for the fiber bridging during mode I fatigue loading, and they further used the curve to determine the variable fracture toughness $G_c(a)$. This normalization lowered the exponent in the Paris relation

and also reduced the spread of the FDG data. However, the magnitude of fiber bridging during the fatigue loading is different from that during the quasi-static loading [26]. For specimens with various interface configurations, higher or lower resistance against the fatigue delamination can be found during the fatigue tests relative to the fracture toughness obtained from the quasi-static tests [27]. Thus, it is doubtful whether the value of $G_c(a)$ obtained from quasi-static tests can present the real changing resistance against the delamination growth during the fatigue loading. Consequently, Zhang et al. [28-30] presented a concept of fatigue delamination resistance $G_{cf}(a)$ to characterize the material resistance during a fatigue loading. They also proposed a re-loading method and a compliance method to determine the fatigue delamination resistance. A formulation of G_{max}/G_{cf} was applied to study the mode I and mixed I/II mode FDG behavior in multidirectional laminates with a stress ratio of $R=0.1$.

Obviously, there is still a lack of consensus on the formulations of G to interpret the FDG behavior better. Potential misinterpretations of the FDG may happen if the formulation of G is not well defined [12]. To reasonably understand the FDG behavior in composite materials, seeking an appropriate formulation is very important and is still a subject of intense current investigations. The major goal of this study is to develop a general understanding of the present FDG models and to propose empirical models with a relatively wide range of application, which could characterize the FDG under a reasonable range of cyclic loadings for multidirectional composite laminates.

2 New FDG models

The development of models based on fracture mechanics for predicting FDG over the past 40 years was summarized in detail by Pascoe et al. [31] and other researchers [19, 32, 33]. Most researchers deem the Paris relation with the form of Eq. (1) to be amongst

the best descriptions of FDG behavior, in which the detailed formulations of G are modified to a greater or lesser extent to address some specific problems, such as the effect of R -ratio, mode-mixity and fiber bridging. Thus, when considering these factors, the idea proposed by Wang et al. [17], in which the FDG is controlled by the crack-driving force and the material resistance, is adopted in the current investigation. Two typical crack-driving force functions are used: a redefined G range $G_{\Delta P} = (\sqrt{G_{\max}} - \sqrt{G_{\min}})^2$ proposed by Rans et al. [12] and the ordinary G range denoted by the arithmetic difference between the maximum and minimum values $\Delta G = G_{\max} - G_{\min}$. In addition, the material resistance during the fatigue loading is characterized by the fatigue delamination resistance, $G_{cf}(a)$, proposed by Zhang et al. [29]. Then, two new FDG models are proposed to represent the FDG behavior in CFRP laminates accurately:

$$\frac{da}{dN} = C \left(\frac{G_{\Delta P}}{G_{cf}(a)} \right)^r = C \left(\frac{(\sqrt{G_{\max}} - \sqrt{G_{\min}})^2}{G_{cf}(a)} \right)^r \quad (2)$$

$$\frac{da}{dN} = C \left(\frac{\Delta G}{G_{cf}(a)} \right)^r = C \left(\frac{G_{\max} - G_{\min}}{G_{cf}(a)} \right)^r \quad (3)$$

where C and r are empirically derived parameters determined by a curve fitting procedure. It is deemed that the two parameters have a non-monotonic relation [8] with mode mixities. Thus, different parameter sets can be obtained for different mode mixities.

In addition, the analogy is obvious between the G used in composite materials and the K used in metal crack growth. Generally, the Paris relations for the fatigue crack growth in metals are usually characterized by the parameter ΔK . Because G is proportional to

K^2 [33] and K is proportional to the applied force P , both $G_{\Delta P}$ and ΔG include the parameter ΔK in the following manner:

$$G_{\Delta P} = \left(\sqrt{G_{\max}} - \sqrt{G_{\min}} \right)^2 \sim (P_{\max} - P_{\min})^2 \sim (K_{\max} - K_{\min})^2 \sim (\Delta K)^2 \quad (4)$$

$$\Delta G = G_{\max} - G_{\min} \sim (P_{\max})^2 - (P_{\min})^2 \sim (K_{\max})^2 - (K_{\min})^2 \sim 2\Delta K K_{\text{mean}} \sim (\Delta K)^2 \frac{1+R}{1-R} \quad (5)$$

Therefore, $G_{\Delta P}$ is a function of the independent variable ΔK in the fatigue analysis without involving the R -ratio effect. However, ΔG has inherently included the effect of R -ratio, which can be concluded from the expression $(\Delta K)^2 \frac{1+R}{1-R}$. When ΔG is expressed as $\Delta K K_{\text{mean}}$, it is consistent with the two-parameter models proposed by Jia and Davalos [34], Atodaria et al. [35] and Khan [36], which correlates to the two components of fatigue loading, i.e., the cyclic and the monotonic load components.

