Particulate Material Fabric Characterization by Rotational Haar Wavelet Transform

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ABSTRACT

A Rotational Haar Wavelet Transform (RHWT) method is developed to characterize the fabric of particulate assemblies from two-dimensional images. A Maximum Energy Ratio Ψ reveals the fabric direction and its intensity. The method is implemented on 12 sand and 3 rice specimens of various shapes. It was shown that Ψ may be expressed in terms of a material's aspect ratio and relative density. A material fabric classification system based on Ψ is proposed. The parameter also defines the fabric tensor for cross anisotropic material. Scanning electron microscope images of kaolinite clay and several rock images are also analyzed.

KEY WORDS

Soil fabric, particle shape, image analysis, wavelet transform, sand

1. Introduction

Because of depositional processes, fabric cross anisotropy (or transverse isotropy) is commonly encountered in earth materials [1-4]. Within the depositional plane, the soil particles' long axes point in random directions thus developing an isotropic fabric within that plane. This contrasts with the fabric established in the direction of deposition. The fabric anisotropy endows earth materials with macroscopic properties including strength, dilatancy, permeability and compressibility that vary with direction. This distinguishes soil from other civil engineering materials such as steel and concrete.

Laboratory tests for investigating soil behavior in different fabric directions were first performed on clays, perhaps because it is relatively easy to prepare anisotropic clay specimens by sampling intact blocks in different directions. The early works of Hansen and Gibson [5], Mitchell [6], Jacobsen [7], Lo [8], and Duncan and Seed [9] all showed that the shear strength of clays varies with the angle between the major principal stress and the deposition plane. Following these early studies, the influence of fabric anisotropy on strength, permeability, and compressibility of clay were systematically studied by Lo and Milligan [10], Saada [11], Lo and Morin [12], Saada and Ou [13], Prevost [14], Yong and Silvestri [15], Graham and Houlsby [16], Kirgord and Lade [2], Dewhurst et al. [17], Kurukulasuriya et al. [18], Nishimura et al. [19], Anantanasakul et al. [20] and many others.

Laboratory studies of the macro behavior of sands at different fabric directions began in the 1970s. It is somewhat difficult to prepare anisotropic sand specimens with varying fabric directions. One way is to freeze a wet specimen or impregnate an air-pluviated specimen with a resin to maintain its fabric and then trim the specimen in different orientations [21-22]. Another way is by using devices that incline a test vessel at various angles during air pluviation [23-25]. Following one of these specimen preparation procedures, various laboratory tests have been performed to study the influence of fabric anisotropy on macro behavior of sands. Studies have included plane strain compression tests by Oda et al. [22], Oda

[26], and Tatsuoka et al. [23]; direct shear tests by Azami et al. [27], Guo [24], and Tong et al. [25]; triaxial tests by Oda [21], Arthur and Menzies [28], Arthur and Phillips [29], Wong and Arthur [30], Arthur et al. [31], Oda et al. [22], Lam and Tatsuoka [32], and Ochiai and Lade [33]; torsion shear tests by Rodriguez and Lade [34] and Yang et al. [35]. The influence of fabric anisotropy on bearing capacity and seismic response of foundations was studied by Meyerhof [36], and Oda and Koishikawa [37]. Besides physical laboratory tests, the effects of fabric anisotropy were also observed in Discrete Element Method (DEM) studies such as in the works of Hosseininia [38], Ng [39], Zhao and Guo [40], Fu and Dafalias [41], Yimsiri and Soga [42]. Based on these laboratory and numerical observations, constitutive models that accounted for fabric anisotropy were developed by Pietruszczak and Mroz [43-44], Li and Dafalias [45], Dafalias et al. [46], Lade [47-48], Schweiger [49], Liu and Indraratna [50], Yao and Kong [51], Kong et al. [52], and Gao and Zhao [53].

Despite an abundance of evidence regarding the importance of fabric anisotropy on mechanical soil behavior, fabric anisotropy is rarely considered in routine geotechnical design and analysis. The primary reason is the lack of an easy, effective and accurate method to quantify it. The fabric anisotropy should be quantified from two aspects, the primary fabric direction and the degree of anisotropy. These two parameters can alternatively be computed based on the orientations of non-spherical particle long axes, from the orientations of interparticle contacts and, or the orientations of non-spherical voids (Oda and Nakayama [1]). The fabric computed from the orientations of non-spherical particle long axes is the focus of this study. This is the most frequently used fabric definition is the because it can be readily determined by visual observation. For example, Oda [21] impregnated resin into sand specimens then cut them into thin sections of thickness between 0.04 and 0.07 mm. He manually measured the long axis orientations of 200 particles to determine fabric orientation. Oda's [21] method was later augmented by digital image processing techniques. Yang et al. [56] cut specimens at different locations (but not thin sections). They polished the surfaces before capturing images. Fonseca et al. [54, 55] captured the

section images of specimens using micro-computed tomography. Instead of manually counting particle orientations, Yang et al. [56] and Fonseca et al. [54, 55] used image thresholding techniques to identify individual particles and compute the orientation distributions of particle long axes. Requisite to this method is that sand particles must have a different color from the surrounding resin. The greatest challenge is to computationally determine the boundaries of contacting particles so that their orientations could be assessed individually.

Existing fabric and anisotropy quantification methods are all based on statistical distributions of individual particles' long axes. Although it is well known that elongated particles will develop stronger fabric anisotropy than spherical particles, particle shape is not typically considered. Guo (2008) showed that direct shear strength systematically increases as particle elongation increases from perfectly round glass beads to sub-rounded Ottawa sand to angular crushed limestone. Similar observations were made by Oda [26], Lade[48], and Tong et al. [25]. Therefore, complete fabric characterization should logically include particle elongation.

To meet the challenge of simultaneously characterizing and quantifying the fabric and shape of particles from images, a new *Rotational Haar Wavelet Transform* (RHWT) technique is proposed in this paper. Instead of identifying individual particles, the RHWT analyzes grayscale distributions and their changes across an image. This allows the method to be utilized on images of contacting particle assemblies; although, it could just as well be used on individual particles. Images of clays and rock will also be analyzed to demonstrate the method's ability to characterize fabric in other earth materials. Finally, in recognition of the wide use of fabric tensors in soil mechanics and constitutive modeling, a method for determining the fabric tensor using the Rotational Haar Wavelet Transform will be presented.

