1 **Thermal Volume Change of Poorly Draining Soils**
2 **II: Model Development and Experimental Validation**

3
4 **Charles James Russell Coccia, Ph.D.**, Research Associate

5 University of Colorado Boulder

6 Department of Civil, Environmental and Architectural Engineering

7 UCB 428, Boulder, CO 80309; coccia@colorado.edu

8 **John S. McCartney, Ph.D., P.E.**, Associate Professor

9 University of California San Diego

10 Department of Structural Engineering

11 9500 Gilman Dr., La Jolla, CA 92093-0085; mccartney@ucsd.edu

12
13 **ABSTRACT:**

14 This study presents the development of an alternative constitutive model for predicting the
15 thermal volume change of saturated and unsaturated soils using the concept of thermally-
16 accelerated secondary compression. The model requires observation of the secondary
17 compression response prior to drained heating to estimate the coefficient of secondary
18 compression. The model was found to successfully capture the trends in thermal volume change
19 arising from accelerated secondary compression resulting from prior mechanical loading for
20 unsaturated, compacted Bonny silt. The model was also used successfully to represent data from
21 the literature for normally consolidated and heavily overconsolidated soils prepared with
22 different loading paths.

23 **KEYWORDS:** thermal volume change behavior; thermal creep; geothermal energy; constitutive
24 modeling

INTRODUCTION

A critical assessment of the mechanisms of thermal volume change in saturated and unsaturated soils was presented in the companion paper by Coccia and McCartney (2016a), and an alternate volume change mechanism related to the effects of thermally-accelerated secondary compression arising from changes in stress prior to heating was found to help explain some discrepancies in data reported in the technical literature. Thermal volume changes are typically estimated using empirical thermal softening equations that predict elastic expansion for overconsolidated soils and plastic contraction for normally consolidated soils during drained heating. However, studies by Towhata et al. (1993) and Burghignoli et al. (2000) observed that clays with an OCR as high as 56 may either expand or contract during drained heating depending on the sequence of loading prior to the change in temperature. Coccia and McCartney (2016a) re-evaluated these results and found a clear relationship between the secondary compression index of soil measured prior to heating and the resulting thermal volume change. This relationship serves as the basis of a new effective-stress based constitutive model for the thermal volume change of saturated and unsaturated soils presented in this paper. The thermal volume change results for unsaturated Bonny silt as well as those for a saturated clay under different stress histories presented by Towhata et al. (1993) are used to validate the newly proposed model.

BACKGROUND

Campanella and Mitchell (1968) developed an early thermo-poroelastic model to estimate the magnitude of thermally-induced excess pore water pressures and accompanying volume change during undrained heating and cooling. This model considered the thermal expansion of the soil constituents (soil pore water and pore solids), but did not evaluate the potential plastic changes in volume of the soil skeleton. To consider the effects of temperature on both volume change and

the shear strength of soils, Hueckel and Borsetto (1990) presented the first thermo-plastic framework for saturated soils by integrating a thermal softening mechanism into the modified Cam-Clay model of Roscoe and Burland (1968). This thermal softening mechanism captures the decreases in mean effective preconsolidation stress, p'_c , with increases in temperature under constant stress that have been observed in several studies on saturated and unsaturated soils (Eriksson 1989; Tidfors and Sällfors 1989; Boudali et al. 1994; Cekerevac and Laloui 2004; Salager et al. 2008; Uchaipichat and Khalili 2009). An important advantage of the framework proposed by Hueckel and Borsetto (1990) is that changes in p'_c can be used to predict the thermal volume change while considering the relative influence of OCR as described in the literature. Accordingly, the use of a thermally modified yield surface as proposed by Hueckel and Borsetto (1990), is typically adopted to serve as the foundation for most current thermo-mechanical constitutive frameworks. Most of these models are focused on the behavior of saturated soils, but there have been recent efforts to consider phenomena encountered in unsaturated soils such as suction hardening (François and Laloui 2008) and degree of saturation effects (Bellia et al. 2015).

In thermo-elasto-plastic models, elastic and plastic deformations may occur due to either changes in mean effective stress or temperature. As this study is focused on thermal volume change, the elastic and plastic volume change components due to mechanical loading are disregarded. Most models define the thermal elastic strain increment due to a change in temperature as follows:

$$\partial \varepsilon_{vT}^e = \alpha_T \partial T \quad \text{EQ. 1}$$

where α_T is the coefficient of thermal volumetric expansion for the bulk soil and ∂T is an incremental change in temperature. The most common discrepancy in the prediction of thermal

elastic expansion among current models is associated with the simplifying assumptions for the value α_T . Cui et al. (2000) assumed that α_T is constant with temperature and mean effective stress while others have presented α_T to vary with temperature and stress history (Laloui 1993; Abuel-Naga et al. 2007; François and Laloui 2008). Specifically, François and Laloui (2008) proposed a coefficient of thermal volumetric expansion that exhibits a non-linear dependency on temperature and overconsolidation ratio, allowing for the elastic thermal volumetric strain to increase with increasing overconsolidation ratio ($OCR = p'_c/p'$) and decrease with increasing temperature. A similar assumption was also proposed by Abuel-Naga et al. (2007). Although these modifications of α_T with temperature and OCR are representative of thermal expansion behaviors observed in the literature (Cekerevac and Laloui 2004), Hong et al. (2013) found that constitutive frameworks that utilize a non-constant α_T provide no significant improvements over models which assume a constant value for α_T .