Furthermore, it is clear that the equation has included the fiber bridging effects through the introduction of the fatigue delamination resistance $G_{\text{cf}}(a)$. Based on the beam theory, the mixed mode I/II loading in MMB tests shown in Fig. 1 can be represented by a superposition of simple mode I (DCB) and mode II (ENF) loading. Thus, the SERR in the MMB tests can be obtained by combining the SERR equations from the literature on DCB and ENF tests. Similarly, the $G_{\text{cf}}(a)$ in mixed mode I/II loading can also be decomposed into mode I and mode II components. Obviously, the fiber bridging doesn't react on the mode II delamination growth at the crack tip. Thus, the effect of fiber bridging on $G_{\text{cf}}(a)$ in MMB tests is actually focused on the mode I delamination. Zhang et al. [28] has investigated the effect of the fiber bridging on the mode I delamination, from which they concluded that the fiber bridging is a dominant factor to make the damage state of FDG different from that of the quasi-static one. Thus,

the introduced fatigue delamination resistance $G_{cf}(a)$ accounts for the difference of fiber bridging damage state between the quasi-static and fatigue delamination with the same crack length. Compared with significant fiber bridging, the effect of delamination migration on $G_{cf}(a)$ is trivial [27, 28].

To determine the fatigue delamination resistance $G_{cf}(a)$, Zhang et al. provided a re-loading method and a compliance method [29, 30]. In the re-loading method, specimens after a certain amount of fatigue loading are firstly quasi-statically loaded until new delamination growth occurs. Then, the $G_{cf}(a)$ of the fatigue specimen is assumed to be equal to the value of the corresponding static specimen, which exhibits the same force-displacement behavior. Obviously, the re-loading method interrupts the fatigue testing, which not only results in incrementing the experimental cost but also impairing the integrality of the collected FDG data. To overcome the disadvantages of the re-loading method, a compliance method, which is theoretically equivalent with the re-loading method, was further proposed. When using the compliance approach, a static delamination test is pre-performed to obtain the variable fracture toughness denoted by $G_c(a)$, which accounts for the R -curve effect. During the fatigue delamination tests, the normalized compliance of the specimen at an arbitrary crack length will be collected. By matching the normalized compliance data obtained from the static tests and the fatigue tests, the corresponding relation between the fatigue delamination resistance $G_{cf}(a)$ and the variable fracture toughness $G_c(a)$ can be established. More details can be found in reference [29].

3 Experimental validation

To validate the presented FDG models, a series of fatigue tests were conducted using mixed mode bending (MMB) specimens to obtain experimental data under I/II loading

with various mode-mixity ratios and stress ratios.

3.1 Specimens

The CFRP laminates used here were made of T700 carbon/bismaleimide unidirectional composites (supplied by Beijing Institute of Aeronautical Material) with a stacking sequence of $(+45/-45/0_6)_s/(-45/+45/0_6)_s$, which effectively minimize the bend-twist coupling and anticlastic effects caused by the central non-symmetric stacking sequence. During the manufacture of the laminate, an initial crack was pre-formed by inserting a thin Teflon-film (with 25 μm thickness and 35 mm length) into the mid plane of the laminate at one edge. After the curing process, the laminate was first ultrasonically scanned to identify imperfections. Then, a series of MMB specimens were cut from the defect-free portions of the laminate with the following geometric dimensions: width of 25 mm, length of 180 mm, and nominal thickness of 4.16 mm. More detailed geometric dimensions are shown in Fig. 2. To improve the visibility of the crack and minimize any surface damage, the edges of each specimen were polished by hand with sandpapers and wiped using alcoholic balls. One side of each specimen was painted with a thin coating of diluted white correction fluid to provide a high-contrast image and thus make the microscopic observations of the crack tip clearer. The elastic properties of the material are the following [29]: $E_{11}=130$ GPa, $E_{22} = E_{33} = 10.4$ GPa, and $\nu_{12} = 0.3$.

3.2 Experimental set-up

Static and FDG tests under mixed I/II mode loading were conducted on an MTS servo-hydraulic fatigue testing machine with a load cell of 1500 N capacity using a displacement control mode and following the ASTM standard D6671M-06 [37]. A set-up originally proposed by Reeder and Crews [38, 39] was used in the tests, whose sketch map and actual application in experiments are illustrated in Fig. 3. This figure

shows the bending fixture used in the tests and how the external opening load was applied at the neutral plane of the specimen arms. A quick-mounted hinges designed by Brandt [40] was adopted, as shown in Fig. 2, because the hinges could avoid extraneous bending moments on the MMB specimens and ensure that the load was effectively applied to the middle plane of the cantilever beams. An instrumented mobile microscope (JCXE-DK), located at one side of the specimen, was connected to a computer system and used to observe the position of crack tips and measure crack lengths at specified intervals. The microscope is equipped with an electronic dial gage, which can record the position of crack tips with a precision of 0.01 mm.