2. Haar Wavelet Transform

The Haar Wavelet Transform (HWT) has been used extensively for digital image compression, denoising, edge detection, feature extraction, texture analysis and image segmentation. The HWT is also not new in geotechnical engineering. It has been used to determine particle size distributions by Shin and Hryciw [57], Hryciw et al. [58] and Ohm and Hryciw [59], and for analysis of surface roughness by Chandan et al. [60]. The rigorous mathematical formulation of the HWT is detailed in many textbooks and will not be repeated here. However, the authors will illustrate the essence of the HWT through a simple example that will illustrate why the HWT is ideally suited for analysis of fabric anisotropy.

The example is the idealized 8×8 pixel grayscale image A_0 shown in Fig. 1. The larger grayscale values shown correspond to brighter areas in the image. They change only in the horizontal direction while staying constant vertically. This reflects a strongly vertical fabric.

Fig. 1. An 8×8 intensity image with a strongly vertical fabric.

The HWT first divides the image into 2×2 subareas as shown in Fig. 2(a). The location of each 2×2 subarea can be represented by $A_0(i, j)$ where i and j range from 1 to 4.

Assuming the four numbers in an (i, j) region are:

$$A_0(i, j) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad (1)$$

the HWT computes four values $A_1(i, j)$, $H_1(i, j)$, $V_1(i, j)$, and $D_1(i, j)$. The $A_1(1, 1)$ is twice the average of the four numbers:

$$A_1(i, j) = \frac{a + b + c + d}{2} \quad (2)$$

The $H_1(i, j)$ is the average difference between two columns:

$$H_1(i, j) = \frac{(a + c) - (b + d)}{2} \quad (3)$$

The $V_1(i, j)$ is the average difference between two rows:

$$V_1(i, j) = \frac{(a + b) - (c + d)}{2} \quad (4)$$

The $D_1(i, j)$ is the average difference between two diagonals:

$$D_1(i, j) = \frac{(a + d) - (b + c)}{2} \quad (5)$$

Fig. 2. The 1st Haar wavelet transform decomposition.

For our example, the computed results for the first subarea $A_0(1,1)$ are: $A_1(1, 1) = 350$, $H_1(1, 1) = 50$, $V_1(1, 1) = 0$, and $D_1(1, 1) = 0$. These computations are repeated for all 16 subareas which yield the 4×4 matrices A_1 , H_1 , V_1 , and D_1 shown in Fig. 2. Only A_1 , H_1 , and V_1 are useful for fabric characterization. The matrix A_1 (which can be thought of as a 4×4 image) is essentially a downscaling of A_0 by a factor of 2. The matrix H_1 quantifies the grayscale change in the horizontal direction while V_1 quantifies the grayscale change in the vertical direction. Since there is no grayscale change in the vertical direction V_1 is a zero matrix.

The *energy* (E) is defined as the sum of the squares of values in a matrix and are shown beneath each matrix in Fig. 2. The energy of the original image is preserved in the four submatrices A_1 , H_1 , V_1 , and D_1 in a HWT:

$$E_{A_0} = E_{A_1} + E_{H_1} + E_{V_1} + E_{D_1} \quad (6)$$

A relatively high E_{H_1} (or E_{V_1}) would indicate the total differences in grayscale values in the horizontal (or vertical) direction is large in the original image A_0 .

The HWT can now be applied to A_1 as shown in Fig. 3(a). This is the second decomposition of A_0 . It yields another four submatrices A_2 , H_2 , V_2 and D_2 . As before, A_2 is a downscaling of A_1 by a factor of 2. The H_2 and V_2 quantify the grayscale changes in the horizontal and vertical directions in A_1 respectively. The A_2 can be further decomposed into A_3 , H_3 , V_3 and D_3 as shown in Fig. 3(b). Again, the energy is preserved at each decomposition. The first, second, and third decompositions are together called a three-level HWT. The maximum number of decomposition levels depends on the original image size. For a $2^L \times 2^L$ image, L decompositions can be performed. At the i -th decomposition level, the computed energy E_{H_i} and E_{V_i} quantify the total differences in grayscale values in the horizontal and vertical directions in the A_{i-1} image. After all of the decompositions are performed, we define the maximum E_{H_i} and the maximum E_{V_i} as the horizontal and vertical energy E_H and E_V of the image:

$$E_H = \max_{i=1 \dots L} (E_{Hi}) \quad (7)$$

$$E_V = \max_{i=1 \dots L} (E_{Vi}) \quad (8)$$

Fig. 3. The 2nd and 3rd Haar wavelet transform decompositions.

In the above example, E_H and E_V are 160,000 and 0 respectively. The physical meaning of Eqs. (7) and (8) will be discussed shortly.

The material fabric is associated with grayscale changes in an image. When there is very little grayscale change in a direction, the energy in this direction will be small. For example, the E_V of A_0 is zero in our

example and this reveals and defines the fabric direction. Meanwhile, perpendicular to the fabric direction, the grayscale changes should be largest and thus produce the maximum energy. As such, to determine fabric direction, we search for the direction that exhibits the minimum energy and the maximum energy perpendicular to it. However, the HWT computes energy only in the horizontal and vertical directions in an image and therefore cannot identify fabric in other directions. Therefore, a Rotational Haar Wavelet Transform (RHWT) is proposed to determine energy values in all directions.

3. Rotational Haar Wavelet Transform

A synthetic image M was generated having a fabric direction at 60° from the horizontal as shown in Fig. 4(a). Two energy terms $E(\theta)$ and $E(\theta+90^\circ)$ are now defined: $E(\theta)$ is the energy in the direction at an angle of θ counterclockwise from the horizontal; it quantifies the grayscale change in the θ direction while $E(\theta+90^\circ)$ quantifies the grayscale change perpendicular to the θ direction. To identify the fabric orientation, we seek the θ value that simultaneously yields the minimum $E(\theta)$ and the maximum $E(\theta+90^\circ)$. We now also define an *energy ratio* ER as:

$$ER(\theta) = \frac{2}{\pi} \arctan \left(\frac{E(\theta+90^\circ)}{E(\theta)} \right) \quad (9)$$

The value of ER will range from 0 to 1. The θ corresponding to the maximum ER is the fabric orientation. It is noted that this maximum ER will be in the direction of minimum E . The need for creating this seeming contradiction in terminology will become apparent later.