Alongside thermal elastic expansion, thermal plastic contraction may also occur during heating. Current thermal constitutive models may predict this behavior by adopting one or more thermally-modified yield surfaces to define the boundary between a thermo-elastic and a thermo-elasto-plastic volumetric response during heating. It is most common to use a single yield surface to capture the relative effects of changes in effective stress and temperature (Hueckel and Baldi 1990; François and Laloui 2008; Laloui and François 2009). Alternatively, some models use two yield surfaces to capture the effects of temperature and effective stress independently (Cui et al. 2000; Abuel-Naga et al. 2007; Tang and Cui 2009). The various yield surfaces used in the literature are shown in Figure 1 in $p' - T$ space.

For the yield surface in Figure 1(a), plastic contraction will be activated through an increase in mean effective stress to p'_{c0} , or by heating the specimen at p'_{c0} (i.e. normally consolidated

conditions), resulting in thermo-elasto-plastic contraction. However, this temperature-independent yield surface does not capture the thermal softening behavior observed in the literature (Salager et al. 2008). Based on the observed decreases in p'_{c0} with increasing temperature, a thermal softening trend may be incorporated into the LC curve as shown in Figure 1(b). This type of yield surface is the most widely utilized approach to model the thermal volume change of saturated soils (Hueckel and Baldi 1990; Laloui and Cekerevac 2003; Abuel-Naga et al. 2007; Laloui and François 2009; Hamidi et al. 2014) and unsaturated soils (Wenhua et al. 2004; François and Laloui 2008; Dumont et al. 2010; Guoqing and Lu 2010). Incorporation of a thermal softening mechanism into the pre-existing LC surface permits the transition from thermo-elastic expansion to thermo-elasto-plastic contraction to be captured for an overconsolidated soil at constant mean effective stress. Further, based on the shape of the LC curve in Figure 1(b), the anticipated transition temperature will increase with OCR as has been observed in the literature (Cekerevac and Laloui 2004). In addition to the ability of capturing thermal softening when modeling drained compression tests at various temperatures, the non-linear behavior of the LC curve introduces a stress state dependency on the thermal volume change behavior of the soil not captured by a yield surface similar to the one in Figure 1(a). According to the yield surface in Figure 1(b), an immediate plastic thermal contraction would only occur at initial heating for a normally-consolidated soil ($p'_{c0} = p'$). For all other stress cases, the model will predict an initial elastic thermal expansion until the yield surface is met at a higher temperature, after which thermo-elasto-plastic contraction occurs. This behavior is in disagreement with that observed in the literature where soils with OCRs of 2 or greater have been observed to exhibit immediate contraction upon heating (Baldi et al. 1991; Cekerevac and Laloui 2004). Further, use of the modified LC curve may result in an over-prediction of the

“transition temperature” for light to medium overconsolidated soils (Cui et al. 2000). In response to this drawback, Cui et al. (2000) implemented a secondary yield curve, referred to as the “thermal yield” (TY) curve, to better capture the observed plastic thermal contraction behavior for soils with high values of OCR, as seen in Figure 1(c).

To highlight the benefits of the TY curve for the thermal volume change prediction of overconsolidated soils, the experimentally observed “transition temperature” at different overconsolidation ratios has been plotted in Figure 2 against the traditional LC curve as defined by Laloui and Cekerevac (2008) for Kaolinite clay using the thermal softening relationship proposed by Salager et al. (2008):

$$p'_c(T) = p'_c(T_0) \cdot \left(1 - \gamma_T \log \frac{T}{T_0}\right) \quad \text{EQ. 2}$$

where $p'_c(T)$ and $p'_c(T_0)$ are the values of mean effective preconsolidation stress at temperatures T and T_0 , respectively, and γ_T is a material parameter that defines the relative impact of temperature on p'_c . The relationship for the TY curve of Cui et al. (2000) is expressed as:

$$T_{yield} = (T_c - T_0)e^{-\gamma_{TY}p'} + T_0 \quad \text{EQ. 3}$$

where T_{yield} is the yielding temperature at a mean effective stress, p' , T_c is a reference temperature at $p' = 0$, and γ_{TY} is a soil parameter which defines the curvature of the TY curve. Abuel-Naga et al. (2007) proposed a curve similar to the TY curve of Cui et al. (2000) as follows:

$$\frac{p'_c(T_0)}{p'_c(T)} = \gamma_{TY} \sqrt{\ln \left(\frac{T}{T_0}\right)} + 1 \quad \text{EQ. 4}$$

The results in Figure 2 indicate that during heating, all overconsolidated soil specimens transition from elastic thermal expansion to elasto-plastic thermal contraction at temperatures much lower than predicted by the LC curve based on the soil’s thermal softening behavior.

Inclusion of the secondary TY curve provides a yield surface capable of predicting the thermal contraction of overconsolidated soils where use of a traditional LC curve would not. However, establishment of the TY curve requires a series of thermal volume change tests at different values of OCR to be performed to properly calibrate the curvature (γ_{TY}) of the TY curve (Abuel-Naga et al. 2007).

Similar to the establishment of a unique yield surface to define the thermo-elastic region, several methods have been used to calculate the plastic thermal strain increment for an incremental change in temperature during yielding. Cui et al. (2000) proposed the following exponential expression to calculate the thermal plastic strain increment, $\partial \varepsilon_{vT}^p$:

$$\partial \varepsilon_{vT}^p = \alpha_p (e^{\alpha_p(T-T_0)} - a) \partial T \quad \text{EQ. 5}$$

where a is a shape parameter, typically close to unity, used to help capture the thermal soil response at higher temperatures, and α_p is related to the slope of the thermal volume change curve. The parameters a and α_p are determined by fitting EQ. 5 to the thermal volume change curve of a normally consolidated soil heated under constant mean effective stress.