3.3 Test procedure

All tests were carried out at room temperature and laboratory ambient conditions (23°C, 50% RH). For the static tests, MMB specimens were quasi-static loaded under the mode-mixity ratios of 0.25, 0.5 and 0.75, respectively, which were realized by adjusting the lever length c of the MMB apparatus, as shown in Fig. 3(a). The static tests were implemented at a low loading rate of 0.1 mm/min. This value was suggested in the study of Sorensen et al. [41], which shows that a high loading rate may induce significant larger fracture toughness than the equilibrium condition, especially for fiber bridging cases. Therefore, specimens were held at a constant displacement during the tests at certain intervals. When the hold was implemented, the crack normally continued to grow for a few millimeters until it stopped at an equilibrium position. A lower G_c value was then measured at this equilibrium position.

In the fatigue tests, the frequency of displacement loading cycling was set at 5 Hz. Similarly, three cases of mode-mixity ratios ($\varphi = 0.25, 0.50$, and 0.75 , respectively) were performed by adjusting the lever length c on the MMB apparatus. For each φ -ratio,

three different values of the R -ratio (0.1, 0.3 and 0.5) were adopted to investigate its effect on the FDG. During each test, the R -ratio stayed constant. The displacement-controlled fatigue tests began with the highest $G_{\max}$. With increasing crack length, $G_{\max}$ decreased gradually to its threshold value. Simultaneously, the specimen compliance increased with the crack length. When the FDG rate was very slow, the applied actuator displacements were purposively added to collect more delamination data from one specimen. Therefore, two or more constant amplitude displacement levels were adopted to obtain a large range of FDG rates from each specimen, in the same way as other work in the literature [42].

To facilitate the observation of the crack tip, the fatigue test was stopped at the maximum fatigue displacement loading after a certain number of cycles. The images of the specimen were recorded by the moveable microscope controlled by the computer system. The corresponding information about load, displacement and number of cycles were also recorded, which enabled the data evaluation after the test. This procedure was repeated throughout the test until enough data about the crack growth rate da/dN were collected.

To determine the fatigue delamination resistance $G_{cf}(a)$, the compliance approach was used here. The fatigue specimens were paused at around the minimum displacement after the crack grew a certain length. Then, a quasi-static test was subsequently implemented at a displacement loading rate of 0.1 mm/min, from which the static load vs. displacement curve was obtained. To avoid further unwanted crack growth, the applied quasi-static displacement loading was no more than the mean value of fatigue displacement. After this, the previous fatigue tests were continued. For each mode-mixity ratio, the compliance approach was applied to only one fatigue specimen

with an R -ratio of 0.1.

A summary of the test matrix is presented in Table 1, which includes test details for all the studied specimens. For each specimen, the applied mixed mode ratio, R -ratio and the value of applied displacements ($d_{\max}$) are outlined. Furthermore, because the maximum load ($P_{\max}$) and the maximum SERR $G_{\max}$ will gradually decrease with the increase of the delamination length (a), their detailed variation range for a certain value of $d_{\max}$ are also provided, which is helpful to better understand the tests.

3.4 Analysis of SERR and fatigue delamination resistance

The MMB test is widely accepted to determine the mixed mode (I/II) fracture with variable mode-mixity ratios. On considering of the shear and bending deformations that occurred at the crack tip of composite laminates, the G values of MMB specimens were derived from Ref [43]. Although the lever and loading apparatus are generally made of lightweight materials, lever weight corrections should be taken into account if the applied load is low. Then, the SERR was calculated by the following equations [37]:

$$G_I = \frac{12[P(3c-L) + P_g(3c_g-L)]^2}{16b^2h^3L^2E_f}(a + \chi_1h)^2 \quad (6)$$

$$G_{II} = \frac{9[P(c+L) + P_g(c_g+L)]^2}{16b^2h^3L^2E_f}(a + 0.42\chi_1h)^2 \quad (7)$$

$$G = G_I + G_{II} \quad (8)$$

where P_g is the weight of the lever and the attached loading apparatus, c_g is the lever length relative to the center of gravity, E_f is the bending modulus and χ is the crack length correction factor for the crack tip deformation, which can be obtained theoretically [44] or experimentally [45]. For a given φ , the value of c could be calculated by combining Eqns. (6) and (7), which guided the setting of the lever length during tests. Illustration of the parameters in the above two equations could be found in

Fig. 3 (a).

