R-LRFD: Load and Resistance Factor Design Considering Robustness

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Abstract

LRFD (Load and Resistance Factor Design) is becoming a design method of choice in geotechnical practice, in lieu of the factor of safety (FS)-based design approach. However, even with LRFD, the need to reduce the variation in the predicted performance of a geotechnical system (or a geotechnical structure), referred to herein as the *system response* under applied loads, is still apparent. This paper presents a novel approach for applying existing LRFD codes, with explicit consideration of design robustness, safety, and cost efficiency. The recently developed reliability-based robust geotechnical design (RGD) approach is modified such that it can be integrated with the standard LRFD approach. This modified RGD approach is termed *R-LRFD*, which stands for *Robust* Load and Resistance Factor Design. In *R-LRFD*, the robustness of the system response against the variation in uncertain parameters is explicitly considered. Unlike the reliability-based design (RBD), the user is not required to conduct a full statistical characterization of uncertain parameters. With *R-LRFD*, the gain in the robustness, as reflected by the reduction in the sensitivity of the system response to the *recognized but unquantified* uncertainty of input parameters, is accompanied by an increase in cost. Thus, the concepts of Pareto front and knee point are introduced to aid in making an informed design decision. Further, a simplified procedure is developed to identify the most preferred design (knee point) in the design space, which is generally solved with multi-objective optimization algorithms. The effectiveness and significance of the proposed *R-LRFD* approach are demonstrated with

two examples: one is the design of drilled shaft in sand (illustrated with a discrete design space) and the other is the design of drilled shaft in clay (illustrated with a continuous design space).

Keywords: LRFD; Robust design; Sensitivity; Pareto front; Knee point; Drilled shaft.

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1. Introduction

The uncertainties in soil parameters, solution model, applied loads, and those caused by the construction, often make it difficult to ascertain the performance of a geotechnical design. In traditional deterministic approaches, these uncertainties are not *explicitly* characterized and included in the design analysis; rather, a conservative factor of safety (FS) is adopted based on the concept of “calculated risk” (Casagrande 1965). This FS-based design approach often leads to an inefficient over-design with an unknown and/or inconsistent safety level, although under-design is also a possibility. To achieve a more rational and consistent assessment of the safety, the reliability-based design (RBD) approach has long been suggested as an alternative (e.g., Honjo et al. 2000; Baecher and Christian 2003; Christian 2004; Phoon et al. 2003a&b; Ang and Tang 2007; Fenton and Griffiths 2008; Najjar and Gilbert 2009; Park et al. 2012; Low and Phoon 2015). The RBD approach for the design of a geotechnical structure is often implemented with a *target* reliability index or failure probability, which is derived from a cost-benefit analysis that balances investment and risk considering the failure probability and consequence.

In the context of RBD approach, the performance of a geotechnical structure is analyzed using probabilistic methods that consider explicitly uncertainties in input parameters and/or solution models. It is noted that although various methods have been investigated to estimate the statistics of soil parameters (e.g., Wang and Cao 2013; Cao and Wang 2014) and model errors (e.g., Zhang et al. 2009&2010), the statistics of soil parameters and those of model errors could not be characterized with certainty due to limited availability of site-specific data. Because of the difficulty in obtaining the accurate statistical characterization of soil parameters and model errors in practice, the RBD approach is not widely applied in geotechnical practice;

rather, the load and resistance factor design (LRFD) approach, which is a simpler variant of the RBD approach by design, is more commonly used. The LRFD code employs partial factors (e.g., resistance factors and load factors), which have been calibrated to achieve a target reliability index approximately over a range of design scenarios covered by the code. From a user's perspective, the partial factors for a given geotechnical design problem using a given solution model are specified in the code (e.g., NSAI 2005; BSI 2007; NEN 2007; Standards Australia 2009; AASHTO 2014); then, the main task of the design involves a proper selection of the nominal values of uncertain parameters (e.g., resistance term and load term). The resulting design is a function of the specified partial factors and selected nominal values, with due consideration of cost (Honjo and Amatya 2005). The application of LRFD is identical to that of the FS-based design approach, involving iterative adjustment of the geometries and other design parameters until an economic design that satisfies safety requirements is achieved.

As is well recognized, LRFD is meant to be a simpler variant of the more demanding RBD; the ideal outcome is that the design obtained by LRFD could achieve the same target reliability index as that obtained by RBD. However, the standard LRFD approach that involves fixed partial factors cannot cover all design scenarios involving different levels of variation of soil parameters and model errors. For a given design scenario, the use of the standard LRFD code may lead to a design that deviates from the target reliability index by an unknown amount, more likely on the conservative side but under-design is also a possibility. This drawback was well recognized (e.g., Honjo et al. 2002; Phoon et al. 2003a; Paikowsky et al. 2004; Honjo and Amatya 2005; Low 2005; Najjar and Gilbert 2009; Stuedlein et al. 2012; Park et al. 2013) and suggestions were made to improve the standard LRFD approach with refined partial factors (e.g., Phoon et al. 2003b; Ching and Phoon 2011; Ching et al. 2013). In this paper, however, a

different approach is taken and a new way to apply the existing LRFD codes with explicit consideration of robustness is proposed.

In general, when applying a LRFD code for the design of a geotechnical structure, the nominal values of uncertain input parameters (e.g., resistance term and load term) should be selected based on the knowledge of how the partial factors are calibrated. In the absence of this knowledge, a conservative estimate may be used in practice based on an evaluation of the mean as well as the level of variation (e.g., Orr 2000; Driscoll and Simpson 2001), or based on local experience and engineering judgment. However, the level of variation of a soil parameter often cannot be quantified with certainty due to the spatial variability and/or limited data availability (Juang et al. 2013). Thus, the selection of the nominal values of soil parameters that are required in the existing LRFD approach is quite uncertain or subjective. Given the uncertain nature in the selection of the nominal values of uncertain parameters, the predicted performance of a design based on the LRFD analysis would be uncertain. Facing the uncertainty in the predicted performance, a conservative design is preferred. However, an overly conservative design is cost-inefficient and it diminishes the advantage of LRFD, which is supposed to perform better than the FS-based design approach in terms of producing more consistent design. Therefore, even with LRFD, the need to reduce the potential uncertainty in the predicted performance is still apparent.

To reduce the uncertainty in the predicted performance of a given geotechnical design in the face of above variations (i.e., the uncertainties in soil parameters, solution model, applied loads, and those caused by construction), a new design philosophy was advanced known as robust geotechnical design (RGD) (Juang et al. 2013). Note that robust design concept has long been utilized in engineering fields such as industrial, mechanical, structural, and aeronautical

engineering (e.g., Doltsinis et al. 2005; Zhang et al. 2005; Park et al. 2006; Lagaros and Fragiadakis 2007; Marano et al. 2008; Lee et al. 2010). The purpose of robust design is to derive a design that effectively accounts for the effect of the variation in “noise factors” (Taguchi 1986) while simultaneously considers the safety and cost efficiency. The term *noise factor* that is typically used in the literature of robust design is referred to herein as the input parameter that is *hard-to-control* (i.e., cannot be easily adjusted by the designer) and *hard-to-characterize* (i.e., the uncertainty is recognized but difficult to quantify).

In the existing RGD approach, the safety is often evaluated with the failure probability or reliability index; thus, the reliability analysis of the *system response* is an integral part of RGD. Note that in this paper the term “system response” is referred to, within the context of robust design, as the response of a structure or a component, or simply as the output of a model. This term is not used in the context of system reliability, where multiple failure modes are involved. Although theoretically sound, the implementation of the existing reliability-based RGD approach (Juang et al. 2013) poses a significant challenge to the practicing engineer, since the tasks of the statistical characterization of noise factors and the reliability analysis of the system response must be performed within the robust design optimization framework. In this paper, we introduce a modified RGD approach that shares the familiar look-and-feel and convenience of the standard LRFD approach. This modified RGD approach is referred to herein as *Robust Load and Resistance Factor Design (R-LRFD)*.

The *R-LRFD* is a new and simpler variant of reliability-based RGD, just like LRFD (if it is calibrated through reliability analysis) is a simpler variant of RBD. It is an important step forward to make RGD more practical, because *R-LRFD* does not require engineers to perform

statistical characterization of noise factors and reliability analysis of the system response (same advantages that LRFD has over RBD). Other significant features of the *R*-LRFD include:

- 1) A modified gradient-based robustness measure is proposed, with which the robustness of a system with recognized but unquantified uncertainties can be evaluated. This new robustness measure is suitable for integration with LRFD.
- 2) A new simplified procedure to identify the most preferred design in the design space is developed. With this new procedure, the optimal design with respect to multiple objectives can easily be identified through a series of single objective optimizations. This reduces the computation effort for robust design optimization and eliminates the need for the more complex multi-objective optimization algorithms.

The rest of this paper is organized as follows. First, the standard LRFD approach is briefly described to set the stage. Second, an updated gradient-based robustness measure, in terms of the sensitivity index (SI), is formulated. Third, the optimization framework of the proposed *R*-LRFD approach is presented. Fourth, the proposed *R*-LRFD approach is applied to two geotechnical problems: the design of drilled shaft in sand (illustrated with a discrete design space) and the design of drilled shaft in clay (illustrated with a continuous design space). They are followed with concluding remarks.