An alternative approach to determine $\partial \varepsilon_{vT}^p$ is by solving the consistency equation formulated from the adopted yield surface. Laloui and Cekerevac (2003) defined the isotropic yield surface, f_{iso} , as follows:

$$f_{iso} = p' - p'_{co} \exp(\beta \varepsilon_v^p) [1 - \gamma_T \log(T/T_0)] \quad \text{EQ. 6}$$

where β is the plastic compressibility modulus. Further, the thermal plastic strain increment for a change in temperature may be defined by the consistency equation as follows:

$$\partial \varepsilon_{vT}^p = \frac{-\frac{\partial f_{iso}}{\partial T} \partial T}{\frac{\partial f_{iso}}{\partial \varepsilon_v^p}} \quad \text{EQ. 7}$$

Substituting EQ. 6 into EQ. 7, the following expression may be used to calculate the thermal plastic strain increment during yielding:

$$\partial \varepsilon_{vT}^p = \frac{\gamma_T \partial T}{\ln 10 \beta T \left[1 - \gamma_T \log \left(T/T_o \right) \right]} \quad \text{EQ. 8}$$

In EQ. 8, the plastic thermal strain increment is determined based on the thermal softening behavior of the soil, as defined by γ_T , and the plastic compressibility modulus, β .

Though these models have been shown to achieve reasonable predictions of the thermal volume change of soil at a target temperature, some limitations exist in the inability to consider the time-dependent behavior of normally consolidated or over consolidated soils that have been prepared via loading, unloading, or reloading (Towhata et al. 1993; Burghignoli et al. 2000). In Figure 3, the thermal volumetric strains for two specimens of MC clay at an OCR of 56 achieved via reloading (Test 56R) and unloading (Test 56U) are shown with the predictions from the model of Cui et al. (2000). Details of the modeling parameters and calibrations for the MC clay can be found in Cui et al. (2000). Prior to heating, secondary compression and expansion was observed in Tests 56R and 56U, respectively. A very strong agreement between the results for Test 56U and the model prediction is observed in Figure 3. However, the model of Cui et al. (2000) is unable to anticipate the thermal volumetric contraction of Test 56R, and predicts the same thermal volume change response for Tests 56R and 56U. This is a result of the model's dependency on the relative value for mean effective stress (or OCR) prior to heating, and not the secondary compression behavior prior to heating. As such, inclusion of a feature to consider the influence of secondary compression will introduce a physical mechanism into the thermal predictive model that is currently not present in available thermo-elasto-plastic models. This may create an advantage over current thermo-elasto-plastic constitutive models that do not consider

the physical mechanisms responsible for thermal volume change in the development of the volume change equation beyond the empirical quantification of thermal softening.

Although the models discussed above were developed for saturated soils, recent modeling efforts have focused on simulation of the thermal volume change response of unsaturated soils. Dumont et al. (2010) developed a model that combines the unsaturated hydro-mechanical framework of Khalili and Loret (2001) with the saturated THM model of Wenhua et al. (2004). However, due to the use of capillary stress to define the effective stress, the soil-water retention curve (SWRC) is not required and changes in degree of saturation due to mechanical loading or changes in matric suction are not captured. François and Laloui (2008) extended the thermo-plasticity model ACMEG-T of Laloui and François (2009) to the unsaturated soils by incorporating a secondary elasto-plastic constitutive framework defining the SWRC and effects of changes in temperature or dry density on the air entry suction. Further, the theory of bounding surface plasticity (Dafalias and Hermann 1980) was utilized to capture the progressive mobilization of irrecoverable strains that occur prior to reaching the preconsolidation stress. Inclusion of the bounding surface approach allows for the modeling of a smooth transition from elastic reloading to plastic yielding. Guoqing and Lu (2010) developed a model extending the existing hydro-mechanical framework of Wheeler et al. (2003) to soils under non-isothermal conditions through the inclusion of a thermal softening mechanism. This model uses the effective degree of saturation as a hardening parameter instead of the matric suction as in the previously-mentioned models. Although these models are enhanced by their capability to consider the advanced hydro-mechanical behavior of unsaturated soils, the drawbacks in their ability to predict thermal volume change are similar to those for saturated thermo-mechanical models.

NEW FRAMEWORK FOR THERMAL VOLUME CHANGE BEHAVIOR OF SOILS

Development

Following the re-examination of the experimental observations of Houston et al. (1985), Towhata et al. (1993), Burghignoli et al. (2000), as well as an independent study performed on Bonny silt by Coccia and McCartney (2016a), the hypothesis of this constitutive model proposed in this study is that the thermal volume change response of saturated and unsaturated soils is due to the thermally-enhanced creep associated with the most recently applied change in effective stress. Particularly, heating of normally consolidated soils will cause an acceleration of the secondary compression process occurring prior to the start of heating, leading to contraction, while heating of overconsolidated soils will lead to either expansion or contraction depending on the direction of the most recent change in effective stress (i.e. reloading or unloading).

In ambient conditions, the change in void ratio with time due to secondary compression, $\Delta e_\alpha(t, T_0)$, may be determined based on the ambient secondary compression index, $C_{\alpha e}(T_0)$, defined as:

$$C_{\alpha e}(T_0) = \frac{\Delta e_\alpha(t, T_0)}{\Delta \log(t)} \quad \text{EQ. 9}$$

For temperatures greater than T_0 , a distinct secondary deformation curve may exist from the secondary compression curve at ambient temperature defined by EQ. 9. Specifically, for the case of a recently loaded normally consolidated soil during heating, the void ratio of the soil specimen may collapse to another curve unique to the applied temperature. An example of this behavior is shown in Figure 4(a) where Towhata et al. (1993) performed a one-dimensional drained heating test on a recently loaded, normally consolidated bentonite clay in stages of 22 to 40 °C, 40 to 60 °C, and 60 to 90 °C. For each heating increment, a rapid decrease in void ratio was observed followed by a gradual decline in the rate of collapse [Figure 4(a)]. To illustrate the concept of a

series of unique thermal secondary compression curves, dotted lines have been included in Figure 4(b) for the temperatures of 22, 40, 60, and 90 °C in reference to the end of primary consolidation.