According to the compliance approach, the fatigue delamination resistance at a certain crack length, a_1 , could be determined by comparing the normalized compliance of fatigue and static specimens. As experimentally measured variables, the normalized compliances of static and fatigue specimens aren't equal to each other mathematically, no matter how close they are. Thus, an artificial tolerance of the compliance deviations is required. Only if the deviation is larger than the tolerance, as shown in Eq. (9), the compliances are considered to be significantly different, and $G_{cf}(a_1)$ needs to be corrected by Eq. (10). Otherwise, at the certain crack length, $G_{cf}(a_1)$ is deemed to be the quasi-static fracture toughness $G_c(a_1)$, as shown in Eq. (11).

$$\left| \frac{\bar{C}_f(a_1)}{\bar{C}_s(a_1)} - 1 \right| > \varepsilon_0 \quad (9)$$

$$G_{cf}(a_1) = G_c \left(a \mid_{\bar{C}_s(a) = \bar{C}_f(a_1)} \right) \quad (10)$$

$$G_{cf}(a_1) = G_c(a_1) \quad (11)$$

In the above equations, ε_0 is the critical tolerance accounting for experimental errors of the compliance. $\bar{C}$ is the normalized compliance of specimens calculated by Eq. (12) and whose subscripts 'f' and 's' indicate the fatigue and static specimens, respectively:

$$\bar{C} = \frac{1}{m} b h^3 \quad (12)$$

where m is the slope of the loading-displacement curve of the specimen.

3.5 Experimental results and FDG model validation

The data of the fatigue delamination resistance (G_{cf}) vs. the crack length of specimens with various mode-mixity ratios were first determined using the compliance approach

described in section 3.4, as shown in Fig. 4. With the increasing crack length, the G_{cf} gradually increases, which exhibits an approximately linear relation for all of the specimens with three types of mode-mixity ratios.

Accompanying the proposed formulations of $G_{\Delta P}/G_{cf}(a)$ and $\Delta G/G_{cf}(a)$, two commonly used ones proposed by other researchers, which include ΔG and $G_{\Delta P}$, are also considered here to examine the experimental fatigue dataset. For different mode-mixity ratios, the measured da/dN data at various R -ratios were corrected with different expression functions of G , as shown in Fig. 5 to Fig. 7, respectively. These figures clearly indicate that different G formulations provide varied understanding of FDG behavior in composites.

The first subfigures in Fig. 5 to Fig. 7 indicate that the FDG rates of tested specimens with various φ - and R -ratios do not have an evident linear relationship with ΔG . The same conclusion can be derived from the top right subfigures in Fig. 5 to Fig. 7, which illustrate the relationship between $G_{\Delta P}$ and da/dN .

The bottom left subfigures in Fig. 5 to Fig. 7 exhibit a strong linear relationship between the FDG rate and $G_{\Delta P}/G_{cf}(a)$ for specimens subjected to fatigue loadings under different φ -ratios. For a given φ -ratio, the data for different R -ratios show good linearity in double logarithmic axes. In addition, for different R -ratios, the distinct fitting lines of da/dN versus $G_{\Delta P}/G_{cf}(a)$ are almost parallel to each other, which means that the R ratio hardly affects the slopes of the fitting lines. The dependence of the FDG rate on the R -ratio is reasonable when the $G_{\Delta P}/G_{cf}(a)$ is used as the control parameter, because the formulation of $G_{\Delta P}$ described in Eq. (4) does not inherently include the R -ratio effect. Moreover, a higher R ratio leads to an increasing FDG rate for a given

$G_{\Delta P}/G_{cf}(a)$, which means that the increasing R ratio will advance the FDG. Similar phenomena were also found in T300 carbon-epoxy laminates under mode I fatigue loading [12]. As a consequence, it follows that the formulation of $G_{\Delta P}/G_{cf}(a)$ is qualified to describe the FDG rate at certain values of R -ratio and φ -ratio.

Furthermore, the relationships between the FDG rate and $\Delta G/G_{cf}(a)$ under fatigue loading with various R -ratios and φ -ratios are illustrated in the last subfigures in Fig. 5 to Fig. 7. Interestingly, when the $\Delta G/G_{cf}(a)$ is used to correct the fatigue data sets, all the data points collapse to a single trend line for the given φ -ratios, and the R -ratio dependency disappears. In addition, the R -squared values, which are used to represent the scatter, are 0.81, 0.85 and 0.81 for different data corresponding to the φ -ratios of 0.25, 0.5 and 0.75, respectively. It follows that the data scatter under different φ -ratios are similar quantitatively. Moreover, for all the φ -ratios considered here, the relationships between $\Delta G/G_{cf}(a)$ and da/dN are also similar for each R -ratio because all the data at a given φ -ratio are within a small scatter band. That means that the FDG rate is independent of the R -ratio. Additional supporting evidence of this conclusion is that it was observed in the literature that for a fixed φ , the R ratio does not affect the fracture surface morphologies [13]. This phenomenon indicates that the R -ratio cannot influence the micro-scale mechanism of FDG. Accordingly, the FDG rate can be characterized by a unique linear relationship on a log-log scale when using $\Delta G/G_{cf}(a)$ as the control parameter for the FDG of multidirectional laminates, which is also logical because the G range has involved the R -ratio effect, as described in Eq. (5). Thus, $\Delta G/G_{cf}(a)$ can be

chosen as a proper control parameter for the FDG of multidirectional laminates when subjected to cyclic loads with different R ratios, and a more generic description of FDG behavior with various R -ratios can be obtained. Although the new power law relationship expressed in Eq. (3) is still empirical in nature, it provides an appealing and simple manner to characterize FDG rates in composite laminates, including the effects of R -ratio and the fiber bridging.