To compute $E(\theta)$ and $E(\theta+90^\circ)$ and thus $ER(\theta)$, the original image M may be thought of as being rotated clockwise by an angle θ as shown in Fig. 4(b). Then, a nine-level HWT is performed on a central 512×512 pixel area of the image. The E_H and E_V are the maximum E_{Hi} and E_{Vi} following Eqs. (7) and (8). If we

rotate M back to its original position in Fig. 4(c), the $E(\theta)$ and $E(\theta+90^\circ)$ will be equal to E_H and E_V from the previously rotated view in Fig. 4(b). Therefore, the ER in the θ and $\theta+90^\circ$ directions can be computed by:

$$ER(\theta) = \frac{2}{\pi} \arctan \left(\frac{E(\theta+90^\circ)}{E(\theta)} \right) = \frac{2}{\pi} \arctan \left(\frac{E_V}{E_H} \right) \quad (10)$$

and

$$ER(\theta+90^\circ) = \frac{2}{\pi} \arctan \left(\frac{E(\theta)}{E(\theta+90^\circ)} \right) = \frac{2}{\pi} \arctan \left(\frac{E_H}{E_V} \right) \quad (11)$$

where E_H and E_V are obtained in the rotated image view in Fig. 4(b).

Fig. 4. The rotational Haar wavelet transform on an Image M

If we rotate the image M from 0° to 89° by increments of 1° , a total of 90 images will be generated. For each rotation, a nine-level HWT is performed in a central 512×512 area to compute E_H and E_V . Taking the computed E_H and E_V into Eq. (10), the ER for $0^\circ \leq \theta \leq 89^\circ$ can be computed. Taking the computed E_H and E_V into Eqs. (11), the ER for $90^\circ \leq \theta \leq 179^\circ$ can be computed. The computed ER versus θ is plotted

in Fig. 5(a). Due to rotational symmetry, $ER(\theta+180^\circ) = ER(\theta)$. Therefore, we can plot ER in polar coordinates over the full range $0^\circ \leq \theta \leq 360^\circ$ in Fig. 5(b).

Fig. 5. The energy ratio of M in different directions.

Fig. 5(b) explains why ER was defined so that $ER_{\max}$ would be in the $E_{\min}$ direction. The ER curve in Fig. 5(b) is equivalent to a rose diagram traditionally developed after statistically counting the number of

particles oriented in each direction. The ER values increase as θ approaches the fabric direction and the peak ER ($ER_{\max}$) identifies the fabric orientation. Meanwhile, orthogonal to the fabric direction, ER will be at a minimum ($ER_{\min}$).

An important property of Eqs. (10) and (11) is that the sum of the computed ER values in any two orthogonal directions equals 1:

$$ER(\theta) + ER(\theta + 90^\circ) = 1 \quad (12)$$

Therefore, the $ER_{\max} + ER_{\min} = 1$ will also always be true.

We can anticipate that for a soil displaying isotropic fabric, the ER plot will be a circle. This is verified by the synthetic image S shown in Fig. 6(a) which has the same grayscale changes in all direction. The image has a size of 724×724 pixels and a nine-level RHWT was performed on a central 512×512 pixel area. The computed ER versus θ is plotted in Fig. 6(b). As expected, the plot is a circle of radius = 0.5 because E_H must be equal to E_V for all θ .

Fig. 6. The energy ratio for an image S having an isotropic fabric.

The two idealized images M and S represent two extreme fabric scenarios. Image M has a completely anisotropic fabric and Image S has a completely isotropic fabric. The RHWT clearly identifies fabric structure for such idealized images. The following sections will explore RHWT results for images of natural particulate materials.

4. Fabric quantification of natural particulate material images by RHWT

Before studying real soil, long-grain rice was used to demonstrate the RHWT. The long-grain rice is used because it is very elongated and therefore the RHWT results will be very distinct. A typical long-grain rice image is shown in Fig. 7. The image size is 1450×1450 pixels. The image was rotated from 0° to 89° in 1° increments. At each rotation step, a ten-level HWT was performed on a central 1024×1024 pixel region. The analyzed regions at rotations $\theta = 0^\circ, 22^\circ, 45^\circ$ and 67° are shown in Fig. 7.

Fig. 7. A long-grain rice image and the analyzed regions.

The HWT decompositions for $\theta = 0^\circ$ are shown in Fig. 8. The A_0 is the original image while A_1 to A_7 are the first through seventh decompositions. The computed E_{Hi} and E_{Vi} values are plotted in Fig. 9. It should be noted that E_{Hi} and E_{Vi} quantify the horizontal and vertical grayscale changes in A_{i-1} .

Fig. 8. The first seven HWT decompositions at $\theta = 0^\circ$.

In Fig. 9, both E_{Hi} and E_{Vi} values of the long-grain rice display bell-shaped curves and have maximum values at $i = 5$. The same trends have generally been observed for other natural particulate materials, although the maximum values may not be at $i = 5$. Using the long-grain rice example, we will explain why both E_{Hi} and E_{Vi} follow bell-shaped curves and why in this example the maximum values are at $i = 5$.

Image grayscale changes occur not only at the particle edges but also within particles interiors due to subtle material textures. However, these internal grayscale texture changes are much weaker than the grayscale changes at particle edges. As the original image is consecutively downsampled to smaller resolutions, the internal grayscale changes are gradually filtered out while the grayscale changes at edges become dominant as shown from A_0 to A_4 in Fig. 8. Correspondingly, both E_{Hi} and E_{Vi} increase and reach their peaks at A_4 meaning the majority of the internal texture is eliminated and the grayscale changes only occur at edges. At higher post-peak decomposition levels the grayscale changes at edges

will also be filtered out. Thus, the entire image becomes progressively blurred as shown in A_5 to A_7 . This reduces both E_{Hi} and E_{Vi} . The maximum E_{Hi} and E_{Vi} values essentially quantify the grayscale change in the horizontal and vertical directions only due to the occurrences of edges. Obviously, only the edges should define the material fabric. This is why Eqs. (7) and (8) use only the maximum E_{Hi} and E_{Vi} values to define the horizontal and vertical energies E_H and E_V .