2. Load and Resistance Factor Design (LRFD)

Given a geotechnical structure with design parameters $\mathbf{d}$ (e.g., geometries) and uncertain parameters $\boldsymbol{\theta}$ (e.g., soil parameters, model errors, applied loads, and construction

variation) as inputs, the system response, denoted as $f(\mathbf{d}, \boldsymbol{\theta})$, is a function of $\mathbf{d}$ and $\boldsymbol{\theta}$, expressed as:

$$f(\mathbf{d}, \boldsymbol{\theta}) = R(\mathbf{d}, \boldsymbol{\theta}) - S(\mathbf{d}, \boldsymbol{\theta}) \quad (1)$$

where $R(\mathbf{d}, \boldsymbol{\theta})$ and $S(\mathbf{d}, \boldsymbol{\theta})$ are the resistance term and the load term, respectively. Generally, the system response function formulated in Eq. (1) is set up using an analytical or empirical model or through a numerical method. To account for the effect of uncertain parameters ($\boldsymbol{\theta}$) on the predicted performance (and thus the final design outcome), the safety, in the context of the standard LRFD approach, is evaluated as:

$$g(\mathbf{d}, \boldsymbol{\theta}^n) = \psi_R R(\mathbf{d}, \boldsymbol{\theta}^n) - \psi_S S(\mathbf{d}, \boldsymbol{\theta}^n) \quad (2)$$

where $R(\mathbf{d}, \boldsymbol{\theta}^n)$ and $S(\mathbf{d}, \boldsymbol{\theta}^n)$ are the nominal values of the resistance term and the load term, respectively; $\boldsymbol{\theta}^n$ are the nominal values of uncertain parameters ($\boldsymbol{\theta}$); and, ψ_R and ψ_S are the resistance factor and the load factor, respectively. Note that the nominal values of the resistance term and the load term are defined in this paper as the functions of the nominal values of uncertain input parameters, although the nominal values of the resistance term and the load term are typically estimated directly in LRFD. The reason is that this paper seeks to show that R-LRFD solutions are insensitive to the uncertain parameters ($\boldsymbol{\theta}$). It is recognized that the application of $\boldsymbol{\theta}^n$ in design is comparable to the characteristic value approach in Eurocode 7, where $\boldsymbol{\theta}^n$ could be defined at 5% for stabilizing parameters. In general, the resistance factor is less than 1.0 and the load factor is greater than 1.0. The nominal values ($\boldsymbol{\theta}^n$) may be selected as the mean or some percentile (e.g., 5% for stabilizing parameter and 95% for destabilizing parameter) of uncertain parameters ($\boldsymbol{\theta}$). It is noted that in the scenario involving multiple load

or resistance terms, the system response function $f(\mathbf{d}, \boldsymbol{\theta})$ and LRFD performance function $g(\mathbf{d}, \boldsymbol{\theta}^n)$, shown in Eq. (1) and Eq. (2), respectively, should be modified as follows.

$$f(\mathbf{d}, \boldsymbol{\theta}) = \sum_i R_i(\mathbf{d}, \boldsymbol{\theta}) - \sum_j S_j(\mathbf{d}, \boldsymbol{\theta}) \quad (3)$$

$$g(\mathbf{d}, \boldsymbol{\theta}^n) = \sum_i \psi_{Ri} R_i(\mathbf{d}, \boldsymbol{\theta}^n) - \sum_j \psi_{Sj} S_j(\mathbf{d}, \boldsymbol{\theta}^n) \quad (4)$$

where $R_i(\mathbf{d}, \boldsymbol{\theta})$ and $S_j(\mathbf{d}, \boldsymbol{\theta})$ are the i^{th} resistance term and the j^{th} load term, respectively; $R_i(\mathbf{d}, \boldsymbol{\theta}^n)$ and $S_j(\mathbf{d}, \boldsymbol{\theta}^n)$ are the nominal values of the i^{th} resistance term and the j^{th} load term, respectively; and, ψ_{Ri} and ψ_{Sj} are the corresponding resistance factor and load factor, respectively.

For a candidate design ($\mathbf{d}$), if $g(\mathbf{d}, \boldsymbol{\theta}^n) > 0$, the safety requirement is satisfied; otherwise, the design is considered infeasible or unsafe. The uncertainties in the environment, whether they can be characterized probabilistically or by other means, or they are hard-to-characterize noises, have been taken care of through a combination of the nominal values and LRFD partial factors. For illustration purposes, the LRFD partial factors are applied to the nominal values of the resistance term and the load term in Eq. (2). Alternatively, the LRFD partial factors can be applied to the nominal values of uncertain input parameters; however, these two different ways of applying the LRFD partial factors lead to similar results.

The LRFD partial factors (e.g., ψ_R and ψ_S) are constants that have been calibrated for the design of a geotechnical structure using a given solution model at a target reliability index. However, the selection of the nominal values for uncertain parameter ($\boldsymbol{\theta}^n$) (and hence the nominal values of the load and resistance terms) is not straightforward. The nominal values of uncertain parameters at a given site under a given design scenario can be rather uncertain, although the standard LRFD approach treats them as constants. Given the *recognized but*

199 *unquantified* uncertainty associated with the nominal values, the potential risk of the design
200 obtained with the existing LRFD code is unknown. Thus, the existing LRFD code has the same
201 dilemma as one facing the traditional FS-based design approach: it may produce an inefficient
202 over-design with an unknown and/or inconsistent safety level, although under-design is also a
203 possibility. In fact, for LRFD codes that are not calibrated using rigorous reliability methods,
204 there is no rational basis to assess if the design outcome is conservative or vice-versa. This is a
205 fundamental limitation faced by all design approaches that rely solely on judgment and
206 experience, though they enjoy the practical advantage of not having to quantify uncertainty.
207 The proposed *R*-LRFD approach retains the advantage of experience-based design approaches
208 while attempting to define a rational basis for optimal design, under the assumption that the
209 data are insufficient for reliability analysis.

210 By introducing the robust design concept into the standard LRFD approach, the effect
211 of the *recognized but unquantified* uncertainty in the nominal values of uncertain parameters on
212 the final design can be explicitly dealt with. The purpose of robust design is to seek an optimal
213 design such that the system response is robust against the uncertainty in noise factors (in this
214 case, the uncertainty in the nominal values). Note that the uncertainty in the nominal values of
215 uncertain parameters may be attributed to either the absence of the knowledge of how the
216 partial factors are derived or the uncertainty in quantifying the level of variation of uncertain
217 parameters. To integrate the robust design concept into the standard LRFD approach, the
218 recently developed gradient-based robustness measure, in terms of the sensitivity index (SI)
219 (Gong et al. 2014b), is modified herein. This modified gradient-based robustness measure does
220 not require statistical characterization of noise factors; however, the effect of the *recognized but*

unquantified uncertainties in the noise factors can be explicitly considered; which is consistent with the spirit of the standard LRFD approach.

3. Updated Gradient-Based Robustness Measure: Sensitivity Index (SI)

In the context of robust design, the input parameters are classified into two categories, in terms of the *design parameters* ($\mathbf{d}$) and the *noise factors* ($\boldsymbol{\theta}$). The design parameters ($\mathbf{d}$) are those that can be easily adjusted by the designer, such as the geometries of the design; the noise factors ($\boldsymbol{\theta}$) are those that cannot be easily adjusted by the designer and that exhibit *recognized but unquantified* uncertainty, such as the soil parameters, model errors, applied loads, and construction variation. The purpose of robust design is to seek an optimal design, represented by a set of *design parameters* ($\mathbf{d}$), in which the system response $[f(\mathbf{d}, \boldsymbol{\theta})]$ is robust against, or insensitive to, the variation in *noise factors* ($\boldsymbol{\theta}$) (Taguchi 1986).

To illustrate the concept of robust design, the response of a system with noise factors is demonstrated in Figure 1. Figure 1(a) shows the large variation in the performance of design $\mathbf{d}_1$ as a result of the combination of the large variation in noise factors $\boldsymbol{\theta}_1$ and the high sensitivity (or low robustness) of the system response to the noise factors, which poses a challenge in the design decision making. Figure 1(b) and 1(c) depict two approaches to reduce the variation in the system response. Figure 1(b) achieves this goal by focusing on reducing the variation in noise factors (e.g., $\boldsymbol{\theta}_2$ in Figure 1b versus $\boldsymbol{\theta}_1$ in Figure 1a), which may be realized by adopting a more comprehensive site characterization program and/or use of a more accurate and precise solution model. Figure 1(c) achieves the same goal with robust design that optimizes the design parameters without reducing the variation in noise factors, (e.g., $\mathbf{d}_2$ in Figure 1c versus $\mathbf{d}_1$ in

Figure 1a). The latter is realized by minimizing the sensitivity, or maximizing the robustness, of the system response to the noise factors.

It is noted that the reduction of the variation in noise factors (θ) is not the focus of this paper; rather, the focus is placed on the optimization of design parameters ($\mathbf{d}$) through robust design in the face of the *recognized but unquantified* uncertainty in geotechnical engineering. The key to the success of the robust design is the measure of robustness. To set the stage for the development of a robustness measure for a system with *recognized but unquantified* uncertainty, the robustness measures for a system with uncertain parameters that have been statistically quantified, as in the reliability-based robust design approach (Juang et al. 2013), are reviewed.