3.6 Crack paths and fracture surfaces

Typical delamination path photos of the front and rear sides of the specimen are presented in Fig. 8(a) and (b). From the view of the front side, the delamination predominantly grew along the mid-plane. However, from the view of the rear side, it migrated from the film insert and upwards through the adjacent $+45^\circ$ fiber, and finally stayed in the $+45/-45$ interface adjacent to mid-plane. In addition, the failed delamination fracture surfaces were explored for specimens under different mode mixities via an optical microscope. Because all the fracture surfaces are similar, only one representative fracture surface is illustrated here, as shown in Fig. 8(c), from which the migration phenomenon can be obviously observed.

Fig. 8(d) to (h) further showed microscopic images taken from different locations on the failed delamination fracture surface. Fig. 8(d) and (e) demonstrated that delamination was along the interface of interlaminar resin and adjacent $+45^\circ$ fibers, in despite of little resin-rich region was found on the fracture surface. The mid-plane delamination grew for a short distance after which point the delamination transmitted into the adjacent $+45^\circ$ fiber layer, with a large amount of $+45^\circ$ fibers exposed as shown in Fig. 8(f). As the delamination grew further, the stress status favored kinking through the $+45^\circ$ fiber layer, leading to eventual migration shown in Fig. 8(g) and resulting in

bridging of ‘a mass of fibers’. The fracture surface close to the rear side finally exhibited a clear interface of the resin and -45° fibers with little amount of -45° fibers, as shown in Fig. 8(h). Totally, most of the fatigue delamination grew along the interface of interlaminar resin and $+45^\circ$ fibers or grew in the $+45^\circ$ fiber layer.

Clearly, complex inter/intra damage process which mainly involved fiber bridging and migration occurred in the specimen, both of which made contributions to the fatigue delamination resistance. It is deemed that the fiber bridging is the dominant factor that results in the different damage status in the static and fatigue delamination because it contributes to most energy absorption while the migration only dissipates a small amount of energy [27, 28]. In addition, because the occurrence of fiber bridging and migration depends on various factors, including the stress states and the stacking sequence, it is worthwhile to point out that the fatigue delamination resistance is not a material constant but specimen dependent.

4 Conclusions

Two Paris-type models involving the crack-driving force relative to the fatigue delamination resistance of composite materials are proposed. The models provide a physical interpretation of the FDG behavior under cyclic loadings, accounting for multiple stress ratios, mode-mixities and the effect of fiber bridging. A series of fatigue tests were carried out with multidirectional MMB laminates under various combinations of mode-mixity ratios and positive R -ratios. Delamination paths and macro-/micro-scale fracture surface photographs of specimens showed the significant fiber bridging and obvious migration in the multidirectional laminates, which provided helpful information for understanding the damage mechanism of the mixed mode fracture. It follows that $G_{\Delta P}/G_{cf}(a)$ as the control parameter of FDG is qualified to describe the FDG rate at

certain values of R -ratio and ϕ -ratio; $\Delta G/G_{cf}(a)$ can further remove the R -ratio dependency and characterize the FDG rate in a perfectly linear relationship on a log-log scale. Thus, both of the proposed FDG models can provide more generic descriptions and a proper interpretation of FDG behavior in CFRP multidirectional laminates. Moreover, the model using $\Delta G/G_{cf}(a)$ as the controlling parameter of FDG has the capability of estimating the FDG rate under an arbitrary R -ratio provided with fatigue delamination tests under one R -ratio, which remarkably reduces the time-consuming and expensive fatigue experiments.

Acknowledgements

The research work is supported by the National Science Foundation of China (11572058, 11372020 and 11202243) and the Innovation Foundation of BUAA for PhD Graduates (YWF-14-YJSY-037). Yu Gong gratefully acknowledges financial support from China Scholarship Council.

