**Permeability Estimation Based on Thin-Section Image Analysis and 2D Flow Modeling in
Grain-Dominated Carbonates**

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Abstract: Permeability estimation based on image analysis is a good alternative when an intact core-plug is not available for laboratory measurement. While there is much research on permeability estimation from 2D and 3D images or models, accurately estimating permeability for carbonates, a type of rock with substantial heterogeneity, is still challenging. In this study, a method for permeability estimation based on thin-section image analysis and 2D permeability simulation is developed for grain-dominated carbonates based on semi-theoretical analysis of 2D and 3D permeability (K2D and K3D) relationship. The mathematical expression of the K2D/K3D is examined and validated with a grainstone carbonate sample, for which both K2D and K3D were obtained through permeability simulation based on micro-CT images. The method was applied to 24 grain-dominated carbonate samples collected from different locations with a large variation in rock textures and fabrics and a broad range of permeability from 0.1 mD to 3200 mD. Representativeness of the thin section and the correct determination of the effective pores are the key for accurate estimation. The estimated permeability was compared to the measured value, and 92% of the results are within a factor of ± 5 , and 46% are with a factor of ± 2 . Potential sources of systematic error are discussed both qualitatively and quantitatively.

Keywords: permeability estimation, carbonate, thin section, 2D flow modeling, K2D and K3D relationship

1. Introduction

The ability to have an accurate estimate of permeability is important in reservoir characterization. Direct laboratory measurements, while more straightforward, are sometimes limited by the availability of usable core plugs. In contrast, smaller samples are more readily

acquired; therefore, estimations based on images from these smaller rock samples is a useful alternative for laboratory measurement. More importantly, permeability estimation based on image analysis provides further insight about pore structure that are fundamental for understanding fluid flow and other transport processes, whereas the core-plug laboratory measurement is more a “black-box” model that only provides output of a single value.

Both 2D and 3D images/reconstructions can be used for permeability estimations. Two-dimensional image can be thin sections or scanning electronic microscopy (SEM) images (Coskun and Wardlaw, 1993; Anselmetti et al., 1998; Schaap and Lebron, 2001; Lock et al., 2002; Jurgawczynski et al., 2004; Al-Bazzaz et al., 2007; Weger et al., 2009; Wong et al., 2012). The 3D model includes images obtained directly from micro-CT scan (Knackstedt et al., 2008; De Boever et al., 2012; Mostaghimi et al., 2012; Peng et al., 2014; Zakirov et al., 2016) or reconstructed from limited amount of 2D training images using multi-point statistical models (Al-Kharus and Blunt, 2008), process-based modeling (Keehm et al., 2004), or an object-based method that involving a third-order Markov mesh (Wu et al., 2008; van der Land et al., 2013). A 3D model generally provides a more realistic representation of the pore network because the real rock sample contains a 3D pore network. However, compared to 2D images, acquisition of 3D images or reconstructions and associated permeability estimation are more complicated, time-consuming, and expensive. Another disadvantage with 3D models is that only a relatively small volume can be characterized because of computational limitation and/or because of a small volume being scanned in order to achieve sufficient high resolution to detect the smaller pores (pore throats) that control connection. The small volume analyzed will be less representative when compared to 1- or 1.5-inch plugs from which the laboratory measurements are conducted, especially for samples with substantial heterogeneity, such as carbonates.

Two-dimensional images are easier to obtain and it is easier to cover larger areas than in 3D. Some research have been conducted about permeability estimation based on 2D thin sections or SEM images. Characteristic pore geometry parameters such as porosity, pore diameter, pore area, perimeter, hydraulic radius, and/or pore shape factor can be obtained either through direct image analysis (Coskun and Wardlaw, 1997; Lock et al., 2002; Al-Bazzaz et al., 2007; Weger et al., 2009) or a two-point correlation function based on the 2D images (Berryman and Blair, 1986; Schaap and Lebron, 2001; Keehm et al., 2004). One method is based on regression analysis between laboratory measured permeability and mean value of selected pore geometry parameters (Coskun and Wardlaw, 1997) or the Kozney-Carman equation (Schaap and Lebron, 2001). In general, good estimations were reported for this type of method with the majority of estimations being within a factor of two; however, by using a mean value of pore size, this method does not allow for the influence of the distribution of pore size and the complexity of pore shape on permeability. In addition, whether the regression-type of equation developed from a particular type of rock samples can be applied to a different type of sample is not known.