These unique curves may be defined by a change in the secondary compression index with temperature. As such, the following expression is proposed to consider the influence of temperature on secondary deformation:

$$\Delta e_{total}(T, t) = C_{\alpha e}(T) \log \left(\frac{t}{t_0} \right) \quad \text{EQ. 10}$$

where $\Delta e_{total}(T, t)$ is the total change in void ratio as a function of time and temperature, $C_{\alpha e}(T)$ is the secondary compression index corresponding to a unique temperature, T , and t is the time at temperature, T (Figure 4). Based on the expression in EQ. 10, a new secondary compression curve has been established, defined by the new value $C_{\alpha e}(T)$, for each temperature greater than the ambient temperature. As such, the secondary compression index should be modified to reflect the influence of temperature. One approach is to consider the viscous nature of secondary compression. Secondary compression in soils has been commonly attributed to the viscous behavior of the soil skeleton (Jain and Nanda 2010). For clays following primary consolidation, in particular, layers of adsorbed water may be found between the solid grains which help carry the applied load to the soil along with any solid grain to grain contact which may exist. During secondary compression, a slow viscous flow will occur as the applied load is transferred from the adsorbed layers to the stronger, solid grain to grain contacts, resulting in a gradual deformation of the loaded soil. As the viscous flow of the soil skeleton is primarily responsible for any secondary deformations, it may be assumed that elevated temperatures will enhance such flow, resulting in an accelerated secondary behavior, particularly based on the influence of temperature on water viscosity [Hillel (1998)]. Evidence of the thermal acceleration

of secondary deformations has been observed by Houston et al. (1985) and Tsutsumi and Tanaka (2012), among others. As such, the evolution of $C_{\alpha e}$ with T may be defined through a function, herein referred to as $H(T)$, which characterizes the relative decrease in pore water viscosity at higher temperatures, shown in EQ. 11:

$$C_{\alpha e}(T) = C_{\alpha e}(T_0)H(T) \quad \text{EQ. 11}$$

The function, $H(T)$, is defined as follows:

$$H(T) = \frac{\eta_k(T_0)}{\eta_k(T)} \quad \text{EQ. 12}$$

where η_k is the kinematic viscosity of water. Data provided by Hillel (1998) indicates the ratio of kinematic viscosity at ambient room temperature ($T_0 = 20^\circ\text{C}$) to the kinematic viscosity at the temperature, T , in $^\circ\text{C}$ to exhibit the following relationship:

$$H(T) = \frac{\eta_k(20^\circ\text{C})}{\eta_k(T)} \approx \frac{C_{\alpha e}(T)}{C_{\alpha e}(T_0)} = 1.013 \times 10^{-4}T^2 + 2.007 \times 10^{-2}T + 0.559 \quad \text{EQ. 13}$$

To assess the use of the function, $H(T)$, to characterize the relationship between the total change in void ratio during heating and that expected due to ambient secondary creep, the ratio $\Delta e_{total}(t, T)/\Delta e_{\alpha}(t, T_0)$ for Bonny silt from Coccia and McCartney (2016a), as well as MC clay and Todi clay from Towhata et al. (1993) and Burghignoli et al. (2000), respectively, has been compared with $H(T)$ as defined in EQ. 13 in Figure 5. In Figure 5, the shape and magnitude of the function $H(T)$ is in very strong agreement with the relationship $\Delta e_{total}(t, T)/\Delta e_{\alpha}(t, T_0)$ observed for Bonny silt, Todi clay, and MC clay. The agreement between the function $H(T)$ and $\Delta e_{total}(t, T)/\Delta e_{\alpha}(t, T_0)$ suggests the utilization of the thermal influence over the viscosity of water to be appropriate to describe the evolution of the rate of secondary creep deformations during a change in temperature. Furthermore, the fit observed in Figure 5 confirms thermal volume change to be a result of the acceleration of secondary creep.

Via the substitution of EQ. 11 into EQ. 10, the change in void ratio due to a change in temperature with time, $\Delta e_{total}(T, t)$, following a recent change in applied load may be determined as follows:

$$\Delta e_{total}(T, t) = C_{\alpha e}(T_0)H(T)\log\left(\frac{t}{t_0}\right) \quad \text{EQ. 14}$$

The relationship in EQ. 14 provides a simple expression to predict the thermal volume change of poorly draining soils. In general, for the thermal volume change prediction of different soil types and/or loading histories, $C_{\alpha e}(T_0)$ may be easily determined experimentally through incremental loading compression tests via a conventional oedometer apparatus, while the remaining parameters (T_0 , T , t_0 , and t_f) may be chosen by the engineer to define the problem. For the function $H(T)$, the relationship defined in EQ. 13 may be utilized when the ambient temperature is equal to 20 °C. When $T_0 \neq 20$ °C, a new function such as that of Hillel (1998) may be used.