3.1 Robustness measure for a system with quantified uncertain parameters

For a design ($\mathbf{d}$) with uncertain input parameters (θ) that have been fully characterized statistically, the robustness of the system response $[f(\mathbf{d}, \theta)]$, expressed in terms of the variability of the system response, can be studied with probabilistic methods. For example, the standard deviation of the system response, denoted as σ_f , may be obtained using the mean value first order second moment method (MFOSM) (Ang and Tang 2007).

$$\sigma_f \approx \sqrt{\left(\mathbf{G}|_{\theta=\mu_\theta}\right) \cdot \mathbf{C}_\theta \cdot \left(\mathbf{G}|_{\theta=\mu_\theta}\right)^T} \quad (5)$$

where $\mathbf{C}_\theta$ is the covariance matrix among the uncertain parameters (θ), and $\mathbf{G}|_{\theta=\mu_\theta}$ is the gradient of the system response $[f(\mathbf{d}, \theta)]$ to the uncertain parameters (θ) evaluated at the mean of uncertain parameters (μ_θ), which is formulated as (Gong et al. 2014b):

$$\mathbf{G}|_{\theta=\mu_\theta} = \left\{ \left. \frac{\partial f(\mathbf{d}, \theta)}{\partial \theta_1} \right|_{\theta=\mu_\theta}, \left. \frac{\partial f(\mathbf{d}, \theta)}{\partial \theta_2} \right|_{\theta=\mu_\theta}, \dots, \left. \frac{\partial f(\mathbf{d}, \theta)}{\partial \theta_n} \right|_{\theta=\mu_\theta} \right\} \quad (6)$$

where n is the number of uncertain parameters (θ).

Further, the mean of the system response [$f(\mathbf{d}, \theta)$], denoted as μ_f , can be approximated using MFOSM.

$$\mu_f \approx f(\mathbf{d}, \mu_\theta) = R(\mathbf{d}, \mu_\theta) - S(\mathbf{d}, \mu_\theta) \quad (7)$$

Then, the variability, expressed in terms of the coefficient of variation (COV), of the system response [$f(\mathbf{d}, \theta)$], denoted as δ_f , can be evaluated as follows.

$$\delta_f = \frac{\sigma_f}{\mu_f} \approx \frac{\sqrt{\left(\mathbf{G}|_{\theta=\mu_\theta}\right) \cdot \mathbf{C}_\theta \cdot \left(\mathbf{G}|_{\theta=\mu_\theta}\right)^T}}{f(\mathbf{d}, \mu_\theta)} \quad (8)$$

Intuitively, a lower variability of the system response (δ_f) signals a higher degree of robustness. For a geotechnical structure with uncertain parameters that have been characterized statistically, the robustness may be formulated in various forms, such as feasibility robustness (Du and Chen 2000; Juang et al. 2013) and standard deviation of the system response of concern (Juang et al. 2014). As has been reported, these robustness measures are all consistent with another popular robustness measure, namely, signal-to-noise-ratio (SNR) (Taguchi 1986; Phadke 1989; Park et al. 2006; Gong et al. 2014a).

3.2 Robustness measure for a system with unquantified uncertain parameters

The traditional robustness measures that are based on probabilistic analyses, such as the coefficient of variation, signal-to-noise-ratio (SNR) and feasibility robustness, are suitable for the system with quantified uncertain parameters. They are, however, not applicable to the system with unquantified uncertain parameters (how do you proceed without the knowledge of the statistics of uncertain parameters?). In other words, the existing robustness measures cannot be integrated with the standard LRFD approach.

The robustness or insensitivity of the system response $[f(\mathbf{d}, \boldsymbol{\theta})]$ to the variation in noise factors $(\boldsymbol{\theta})$ can be measured with its gradient $(\mathbf{G})$, which is formulated as the variation of the system response caused by one unit change in noise factors (Gong et al. 2014b). The plots in Figure 1(a) and 1(c) illustrate the gradient-based robustness concept: two designs (i.e., $\mathbf{d}_1$ and $\mathbf{d}_2$) with the same noise factors $(\boldsymbol{\theta}_1)$ exhibit different patterns of system response, one (i.e., $\mathbf{d}_1$ in Figure 1a) yields high variation in the system response and the other (i.e., $\mathbf{d}_2$ in Figure 1c) yields low variation in the system response. The design that yields lower variation in the system response, known as a more robust design by definition, is shown to have a lower gradient $(\mathbf{G})$. Thus, the gradient $(\mathbf{G})$ is an effective measure of robustness, and a robust design can be found by lowering the gradient of the system response to the noise factors without quantifying the uncertainties of noise factors. This unique characteristic of the gradient-based robustness measure is a perfect match with LRFD, in which the uncertainties in input parameters and the solution model are recognized but unquantified. Note that while the existing gradient-based robustness measure (Gong et al. 2014b) is applicable for the scenario involving a system with quantified uncertain parameters, the modified gradient-based robustness measure, outlined below, is intended for the scenario involving a system with recognized but unquantified uncertainty. The latter is uniquely suitable for integration with the standard LRFD.

Similar to Eq. (6), the gradient of the system response $[f(\mathbf{d}, \boldsymbol{\theta})]$ to the noise factors at a checkpoint of $\boldsymbol{\theta}'$, denoted as $\mathbf{G}|_{\boldsymbol{\theta}=\boldsymbol{\theta}'}$, can be expressed as follows.

$$\mathbf{G}|_{\boldsymbol{\theta}=\boldsymbol{\theta}'} = \left\{ \left. \frac{\partial f(\mathbf{d}, \boldsymbol{\theta})}{\partial \theta_1} \right|_{\boldsymbol{\theta}=\boldsymbol{\theta}'}, \left. \frac{\partial f(\mathbf{d}, \boldsymbol{\theta})}{\partial \theta_2} \right|_{\boldsymbol{\theta}=\boldsymbol{\theta}'}, \dots, \left. \frac{\partial f(\mathbf{d}, \boldsymbol{\theta})}{\partial \theta_n} \right|_{\boldsymbol{\theta}=\boldsymbol{\theta}'} \right\} \quad (9)$$

Within the framework of the standard LRFD approach using fixed nominal values of uncertain parameters (θ^n) as inputs, it is intuitive to take these nominal values (θ^n) as the checkpoint of noise factors (θ'), expressed as: $\theta' = \theta^n$. This formulation is consistent with the requirement of using fixed nominal values in the standard LRFD. In addition, the *R*-LRFD framework is predicated on unquantifiable uncertainties and it is thus compelled to minimize reliance on statistics such as mean values.

Although the gradient ($\mathbf{G}|_{\theta=\theta'}$) in Eq. (9) is an effective indicator of robustness, the following two problems need to be resolved before the gradient-based robustness measure can be applied for the general robust design optimization. First, the gradient is a dimensional vector. The units of noise factors are different, thus the optimization operation of the vector can be a problem. Second, the gradient is a vector rather than a scalar, it is inconvenient to be used for screening candidate designs in an optimization framework. That is to say, the gradient vector in Eq. (9) must be normalized with scaling factors and then converted into a scalar for ease of application in the robust design optimization.

To overcome the first problem, each partial derivative of the gradient vector ($\mathbf{G}|_{\theta=\theta'}$), in terms of $\left. \frac{\partial f(\mathbf{d}, \theta)}{\partial \theta_i} \right|_{\theta=\theta'}$, is multiplied by a scaling factor of $w_i \frac{\theta'_i}{f(\mathbf{d}, \theta')}$ such that it becomes a dimensionless parameter. Similar normalization methods can be found in Gonsamo (2011). The gradient vector ($\mathbf{G}|_{\theta=\theta'}$) in Eq. (9) is then re-written as a dimensionless vector ($\mathbf{J}|_{\theta=\theta'}$).

$$\mathbf{J}|_{\theta=\theta'} = \left\{ w_1 \frac{\theta'_1}{f(\mathbf{d}, \theta')} \left. \frac{\partial f(\mathbf{d}, \theta)}{\partial \theta_1} \right|_{\theta=\theta'}, w_2 \frac{\theta'_2}{f(\mathbf{d}, \theta')} \left. \frac{\partial f(\mathbf{d}, \theta)}{\partial \theta_2} \right|_{\theta=\theta'}, \dots, w_n \frac{\theta'_n}{f(\mathbf{d}, \theta')} \left. \frac{\partial f(\mathbf{d}, \theta)}{\partial \theta_n} \right|_{\theta=\theta'} \right\} \quad (10)$$

where w_i is the weighting factor of the i^{th} noise factor (θ_i), which signals the contribution of the i^{th} noise factor (θ_i) to the robustness. For instance, a noise factor that exhibits higher variability would contribute more to the robustness measure defined herein. Furthermore, the above formulation is quite similar to the concept of elasticity in economics (Gillespie 2007).