Lock et al (2002) and other researchers (e.g., Wong et al., 2012) developed another type of method based on 2D images using effective-medium theory (Kirkpatrick, 1973). The effective-medium theory is a counter-approach of pore network modeling. A pore network model is used to represent the pore structure, but calculated permeability is based on an effective single-tube conductance. The effective single-tube conductance was obtained on the basis of individual conductance of each tube that is estimated from pore areas and perimeters (Lock et al., 2002). There are several assumptions in their algorithm, such as constant coordination number, fixed ratio of minimum-to-maximum pore radius, and a fixed stereological correction factor. Such assumptions may work out well with a particular set of samples, but may not be appropriate for

another set of samples. Also, many of the previous estimations are for sandstones (Coskun and Wardlaw, 1997; Lock et al., 2002; Keehm et al., 2004) or other relatively homogeneous porous media (Schaap and Lebron, 2001). For carbonates, that have greater extent of heterogeneity, ineffective permeability estimation with low coefficient of determination ($R^2 < 0.5$) (Weger et al., 2009) or larger deviations of estimations from measured value up to two orders of magnitude (Jaasdfasd et al., 2004) are reported. Anselmetti et al. (1998) used combined pore geometry information (total porosity, macroporosity, and macropore shape parameter) from both thin section and high-resolution SEM images that cover both macro- and micropores for permeability analysis in carbonate rocks. They found poor correlation between porosity (either total porosity or macroporosity) and permeability, but good correlation between macropore shape parameter and permeability. An artificial neural network model was trained to obtain good permeability estimation based on image analysis. For such type of modeling, different database may lead to different estimation. Knackstedt et al. (2008) conducted permeability simulation for a few carbonate samples with different textures and rock fabrics based on micro-CT images; however, they only compared the porosity-and-permeability relationship from the simulation and the laboratory experiment. It is not made clear how the simulated permeability compare to the measured values.

Lucia (1983) focused on carbonates and developed a method for permeability estimation based on porosity-permeability relationships through analyzing a large variety of carbonate samples. Based on thin-section analysis and porosity and permeability measurements, Lucia (1983) defined three permeability fields according to particle size range, i.e., $< 20 \mu\text{m}$, $20\text{-}100 \mu\text{m}$, and $100\text{-}500 \mu\text{m}$. Each permeability field has a permeability-porosity equation calculated by regression analysis. Later he defined three classes of carbonate rock types based on rock fabrics

and provided corresponding porosity-permeability relationships (Lucia, 1995). According to his method, permeability can be estimated using his rock-fabric porosity-permeability relationships. This method provides a quick and simple way to estimate permeability based on porosity and rock fabric; however, Lucia (1995) also noted that considerable scatters exist when comparing the measurement and estimated results (Lucia, 1995). The deviation of measured permeability from the pre-defined porosity-permeability relationships can be as large as two orders of magnitude. This is not unexpected, giving the complicated depositional and diagenesis controls on the pore structure and heterogeneity of the sample. It is also well understood that permeability is not only dependent on porosity, or effective porosity (predominantly interparticle pores, Lucia, 1983), but also is dependent on pore geometry, pore size distribution, and pore connectivity. Therefore, a simple porosity-permeability relationship is not adequate to make accurate estimations for all samples.

In this paper, a method for permeability estimation based on thin-section image analysis and 2D flow modeling is introduced and was applied to 24 grain-dominated carbonate samples. The study starts with semi-theoretical analysis of the ratio of 2D and 3D permeability (K_{2D}/K_{3D}) based on Kozeny-Carman Equation and a constriction factor (Lock et al., 2002). In the context of this paper, we define K_{2D} to refer to the permeability obtained from 2D images and modeling, and K_{3D} denotes permeability either obtained from 3D modeling, or measured/actual permeability for a plug sample. The K_{2D}/K_{3D} mathematical expression is examined using a carbonate grainstone sample. After that, K_{2D} was obtained based on thin section analysis and 2D pore modeling for all 24 carbonate samples. Finally, K_{3D} was estimated based on the K_{2D} and the K_{2D}/K_{3D} ratio for the 24 samples. The results were compared with laboratory measured permeability values. This study aims to develop a quick yet reliable method

for permeability estimation for grain-dominated carbonates using thin sections and flow simulation based on 2D pore structure which is relatively easy and fast, but in the same time inherently includes the complex pore characteristics, e.g., pore shape and pore size distribution.