References

- [1] Kardomateas GA, Pelegri AA, Malik B. Growth of internal delaminations under cyclic compression in composite plates. *J Mech Phys Solids* 1995; 43(6): 847-68.
- [2] Mortell DJ, Tanner DA, McCarthy CT. In-situ SEM study of transverse cracking and delamination in laminated composite materials. *Compos Sci Technol* 2014; 105: 118-26.
- [3] Canturri C, Greenhalgh ES, Pinho ST, The relationship between mixed-mode II/III delamination and delamination migration in composite laminates. *Compos Sci Technol* 2014; 105: 102-9.
- [4] Zhao L, Gong Y, Zhang J, Chen Y, Fei B. Simulation of delamination growth in multidirectional laminates under mode I and mixed mode I/II loadings using cohesive elements. *Compos Struct* 2014; 116: 509-22.
- [5] Elmarakbi AM, Hu N, Fukunaga H. Finite element simulation of delamination growth in composite materials using LS-DYNA. *Compos Sci Technol* 2009; 69(14): 2383-91.
- [6] Andersons J, Hojo M, Ochiai S, Empirical model for stress ratio effect on fatigue delamination growth rate in composite laminates. *Int J Fatigue* 2004; 26(6): 597-604.

- [7] Khan R, Alderliesten R, Yao L, Benedictus R. Crack closure and fibre bridging during delamination growth in carbon fibre/epoxy laminates under mode I fatigue loading. *Compos Part A: Appl Sci Manuf* 2014; 67: 201-11.
- [8] Blanco N, Gamstedt EK, Asp LE, Costa J. Mixed-mode delamination growth in carbon-fibre composite laminates under cyclic loading. *Int J Solids Struct* 2004; 41(15): 4219-35.
- [9] Dahlen C, Springer GS. Delamination growth in composites under cyclic loads. *J Compos Mater* 1994; 28(8): 732-81.
- [10] Kenane M, Benzeggagh ML. Mixed-mode delamination fracture toughness of unidirectional glass/epoxy composites under fatigue loading. *Compos Sci Technol* 1997; 57(5): 597-605.
- [11] Tumino D, Cappello F. Simulation of fatigue delamination growth in composites with different mode mixtures. *J Compos Mater* 2007; 41(20): 2415-41.
- [12] Rans C, Alderliesten R, Benedictus R. Misinterpreting the results: How similitude can improve our understanding of fatigue delamination growth. *Compos Sci Technol* 2011; 71(2): 230-8.
- [13] Hojo M, Tanaka K, Gustafson CG, Hayashi R. Effect of stress ratio on near-threshold propagation of delamination fatigue cracks in unidirectional CFRP. *Compos Sci Technol* 1987; 29(4): 273-92.
- [14] Alderliesten RC, Schijve J, Zwaag SVD. Application of the energy release rate approach for delamination growth in Glare. *Eng Fract Mech* 2006; 73(6): 697-709.
- [15] Jones R, Pitt S, Bunner AJ, Hui D. Application of the Hartman-Schijve equation to represent Mode I and Mode II fatigue delamination growth in composites. *Compos Struct* 2012; 94(4): 1343-51.
- [16] Yao L, Alderliesten R, Zhao M, Benedictus R. Bridging effect on mode I fatigue delamination behavior in composite laminates. *Compos Part A: Appl Sci Manuf* 2014; 63: 103-9.
- [17] Wang A, Slomiana M, Bucinell RB. Delamination crack growth in composite laminates. *Delamination and debonding of materials*, ASTM STP 1985.
- [18] Allegri G, Jones MI, Wisnom MR, Hallett SR. A new semi-empirical model for stress ratio effect on mode II fatigue delamination growth. *Compos Part A: Appl Sci Manuf* 2011; 42(7): 733-40.
- [19] Blanco N, Gamstedt EK, Asp LE, Costa J. Mixed-mode delamination growth in carbon-fibre composite laminates under cyclic loading. *Int J Solids Struct* 2004; 41(15): 4219-35.
- [20] Ramkumar RL, Whitcomb JD. Characterization of mode I and mixed-mode delamination growth in T300/5208 graphite/epoxy. *Delamination and debonding of materials*, ASTM STP 1985; 876(985): 315-35.
- [21] Russell AJ, Street KN. Predicting interlaminar fatigue crack growth rates in compressively loaded laminates. *Delamination and Debonding of Materials*, American Society for Testing and Materials, Philadelphia 1989: 162-78.
- [22] Allegri G, Wisnom MR. A non-linear damage evolution model for mode II fatigue delamination onset and growth. *Int J Fatigue* 2012; 43: 226-34.