To solve the second problem, the “length” of the scaled dimensionless vector ($\mathbf{J}|_{\theta=\theta^*}$) is computed and defined herein as *sensitivity index* (SI).

$$SI = \sqrt{\left| \left(\mathbf{J}|_{\theta=\theta^*} \right) \cdot \mathbf{R}_{\theta} \cdot \left(\mathbf{J}|_{\theta=\theta^*} \right)^T \right|} \quad (11)$$

where $\mathbf{R}_{\theta}$ is a n by n matrix depicting the correlation structure among the noise factors (θ). Note that the estimation of $\mathbf{R}_{\theta}$ does not require full statistical characterization (i.e., knowledge of covariance $\mathbf{C}_{\theta}$). In the scenario that lacks the knowledge of $\mathbf{C}_{\theta}$, as in the system with recognized but unquantified uncertainties, $\mathbf{R}_{\theta}$ may be estimated based on experience and engineering judgment. As will be discussed later, a rough estimate of $\mathbf{R}_{\theta}$ is sufficient to obtain consistent design outcome. The sensitivity index (SI) offers a single value representation of the robustness. A higher SI value signals a lower degree of robustness, as it suggests a greater level of variation of the system response $[f(\mathbf{d}, \theta)]$.

It is noted that if the uncertainty of each noise factor could be *quantified*, the weighting factor w (Eq. 10) may be replaced by the COV of the noise factor and the nominal value (θ^n) may be replaced by the mean of the noise factor. Then, the sensitivity index (SI) defined in Eq. (11) becomes the mean-centered first-order approximation of the COV of the system response (δ_f). In such a scenario, this new robustness measure is seen as equivalent to the existing robustness measures that based on probabilistic analyses. However, this new robustness

measure is more useful under the pivotal assumption of unquantifiable noises. In the scenario of unquantifiable noises, the proposed new robustness measure is a significant improvement over the existing robustness measures. Finally, there is a question of whether the computed SI (and thus the resulting design) is sensitive to the estimate of the weighting factors (w) and the correlation structure ($\mathbf{R}_\theta$). As is demonstrated later, the results of the robust design optimizations are not sensitive to the estimate of the weighting factors (w) and the correlation structure ($\mathbf{R}_\theta$).

4. Robust Load and Resistance Factor Design (*R*-LRFD)

A key benefit of the RGD approach is that it treats the geotechnical design as a multi-objective optimization problem that considers safety, robustness, and cost efficiency simultaneously (Juang et al. 2013; Gong et al. 2014a&b). In this approach, the safety is a compulsory constraint that must be satisfied, while the robustness and the cost efficiency are the objectives to be optimized. A possible optimization setting of the proposed *R*-LRFD approach is shown as follows:

$$\begin{aligned}
 &\text{Find:} && \mathbf{d} \text{ (design parameters)} \\
 &\text{Subject to:} && \mathbf{d} \in \mathbf{DS} \text{ (design space)} \\
 &&& g(\mathbf{d}, \boldsymbol{\theta}^n) > 0 \text{ (safety)} \\
 &&& \text{Other constraints} \\
 &\text{Objectives:} && \text{Minimize SI (a function of } f(\mathbf{d}, \boldsymbol{\theta})) \\
 &&& \text{Minimize } C \text{ (cost)}
 \end{aligned} \tag{12}$$

where the constraint “ $g(\mathbf{d}, \boldsymbol{\theta}^n) > 0$ ” means “safe”. In reference to Eq. (12), *R*-LRFD is to seek an optimal design, represented by a set of design parameters ($\mathbf{d}$), in the design space ($\mathbf{DS}$) such that the safety is satisfied, in terms of $g(\mathbf{d}, \boldsymbol{\theta}^n) > 0$, as in the standard LRFD approach, and the

sensitivity index (SI) and the cost (C) are minimized simultaneously. Note that the system response function $f(\mathbf{d}, \boldsymbol{\theta})$, defined in Eq. (1), not $g(\mathbf{d}, \boldsymbol{\theta}^n)$ defined in Eq. (2), is adopted for evaluating the sensitivity index (SI) (Eqs. 9-11), which is an index for robustness in this paper; this is the “ R ” part of the proposed R -LRFD approach. Whereas, the system response function $g(\mathbf{d}, \boldsymbol{\theta}^n)$, defined in Eq. (2), is adopted for assessing the safety; this is the “LRFD” part of the proposed R -LRFD approach.

4.1 Most preferred design based on the results of robust design optimization

The objective to maximize the robustness (i.e., minimize the sensitivity index SI) and the objective to minimize the cost are generally conflicting with each other. Thus, a design that is optimal with respect to both objectives simultaneously is not attainable within the framework of the proposed R -LRFD approach. Instead, the robust design optimization usually leads to a set of non-dominated designs, which collectively defines a Pareto front that shows a tradeoff between robustness and cost efficiency. The Pareto front might be obtained using genetic algorithms such as Non-dominated Sorting Genetic Algorithm version II, NSGA-II (Deb et al. 2002; Juang et al. 2013). Alternatively, the Pareto front may be obtained through a series of single-objective optimization procedures (Chen et al. 1999). Both approaches are employed in this study; however, the focus is to develop a simplified procedure for obtaining the most preferred design in the design space that can be integrated smoothly with R -LRFD without the need to perform multi-objective optimization. The simplified procedure will be presented and elaborated later.

By presenting a tradeoff between robustness and cost efficiency, the Pareto front can aid in making an informed design decision. For example, either the least cost design that is above a target robustness level or the most robust design that falls within a target cost level may be

selected as the most preferred design in the design space. The determination of an appropriate target level of robustness or cost, however, is problem-specific. When a strong preference is not specified by the owner or client, the knee point on the Pareto front, which yields the best compromise solution among the conflicting objectives, may be taken as the most preferred design in the design space (Branke et al. 2004; Deb and Gupta 2011; Gong et al. 2014b). Once the Pareto front is established, the knee point can be readily identified using the marginal utility function approach (Branke et al. 2004; Gong et al. 2014b), normal boundary intersection approach (Deb and Gupta 2011), or reflex angle approach (Branke et al. 2004; Deb and Gupta 2011). Figure 2(a) shows a conceptual illustration of the Pareto front and the knee point in a bi-objective optimization problem that minimizes Objective 1 and Objective 2. In Figure 2(a), the feasible domain is defined as a domain that consists of feasible or safe designs.

In most cases, the owner or client may only be interested in the final design or the most preferred design in the design space, and not necessarily the whole Pareto front; further, the designer would not like to perform multi-objective optimizations. In fact, if the most preferred design in the design space can be identified directly, the sorting of Pareto front may not be needed. To this end, a simplified procedure is developed herein to identify the most preferred design in the design space. Coupling with this simplified procedure for the most preferred design, the proposed *R*-LRFD approach has the potential as a practical design tool for the general geotechnical engineering application.

The simplified procedure for identifying the most preferred design in the design space that requires no multi-objective optimization is summarized as follows:

Step 1: Select a design space (**DS**) based on local experience and engineering judgment, and screen for feasible (safe) designs based on the design constraints.

Step 2: Conduct a single-objective optimization among all the feasible (safe) designs.

This step is repeated for each and every design objective. For example, in Figure 2(b), the most optimal design with respect to Objective 1 and that with respect to Objective 2 are identified separately.

Step 3: Identify a utopia design (in the space of the objective functions) for this multi-objective optimization problem. Here, the utopia design is a “design” that is optimal with respect to each and every design objective, as plotted in Figure 2(b). Note that this utopia design is in reality not attainable if the objectives are conflicting with each other.

Step 4: Normalize the objective values of all feasible designs into a range of 0.0 to 1.0. For example, the i^{th} design objective, denoted as $b_i(\mathbf{d})$, can be normalized into $b_i^n(\mathbf{d})$ using the following transformation.

$$b_i^n(\mathbf{d}) = \frac{b_i(\mathbf{d}) - [b_i(\mathbf{d})]_{\min}}{[b_i(\mathbf{d})]_{\max} - [b_i(\mathbf{d})]_{\min}} \quad (13)$$

where $[b_i(\mathbf{d})]_{\max}$ and $[b_i(\mathbf{d})]_{\min}$ are the maximum and minimum objective values, respectively, among all the optimal designs (obtained in Step 2) based on the i^{th} design objective $[b_i(\mathbf{d})]$. This normalization step is repeated for each and every design objective of concern.

Step 5: Calculate the distance from the utopia design to the candidate design (in the feasible domain) in the normalized space of the objective functions. The design that yields the minimum distance is taken as the most preferred design in the design space, as plotted in Figure 2(b).

Theoretically, the computed distance from the utopia design to a candidate design may be interpreted as the additional “price”, in terms of a combination of robustness and cost efficiency, the owner or client has to pay to select this candidate design as the final design.

Although this idea can be traced back to the marginal utility concept (Branke et al. 2004) and the compromise programming approach (Chen et al. 1999), the simplified procedure using the “minimum distance” concept is a new contribution. As will be shown later, the most preferred design in the design space obtained using this simplified procedure is practically the same as the knee point on the Pareto front that is obtained based on the more comprehensive procedures of multi-objective optimization. It is worth noting that the simplified procedure is applicable to problems with more than two objectives and can be implemented with Microsoft Excel. If the explicit system response function is not available, the response surface method (Zhang et al. 2015) may be adopted to implement the proposed procedure with Excel.