2. Materials and Methods

Thin sections were prepared for 24 grain-dominated carbonate samples collected from six locations (Table 1). Petrographic images were taken with an optical (light) microscopy of the polished thin sections in order to study the pore-space attributes and the link between these attributes and permeability values. Fourteen samples are from oil fields producing from the prolific Lower Cretaceous Thamama carbonates in Abu Dhabi, United Arab Emirates. This suite of samples spans a wide spectrum of sedimentary textures ranging from well-sorted ooid grainstones, poorly-sorted intraclastic rudstone, to skeletal peloidal grain-dominated packstones. Measured plug porosity and permeability range from 9 to 23% and 0.1 to 2621 md, respectively. The other 10 samples are from various carbonate fields in Texas and Saudi Arabia with fabrics including grainstones, grain-dominated packstones, and dolostones. Air permeability was measured for these samples by commercial laboratories. Table 1 lists the sample descriptions. Sample S16, a foram-ooid grainstone, was selected for micro-CT scanning with synchrotron x-ray microtomography at the Advanced Photon Source at Argonne National Laboratory. A mini-plug with diameter of approximately 2 mm was drilled from the core to achieve a high resolution and at the same time include the entire width of the plug in the field-of-view. The sample was mounted with plasticine on top of the standard beamline sample holder. The beam energy was set to 21 keV. X-rays were converted to visible light by a 10- μ m thick Ce-doped Lutetium Aluminum Garnet (LuAG) scintillator. The images were magnified by a 10 $\times$ Zeiss microscope

lens and recorded by a CoolSNAP K4 camera (Photometrics, Tucson, AZ). The resulting pixel size was 1.0 μm . More details on micro-CT scan operational parameters and data analysis procedures are included in a previous study (Peng et al., 2015).

Pore segmentation was conducted for the micro-CT images for Sample S16 to separate inter- and intraparticle pores using top-hat and watershed segmentation tools in Avizo 9.0 (FEI). Permeability simulation on both 3D and 2D images were conducted with Avizo 9.0 (Section 4.1). In the 2D permeability simulation, flow was modeled along the direction perpendicular to the plane of the thin section. More description on the permeability simulation is included in Section 4.1. Another image processing and modeling software, 3DMA-Rock (Lindquist et al., 1999), was used for shortest path and pore-throat statistical analysis on the 3D pore structure for Sample S16 (Section 4.2).

Pore segmentation for thin sections of the 24 samples was processed using Avizo 9.0 as well. Because the thin sections are colored and have pores mostly in blueish color, pore segmentation was conducted using decorrelation stretch filtering followed by interactive thresholding based on intensity of the blue channel using Avizo 9.0. An important step in pore segmentation is to eliminate pores that do not contribute to permeability proportionally to their porosity, basically vuggy pores for the grain-dominated carbonates. This step is conducted with the manual segmentation tool in Avizo 9.0.

3. K2D/K3D Ratio

Understanding the relationship between K2D and K3D is necessary for estimation of the actual permeability (K3D) based on permeability from 2D images of thin sections (K2D). K2D/K3D

ratio can be evaluated based on the Kozeny-Carman (K-C) equation. Even though the K-C equation is only a semi-theoretical equation, predictions based on it has been found to be in excellent agreement with measured values for various type of porous media from well-sorted and rounded sands to platy or rounded cemented/uncemented granular rocks and soils (Carman, 1956; Van Brakel, 1975; Witt and Braus, 1982; Cancelliere et al., 1990; Dullien, 1992;). One form of K-C equation is:

$$K = \frac{\phi r_h^2}{\beta \tau^2} \quad (1)$$

where ϕ is porosity, r_h is hydraulic radius, β is a shape factor that accounts for the influence of different shapes of the cross-sectional area, and τ is hydraulic tortuosity (L_{eh}/L_m), which is calculated as the ratio of effective hydraulic path length (L_{eh}) to minimum length along the flow path (L_m). The 2D and 3D versions of Equation 1 can be written as:

$$K_{2D} = \frac{\phi_{2D} r_{h_{2D}}^2}{\beta_{2D}} \quad (2a)$$

$$K_{3D} = \frac{\phi_{3D} r_{h_{3D}}^2}{\beta_{3D} \tau^2} \quad (2b)$$

Subscriptions of 2D or 3D in equations 2a and 2b denote the corresponding parameters in 2D or 3D. In K2D simulation in the direction perpendicular into the image, it is essentially assumed that water flows along straight paths with a cross-section-area defined by the shapes of pores in the 2D image. Therefore, tortuosity in 2D simulation has a value of one and thus omitted in Equation 2a. Hydraulic radius is defined as pore cross section area divided by cross section perimeter in 2D and pore volume divided by pore surface area in 3D, respectively:

$$r_{h_{2D}} = \frac{\pi r_{2D}^2}{2\pi r_{2D}} \quad (3a)$$

$$r_{h_3D} = \frac{\frac{4}{3}\pi r_{3D}^3}{4\pi r_{3D}^2} \quad (3b)$$

in which r_{2D} and r_{3D} are the equivalent pore radius from 2D and 3D image analysis, respectively.

Combining equations 2 and 3 yields:

$$\frac{K_{2D}}{K_{3D}} = \frac{3}{2} \frac{\beta_{3D}}{\beta_{2D}} \frac{\phi_{2D}}{\phi_{3D}} \tau^2 \frac{r_{2D}}{r_{3D}} \quad (4)$$

For 2D and 3D images acquired from the same part of sample, parameters such as pore size and pore shape should be statistically the same. Therefore, $\langle\beta_{3D}\rangle = \langle\beta_{2D}\rangle$ and $\langle r_{2D}\rangle = \langle r_{3D}\rangle$ should be reasonable approximations, where $\langle\cdot\rangle$ denotes an average taken over a 2D or 3D image. It is important to note that both ϕ_{2D} and ϕ_{3D} in Kozeny-Carman Equation should be effective porosity, which is mostly consist of interparticle pores in grain-dominated carbonates. Poorly-connected pores, mostly vuggy pores, may account for substantial porosity, but do not affect permeability significantly (more discussion in Section 4.1). Therefore, such poorly-connected pores should not be counted in effective porosity and need be excluded for permeability calculation. Consequently, only effective pores, which are interpreted to be interparticle pores in carbonates, should be included in both 2D and 3D permeability simulation. With this being carefully taken into account, ϕ_{2D} and ϕ_{3D} should have values that are similar, with some difference related to the representativeness of the 2D image when compared to the whole 3D volume. The similar values of ϕ_{2D} and ϕ_{3D} (11.3% vs. 11.6%) are discussed in Section 4.1 for sample S16. Equation 4 can thus be simplified to

$$\frac{K_{2D}}{K_{3D}} = \frac{3}{2} \tau^2 \quad (5)$$

Weger et al. (2009) applied another form of the K-C equation based on specific surface area (S) and Carman constant (c), which is $K = \frac{\phi^3}{cS^2}$, to permeability estimation in carbonates.

Applicability of the K-C equation was not discussed in Weger et al. (2009), whereas other research reported overestimation of permeability by K-C Equation ($K = \frac{\phi^3}{cS^2}$) for carbonates (Mostaghimi et al., 2012; Zakirov et al., 2016). Carman constant (c) is related to two factors: a shape factor that characterizes the cross-section of the flow channel and a tortuosity factor (Ben Clennell, 1997). Carman constant generally has a range of 4.0-8.3 for sandstone and an average value of 5.0 is often used for permeability estimation (Zakirov et al., 2016). However, both Mostaghimi et al. (2012) and Zakirov et al. (2016) found the Carman constant for carbonates has a larger value (up to 355.1) that is far beyond the values for sandstone (4.0-8.3). One reason is K-C Equation was originally developed based on sphere packings, and is lack of the ability to account for more complex and tortuous pore network in carbonates (Mostaghimi et al., 2012). Nevertheless, the method in this study does not make permeability estimation directly based on K-C equation. Instead, it relies on the ratio of K2D/K3D, in which part of the Carman constant effect (pore shape factor) is cancelled, and the effect of tortuosity is explicitly considered as in Equation 5.