Fig. 9. The computed horizontal and vertical energies at $\theta = 0^\circ$

In the synthetic images in Figs. 1 and 4, the grayscale did not change along the fabric direction, therefore the energy was zero in this direction. However, in images of natural materials such as Fig. 8, the grayscale will also change in the fabric direction due to the occurrence of short edge segments normal to the fabric direction. Nevertheless, the occurrence of long edges in the direction parallel to the fabric direction will be more frequent than the occurrence of short edges normal to the fabric direction.

For the long-grain rice example, the HWT was performed at each rotation angle and the resulting ER plot is shown in Fig. 10(a). It confirms a horizontal fabric orientation in agreement with a visual observation

of Fig. 7. However, unlike for the previous perfect synthetic images M and S , the ER plot is zig-zagged because of the non-perfectly aligned particle orientations and somewhat due to image noise. To smooth out this “zig-zaggedness”, a second order Fourier series is used to fit the ER curve:

$$ER(\theta) = a_0 + a_1 \cos(\theta) + b_1 \sin(\theta) + a_2 \cos(2\theta) + b_2 \sin(2\theta) \quad (13)$$

Since $ER(\theta+180^\circ) = ER(\theta)$, a_1 and b_1 must be zero. Equation (13) must also satisfy Eq. (12). Therefore, a_0 must be 0.5. With these two observations, Eq. (13) simplifies to:

$$ER(\theta) = 0.50 + a_2 \cos(2\theta) + b_2 \sin(2\theta) \quad (14)$$

The coefficients a_2 and b_2 can be determined by fitting Eq. (14) to the original ER plot as shown in Fig. 10(b). For the long-grain rice image the parameters were found to be: $a_2 = 0.350$, and $b_2 = -0.077$. The $ER_{\max}$ and $ER_{\min}$ are computed as:

$$\begin{bmatrix} ER_{\max} \\ ER_{\min} \end{bmatrix} = 0.50 \pm \sqrt{a_2^2 + b_2^2} \quad (15)$$

For the plot in Fig. 10(b) $ER_{\max} = 0.86$ and $ER_{\min} = 0.14$. The fabric direction (direction of $ER_{\max}$) is 174° .

A rotational increment of 1° was used in the above computation in above computation but other increments could also be used. Figs. 10(c) and (d) show the results of using a rotational increment of 0.1° while Figs. 10 (e) and (f) show the results for an increment of 10° . As the rotational increment increases, the original the original ER plot becomes smoother. However, the computed Fourier plots are very close in all cases. The Fourier plots depend only on a_2 and b_2 as shown in Eq. (14). Therefore, despite having fewer ER values computed using larger rotational increments, the a_2 and b_2 from regression are very close. This study found that once the rotational increment is smaller than 1° , the computed Fourier ER plots are unaffected by the increment. Therefore, a rotational increment of 1° is suggested.

Fig. 10. The original ER data and the smoothed Fourier ER plot for the long-grain rice in Fig. 7.

4.2 Effects of *PPS* on RHWT results

In the previous example, the image size was 1450×1450 of which its central 1024×1024 area was analyzed. Generally, if the size of the analyzed central region is $2^L \times 2^L$ on which an L -level HWT is performed, the corresponding image size must be at least $2^{L+0.5} \times 2^{L+0.5}$. For example, if the analyzed central area is to be 128×128 , the original image size must be at least 181×181 . This raises an important question: what is the appropriate image magnification to ensure that accurate ER values are obtained. The number of pixels corresponding to the shorter particle dimension is the appropriate criteria to study this problem. We refer to this value as the “pixels per smaller dimension” (*PPS*). For example, The *PPS* of the long-grain rice in Fig. 7 is about 55 pixels. For a fixed particle size, we can anticipate that too small a *PPS* will suppress the grayscale change at the particle edges (such as A_4 to A_7 in Fig. 8) and therefore the E_H and E_V will approach each other as shown in Fig. 9. Consequently, the degree of fabric anisotropy will be underestimated.

The long-grain rice can develop a strong fabric, Fig. 7 is used to investigate the effects of *PPS*. If we could determine the minimum *PPS* that would correctly characterize such a strong fabric anisotropy we would be able to use this minimum *PPS* as the criterion for other natural materials, such as sands, whose particles are less elongated. The long-grain rice image was upscaled and downscaled to generate a series of images having different *PPS* values. The images were 46×46 , 91×91 , 181×181 , 362×362 , 724×724 , and 2897×2897 pixels. The analyzed central areas were correspondingly $2^5 \times 2^5$, $2^6 \times 2^6$, $2^7 \times 2^7$, $2^8 \times 2^8$, $2^9 \times 2^9$, and $2^{11} \times 2^{11}$ pixels. The corresponding *PPS* values are 2, 4, 7, 14, 28, and 110 pixels.

The computed original and Fourier ER plots for different *PPS*s are shown in Fig. 11. The results are stable and consistent once *PPS* exceeds 7 pixels. Therefore, the required minimum *PPS* to achieve reliable results is 7 pixels. In some circumstances, large *PPS* images may be available using very high resolution

devices. For such images, users can either use more powerful computers to implement the RHWT or simply downscale the image to an appropriate PPS value (must be larger than 7 pixels). Such downscaling does not change the computed fabric structure as shown in Fig. 11.

Fig. 11. Effect of image magnification on the computed ER plot

For images of natural materials, the computed ER plots must be somewhere between the two extreme cases shown in Figs. 5(b) and 6(b). The computed ER_{max} will be in the range between 0.5 and 1.0 while the computed ER_{min} will be between 0.0 and 0.5. The degree of fabric anisotropy can be quantified by the shape of the ER plot. Therefore, the ER_{max} can serve as a quantifiable metric of the degree of fabric anisotropy. For brevity, the ER_{max} is hereafter symbolized by Ψ .