4.2 Procedures for implementing the proposed *R*-LRFD approach

The procedures to implement the proposed *R*-LRFD approach are summarized in the following main steps:

Step 1: Characterize the geotechnical problem of interest. In this step, the system response of concern [i.e., $f(\mathbf{d}, \boldsymbol{\theta})$ and $g(\mathbf{d}, \boldsymbol{\theta}^n)$], design parameters ($\mathbf{d}$), and noise factors ($\boldsymbol{\theta}$) are identified, and the models for the safety, robustness, and cost are chosen. Further, the nominal values of uncertain parameters ($\boldsymbol{\theta}^n$) in the standard LRFD approach are selected along with the design space (**DS**). Finally, the weighting factors (w) and the correlation structure ($\mathbf{R}_{\boldsymbol{\theta}}$) are estimated. For design of ordinary geotechnical structures, all weighting factors may be simply set to 1.0 and the correlation structure set as an n -by- n identity matrix.

Step 2: Evaluate the safety, in terms of $g(\mathbf{d}, \boldsymbol{\theta}^n) > 0$ (Eq. 2), robustness, in term of the sensitivity index (SI) (Eqs. 9-11), and cost (C) for each and every candidate design in the design space (**DS**).

Step 3: Obtain the most preferred design in the design space (**DS**) using the simplified procedure that is based on the minimum distance from the utopia design.

Step 4 (optional): Derive the Pareto front with the multi-objective optimization setting in Eq. (12). The most preferred design is then selected based on the preference of the owner or the knee point concept. This design may be used to verify the design obtained in Step 3.

5. Illustrative Example 1 – Robust Design of a Drilled Shaft in Sand

The first illustrative example considers the design of a drilled shaft in sand (Wang et al. 2011; Juang et al. 2013). As shown in Figure 3, this drilled shaft is designed to bear a 50-year design load of 800 kN and the maximum settlement of this drilled shaft is required to be less than 25 mm. Symbolically, they are expressed as: $F_{50} = 800$ kN and $y_a = 25$ mm, respectively.

5.1 Geotechnical characterization for this drilled shaft problem

Two fundamental requirements of concern in the design of drilled shafts are the bearing capacity and the settlement. The former is referred to herein as the ultimate limit state (ULS) requirement and the latter the serviceability limit state (SLS) requirement. For this drilled shaft problem, the studies conducted by Wang et al. (2011) and Juang et al. (2013) found that the design is controlled by the SLS requirement. Thus, only the SLS behavior of this drilled shaft is considered herein and the corresponding system response function is formulated as follows.

$$f(\mathbf{d}, \boldsymbol{\theta}) = Q_{\text{SLS}} - F_{\text{load}} \quad (14)$$

where Q_{SLS} is the SLS-based bearing capacity of the drilled shaft, detailed formulation of which can be found in Appendix A; and, F_{load} is the unfactored load acting on the drilled shaft while F_{50} is the factored load.

In the context of the standard LRFD approach, the safety of the drilled shaft with respect to the SLS requirement is assessed with the following equation (Phoon et al. 1995):

$$g(\mathbf{d}, \boldsymbol{\theta}^n) = \psi_c Q_{\text{SLS}} - F_{50} \quad (15)$$

where ψ_c is the resistance factor, which is taken as 0.56 in this example (Phoon et al. 1995). If $g(\mathbf{d}, \boldsymbol{\theta}^n) > 0$, the safety is satisfied; otherwise, the safety is not satisfied. In the standard LRFD approach, the resistance factor is adopted to consider the effect of the *recognized but unquantified* uncertainty in the perceived uncertain parameters (i.e., effective friction angle ϕ' and coefficient of earth pressure at rest K_0).

In the proposed R-LRFD approach, the nominal values of uncertain parameters ϕ' and K_0 are referred to herein as the noise factors ($\boldsymbol{\theta}$), while the dimensions of the drilled shaft (i.e., shaft diameter B and shaft length D) are referred to herein as the design parameters ($\mathbf{d}$). The cost of this drilled shaft (C), in US dollars (USD), is expressed as a function of the design parameters ($\mathbf{d}$) as follows (R.S. Means Co. 2007; Wang et al. 2011).

$$C(\mathbf{d}) = \begin{cases} 77.5 \times \frac{D}{0.3} & \text{for } B = 0.9 \text{ m} \\ 116.0 \times \frac{D}{0.3} & \text{for } B = 1.2 \text{ m} \\ 157.0 \times \frac{D}{0.3} & \text{for } B = 1.5 \text{ m} \end{cases} \quad (16)$$

In the example of the reliability-based RGD of this drilled shaft by Juang et al. (2013), the mean values of ϕ' and K_0 were estimated as 32° and 1.0, respectively; the COVs of ϕ' and K_0 were estimated as 5% to 10% and 20% to 90%, respectively; and, the correlation coefficient between ϕ' and K_0 was estimated as -0.60 to -0.90. To compare with the reliability-based RGD reported in Juang et al. (2013), the nominal values of ϕ' and K_0 , in this example, are taken as

32° and 1.0, respectively. As discussed previously, the knowledge of COVs of ϕ and K_0 and the correlation coefficient between ϕ and K_0 is not needed in the *R*-LRFD approach. However, to analyze the effect of the weighting factors and the correlation structure, which are included in the formulation of the sensitivity index, on the final design, the COVs of 7% and 50% for ϕ and K_0 , respectively, and the correlation coefficient between ϕ and K_0 of -0.75 are assumed.

Of course, the safety and the robustness of this drilled shaft are also dependent upon other parameters, such as the unit weight of sand (γ_s), unit weight of water (γ_w), unit weight of concrete (γ_c), ground water table (H), and nominal in situ horizontal stress coefficient ratio $[(K/K_0)_n]$. These parameters are assumed to have fixed values as their variations are relatively small and negligible. Here, they are assumed as: $\gamma_s = 20 \text{ kN/m}^3$, $\gamma_w = 9.81 \text{ kN/m}^3$, $\gamma_c = 24 \text{ kN/m}^3$, $H = 0.0 \text{ m}$, and $(K/K_0)_n = 1.0$; these values are the same as those in Juang et al. (2013). Similar to that in Juang et al. (2013), a discrete design space (**DS**) is used in this example: **DS** = $\{(B, D) | B = \{0.9 \text{ m}, 1.2 \text{ m}, 1.5 \text{ m}\} \text{ and } D = \{2.0 \text{ m}, 2.2 \text{ m}, 2.4 \text{ m}, \dots, 8.0 \text{ m}\}\}$. In the selected design space (**DS**), a total number of 93 discrete candidate designs are possible. Further, an additional constraint of $D/B \geq 4$ is adopted such that the derived drilled shaft design is consistent with the perception of deep foundation. Note that while a discrete design space is used in this example, a continuous design space could also be handled, and similar results would be achieved, as is demonstrated later in Example 2.

5.2 Results of robust design of this drilled shaft problem

The safety, robustness, and cost of each and every candidate design in the design space (**DS**) are first evaluated. Then, the most preferred design in the design space (**DS**) of this drilled shaft is identified using the simplified procedure based on the minimum distance concept

described previously. The design parameters of the obtained most preferred design are: $B = 0.9$ m and $D = 8.0$ m, and the corresponding cost and sensitivity index are: $C = 2,067$ USD (raw value) and $SI = 0.340$ (raw value), respectively. Listed in Table 1 are the detailed analysis results of all these candidate designs. As can be seen in Table 1, the least cost design in the feasible domain yields $B = 0.9$ m and $D = 6.2$ m, which is associated with a sensitivity index of $SI = 0.482$ (raw value) and a cost of $C = 1,602$ USD (raw value). On the other hand, the most robust design in the feasible domain yields $B = 1.5$ m and $D = 8.0$ m, which is associated with a sensitivity index of $SI = 0.289$ (raw value) and a cost of $C = 4,187$ USD (raw value).

Compared to the least cost design in the design space, the most preferred design yields a significantly higher degree of robustness (normalized $SI^n = 0.262$ versus 1.0) while the increase in cost is only modest (normalized $C^n = 0.180$ versus 0.0). Compared to the most robust design, the most preferred design yields significant savings in cost (normalized $C^n = 0.180$ versus 1.0) while the robustness is lowered only slightly (normalized $SI^n = 0.262$ versus 0.0). Therefore, the design by *R*-LRFD (i.e., $B = 0.9$ m and $D = 8.0$ m) yields the best compromise between cost efficiency and robustness.

A Pareto front is further obtained by taking the optional step, which involves a multi-objective optimization algorithm (Deb et al. 2002). Figure 4 shows the derived Pareto front for this drilled shaft problem, in both normalized objectives space (Figure 4a) and unnormalized objectives space (Figure 4b). It is noted that the existence of three segments in this Pareto front, as opposed to a single smooth curve, is caused by the adoption of a discrete design space (i.e., with only three choices of shaft diameter B). The knee point on the Pareto front is identified using the marginal utility function approach (Branke et al. 2004). The design parameters of the derived knee point are: $B = 0.9$ m and $D = 8.0$ m, which are the same as those of the most

preferred design obtained using the simplified procedure. Thus, the optional step to obtain the Pareto front is unnecessary if the intent of the designer is just to identify the most preferred design in the design space.