- [23] Foote RM, Mai Y, Cotterell B. Crack growth resistance curves in strain-softening materials. *J Mech Phys Solids* 1986; 34(6): 593-607.
- [24] Shivakumar K, Chen H, Abali F, Le D, Davis C. A total fatigue life model for mode I delaminated composite laminates. *Int J Fatigue* 2006; 28(1): 33-42.
- [25] Chen H, Shivakumar K, Abali F. A comparison of total fatigue life models for composite laminates. *Fatigue Fract Eng M* 2006; 29(1): 31-9.
- [26] Murri GB. Evaluation of Delamination Growth Characterization Methods Under Mode I Fatigue Loading. in *ASC 27th technical conference/15th US-Japan conference on composites/ASTM D30 Meeting* October. 2012.
- [27] Yao L, Alderliesten RC, Zhao M, Benedictus R. Discussion on the use of the strain energy release rate for fatigue delamination characterization. *Compos Part A: Appl Sci Manuf* 2014; 66: 65-72.
- [28] Peng L, Zhang J, Zhao L, Bao R, Yang H, Fei B, Mode I delamination growth of multidirectional composite laminates under fatigue loading. *J Compos Mater* 2011; 45(10). 1077-90.
- [29] Zhang J, Peng L, Zhao L, Fei B. Fatigue delamination growth rates and thresholds of composite laminates under mixed mode loading. *Int J Fatigue* 2012; 40: 7-15.
- [30] Peng L, Xu J, Zhang J, Zhao L. Mixed mode delamination growth of multidirectional composite laminates under fatigue loading. *Eng Fract Mech* 2012; 96: 676-86.
- [31] Pascoe JA, Alderliesten RC, Benedictus R. Methods for the prediction of fatigue delamination growth in composites and adhesive bonds-A critical review. *Eng Fract Mech* 2013; 112: 72-96.
- [32] Jia J, Davalos JF. Loading variable effects on mode-I fatigue of wood-FRP composite bonded interface. *Compos Sci Technol* 2004; 64(1): 99-107.
- [33] Azari S, Jhin G, Papini M, Spelt JK. Fatigue threshold and crack growth rate of adhesively bonded joints as a function of load/displacement ratio. *Compos Part A: Appl Sci Manuf* 2014; 57: 59-66.
- [34] Jia J, Davalos JF. Study of load ratio for mode-I fatigue fracture of wood-FRP-bonded interfaces. *J Compos Mater* 2004; 38(14): 1211-30.
- [35] Atodaria DR, Putatunda SK, Mallick PK. Delamination growth behavior of a fabric reinforced laminated composite under mode I fatigue. *Journal of engineering materials and technology* 1999; 121(3): 381-5.
- [36] Khan R, Alderliesten R, Benedictus R. Two-parameter model for delamination growth under mode I fatigue loading (Part A: Experimental study). *Compos Part A: Appl Sci Manuf* 2014; 65: 192-200.
- [37] ASTM, Standard test method for mixed mode I-mode II interlaminar fracture toughness of unidirectional fiber reinforced polymer matrix composites, in *ASTM D6671/D6671M-06*. 2006.
- [38] Reeder JR, Crews JH. Mixed-mode bending method for delamination testing. *Aiaa J* 1990; 28(7): 1270-6.
- [39] Reeder JR, Crews JH, Nonlinear analysis and redesign of the mixed-mode bending delamination test. 1991: National Aeronautics and Space Administration, Langley

Research Center.

[40] Brandt F. New load introduction concept for improved and simplified delamination beam testing. *Exp Techniques* 1998; 22(1): 17-20.

[41] Sørensen L, Botsis J, Gmür T, Humbert L. Bridging tractions in mode I delamination: measurements and simulations. *Compos Sci Technol* 2008; 68(12): 2350-8.

[42] Mall S, Ramamurthy G, Rezaizadeh MA. Stress ratio effect on cyclic debonding in adhesively bonded composite joints. *Compos Struct* 1987; 8(1): 31-45.

[43] Kinloch AJ, Wang Y, Williams JG, Yajla P. The mixed-mode delamination of fibre composite materials. *Compos Sci Technol* 1993; 47(3): 225-37.

[44] Williams JG. End corrections for orthotropic DCB specimens. *Compos Sci Technol* 1989; 35(4): 367-76.

[45] Hashemi S, Kinloch AJ, Williams JG. Corrections needed in double-cantilever beam tests for assessing the interlaminar failure of fibre-composites. *J Mater Sci Lett* 1989; 8(2): 125-9.

Figure Captions

Fig. 1 Superposition analysis of loading on the MMB specimen

Fig. 2 Geometric dimensions of specimen and schematic map of hinge

Fig. 3 MMB test set-up

Fig. 4 Fatigue delamination resistance vs. crack length

Fig. 5 da/dN vs. different G formulations for various R -ratios at $\varphi=0.25$

Fig. 6 da/dN vs. different G formulations for various R -ratios at $\varphi=0.5$

Fig. 7 da/dN vs. different G formulations for various R -ratios at $\varphi=0.75$

Fig. 8 Typical crack paths (a) front side (b) rear side and macro-/micro-scale fracture surface pictures (c)-(h) of studied specimens (Dot lines indicate the position of the midplane, while dash lines indicate adjacent ply interfaces, 'A' indicates the interface of interlaminar resin and +45° fibers and 'B' indicates the interface of resin and -45° fibers)