The relationships established in EQ. 13 and 14 have been utilized to present the general behavior of the newly proposed method for the prediction of thermal volume change. In Figure 6, a hypothetical soil has been heated 24 hours following the end of primary consolidation while exhibiting a rate of ambient secondary compression of $C_{\alpha e}(T_0) = 0.01$. To show the predicted behavior of the soil during slow heating, the soil was heated at a rate of 0.3 °C/hr from an ambient temperature of 20 °C to 40 °C in Figures 6(a) and (b). Typical of most experimental studies, the soil is allowed to deform under constant temperature for a period of 24 hours and is then heated to 60 °C, where it is allowed to rest for another 24 hours before equilibrium has been established. The unique secondary compression curves for the target soil temperatures of 20, 40, and 60 °C are included in Figure 6(a) as a dotted line, while the void ratio with time is shown by

a solid line. The soil volume is observed to collapse to the corresponding thermal secondary curve during heating in Figure 6(a). The associated evolution in void ratio against temperature is shown in Figure 6(b). Hollow circles have been included in Figures 6(a) and 6(b) to illustrate the equilibrium points commonly chosen for analysis during thermal volume change tests to represent the ultimate change in volume related to a particular change in temperature. In most cases, the soil is assumed to be in equilibrium once the soil has achieved a rate of strain less than a pre-determined value.

Comparison of the hypothetical thermal collapse shown in Figure 6(a) with that measured for bentonite clay in Figure 4 indicates a difference in trend for the thermal volume change response. For the case of the bentonite clay, the void ratio is observed to collapse immediately, while a slow gradual collapse is observed for the hypothetical case in Figure 6(a). This discrepancy is due to the difference in the applied heating rate, as the bentonite clay was heated by Towhata et al. (1993) in one large rapid increment, compared to a rate of 0.3 °C/hr as simulated in Figures 6(a) and 6(b). To showcase the ability of the newly proposed model to capture the thermal collapse behavior at fast heating rates, the hypothetical soil was heated at a maximum heating rate of 15 °C/hr followed by a gradual decrease in the rate of heating until the target temperature of 40 and 60 °C was achieved. The results of this simulation are shown in Figures 6(c) and 6(d). In Figure 6(c), a collapse behavior similar to that for the bentonite clay is observed as a result of the accelerated heating rate. The evolution in void ratio against temperature for the faster simulation is shown in Figure 6(d) where a smaller overall change in void ratio is observed compared to that seen in Figure 6(b). Comparison of these two figures highlights the influence of time and heating rate on the ultimate thermal volume change that may be reported after testing.

Model Validation

Based on the experimental results of this study as well as those of Towhata et al. (1993) and Burghignoli et al. (2000), the recent stress history and secondary compression behavior, prior to heating, appears to dominate the overall thermal volume change response. As such, an elegant approach to predict the thermal volume change of poorly draining soils during heating has been proposed. The model expression defined in EQ. 13 and EQ. 14 will be used to predict the thermal volume change response observed for the compacted Bonny silt from the companion study of Coccia and McCartney (2016a). The values of $C_{\alpha e}(T_0)$ for Tests S-0, US-15, US-20, US-30, and US-40 have been summarized in Table 4 in Coccia and McCartney (2016a). In EQ. 14, t_0 represents the time at initial heating following drained mechanical loading. The total change in void ratio during heating was predicted using the relationships proposed in EQ. 13 and EQ. 14 and is shown with the experimental results for Tests S-0, US-15, US-20, US-30, and US-40 in Figures 7 and 8 in terms of the total change in void ratio versus temperature and time, respectively.

Good agreement between the model predictions and the experimental results in Figure 7 is observed. This indicates the thermal dependency of the secondary deformation behavior of soil to be a suitable mechanism to explain thermal volume change. Further, the model predictions are calculated using only one unique parameter ($C_{\alpha e}$) that may be easily measured both in the lab and in the field, and four others (T_r , T , t_0 , and t_f) chosen by the user. To analyze the ability of the new model to predict thermal volume changes during a heating and cooling cycle, EQ. 14 was used to simulate the heating-cooling volume change results for saturated Test S-0 and is shown in Figure 7(a). Excellent agreement between the test results for Test S-0 and the model prediction during heating and cooling is observed. The ability of EQ. 7 to predict the thermal

volume change response during cooling indicates the additional contraction observed during cooling to be primarily a result of additional secondary compression, alongside with any thermal contraction of the soil constituents, which occurs during the time of cooling. However, the magnitude of secondary compression with time during cooling is observed to be smaller than during heating as the secondary compression index will decrease with temperature.

It is important to acknowledge the reliance of this new method for the prediction of thermal volume change on the viscous properties of pore water with temperature and its influence on secondary deformations. The presence of water has been observed to aid in the rate of secondary compression (Oldecop and Alonso 2007; Priol et al. 2007; Ye et al. 2014). Ye et al. (2014) performed high-pressure suction controlled oedometer tests on compacted GMZ01 bentonite to investigate the influence of vertical stress and matric suction on secondary compression. $C_{\alpha e}$ was observed to decrease with increases in matric suction and decreases in moisture content. Similar behavior has also been observed by Oldecop and Alonso (2007) and Priol et al. (2007) on unsaturated rockfill material and unsaturated Lixhe chalk, respectively. Alongside a decrease in the secondary compression index, as the volume of pore water is reduced, the impact of variations of viscosity with temperature on the interparticle friction, and therefor secondary movement, may also reduce. As such, the current model proposed may begin to over-predict the thermal acceleration of secondary compression for soils of lower degrees of saturation, for example dry to nearly dry soils. To investigate this potential issue, thermal volume change tests on dry soils must be performed. However, for the simulations performed for the Bonny silt, the proposed model appears to capture the phenomena of thermally accelerated secondary compression for degrees of saturation ranging from 1 to 0.5.