5.3 Effect of weighting factors and correlation structure on the final design

A key component of the proposed *R*-LRFD approach is the updated robustness measure, in terms of the modified sensitivity index (SI). According to Eqs. (9-11), this index appears to depend upon the weighting factors (w) and the correlation structure ($\mathbf{R}_\theta$) of noise factors. The question is whether the results of the proposed *R*-LRFD approach are affected by the estimate of these parameters. To answer this question, let's recall the purpose of robust design, which is to find the most preferred design in the design space. Is the contribution of the gradient alone, as illustrated in Figure 1 and discussed in Eq. (9), which is represented by the scenario of equal weights and no correlation, sufficient to constrain the search for the most preferred design? Or, is the quantitative knowledge of the statistical characterization of noise factors crucial in locating the most preferred design? A series of parametric analyses are performed herein to investigate the effect of the assumption of the COVs (and thus weighting factors) and the correlation structure on the outcome (i.e., the most preferred design) of the proposed *R*-LRFD approach.

Tabulated in Table 2 are the *R*-LRFD outcomes obtained with different estimates of the weighting factors (expressed in terms of the COV) and the correlation structure of noise factors. It is found that the *R*-LRFD outcome is insensitive to the weighting factors and the correlation structure of noise factors over a wide range of possibilities. Thus, the effect of the weighting factors and the correlation structure on the final robust design is dominated by the influence of the gradient term ($\mathbf{G}$).

In addition to the results presented in Table 2, which are obtained based on limited cases, 1,000 Monte Carlo simulations have been further conducted to generate the weighting factors and the correlation structure. For these MCS runs, w_1 and w_2 are assumed uniformly distributed over the possible ranges of [5%, 10%] and [20%, 90%], respectively; and, the correlation coefficient between ϕ and K_0 is assumed uniformly distributed in the range of [-0.6, -0.9]. In each of these MCS samples, the simplified procedure of the proposed *R*-LRFD approach is applied and an optimal design is obtained. It is found that the variation of the final designs among the 1000 simulations is relatively small and negligible. For example, the mean of the resulting shaft diameter (B) and shaft depth (D) are 0.9 m and 8.0 m, respectively; whereas, the COV of the resulting B and D values are only 1.05% and 0.75%, respectively.

Further, given the statistical characterization of uncertain parameters in Table 2, this drilled shaft is readily designed with RBD and reliability-based RGD (Gong et al. 2014b); and, the derived designs are also included in Table 2. For illustration purposes, the target reliability index (β_T) is set to 2.6 and the reliability index (β) of each candidate design is analyzed with first order reliability method (FORM) (Low and Tang 2007). As can be seen in Table 2, the outcomes obtained by RBD and reliability-based RGD are sensitive to the assumption of the statistical information of uncertain parameters; whereas, the *R*-LRFD outcome is insensitive to the assumption of the statistical information of uncertain parameters. The implication is that the outcomes obtained by RBD and reliability-based RGD are meaningful only if the statistics of uncertain parameters are accurately quantified; whereas, the proposed *R*-LRFD always yields a consistent design in the scenario that involves the recognized but unquantified uncertainty. The results shown in Table 2 also depict that the *R*-LRFD outcome and the reliability-based RGD outcome are more conservative than the RBD outcome (indicated by a larger reliability index).

In the case of *R*-LRFD, the conservativeness may be due to the adopted LRFD partial factors, which are generally conservative. In the case of the reliability-based RGD, the conservativeness is attributed to the consideration of design robustness, as reported by Gong et al. (2014b).

In summary, when applying the proposed *R*-LRFD approach, the most preferred design can be obtained with a design scenario that sets all weighting factors to 1.0 and the correlation structure as an n -by- n identity matrix. Although this approach might be viewed as an approximation, it yields a fairly accurate outcome (i.e., final design) and it is consistent with the spirit of the standard LRFD approach that does not require a full statistical characterization of uncertain parameters on the part of the user. Nevertheless, the formulations presented in Eqs. (9-11) allow for an assessment and use of the statistical characterization of noise factors in the evaluation of the robustness if so desired.

5.4 Effectiveness of the sensitivity index (SI) as a robustness measure

In the proposed *R*-LRFD approach, the sensitivity index (SI) of the system response to the noise factors is used to measure the robustness. Here, the sensitivity index (SI) is compared with the more traditional probabilistic analysis based robustness measures (i.e., standard deviation of the failure probability σ_{p_f} and feasibility robustness β_β), which were reported previously in the reliability-based robust design of drilled shafts (Juang et al. 2013).

In order to calculate the more rigorous probability-based robustness measures, a full statistical characterization of noise factors is needed. Given the full statistical characterization of noise factors in Table 3, the standard deviation of the failure probability (σ_{p_f}) and the feasibility robustness index (β_β) can readily be computed for all the non-dominated designs

(i.e., designs on the Pareto front) plotted in Figure 4. Comparisons of these robustness measures with the new robustness measure (SI) are shown in Figure 5.

Detailed procedures for computing the standard deviation of the failure probability (σ_{P_f}) due to the variation of the COVs are documented in Juang et al. (2013). The feasibility robustness index (β_β) is defined as follows (Juang et al. 2013):

$$\beta_\beta = \Phi^{-1} \left[\Pr(P_f - P_{fT} \leq 0) \right] \quad (17)$$

where P_{fT} is the target failure probability, which is taken as 0.0047 in this example; the term $\Pr(P_f - P_{fT} \leq 0)$ is the probability that the computed P_f is not greater than P_{fT} ; and, $\Phi(\cdot)$ is the cumulative distribution function of the standard normal variable. Note that the feasibility robustness index measures the confidence level (or probability) that the safety constraint, in the form of the target failure probability, can still be satisfied even with the uncertainty (variation) in the characterized statistics of uncertain parameters.

Figure 5(a) shows that the standard deviation of the failure probability (σ_{P_f}) increases with the normalized sensitivity index (SI^n); that is to say, a design with a lower sensitivity index (indicating a higher degree of robustness) generally exhibits less variation in the computed failure probability. This is consistent with the concept of a robustly designed system. Similarly, Figure 5(b) depicts that the computed feasibility robustness index (β_β) decreases with the normalized sensitivity index (SI^n); that is to say, a design with a higher sensitivity index (indicating a lower degree of robustness) yields a lower feasibility robustness index, indicating a less robust design. This is, again, consistent with the concept of robust design. As both the standard deviation of the failure probability and the feasibility robustness have been proven as effective robustness measures (Juang et al 2013), the results shown in Figure 5 support the

claim that the gradient-based sensitivity index (SI) is an effective robustness measure. The latter, however, requires only a deterministic analysis and thus is much simpler and more computationally efficient than the former. This deterministic nature makes the sensitivity index (SI) an ideal robustness measure to be used in the proposed *R*-LRFD approach.

6. Illustrative Example 2– Robust Design of a Drilled Shaft in Clay

The second example considers the design of a drilled shaft in clay (ISSMGE 2009). As shown in Figure 6, this drilled shaft is designed to support a dead load (F_d) and a live load (F_l); and, the maximum settlement of this drilled shaft is required to be less than 20 mm, expressed as: $y_a = 20$ mm. One main difference between this example and Example 1 is that a continuous design space is adopted instead of a discrete design space (Example 1).

6.1 Geotechnical characterization for this drilled shaft problem

For this drilled shaft in clay, the studies conducted by Orr et al. (2011) and Khoshnevisan et al. (2015) found that critical factor that controls the design is the ULS requirement. Thus, only the ULS behavior of this drilled shaft is considered and the corresponding system response is evaluated as follows:

$$f(\mathbf{d}, \boldsymbol{\theta}) = Q_s + Q_b - F_d - F_l \quad (18)$$

where Q_s and Q_b are the side resistance and the base resistance of the drilled shaft, respectively. These two terms can be determined as follows:

$$Q_s = \pi B \alpha \int_0^D c_{u1} dz \quad (19)$$

$$Q_b = \frac{\pi}{4} B^2 N_c c_{u2} \quad (20)$$

where c_{u1} is the undrained shear strength along the shaft length (kPa); α is an empirical adhesion coefficient, taken as 0.5 (e.g., Meyerhof 1976); z is the depth of clay stratum (m); N_c is a base bearing capacity factor, taken as 9 (e.g., Meyerhof 1976); and, c_{u2} is the undrained shear strength at the shaft base (kPa).

In the context of the standard LRFD approach, the safety of the drilled shaft with respect to the ULS requirement is assessed with the following equation (AASHTO 2014):

$$g(\mathbf{d}, \boldsymbol{\theta}^n) = \psi_{Q_s} Q_s + \psi_{Q_b} Q_b - \psi_{F_d} F_d - \psi_{F_l} F_l \quad (21)$$

where ψ_{Q_s} and ψ_{Q_b} are the resistance factors with respect to the side resistance (Q_s) and the base resistance (Q_b), respectively; and, ψ_{F_d} and ψ_{F_l} are the load factors with respect to the dead load (F_d) and the live load (F_l), respectively. In this example, $\psi_{Q_s} = 0.45$, $\psi_{Q_b} = 0.4$, $\psi_{F_d} = 1.25$, and $\psi_{F_l} = 1.75$ (AASHTO 2014).