Fig. 1 Superposition analysis of loading on the MMB specimen

Fig. 2 Geometric dimensions of specimen and schematic map of hinge

(a) Sketch of MMB test set-up

(b) Physical MMB test set-up

Fig. 3 MMB test set-up

Fig. 4 Fatigue delamination resistance vs. crack length

Fig. 5 da/dN vs. different G formulations for various R -ratios at $\varphi=0.25$

Fig. 6 da/dN vs. different G formulations for various R -ratios at $\varphi=0.5$

Fig. 7 da/dN vs. different G formulations for various R -ratios at $\varphi=0.75$

Fig. 8 Typical crack paths (a) front side (b) rear side and macro-/micro-scale fracture surface pictures (c)-(h) of studied specimens (Dot lines indicate the position of the midplane, while dash lines indicate adjacent ply interfaces, 'A' indicates the interface of interlaminar resin and +45° fibers and 'B' indicates the interface of resin and -45° fibers)

Table 1 Static and fatigue test matrix.

Fatigue specimen	F1		F2		F3			F4									
φ -ratio	0.25																
R -ratio	0.1				0.3			0.5									
$d_{\max}$ (mm)	2.0	3.8	3.0	4.6	3.1	4.4	5.5	3.0	4.1								
Range of a (mm)	35.1-37.4	38.0-42.2	35.1-39.0	39.3-45.6	38.2-40.5	40.6-43.8	43.8-46.2	35.0-37.8	37.8-41.0								
Range of $P_{\max}$ (N)	40.5-35.7	62.0-48.2	49.8-40.0	64.2-48.8	51.3-42.8	60.1-51.5	66-58.7	53.1-47.2	61.7-54.4								
Range of $G_{\max}$ (J/m ²)	108.3-95.4	266.7-203.5	170.7-130.5	322.3-253.6	192.9-152.3	290.7-243.4	389.7-338.9	199.1-180.1	300.6-268.1								
Fatigue specimen	F4		F5		F6	F7	F8		F9								
φ -ratio	0.25		0.5														
R -ratio	0.5		0.1				0.3		0.5								
$d_{\max}$ (mm)	5.4	6.7	1.6	2.4	3.2	2.2	2.3	3.2	2.4								
Range of a (mm)	41.1-44.1	44.1-46.6	35.5-38.5	38.9-43.9	39.5-46.5	35.8-41	37.7-41.4	41.5-46.6	35.9-39.7								
Range of $P_{\max}$ (N)	70.7-60.3	74.1-65.0	87.2-77.6	117.2-95.5	153.5-107.4	114.7-86.8	126.8-107.4	149.2-118.9	126.8-107.3								
Range of $G_{\max}$ (J/m ²)	433.7-363.7	535.2-456.6	110.9-104.3	226.2-174.4	372.9-240.3	202.3-150.7	246.1-210.1	382.2-294.7	234.3-200.8								
Fatigue specimen	F9		F10		F11		F12		F13								
φ -ratio	0.5		0.75														
R -ratio	0.5		0.1				0.3										
$d_{\max}$ (mm)	3.4	4.0	2.2	2.9	2.5	3.3	2.0	2.9	2.5								
Range of a (mm)	39.7-43.3	43.3-45.1	35.2-40.3	40.9-44.5	36.6-41.3	43.7-46.9	37.8-40.0	40.0-47.0	36.6-43.9								
Range of $P_{\max}$ (N)	154.1-130.3	155.3-140.3	216.0-140.7	246-164.8	215.5-146.3	236.5-145.8	165-153.8	226.7-170.1	199.1-165.8								
Range of $G_{\max}$ (J/m ²)	400.6-336.6	472.2-414.4	289.5-158.1	471.1-265.8	335.9-190.7	529.7-238.8	195.7-187.0	395.4-288.1	298.4-238.9								
Fatigue specimen	F13	F14			<table><tr><th>Static specimen</th><th>S1</th><th>S2</th><th>S3</th></tr><tr><td>φ-ratio</td><td>0.25</td><td>0.5</td><td>0.75</td></tr></table>					Static specimen	S1	S2	S3	φ -ratio	0.25	0.5	0.75
Static specimen	S1	S2	S3														
φ -ratio	0.25	0.5	0.75														
φ -ratio	0.75																
R -ratio	0.3	0.5															
$d_{\max}$ (mm)	3.8	2.4	3.1	3.8													
Range of a (mm)	43.9-48.1	37.2-41.6	41.6-45.0	45.0-46.8													
Range of $P_{\max}$ (N)	252.8-212.3	191.5-170.2	223.3-186.4	236.6-222.2													
Range of $G_{\max}$ (J/m ²)	530.3-443.6	279.6-259.4	436.4-347.6	548.0-521.0													