To assess the ability of the proposed model to predict thermal volume change for higher plasticity clays, EQ. 14 was used to predict the thermal change for MC clay as tested by Towhata et al. (1993). Several tests from Towhata et al. (1993) were selected to explore a wide range of OCRs (1 to 56) achieved via unloading and reloading as designated by a U or R, respectively. A summary of the final OCR and applied stress for the specimens selected for model validation are summarized in Table 1. Values for $C_{\alpha e}(T_0)$ were calculated between the time, t_α , and time, t_0 , from the time series results provided by Towhata et al. (1993). An example is shown in Figure 9. Values for $C_{\alpha e}(T_0)$ and the times utilized for calculation (t_α to t_0) are shown in Table 1.

From the time series data provided in Towhata et al. (1993), the final time for each temperature increment, t_f , from 22 to 40 °C, 40 to 60 °C, and 60 to 80 °C was selected to predict the final thermal volume change for each heating increment. This was the most suitable approach for model validation as the actual temperature time series for each test was not provided in the literature. The predicted void ratio using EQ. 14 is presented with the void ratio reported in the literature for the MC clay for several loading conditions in Figure 10. The model predicts the thermally induced change in void ratio reasonably well for the MC clay (Figure 10). However, discrepancies between the model and experimental data is observed for Test 1.7U and 56R. Further, the transition from thermal expansion to contraction is not captured by the model for Test 1.7U.

The model comparison with the results from this study on Bonny silt as well as for MC clay from Towhata et al. (1993) indicates that the proposed model can reasonably predict the thermal volume change response during the heating and cooling of saturated and unsaturated silts and clays under various stress states, specifically normally-consolidated and overconsolidated soils achieved via loading, unloading, or reloading. Further, it can achieve these predictions while

requiring only one unique soil parameter, $C_{\alpha e}(T_0)$, that may be obtained from assessment of the prior mechanical loading or potentially from empirical databases. Further, no previous thermal volume change tests are required to calibrate the thermal model as typically required of most current thermo-mechanical constitutive models. A drawback of the proposed model is the assumption of a constant $C_{\alpha e}(T_0)$ with time, which may lead to an over prediction of the thermal volume change at higher temperatures (Figure 7) and larger times in some cases (Figure 8). In addition, the model cannot capture the transition from thermal volumetric expansion to thermal contraction for lightly overconsolidated soils [Figure 10(c)], but this behavior is not necessarily observed for all soils.

CONCLUSIONS

This paper presents an alternative approach to modeling the thermal volume change response of poorly draining soils. Specifically, the thermal enhancement of secondary compression was utilized to explain thermal volume change and a new mathematical expression was developed. The results from an experimental study on Bonny silt, performed in the companion paper, as well as that from Towhata et al. (1993) for MC clay was used to validate the new model. Overall, the model was observed to reasonably predict the thermal volume change response of saturated and unsaturated silt and clay under various stress states for degrees of saturation ranging from 1 to 0.5, subjected to both fast and slow rates of drained heating. Finally, only one soil parameter that may be determined experimentally or through field monitoring is required for the prediction of thermal volume change.

ACKNOWLEDGEMENTS

The authors appreciate the support from National Science Foundation grant CMMI-0928159. The views in this paper are those of the authors alone.

REFERENCES

- Abuel-Naga, H.M., Bergado, D.T., Bouazza, A. and Ramana, G.V. (2007). "Volume Change Behaviour of Saturated Clays under Drained Heating Conditions: Experimental Results and Constitutive Modeling." *Canadian Geotechnical Journal*. 44: 942-956.
- Alsherif, N.A. and McCartney, J.S. (2015). "Nonisothermal Behavior of Compacted Silt at Low Degrees of Saturation." *Géotechnique*. 65(9), 703-716.
- Alsherif, N. and McCartney, J.S. (2013). "Triaxial Cell for Nonisothermal Shear Strength of Compacted Silt under High Suction Magnitudes." *Proceedings of the 1st Pan-American Conference on Unsaturated Soils*. Cartagena de Indias. 147-152.
- Baldi, G., Hueckel, T., Peano, A. and Pellegrini, R. (1991). "Developments in Modelling of Thermo-Hydro-Geomechanical Behaviour of Boom clay and Clay-Based Buffer Materials." *CEC Report EUR 13365*.
- Bellia, Z., Ghembaza, M.S. and Belal, T. (2015). "A Thermo-Hydro-Mechanical Model of Unsaturated Soils Based on Bounding Surface Plasticity." *Computers and Geotechnics*. 69: 58-69.
- Bishop, A.W. and Blight, G.E. (1963). "Some Aspects of Effective Stress in Saturated and Unsaturated Soils." *Géotechnique*. 13(3): 177-197.
- Boudali, M., Leroueil, S. and Murthy, B.R.S. (1994). "Viscous Behaviour of Natural Clays". In *Proceedings of the 13th International Conference on Soil Mechanics and Foundation Engineering*. New Delhi. 1: 411-416.
- Burghignoli, A., Desideri, A. and Miliziano, S. (1995). "Volume Change of Clays Induced by Heating as Observed in Consolidation Tests." *Soils and Foundations*. 35(3): 122-124.

419 Campanella, R.G. and Mitchell, J.K. (1968). "Influence of Temperature Variations on Soil
 420 Behavior." *Journal of the Soil Mechanics and Foundation Engineering Div.* 94(SM3): 9-22.

421 Cekerevac, C. and Laloui, L. (2004). "Experimental Study of Thermal Effects on the Mechanical
 422 Behavior of a Clay." *International Journal for Numerical Analytical Methods Geomechanics.*
 423 28: 209–228.