The nominal values of the undrained shear strength (i.e., c_{u1} and c_{u2}), dead load (F_d), and live load (F_l) are referred to herein as the noise factors ($\boldsymbol{\theta}$). In Khoshnevisan et al. (2015), the normalized undrained shear strength of the clay, denoted as $c_u^n = c_u / z$, is characterized as a lognormal random variable with a mean of 13.04 and a standard deviation of 4.15. Thus, the nominal value of the normalized undrained shear strength (c_u^n) is taken as 13.04 for simplicity. In ISSMGE (2009), the nominal values of the dead load (F_d) and the live load (F_l) are specified as 300 kN and 150 kN, respectively; and, the COVs of the dead load and the live load are assumed as 0.1 and 0.18, respectively, based on the study by Zhang et al. (2011). Further, the correlation structure among the normalized undrained shear strength (c_u^n), dead load (F_d), and live load (F_l) is assumed to be a 3 by 3 identity matrix.

Similar to that in Example 1, the dimensions of the drilled shaft (i.e., shaft diameter B and shaft length D) are referred to herein as the design parameters ($\mathbf{d}$). In this example, a continuous design space ($\mathbf{DS}$) is considered: $\mathbf{DS} = \{(B, D) | 0.3 \text{ m} \leq B \leq 0.6 \text{ m and } 10 \text{ m} \leq D \leq 25 \text{ m}\}$. For ease of illustration, the cost of this drilled shaft (C) is expressed with the volume of the concrete, expressed as:

$$C(\mathbf{d}) = \frac{\pi B^2}{4} D \quad (22)$$

6.2 Results of robust design of this drilled shaft problem

Using the R -LRFD approach described previously, the final design, in terms of the most preferred design in the design space ($\mathbf{DS}$), of this drilled shaft can easily be identified. The process involves three single-objective optimizations: one is to identify the least cost design (based on cost C), another one is to identify the most robust design (based on sensitivity index SI), and the last one is to identify the most preferred design using the simplified procedure (based on the minimum distance concept). The design parameters of the final design outcome using the R -LRFD approach are: $B = 0.37 \text{ m}$ and $D = 25.00 \text{ m}$, and the corresponding cost and sensitivity index are: $C = 2.66 \text{ m}^3$ (raw value) and $SI = 0.383$ (raw value), respectively. This most preferred design is listed in Table 4 along with the least cost design and the most robust design obtained for this drill shaft problem.

As can be seen in Table 4, the least cost design in the feasible domain yields $B = 0.30 \text{ m}$ and $D = 20.30 \text{ m}$, which is associated with a sensitivity index of $SI = 0.465$ (raw value) and a cost of $C = 1.43 \text{ m}^3$ (raw value). On the other hand, the most robust design in the feasible domain yields $B = 0.60 \text{ m}$ and $D = 25.00 \text{ m}$, which is associated with a sensitivity index of $SI = 0.352$ (raw value) and a cost of $C = 7.07 \text{ m}^3$ (raw value). Thus, compared to the least cost

design in the design space, the most preferred design yields a significantly higher degree of robustness (normalized $SI^n = 0.273$ versus 1.0) while the increase in cost is only modest (normalized $C^n = 0.217$ versus 0.0). Compared to the most robust design, the most preferred design yields significant savings in cost (normalized $C^n = 0.217$ versus 1.0) while the robustness is lowered only slightly (normalized $SI^n = 0.273$ versus 0.0). Thus, the *R*-LRFD design (i.e., $B = 0.37$ m and $D = 25.00$) yields the best compromise between cost efficiency and robustness.

A Pareto front is further obtained by taking the optional step, which involves a multi-objective optimization algorithm (Deb et al. 2002). Figure 7 shows the derived Pareto front for this drilled shaft problem, in both normalized objectives space (Figure 7a) and unnormalized objectives space (Figure 7b). Since a continuous design space (**DS**) is adopted in this example, the resulting Pareto front shown in Figure 7 is a single smooth curve. The knee point on the Pareto front is identified using the marginal utility function approach (Branke et al. 2004). The design parameters of the derived knee point are: $B = 0.37$ m and $D = 25.00$ m, which are the same as the most preferred design obtained using the simplified procedure of the *R*-LRFD approach. Thus, the optional step to obtain the Pareto front is unnecessary if the intent of the designer is just to identify the most preferred design in the design space.

Furthermore, Table 5 compares the computational efforts of the simplified procedure with existing multi-objective optimization procedure. The computation time is recorded using the same Windows 7® PC equipped with a 8.0 GB RAM and an Intel® Core™ i7-3630QM CPU running at 2.40GHz. The RGD approach with the simplified procedure for knee point reduces the computation time drastically.

6.3 Effect of weighting factors on the final design

Similar to Example 1, a parametric study is conducted herein to investigate the effect of the assumption of the COVs on the outcome of the proposed *R*-LRFD approach. For this drilled shaft problem, a design scenario is assumed with all weighting factors setting to 1.0 and the correlation structure setting as a 3 by 3 identity matrix. The robust design using the *R*-LRFD approach with this design scenario yields exactly the same results: $B = 0.37$ m and $D = 25.00$ m. The conclusion reached in Example 1 is confirmed in Example 2. The effect of the weighting factors on the final design is dominated by the gradient.

6.4 Effectiveness of the sensitivity index (SI) as a robustness measure

Here, the sensitivity index (SI) is compared with another popular robustness measure, namely, signal-to-noise ratio (SNR) (Phadke 1989). For this illustrative example, the SNR of a design $\mathbf{d}$ is computed as follows (Gong et al. 2015):

$$\text{SNR}(\mathbf{d}) = 10 \log_{10} \left(\frac{\mu_f^2}{\sigma_f^2} \right) \quad (23)$$

where μ_f and σ_f are the mean and the standard deviation of the system response $[f(\mathbf{d}, \boldsymbol{\theta})]$, respectively, which can readily be estimated using MFOSM (see Eq. 5 and Eq. 7).

The plots in Figure 8 indicate that the SNR decreases with the normalized sensitivity index (SI^n); thus, a design with a lower sensitivity index (indicating a higher degree of robustness) generally exhibits a higher SNR value. This is consistent with the concept of robust design. Since the SNR is one of the most well accepted robustness measure in the literature (Taguchi 1986; Phadke 1989; Park et al. 2006), the results shown in Figure 8 support the claim that the updated gradient-based sensitivity index (SI) is an effective robustness measure. The latter, however, requires only a deterministic analysis and thus is much simpler and more computationally efficient than the former.

7. Further Discussion: Comparison between LRFD and *R*-LRFD

The *R*-LRFD approach retains the simplicity in the design process that makes the LRFD approach a preference (from usability standpoint) over the RBD, and yet it allows for a more informed design decision based on a simultaneous consideration of the safety, robustness, and cost efficiency. Since the *R*-LRFD approach can remove some of the shortcomings of the standard LRFD approach, and it requires little additional effort, it appears to be a method of choice. Figure 9 shows the optimal design (typically based on cost) obtained by LRFD along with the most preferred design obtained by the *R*-LRFD. The design by LRFD happens to be the least cost design on the Pareto front. This is easily understood, as the Pareto front consists of all non-dominated designs that satisfy the requirements of LRFD. Note that the least cost design on the Pareto front is found to have the lowest robustness (i.e., with the highest SI^n value). The design (i.e., most preferred design) obtained by the *R*-LRFD yields the best compromise between robustness (represented by SI^n) and cost efficiency (represented by C^n).

8. Summary and Conclusions

This paper presents a new design approach, termed *Robust* Load and Resistance Factor Design (*R*-LRFD). The proposed *R*-LRFD approach treats the geotechnical design as a multi-objective optimization problem that considers explicitly the robustness, in addition to the safety and the cost efficiency. A simplified version of *R*-LRFD is also developed, which does not require the use of multi-objective optimization algorithms. The following conclusions are reached based on the results presented:

1) The proposed *R*-LRFD approach is found effective and capable of producing a design that yields the best compromise between robustness and cost efficiency in the design space. This *R*-LRFD approach is seen as an effective integration of the standard LRFD approach with the robust geotechnical design (RGD) concept. *R*-LRFD is applicable to all scenarios of data availability, from one extreme that sufficient data is available to characterize noise factors statistically to another that no data is available to quantify noise factors, although it is most useful under the pivotal assumption of unquantifiable noises.

2) The developed sensitivity index (SI) is shown as an effective robustness measure. The SI is found valid and suitable for the purpose of robust design regardless of data availability (i.e., whether data is sufficient to quantify noise factors statistically). While the SI is formulated in a deterministic manner, it is consistent with the existing robustness measures (e.g., standard deviation of the failure probability, feasibility robustness, and signal-to-noise ratio SNR). A design with a higher sensitivity index always yields a larger variation of the failure probability, a lower feasibility robustness index, and a lower SNR value, all indicating a lower degree of robustness.

3) The multi-objective optimization of the proposed *R*-LRFD approach generally results in a Pareto front that depicts the tradeoff between robustness and cost efficiency, which can aid in making an informed design decision. Without a specified preference, the knee point on the Pareto front that yields the best compromise between robustness and cost efficiency is usually selected as the final design. A simplified version of the *R*-LRFD approach is developed, which is found to be able to produce a final design identical to the knee point on the Pareto front. The simplified version of the proposed *R*-LRFD approach, which does not require a multi-objective

optimization, can be implemented in Microsoft Excel using the Solver feature. Thus, *R*-LRFD has a potential as a practical geotechnical design approach.