5. Particulate material fabric by RHWT

The degree of fabric anisotropy developed in particulate materials depends on both the orientation distribution of the particles' long axes and the particle Aspect Ratio (AR). The AR is defined as the ratio of particle length to particle width, which can be readily determined by computational geometry techniques proposed by Zheng and Hryciw [61] or by caliper measurements. The AR is an intrinsic property that brackets the range of fabric anisotropies which a material can develop while the orientation distribution of particle long axes is a state property. Those two properties will be investigated in this section.

5.1 Orientation distributions of particle long axes

A series of long-grain rice specimens displaying different degrees of fabric anisotropy were prepared as shown in Figs. 12(a) to (k). The image sizes were 1450×1450 pixels in which their central 1024×1024 pixel areas were analyzed by RHWT. The computed original and Fourier ER plots are shown in Figs. 12(b) to (l) in which the Ψ values and fabric orientations are also provided. For reference, the dashed circles at $ER = 0.5$ shown in each figure represent the isotropic fabric state. The computed results agree well with visual observations.

The particles in Fig. 12(a) display completely random orientations. Therefore, the computed ER plot overlaps the isotropic circle. From Fig. 12(b) to (k), the long axes of the particles gradually manifest stronger preferred orientations. Therefore, the ER plots sequentially depart from the isotropic circle and approach a strongly anisotropic fabric plot. The range of degrees of fabric anisotropy that were

observed for the long-grain rice was from near perfect isotropy ($\Psi = 0.51$) in Fig. 12(a) to extreme anisotropy ($\Psi = 0.86$) in Fig. 7.

Fig. 12. A series of rice images displaying different degrees of fabric anisotropy.

5.2 Particle aspect ratio

The range of fabric anisotropy that can develop in a sand depends on the particle Aspect Ratio (AR). Particles having larger AR s tend to develop stronger fabric anisotropy and thus display a wider fabric range. Different sands with a range of AR values will be tested to explore the relationship between AR and degrees of fabric anisotropy.

A total of 15 sands representing a range of particle sizes, gradations, roundnesses, aspect ratios and particle colors as listed in Table 1 were investigated. The particle size distributions were determined using sieving tests while the index void ratios were determined following ASTM D4253-14 [62] and ASTM D4254-14 [63]. For each soil, the average *AR* value was computed after analyzing 200 particles using Zheng and Hryciw [61]’s computational geometry techniques. As shown in Table 1, the average *AR* values of natural soils are in a relative narrow range of 1.22 (Ottawa Sand) to 1.78 (Crushed Gabbro). Zheng and Hryciw [64] collected the *AR*s of 167 sands (including natural sands and crushed rocks). They found that the average *AR*s of most sands range between 1.2 and 1.8. More elongated soil particles are rare in nature because they are vulnerable to breakage. While occasional individual particles could have larger *AR* values, it is rare for natural sands to possess average *AR* values larger than 1.8. As such, short-grain rice, medium–grain rice, and long–grain rice were added to expand the studied *AR* range.

Air pluviation was used to generate the cross–anisotropic fabric structure. The laboratory equipment included a 5 cm × 5 cm square glass tube of 30 cm height, a funnel, a 2.5 kg weight for surcharge, a shaking table and a Nikon D800 camera with a magnification lens. The funnel and glass tube were used to prepare the loose specimen to replicate a previously determined $e_{\max}$. Images of 2900 × 5800 pixel size were captured through the side of the square glass tube. The captured images of five typical sands including Ottawa, Indiana Beach, Crushed Gabbro, short-grain rice and medium-grain rice are shown in Figs. 13(a) to 17(a). The glass tube was mounted on a shaking table and a surcharge weight was placed on top of the material. The specimens were vibrated until they achieved their respective independently established ASTM values of $e_{\min}$. Images were again captured after densification to $e_{\min}$. They are shown in Figs. 13(b) to 17(b).

To generate isotropic fabrics, the particles from each specimen were spread on a flat surface to ensure that the long axes of particles were randomly oriented. The images were captured from above the specimens. They are shown in Figs. 13(c) to 17(c). The image sizes were also 2900 × 5800 pixels.

The captured images were divided into 8 sub-images of 1450×1450 pixel size. For each sub-image, a ten decomposition level RHWT was performed on its central 1024×1024 pixel region. The horizontal and vertical energies of the original images at the i -th decomposition levels were averaged from all eight sub-squares:

$$E_{Hi}(\theta) = \frac{1}{m} \sum_{j=1}^m E_{Hi,j}(\theta) \quad (16)$$

and

$$E_{Vi}(\theta) = \frac{1}{m} \sum_{j=1}^m E_{Vi,j}(\theta) \quad (17)$$

where m is the total number of sub-squares; $E_{Hi,j}(\theta)$ and $E_{Vi,j}(\theta)$ are the horizontal and vertical energy of the j -th sub-square at rotation θ . Combining Eqs. (16), (17), (7), (8), (10) and (11), the $ER(\theta)$ of the original image was computed and fitted with a Fourier ER plot. Only the Fourier ER plots are shown in Figs. 13(d) to 17(d) in which the isotropic circle is also shown for reference. The original ER plots are close to their fitted Fourier plots and therefore are not shown. The computed degrees of fabric in loose condition, dense condition and, flat surface tests are represented by Ψ_L , Ψ_D , and Ψ_F respectively. The fabric directions of the loose and dense conditions are also shown.

As expected, the ER plots of the flat surface tests were very close to the reference isotropic circles. The ER plots for the dense condition display stronger anisotropy than the plots for the loose condition. The fabric directions for both the dense and loose conditions are very close to the horizontal direction. The Ψ_L , Ψ_D , and Ψ_F values of all the soils are summarized in Table 1.

Fig. 13. Fabric anisotropy of Ottawa sand: (a) loose condition, (b) dense condition, (c) flat surface test, (d) computed ER plots.

Fig. 14. Fabric anisotropy of Indiana Beach sand: (a) loose condition, (b) dense condition, (c) flat surface test, (d) computed ER plots.

Fig. 15. Fabric anisotropy of Crushed Gabbro sand: (a) loose condition, (b) dense condition, (c) flat surface test, (d) computed *ER* plots.