However, there is one factor that K-C Equation does not consider. K-C Equation was developed based on capillary tube bundle model, in which the actual pore network was simplified as a bundle of sinuous capillary tubes with a specified radius for each tube. However, the actual “tube” that can better represent the actual pore flow path would have various radius (from pore body to pore throat), and the pore throat will have greater constriction on the flow because of its smaller size. In 2D permeability simulation based on thin sections, larger pores, which can be interpreted as pore bodies, dominate. Therefore, if we use K2D from thin section, the actual permeability will be overestimated by a factor of $1/f$, where f is less than one and is

defined as constriction factor by Lock et al. (2002). With the consideration of constriction factor, Equation 5 becomes:

$$\frac{K_{2D}}{K_{3D}} = \frac{3}{2} \tau^2 \frac{1}{f} \quad (6a)$$

in which

$$f = \frac{256\rho^{3.5}}{(1+\rho)^4(5\rho^3+3\rho^2+3\rho+5)} \quad (6b)$$

where $\rho = r_{min}/r_{max}$, a pore radius ratio that can be interpreted as the pore throat/pore body ratio. The definition of f according to Equation 6b is based on the assumption that the variation of the pore radius is sinusoidal and pores have circular-shaped cross-section (Lock et al., 2002), which are considered as a reasonable approximation for real pore systems. Although Lock et al. (2002) developed their method for sandstones, the derivation of f is essentially based on a general conceptual model of a pore channel having variable radius. Therefore, Equation 6b should also be applicable for carbonates.

4. Examination of the K2D/K3D Relationship Based on Sample S16

4.1 Porosity and Permeability Analysis

Equation 6a on the K2D/K3D ratio is examined for the test Sample S16 based on its 2D and 3D image analysis and permeability simulations. Figure 1A shows the reconstructed digital mini-plug for Sample S16. Red and blue shaded areas (Figure 1B) represent inter- and intraparticle pores, respectively. Interparticle pores were extracted through watershed segmentation using relatively larger markers as seeds, whereas the intraparticle pores were obtained through top-hat

segmentation using small kernel size in Avizo. The voxel size for the 3D image is 1.0 μm . In Figure 1C, interparticle pores that are between the ooid grains are labelled in yellow, and they are well connected; intraparticle pores, within the ooid, are in red, and are poorly connected to the main pore network. Porosity can be readily measured from the segmented pore volumes. Inter- and intraparticle porosity was calculated as 11.6% and 2.8%, respectively. Distinct pore-size differences can be observed from Figure 1 between inter- and intraparticle pores. Combined with the pore throat size distribution obtained from mercury intrusion (MICP; Figure 1D), it is clear that areas below the right and left peaks with pore throat boundary at $\sim 10 \mu\text{m}$ represent the pore volume of inter- and intraparticle pores, respectively. Total porosity obtained through MICP analysis is 19.2%. Divided into inter- and intraparticle porosity, the results are 12.1% and 7.1%, respectively. Comparison of the image and MICP porosities shows similar interparticle porosity (11.6% vs. 12.1%), whereas approximately three fold difference in intraparticle porosity (2.8% vs. 7.1%). The smaller image-based intraparticle porosity indicates that micro-CT scan with voxel size of 1 μm cannot fully measure intraparticle porosity.