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Table 1. Results of the robust design optimization for Example 1

No.	Design parameters		Safety, $g(\mathbf{d}, \mathbf{\theta}^n)$	Cost, $C \text{ (m}^3\text{)}$	Sensitivity index, SI	Feasible or safe?	Normalized objectives		Computed distance
	$B \text{ (m)}$	$D \text{ (m)}$					C^n	SI^n	
1	0.9	2.0	-561.64	516.67	0.309	No	—	—	—
2	0.9	2.2	-536.54	568.33	0.383	No	—	—	—
3	0.9	2.4	-511.15	620.00	0.482	No	—	—	—
4	0.9	2.6	-485.50	671.67	0.618	No	—	—	—
5	0.9	2.8	-459.58	723.33	0.821	No	—	—	—
...									
20	0.9	5.8	-40.95	1498.33	0.543	No	—	—	—
21	0.9	6.0	-11.10	1550.00	0.510	No	—	—	—
22 ⁽¹⁾	0.9	6.2	19.00	1601.67	0.482	Yes	0.000	1.000	1.000
23	0.9	6.4	49.34	1653.33	0.458	Yes	0.020	0.874	0.875
24	0.9	6.6	79.92	1705.00	0.437	Yes	0.040	0.765	0.766
...									
29	0.9	7.6	236.42	1963.33	0.361	Yes	0.140	0.373	0.398
30	0.9	7.8	268.45	2015.00	0.350	Yes	0.160	0.315	0.353
31 ⁽²⁾	0.9	8.0	300.71	2066.67	0.340	Yes	0.180	0.262	0.318
32	1.2	2.0	-426.13	773.33	1.419	No	—	—	—
33	1.2	2.2	-387.68	850.67	3.217	No	—	—	—
...									
89	1.5	7.2	1259.29	3768.00	0.306	Yes	0.838	0.084	0.842
90	1.5	7.4	1322.73	3872.67	0.301	Yes	0.879	0.061	0.881
91	1.5	7.6	1386.49	3977.33	0.297	Yes	0.919	0.040	0.920
92	1.5	7.8	1450.58	4082.00	0.293	Yes	0.960	0.019	0.960
93 ⁽³⁾	1.5	8.0	1515.01	4186.67	0.289	Yes	1.000	0.000	1.000

Note:

- (1) The least cost design in the feasible domain.
(2) The most preferred design in the design space.
(3) The most robust design in the feasible domain.

Table 2. Final designs obtained by *R*-LRFD, RBD, and reliability-based RGD with different design scenarios of the statistical characterization of uncertain input parameters for Example 1

Design scenario	COV		Correlation structure $\mathbf{R}_\theta$	<i>R</i> -LRFD			RBD			Reliability-based RGD		
	ϕ	K_0		B (m)	D (m)	β	B (m)	D (m)	β	B (m)	D (m)	β
0 ⁽¹⁾	7%	50%	$\begin{bmatrix} 1.0 & -0.75 \\ -0.75 & 1.0 \end{bmatrix}$	0.9	8.0	6.29	0.9	5.6	2.65	0.9	8.0	6.29
1 ⁽¹⁾	5%	30%	$\begin{bmatrix} 1.0 & -0.60 \\ -0.60 & 1.0 \end{bmatrix}$	0.9	8.0	7.48	0.9	5.2	2.65	0.9	8.0	7.48
2 ⁽¹⁾	5%	30%	$\begin{bmatrix} 1.0 & -0.90 \\ -0.90 & 1.0 \end{bmatrix}$	0.9	8.0	13.60	0.9	5.0	2.85	0.9	7.6	12.22
3 ⁽¹⁾	5%	70%	$\begin{bmatrix} 1.0 & -0.60 \\ -0.60 & 1.0 \end{bmatrix}$	0.9	8.0	6.77	0.9	5.2	2.63	0.9	8.0	6.77
4 ⁽¹⁾	5%	70%	$\begin{bmatrix} 1.0 & -0.90 \\ -0.90 & 1.0 \end{bmatrix}$	0.9	7.8	12.30	0.9	4.8	3.10	0.9	7.4	11.24
5 ⁽¹⁾	10%	30%	$\begin{bmatrix} 1.0 & -0.60 \\ -0.60 & 1.0 \end{bmatrix}$	0.9	8.0	3.60	0.9	7.0	2.70	1.2	7.0	4.62
6 ⁽¹⁾	10%	30%	$\begin{bmatrix} 1.0 & -0.90 \\ -0.90 & 1.0 \end{bmatrix}$	0.9	8.0	5.36	0.9	6.6	2.87	1.2	6.6	5.86
7 ⁽¹⁾	10%	70%	$\begin{bmatrix} 1.0 & -0.60 \\ -0.60 & 1.0 \end{bmatrix}$	0.9	8.0	3.50	0.9	7.0	2.70	1.2	7.0	4.63
8 ⁽¹⁾	10%	70%	$\begin{bmatrix} 1.0 & -0.90 \\ -0.90 & 1.0 \end{bmatrix}$	0.9	8.0	6.27	0.9	6.0	2.79	1.2	6.2	6.59
9 ⁽²⁾	–	–	$\begin{bmatrix} 1.0 & 0 \\ 0 & 1.0 \end{bmatrix}$	0.9	8.0	–	–	–	–	–	–	–

Note:

(1) For noise factors, the weighting factors are taken as the COVs of noise factors for *R*-LRFD.

(2) Equal weights and an identity correlation matrix are adopted for *R*-LRFD.

Table 3. A full statistical characterization of uncertain input parameters for Example 1

Soil parameter	Type of distribution	Mean	Coefficient of variation (COV)		
			Approximate range	Mean of COV	Std. Dev. of COV
Effective friction angle, ϕ'	Lognormal	32°	5% to 10%	7%	1.25%
Coefficient of earth pressure at rest, K_0	Lognormal	1.0	20% to 90%	50%	17.50%

Note:

The correlation coefficient between ϕ' and K_0 is assumed normally distributed with a mean of -0.75 and a standard deviation (Std. Dev.) of 0.075.

Table 4. Results of the robust design optimization for Example 2

Item	Design parameters		Cost, $C \text{ (m}^3\text{)}$	Sensitivity index, SI	Normalized objectives	
	$B \text{ (m)}$	$D \text{ (m)}$			C^n	SI^n
Least cost design ⁽¹⁾	0.30	20.30	1.43	0.465	0.0	1.0
Most robust design ⁽²⁾	0.60	25.00	7.07	0.352	1.0	0.0
Most preferred design ⁽³⁾	0.37	25.00	2.66	0.383	0.217	0.273

Note:

- (1) The least cost design is identified by minimizing the cost (C).
- (2) The most robust design is identified by minimizing the sensitivity index (SI).
- (3) The most preferred design is identified by minimizing the distance measured from the utopia design to each of the candidate designs.

Table 5. Comparison of the computational efficiency between the simplified procedure and the existing multi-objective optimization procedure

Adopted approach	Analysis steps	Computation time (second)
Proposed procedure	1) Locate the least cost design ⁽¹⁾	0.0200
	2) Locate the most robust design ⁽¹⁾	0.0080
	3) Locate the most preferred design ⁽¹⁾	0.0140
Existing procedure	1) Locate the Pareto front with NSGA-II ⁽²⁾	68.1057
	2) Locate the knee point using marginal utility function approach ⁽³⁾	7.9290

Note:

(1) The optimization setting is set at: options=optimset('Algorithm','active-set','LargeScale','on','Display','off','TolFun',1e-3,'MaxIter',50,'MaxFunEvals',1e7,'TolX',1e-3,'TolCon',1e-3).

(2) The population size and generation number in NSGA-II are set at 100 and 20,000, respectively; and, four parallel computations are utilized within Matlab.

(3) 10,000 MCS samples are employed for evaluating the marginal utility function.

Figure 1. Variation in the system response: (a) System response of a non-robust design $\mathbf{d}_1$ with noise factors as its input; (b) Reducing the variation in the system response by reducing the variation in noise factors; (c) Reducing the variation in the system response by adopting a robust design $\mathbf{d}_2$ without reducing the variation in noise factors.

(a) Pareto front and knee point

(b) Simplified procedure to identify the most preferred design in the design space

Figure 2. Conceptual illustration of a bi-objective optimization problem

Figure 3. Schematic diagram of the design of a drilled shaft in sand (Example 1)

(a) Normalized design objectives

(b) Unnormalized design objectives

Figure 4. The Pareto front and knee point obtained for the drilled shaft in sand (Example 1)

Figure 5. Effectiveness of the sensitivity index as a robustness measure (Example 1)

Figure 6. Schematic diagram of the design of a drilled shaft in clay (Example 2)

(a) Normalized design objectives

(b) Unnormalized design objectives

Figure 7. The Pareto front and knee point obtained for the drilled shaft in clay (Example 2)

Figure 8. Effectiveness of the sensitivity index as a robustness measure (Example 2)

(a) Example 1

(b) Example 2

Figure 9. Comparison of designs obtained with LRFD and *R*-LRFD