424 Coccia, C.J.R. (2015). Mechanisms of Thermal Volume Change in Unsaturated Silt. Ph.D.
 425 Thesis. University of Colorado Boulder. Boulder, CO, USA.

426 Coccia, C.J.R. and McCartney, J.S. (2016a). "Thermal Volume Change of Poorly Draining Soils
 427 I: Critical Assessment of Volume Change Mechanisms." *Computers and Geotechnics. In*
 428 *Review.*

429 Coccia, C.J.R. and McCartney, J.S. (2016b). "High-Pressure Thermal Isotropic Cell for
 430 Evaluation of Thermal Volume Change of Soils." *Geotechnical Testing Journal.* 39(2): 217-
 431 234.

432 Cui, Y. J., Sultan, N. and Delage, P. (2000). "A Thermomechanical Model for Clays." *Canadian*
 433 *Geotechnical Journal.* 37(3): 607–620.

434 Dafalias, Y. and Herrmann, L. (1980). "A Bounding Surface Soil Plasticity Model."
 435 *International Symposium on Soils Under Cyclic and Transient Loading.* Swansea. 335–345.

436 Dumont, M., Taibi, S., Flureau, J.M., Bekr, N.A. and Saouab, A. (2010). "Modelling the Effect
 437 of Temperature on Unsaturated Soil Behaviour." *Comptes Rendus Geoscience.* 342: 892-900.

438 Eriksson, L.G. (1989). "Temperature Effects on Consolidation Properties of Sulphide Clays." *In*
 439 *Proceedings of the 12th International Conference on Soil Mechanics and Foundation*
 440 *Engineering.* Rio de Janeiro. 12: 2087-2090.

441 François, B. and Laloui, L. (2008). "ACMEG-TS: A Constitutive Model for Unsaturated Soils
 442 Under Non-Isothermal Conditions." *International Journal for Numerical and Analytical*
 443 *Methods in Geomechanics*. 32: 1955-1988.

444 Guoqing, C. and Lu, H. (2010). "Constitutive Modelling Framework of Thermo-Hydro-
 445 Mechanical Coupling for Unsaturated Clays." *Proceedings of the 2010 International*
 446 *Symposium on Multi-Field Coupling Theory of Rock and Soil Media and Its Applications*.
 447 Chengdu, City. 60-64.

448 Hamidi, A., Turchi, S. and Khazaei, C. (2014). "Thermomechanical Constitutive Model for
 449 Saturated Clays Based on Critical State Theory." *International Journal of Geomechanics*.

450 Hillel, D. (1998). *Environmental Soil Physics*. Academic Press. San Diego, California.

451 Hong, P.Y., Pereira, J.M., Tang, A.M. and Cui, Y.J. (2013). "On Some Advanced Thermo-
 452 Mechanical Models for Saturated Clays." *International Journal for Numerical and Analytical*
 453 *Methods in Geomechanics*. 37: 2952-2971.

454 Houston, S.L., Houston, W.N. and Williams, N.D. (1985). "Thermo-Mechanical Behavior of
 455 Seafloor Sediments." *Journal of Geotechnical Engineering*. 111(11): 1249-1263.

456 Houston, S.L. and Lin, H.D. (1987). "Thermal Consolidation Model for Pelagic Clays." *Marine*
 457 *Geotechnology*. 7: 79-98.

458 Hueckel, T. and Baldi, M. (1990). "Thermoplasticity of Saturated Clays: Experimental
 459 Constitutive Study." *Journal of Geotechnical Engineering*. 116(12): 1778–1796.

460 Hueckel, T. and Borsetto, M. (1990). "Thermoplasticity of Saturated Soils and Shales:
 461 Constitutive Equations." *Journal of Geotechnical Engineering*. 116: 1765–1777.

462 Jain, S.K. and Nanda, A. (2010). "On the Nature of Secondary Compression in Soils."
 463 *Proceedings of the 2010 Indian Geotechnical Conference*. Mumbai, India. 1121-1124.

464 Khalili, N. and Loret, B. (2001). “An Elastoplastic Model for Non-Isothermal Analysis of Flow
 465 and Deformation in Unsaturated Porous Media: Formulation.” *International Journal of*
 466 *Solids and Structures*. 38: 8305–8330.

467 Laloui, L. (1993). Modélisation du Comportement Thermo-hydro-mécanique des Milieux Poreux
 468 Anélastiques. Ph.D. Thesis. Ecole Centrale. Paris, France.

469 Laloui, L. and Cekerevac, C. (2003). “Thermo-Plasticity of Clays: An Isotropic Yield
 470 Mechanism.” *Computers and Geotechnics*. 30: 649-660.

471 Laloui, L. and Cekerevac, C. (2008). “Non-Isothermal Plasticity Model for Cyclic Behaviour of
 472 Soils.” *Numerical and Analytical Methods in Geomechanics*. 32: 437-460.

473 Laloui, L. and François, B. (2009). “ACMEG-T: Soil Thermoplasticity Model.” *Journal of*
 474 *Engineering Mechanics*. 135(9): 932-944.

475 Oldecop, L.A. and Alonso, E.E. (2007). “Theoretical Investigation of the Time-Dependent
 476 Behaviour of Rockfill.” *Géotechnique*. 57: 289-301.

477 Priol, G., De Gennaro, V., Delage, P. and Servant, T. (2007). “Experimental Investigation on the
 478 Time Dependent Behavior of a Multiphase Chalk.” *Springer Proceedings in Physics*. 112:
 479 161-167.

480 Romero, E., Gens, A. and Lloret, A. (2001). “Temperature Effects on the Hydraulic Behaviour of
 481 an Unsaturated Clay.” *Geotechnical and Geological Engineering*. 19: 311-332.