Fig. 16. Fabric anisotropy of short-grain rice: (a) loose condition, (b) dense condition, (c) flat surface test, (d) computed *ER* plots.

Fig. 17. Fabric anisotropy of medium-grain rice: (a) loose condition, (b) dense condition, (c) flat surface test, (d) computed ER plots.

Table. 1. The intrinsic properties and maximum fabric anisotropy of the tested materials

Soils	D_{50}	C_u	e_{max}	e_{min}	AR	Ψ_L	Ψ_D	Ψ_F
Ottawa 20 – 30	0.71	1.4	0.74	0.49	1.23	0.57	0.58	0.51
Michigan Dunes	0.30	1.5	0.85	0.56	1.32	0.58	0.60	0.52
Treasure Island, California	0.25	1.8	0.84	0.56	1.39	0.59	0.61	0.53
Oakland County, Michigan	0.31	1.6	0.86	0.53	1.45	0.58	0.61	0.51
Class IIA, Michigan	0.21	1.9	0.86	0.56	1.45	0.60	0.62	0.50
Capitola, California	0.33	1.5	0.94	0.60	1.39	0.58	0.60	0.53
New Madrid, Missouri	0.32	2.2	0.81	0.52	1.47	0.61	0.63	0.52
Chesterton, Indiana beach	0.64	1.9	0.85	0.54	1.40	0.59	0.63	0.54
Upper Peninsula, Michigan	0.60	2.8	0.85	0.54	1.45	0.60	0.62	0.54
Michigan 2NS	0.50	2.3	0.82	0.54	1.49	0.59	0.62	0.53
Griffin, Indiana	0.74	3.3	0.79	0.51	1.54	0.61	0.64	0.52
Crushed Gabbro	0.80	1.5	0.98	0.60	1.78	0.63	0.68	0.54
Short - grain rice	1.91	1.1	0.97	0.65	1.82	0.64	0.71	0.50
Medium - grain rice	1.62	1.1	1.01	0.72	2.15	0.66	0.76	0.53
Long -grain rice	1.51	1.1	1.08	0.85	3.00	0.70	0.86	0.51

Note: C_u = coefficient of uniformity; D_{50} = 50% size (mm); e_{max} and e_{min} = maximum and minimum index void ratios; NS = natural sand; AR = aspect ratio.

The relationships between AR and Ψ_L , Ψ_D , and Ψ_F are shown in Fig. 18. Excluding the rightmost three data points which are from the rice specimens, most of the data points are concentrated in the narrow range of $1.2 < AR < 1.8$. As expected, all of the Ψ_F values are below 0.55 and are independent of the AR s. Both Ψ_L and Ψ_D values increase with increasing AR reflecting the stronger fabric anisotropy developed in soils with more elongated particles. The relative density will affect the orientation distribution of particle long axes. Following shakedown to their dense states, soil particles display stronger preferred orientations and therefore stronger fabric anisotropy. Therefore, the Ψ_D values are larger than Ψ_L values in Fig. 18.

The Ψ_L and Ψ_D values establish the degree of cross-anisotropic fabric in soils at $e_{\max}$ and $e_{\min}$ states respectively. The relationships can be described by:

$$\Psi_D = 0.33 \ln(AR) + 0.50 \quad (18)$$

$$\Psi_L = 0.25 \ln(AR) + 0.50 \quad (19)$$

In the above two equations, the constant “0.50” is the reference isotropic fabric value that would be expected for perfectly spherical particles such as glass beads. The terms “ $0.33 \ln(AR)$ ” and “ $0.25 \ln(AR)$ ” reflect the divergence from isotropic fabric due to the particle AR s. Those two equations are also plotted in Figs. 18. Eq. (18) fits the Ψ_D values very well while Eq. (19) fits the Ψ_L values well when $AR < 1.8$ but overpredicts Ψ_D when $AR > 1.8$. However, as mentioned before, the average AR values for the majority of natural sands are below 1.8. Therefore, Eq. (19) should be valid for most sands.

The difference between Eqs. (18) and (19) is $0.08 \ln(AR)$ indicating how much the range of Ψ depends on AR . Elongated soils tend to exhibit a wider range of Ψ . The Ψ value at any relative density $D_r = (e_{\max} - e)/(e_{\max} - e_{\min})$ can be computed by linear interpolation between Eqs. (18) and (19):

$$\Psi = D_r(\Psi_D - \Psi_L) + \Psi_L = (0.08D_r + 0.25)\ln(AR) + 0.5 \quad (20)$$

Although users can always determine Ψ values through the RHWT presented in this study, Eq. (20) provides an alternative way to estimate Ψ if the D_r and AR of a soil are known.

Fig. 18. The relationship between aspect ratio and degree of fabric anisotropy.

The degrees of fabric anisotropy may be divided into five levels based on Ψ values. The Ψ values of flat surface tests are below 0.55, so $\Psi = [0.50, 0.55]$ is classified as an isotropic fabric. For Ψ values larger than 0.55, the degrees of fabric anisotropy may be divided into four additional levels: weak anisotropy ($0.55 < \Psi \leq 0.60$), moderate anisotropy ($0.60 < \Psi \leq 0.65$), strong anisotropy ($0.65 < \Psi \leq 0.70$), and extreme anisotropy ($0.70 < \Psi$) as shown in Fig. 18.

6. Fabric of Clay and Rock by RHWT

The Rotational Haar Wavelet Transform technique is not limited to analysis of sand fabric. It can analyze the fabric of any material that manifests spatial grayscale changes in an image. Images can be captured by off-the-shelf cameras, cellphone cameras, digital microscopes, scanning electron microscopes, airborne drones, satellite cameras and other devices. The developed method can be used in many disciplines in which fabric analysis is needed including material science, soil science, geology, powder technology, pavement engineering agriculture, medicine and many others. Building on this paper's theme of earth materials, example analyses of clay and rock fabric will be presented in this section.