Fluid flow simulation was carried out with the permeability tensor measurement module in Avizo 9.0, which applies finite volume method to solve Stokes equation with periodic boundary conditions based on the pore structures (Zhang et al., 2012). Permeability was then calculated as the mean value of velocity over the studied volume in a specific direction. Permeability simulation was conducted separately for the whole interparticle pore network and a selected intraparticle pore volume as indicated in the white circle in Figure 1C. The dimension of the intraparticle pore volume is $468 \times 456 \times 353 \mu\text{m}^3$. Simulated permeability for the selected intraparticle pore volume is 0.67 mD, more than three-order-of-magnitude smaller than that for the interparticle pore network (2.45 D). For the intraparticle pore network, the actual contribution

to the total permeability can be even smaller because they are connected to the interparticle pore network through smaller pores that were not detected in the micro-CT scan as discussed previously. The simulated interparticle pore permeability (2.45 D) is close to laboratory measured permeability value of 1.98 D, suggesting that interparticle pores dominate in producing permeability in this sample, whereas intraparticle pores is negligible as to contributing to permeability even though they (7.1% intra-particle porosity) accounts for ~40% of the total porosity (19.2%).

Permeability simulation based on the 3D interparticle pore structure produced results close to the laboratory measured permeability, suggesting that the scanned volume with micro-CT is representative of sample S16 and that the flow simulation based on the 3D interparticle pore network is valid for this sample. Figure 2A shows the interparticle pore network without intraparticle pores. In order to examine the difference between results from 2D and 3D permeability simulations, six 2D images from the same pore structure were selected along depth of the plug with equidistance for 2D permeability simulation (Figure 2A). The 2D permeability simulation was conducted along the direction perpendicular to the image. Porosity along the depth was also calculated based on the 2D images (Figure 2B).

The 2D permeability and porosity results are presented in Figure 2C. Observation reveals that porosity varies along the depth, with an average value of 11.6%, whereas permeability (orange dots in Figure 2C) is closely correlated to porosity and have values from 70 to 190 D. Harmonic mean of these six permeability values is 116.0 D. The average porosity of these six images (ϕ_{2D}) is 11.3%, close to the overall average porosity (ϕ_{3D}) of 11.6%. Given the simulated K3D ($K_{3D_{sim}}$) of 2.45 D, a K2D/K3D ratio of 47.3 is obtained. If using the measured permeability as K3D ($K_{3D_{measured}}$), which is 1.98 D, the K2D/K3D ratio is 58.6.

4.2 Tortuosity and Pore Throat/Body Ratio

In order to use Equation 6A to calculate K2D/K3D, hydraulic tortuosity and pore throat/pore body ratio analysis are needed. In Kozeny-Carman equation, hydraulic tortuosity (τ_h) is used to make a more accurate estimation on permeability because stream line tortuosity is more accurate to describe the effect of sinuous path on fluid flow (Ben Clennell, 1997). However, τ_h is very difficult to measure experimentally; therefore, the more-readily-measured geometric tortuosity (τ_g) is used. Shortest path analysis was conducted using 3DMA-Rock. Figure 3A shows the shortest paths in Z direction. Geometric tortuosity is calculated based on its definition:

$$\tau_g = \frac{L_{ij}}{d_{ij}} \quad (7)$$

where L_{ij} is the shortest path length between point i (entrance face) and point j (exit face), d_{ij} is the Euclidean distance (straight line distance) between points i and j . Distribution of calculated geometric tortuosity is presented in Figure 3B. Geometric tortuosity ranges from approximately 1.5 to 2.5, with a mean value of 1.9.

Pore throat/body statistics analysis was conducted based on media axis of the interparticle pore network using 3DMA-Rock (Figure 4A). The medial axis is a 26-connected centrally-located skeleton which preserves the original topology and geometry of the rock sample. The rainbow color scale represents the distance of the path to the closest grain voxels, with red denoting short distance and blue long distance. Pore throats are located as the cross sectional surfaces with locally minimal areas. With throats located, the pore space can be divided into a network of pores separated by throat surfaces. Every pore is cross-indexed with its connecting throats and adjoining pores, and every throat is cross-indexed with the pores it connects

(Lindquist, 1999). Figures 4B and 4C show the pore throat and pore body radius statistics. With the majority part of the pore throat radius in the range of 0 to 100 μm , the adjoining pores have a radius range between 0 and 200 μm (Figure 4B). Dominating throat/pore ratio ranges from 0.1 to 0.6 and accounts for 80% of the total throat/pore ratio frequency. The mean throat/pore ratio is approximately 0.3 (Figure 4C).