482 Roscoe, K. and Burland, J. (1986). “On Generalized Stress Strain Behaviour of Wet Clay.”
 483 *Engineering Plasticity*. Cambridge University Press. Cambridge, England. 535–609.

484 Salager, S., François, B., El Youssoufi, M.S., Laloui, L. and Saix, C. (2008). “Experimental
 485 Investigations of Temperature and Suction Effects on Compressibility and Pre-Consolidation
 486 Pressure of a Sandy Silt.” *Soils and Foundations*. 48(4): 453-466.

487 Sultan, N., Delage, P. and Cui, Y.J. (2002). "Temperature Effects on the Volume Change
488 Behavior of Boom Clay." *Engineering Geology*. 64, 135-145.

489 Tang, A.M. and Cui, Y.J. (2009). "Modelling the Thermomechanical Volume Change Behaviour
490 of Compacted Expansive Clays." *Géotechnique*. 59(3): 185-195.

491 Tidfors, M. and Sällfors, G. (1989). "Temperature Effect on Preconsolidation Pressure."
492 *Geotechnical Testing Journal*. 12(1): 93-97.

493 Towhata, I., Kuntiwattanakul, P., Seko, I. and Ohishi, K. (1993). "Volume Change of Clays
494 Induced by Heating as Observed in Consolidation Tests." *Soils and Foundations*. 33(4): 170-
495 183.

496 Tsutsumi, A. and Tanaka, H. (2012). "Combined Effects of Strain Rate and Temperature on
497 Consolidation Behavior of Clayey Soils." *Soils and Foundations*. 52(2): 207-215.

498 Uchaipichat, A. and Khalili, N. (2009). "Experimental Investigation of Thermo-Hydro-
499 Mechanical Behaviour of an Unsaturated Silt." *Géotechnique*. 59(4): 339-353.

500 Wenhua, W., Xikui, L., Charlier, R. and Collin, F. (2004). "A Thermo-Hydro-Mechanical
501 Constitutive Model and its Numerical Modelling for Unsaturated Soils." *Computers and*
502 *Geotechnics*. 31: 155-167.

503 Wheeler, S.J., Sharma, R.J. and Buisson, M.S.R. (2003). "Coupling of Hydraulic Hysteresis and
504 Stress-Strain Behaviour in Unsaturated Soils." *Géotechnique*. 53(1): 41-54.

505 Ye, W.M., Lai, X.L., Wang, Q., Chen, Y.G., Chen, B. and Cui, Y.J. (2014). "An Experimental
506 Investigation on the Secondary Compression of Unsaturated GMZ01 Bentonite." *Applied*
507 *Clay Science*. 98: 104-109.

508

LIST OF TABLE CAPTIONS

TABLE 1: Summary of data for MC Clay from Towhata et al. (1993)

LIST OF FIGURE CAPTIONS

FIGURE 1: Yield surface comparison for: (a) LC curve w/o thermal softening mechanism; (b) LC curve w/ thermal softening mechanism; (c) LC with thermal softening mechanism and TY curve

FIGURE 2: LC curve thermal yield predictions with experimental data for Kaolinite clay (Cekerevac and Laloui 2004)

FIGURE 3: Thermal volume change prediction of Cui et al. (2000) for heavily overconsolidated MC clay (Towhata et al. 1993)

FIGURE 4: Thermal consolidation time series behavior for Bentonite (after Towhata et al. 1993): (a) Thermal collapse during heating; (b) Thermal collapse in reference to end of primary consolidation

FIGURE 5: Comparison of $H(T)$ function with thermal volume change data for various soils – [1] Burghignoli et al. (2000); [2] Towhata et al. (1993)

FIGURE 6: Thermal volume change simulation for hypothetical soil subjected to various heating rates: (a) Slow heating rate – void ratio versus time after primary consolidation; (b) Slow heating rate – void ratio versus temperature; (c) Fast heating rate – void ratio versus time after primary consolidation; (d) Fast heating rate – void ratio versus temperature

FIGURE 7: Model comparison with thermal volume change results with temperature for: (a) Test S-0; (b) Test US-15; (c) Test US-20; (d) Test US-30; (e) Test US-40

FIGURE 8: Model comparison with thermal volume change results with time for: (a) Test S-0; (b) Test US-15; (c) Test US-20; (d) Test US-30; (e) Test US-40

FIGURE 9: Approach to calculate $C_{\alpha e}(T_0)$ and predict thermal volume change for the data of Towhata et al. (1993)

FIGURE 10: Model comparison with thermal volume change results from Towhata et al. (1993) for MC clay: (a) OCR = 1, $\sigma = 2230$ kPa; (b) OCR = 1, $\sigma = 160$ kPa; (c) OCR = 1.7U, $\sigma = 1280$ kPa; (d) OCR = 3.5U, $\sigma = 640$ kPa; (e) OCR = 56U, $\sigma = 40$ kPa; (f) OCR = 56R, $\sigma = 40$ kPa

539 TABLE 1: Summary of data for MC Clay from Towhata et al. (1993)

OCR	σ'_v	$C_{\alpha e}(T_0)$	t_α	t_0
(-)	(kPa)	(-)	(minutes)	(minutes)
1	2230	0.0118	15	60
1	160	0.0088	31	60
1.7U	1280	-0.0016	20	58
3.5R	640	0.0079	20	58
56U	40	-0.0084	40	98
56R	40	0.0012	20	58

540

Figure 1

Figure 2

Figure 3

Figure 4

(a)

(b)

Figure 5

Figure 6

Figure 7

Figure 8

Figure 9

Figure 10