Stone et al. [65] captured a series of kaolinite clay images using an LEO (Zeiss) 1550 field emission scanning electron microscope (FESEM). The kaolinite has a size distribution of 97.5% finer than the No. 200 sieve, 68% finer than 2 microns, a liquid limit of 76 and a plastic limit of 33. The specimen was compacted at optimum moisture content in a standard Proctor test, then freeze dried and fractured. The fracture surface was sputter coated with iridium for imaging. The clay particles on the fractured surface should be distributed somewhat randomly as they were in the flat surface tests on sands. As such, the fabric should be isotropic. Figs. 19(a) and (c) present two images of the same fracture surface at different magnifications. The RHWT method was used to evaluate the fabric anisotropy in both images. The computed Ψ values were 0.54 and 0.52 respectively indicating an isotropic fabric as shown in Figs. 19(b) and (d).

In another group of tests, the clay was compacted at optimum moisture then failed in unconfined compression. After freeze drying and sputter coating with iridium, an image was taken of the failure surface. During shearing, the particle long axes become oriented in the sliding direction generating an anisotropic fabric. Figs. 20(a) and (c) capture this failure surface at two magnifications. Based on visual observation, the sliding direction of the clay particles is about 100° . Fig. 20 (b) and (d) provide the ER plots. The computed fabric orientations are 106° and 102° which agree well with visual observations.

The computed Ψ values are 0.57 and 0.62 indicating that weak to moderate anisotropy was developed as a result of shearing.

Fig. 19. Fabric anisotropy of kaolinite clay caused by fracturing.

Fig. 20. Fabric anisotropy of kaolinite clay caused by shearing.

Fabric anisotropy also exists in rock. In sedimentary rocks, the fabric can arise from orientation of mineral grains during deposition. In igneous rocks, fabrics can be developed by magmatic flow orientation. In metamorphic rocks, fabrics can arise due to either deviatoric stress, or relicts inherited from their protoliths. Two rock images from the online Imperial College Rock Image Library [66] are shown in Fig. 21. Fig. 21(a) is an igneous rock showing a horizontal magmatic flow. The computed $\Psi = 0.65$ indicates a strong anisotropic fabric. As shown in Fig. 21(b), the computed fabric orientation of 6° agrees well with visual observation. Fig. 21(c) shows a metamorphic rock composed mainly of antigorite with some minor opaque hematite. The antigorite was compressed under high stress in the diagonal

direction from upper right to lower left to form the fabric. The computational results in Fig. 21(d) show the rock has an extreme fabric with $ER_{\max} = 0.73$. The computed fabric orientation of 133° agrees well with visual observation.

Fig. 21. Examples of fabric anisotropy in rocks.

7. Fabric tensor from RHWT results

In soil mechanics and constitutive modeling, the fabric of soil is usually quantified by a fabric tensor [43-48, 53]:

$$\phi_{ij} = \int_{\Omega} P(\Omega) n_i n_j d\Omega \quad (i, j = 1, 2, 3) \quad (21)$$

where the n_i ($i = 1, 2, 3$) are the components of a unit vector $\mathbf{n}$ describing the orientation of particle long axes; the $P(\Omega)$ is a probability density function describing the distribution of $\mathbf{n}$; Ω is the solid angle corresponding to the entire surface of a unit sphere; and $P(\Omega)d\Omega$ is the fraction of $\mathbf{n}$ oriented within a small solid angle $d\Omega$. The computed fabric tensor by Eq. (21) is a symmetric, third-rank matrix, which can be represented by three principal values φ_1 , φ_2 and φ_3 at corresponding principal directions θ_1 , θ_2 and θ_3 .

The sum of the three principal values of the fabric tensor φ_1 , φ_2 and φ_3 is unity and for a cross-anisotropic fabric, φ_1 must equal to φ_2 . Therefore, if we know the relationship between φ_1 and φ_3 , the three principal values can be computed. Such a relationship between φ_1 and φ_3 can be obtained from the RHWT of two-dimensional side images along the depositional direction such as the images in Figs. 13(a) to 17(a).

For a two - dimensional image, Eq. (21) can be simplified to

$$\bar{\varphi}_{ij} = \int_0^{2\pi} P(\theta) n_i n_j d\theta \quad (i, j = 1, 2) \quad (22)$$

where $(n_1, n_2) = (\cos\theta, \sin\theta)$ and the bar indicates that it is for a 2D image. The $P(\theta)$ can be any function that satisfies:

$$\int_0^{2\pi} P(\theta) d\theta = 1 \quad (23)$$

With a minor mathematical adjustment to satisfy Eq. (23), our Fourier idealization of the ER distribution given by Eq. (14) becomes the function $P(\theta)$:

$$P(\theta) = \frac{2}{\pi} ER(\theta) - \frac{1}{2\pi} = \frac{1}{2\pi} + \frac{2a_2}{\pi} \cos(2\theta) + \frac{2b_2}{\pi} \sin(2\theta) \quad (24)$$

Combining Eqs. (24) and (22), the two - dimensional fabric tensor is computed to be:

$$\bar{\varphi} = \begin{bmatrix} 0.5 + a_2 & b_2 \\ b_2 & 0.5 - a_2 \end{bmatrix} \quad (25)$$

The $\bar{\varphi}$ is a symmetric, second-rank tensor and its two principal values $\bar{\varphi}_1$ and $\bar{\varphi}_2$ are:

$$\begin{Bmatrix} \bar{\varphi}_1 \\ \bar{\varphi}_3 \end{Bmatrix} = \frac{(\bar{\varphi}_{11} + \bar{\varphi}_{33}) \pm \sqrt{(\bar{\varphi}_{11} - \bar{\varphi}_{33})^2 + 4\bar{\varphi}_{13}^2}}{2} = 0.5 \pm \sqrt{a_2^2 + b_2^2} \quad (26)$$

Comparing Eq. (26) and Eq. (15), the two principal values $\bar{\varphi}_1$ and $\bar{\varphi}_3$ are exactly $ER_{\max}$ and $ER_{\min}$. If $ER_{\max}$ and $ER_{\min}$ are replaced by Ψ and $1-\Psi$ respectively, we have $\bar{\varphi}_1 = \Psi$ and $\bar{\varphi}_3 = 1-\Psi$.