Using mean τ_g of 1.9 as τ and mean ρ of 0.3 in Equations 6a-b, K2D/K3D is calculated as 24, which is 42% smaller than K2D/K3D of 47.5 obtained from permeability simulation. The underestimation of K2D/K3D from image analysis is expected because the use of τ_g in replacement of τ_h . By definition, geometric tortuosity is based on the shortest path between inflow and outflow points, whereas hydraulic tortuosity is based on the effective flow path. Fluid passes along a smoothed route through porous media so that τ is always larger than τ_g (Ben Clennell, 1997). This will lead to underestimation of K2D/K3D when using τ_g in replacement of τ in Equation 6a.

Another way to estimate hydraulic tortuosity is based on Archie's law and formation factor (Archie, 1942). Formation factor (F_m) is a power law equation of porosity and formation factor and is related to hydraulic tortuosity as follows:

$$F_m = \phi^{-m} \quad (8a)$$

$$\tau_h^2 = F_m \phi \quad (8b)$$

Therefore,

$$\tau_h^2 = \phi^{1-m} \quad (8c)$$

351 where m is commonly known as cementation exponent. Despite wide use of Archie's law, the
352 value of m and how it is dependent of the type of reservoir have remained controversial
353 (Dashtian et al., 2015). Madden (1976) mentioned a constant m value of 2.0 for a few low
354 porosity rocks. Focke and Munn (1987) showed an approximate m value of 2.0 based on
355 extensive laboratory measurement on sucrosic dolomites with intercrystalline porosity and open
356 textures. Dullien (1992) reported the range of m value of 1.3-2.5 for various types of rocks.
357 Knackstedt et al. (2008) obtained m value of 1.5-2.1 for a few sucrosic dolomites. Dashtian et al.
358 (2015) noted a lower m value of 0.21-1.75 for carbonates with substantial clay minerals, but a
359 range of 1.6-2.5 for carbonates without clay minerals. For rocks without substantial influence
360 from clay minerals, although the reported m values are not the same, they basically have similar
361 range between 1.3 and 2.5. Ben Clennel (1997) argued that the value of m is related to the
362 importance of a random pore on pore connection. Higher values of m mean greater importance of
363 a random pore on pore connection, such as in a poorly connected pore system in which the loss
364 of a random pore can change the pore connectivity significantly. Lower values of m indicate less
365 importance on pore connection, such as in a well-connected pore system in which the removal of
366 a random pore will not affect the overall connection significantly. Dead-end pores will not affect
367 overall pore connection either. Therefore, higher values of m indicate poorer pore connectivity
368 and lower values of m indicate better pore connectivity as observed in many literature (e.g.,
369 Dullien, 1992, Dashtian et al., 2015). An m value of 2.0 indicates equal importance of random
370 pores and moderate pore connectivity without much dead-end pores. Since m value is not
371 measured in this study, an m value of 2.0 is used. This can cause error in permeability estimation
372 because a universal m value may not be valid for every rock – more detailed discussion is
373 presented in Section 5.2. Using m value of 2.0 and with porosity of 11.6%, hydraulic tortuosity is

3.0, and the K2D/K3D based on Equation 6a is calculated as 61.8, which is larger than K2D/K3D_{sim} value of 47.5 by 30%, but is only 5.4% larger than the K2D/K3D_{measured} value of 58.6.

From the discussions above we can see that the uncertainty in hydraulic tortuosity estimation can be a main source of error for the K2D/K3D estimation based on Equation 6a. Using geometric tortuosity can lead to underestimation of K2D/K3D, whereas using Archie's law and formation factor (Equations 8a-b) resulted in overestimation of K2D/K3D for the sample S16. However, the extent of overestimation with the latter is smaller – only 5.4% when compared to K2D/K3D_{measured}. More discussion on source of error will be included in Section 5 for thin-section based estimation. Nevertheless, the evaluation on K2D/K3D for sample S16 suggests that Equations 6a, 6b can be used as a semi-theoretical equation for K2D/K3D determination. This also provides a semi-theoretical basis for permeability estimation based on simple 2D images rather than complicated 3D images/models.