Assuming principal values $\bar{\varphi}_1$ and $\bar{\varphi}_3$ in two - dimensional images are proportional to the principal values φ_1 and φ_3 in three dimensions for cross-anisotropic soil (Oda and Nakayama 1989):

$$\frac{\varphi_1}{\varphi_3} = \frac{\bar{\varphi}_1}{\bar{\varphi}_3} = \frac{\Psi}{1-\Psi} \quad (27)$$

Since $\varphi_1 = \varphi_2$ and $\varphi_1 + \varphi_2 + \varphi_3 = 1$, the three principal values of the fabric tensor φ_1 , φ_2 and φ_3 can be obtained. The complete fabric tensor defined by Eq. (21) can finally be rewritten as:

$$\varphi = \begin{bmatrix} \varphi_1 & 0 & 0 \\ 0 & \varphi_2 & 0 \\ 0 & 0 & \varphi_3 \end{bmatrix} = \frac{1}{1+\Psi} \begin{bmatrix} \Psi & 0 & 0 \\ 0 & \Psi & 0 \\ 0 & 0 & 1-\Psi \end{bmatrix} \quad (28)$$

The Ψ can be determined using either the RHWT technique detailed in this paper or estimated by Eq. (20) with known values of AR and D_r . Equation (28) can be readily used in development of constitutive soil models and statistically correlated to various anisotropic soil properties. Fig. 22 plots φ_1 , φ_2 , and φ_3 versus Ψ in which the φ_1 and φ_2 curves overlap. For isotropic fabric, $\Psi = 0.5$ and therefore, $\varphi_1 = \varphi_2 = \varphi_3$

= 1/3. For complete anisotropy, $\Psi = 1.0$ and then $\varphi_1 = \varphi_2 = 0.5$ and $\varphi_3 = 0$. As Ψ increases from 0.5 to 1, the φ_1 and φ_2 increase from 1/3 to 0.5 while φ_3 decreases from 1/3 to 0.

The degree of cross-anisotropic fabric can also be quantified by a deviatoric fabric parameter defined by Thornton [67] as $\varphi_d = \varphi_1 - \varphi_3$. The evolution of φ_d in the shearing process has been extensively used in DEM modeling by Batteto and O'Sullivan [68] and Huang et al. [69]. Fig. 22 also plots the relationship between φ_d and Ψ . As Ψ increases from 0.5 to 1, φ_d increases from 0 to 0.5.

Fig. 22. Relationship between the fabric tensor and Ψ .

Conclusions

A rotational Haar wavelet transform (RHWT) method has been developed and implemented to characterize the fabric of particulate materials from two-dimensional images. To demonstrate the RHWT,

the method was first used on synthetic purely anisotropic and purely isotropic grayscale images. The Energy of the Haar wavelet was shown to be minimum in the fabric direction and maximum in the direction normal to the fabric direction. An Energy Ratio, ER was defined that ranges from 0 to 1.0 and is maximum in the fabric direction. This ER is plotted in a traditional polar rose diagram that shows lobes elongated in the fabric direction. For a purely isotropic material the ER plot is a circle having a value of 0.5 in all directions. As a material displays stronger fabric anisotropy the maximum ER extends in the fabric direction. Images of long-grain rice were taken at various degrees of particle alignment to show that $ER_{\max}$ increases as rice grain alignment increases. Because of its significance, $ER_{\max}$ was given the symbol Ψ which has a theoretical range of 0.5 to 1.0.

The RHWT was utilized on 12 sands of various geologic origins, particle sizes, coefficients of uniformity, index void ratios and most importantly, aspect ratios (AR). For each material, specimens were prepared on a flat surface, in a square glass vessel at $e_{\max}$ and then densified to $e_{\min}$. As expected, images taken of the specimens prepared on a flat surface all exhibited isotropic fabric defined by $\Psi < 0.55$. Images of the specimens taken through the vertical walls of the square vessel exhibited increasing fabric anisotropy as quantified by increased Ψ . The fabric anisotropy increased with AR . The Ψ further increased with densification in the vessel from $e_{\max}$ to $e_{\min}$. A simple expression for Ψ as a function of AR and relative density, D_r was presented. A five-level fabric anisotropy classification system based on Ψ was proposed.

The RHWT was also employed on scanning electron images of fractured and sheared clay surfaces as well as on several rock images to demonstrate the versatility of the method. Finally, it was shown that the commonly used fabric tensor can be elegantly defined for transversely isotropic materials in terms of the RHWT based value of Ψ .

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Oakland County, Michigan	0.31	1.6	0.86	0.53	1.45	0.58	0.61	0.51
Class IIA, Michigan	0.21	1.9	0.86	0.56	1.45	0.60	0.62	0.50
Capitola, California	0.33	1.5	0.94	0.60	1.39	0.58	0.60	0.53
New Madrid, Missouri	0.32	2.2	0.81	0.52	1.47	0.61	0.63	0.52
Chesterton, Indiana beach	0.64	1.9	0.85	0.54	1.40	0.59	0.63	0.54
Upper Peninsula, Michigan	0.60	2.8	0.85	0.54	1.45	0.60	0.62	0.54
Michigan 2NS	0.50	2.3	0.82	0.54	1.49	0.59	0.62	0.53
Griffin, Indiana	0.74	3.3	0.79	0.51	1.54	0.61	0.64	0.52
Crushed Gabbro	0.80	1.5	0.98	0.60	1.78	0.63	0.68	0.54
Short - grain rice	1.91	1.1	0.97	0.65	1.82	0.64	0.71	0.50
Medium - grain rice	1.62	1.1	1.01	0.72	2.15	0.66	0.76	0.53
Long -grain rice	1.51	1.1	1.08	0.85	3.00	0.70	0.86	0.51

Note: C_u = coefficient of uniformity; D_{50} = 50% size (mm); $e_{\max}$ and $e_{\min}$ = maximum and minimum index void ratios; NS = natural sand; AR = aspect ratio.

Figure 1

[illegible]

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

(a)

(b)

Figure 7

Figure 8

Figure 9

Figure 10

Figure 11

Figure 12

Figure 13

Figure 14

Figure 15

Figure 16

Figure 17

Figure 18

Figure 19

Figure 20

Figure 21

Figure 22