5. Thin-Sections Image Analysis and Permeability Estimation

5.1 Thin-Section Pore Segmentation and Image Analysis

Pore segmentation is the first and also the key step for permeability estimation based on thin sections. According to Lucia (1983), carbonate pores can be classified as interparticle pores or vuggy pores. Vuggy pores are divided into separate vugs and touching vugs. Touching vugs form their own connected pore system and are not the subject of this study – samples in this study do not include touching vugs. This study focuses on separate-vugs that include various types of intraparticle pores including moldic pores and intragrain micropores in ooid grainstone

plus pores that are substantially larger than the particle size. Separate vugs generally do not contribute to permeability significantly even though they may account for substantial porosity. For example, the permeability from intraparticle pores of the sample S16 is more than three-order-of-magnitude smaller than that from interparticle pores, as discussed in Section 4.1, although the porosity from the intraparticle pores accounts for ~40% of the total porosity. Therefore, separate vugs, either intraparticle or moldic pores, need to be excluded in pore segmentation before permeability simulation.

Figures 5 to 7 are examples, with different texture, of thin section samples analyzed and the associated pore-segmentation. Measured core-plug permeability for these samples range from 0.12 to 2000 mD. Figures 5A-D are examples of two grainstone samples (S16 and S18) with relatively homogeneous spatial pore distribution. Most obvious pores are interparticle pores for both of these samples. Few small visible intraparticle pores in these images were removed manually during the pore segmentation, and only interparticle pores remain (red areas in Figures 5B, D). All the interparticle pores are considered effective porosity in that they all contribute to permeability. Pore size analysis was conducted based on the separated interparticle pores. Figure 8 shows the pore size distribution based on pore areas for samples S16 and S18. In Figure 8, x-axis is the equivalent pore diameter, and y-axis is the incremental or cumulative area fraction. Effective porosity is calculated based on the segmented interparticle pore areas. Comparison of S16 and S18 reveals that S16 has larger pore size and effective porosity than S18. The mean pore diameter (D50) is 260 and 126 μm for S16 and S18, respectively. The effective porosity is 12.6% and 7.9%. Both larger pore size and higher porosity will produce higher permeability. This is consistent with the comparison of the measured permeability for sample S16 and S18, which is 1.98 D and 118 mD, respectively.

Figure 6A shows a fossiliferous/peloid grainstone with minor pore-filling cement, large interparticle pore spaces, and irregularly shaped and sized intraparticle pores. Size of the thin section is approximately 5.8 mm × 4 mm. The intraparticle pores are poorly connected through limited and narrow pore throats and therefore do not contribute significantly to permeability. Although ignoring these types of pores may cause a slight underestimation of permeability, inclusion of them can result in significant overestimation. Therefore, pores within grains are removed and excluded from effective porosity (Figure 6B).

Figure 7A is a thin-section (4X) of sample SN51 as an example of grain-dominated packstone with well-defined rounded peloids, having an estimated average pore size of 20 to 60 μm, and creating a network of visible but small interparticle pores and pore-throats. Most of the pores in SN51 are interpreted as peloid-related interparticle pores and thus contribute to effective porosity and permeability. Red areas in Figures 7B illustrate the interparticle pores that are used for 2D permeability (K2D) simulation for sample SN51.

5.2 Permeability Estimation and Discussion on Sources of Error

Pore segmentation, pore size, and effective porosity analysis based on thin section were conducted on all the 24 samples. Two-dimensional permeability simulation was then conducted based on the effective pores to obtain K2D. Subsequently, Equations 6a-b were used to convert K2D to K3D, the final estimate of the permeability of the samples. With 2D thin-section as the only input, hydraulic tortuosity is estimated using Archie's law based on Equations 8c. Effective porosity from thin section image analysis is used as ϕ , and the m value of 2.0 is used in Equation 8c. The constriction factor is dependent on pore throat/pore body ratio (Equation 6b). With a 3D pore network, pore throat/pore body ratio can be obtained explicitly through pore throat analysis as described in Section 4.2. However, with thin sections as the only input, the pore throat/pore

body ratio needs to be determined from 2D pore analysis. A thin section is a 2D slice of a 3D rock volume; pores observed in a thin section include not only pore bodies, but also pore throats. Here we use D_{10}/D_{60} as an approximation of the mean pore throat/pore body size ratio after preliminary screening of other pore diameter ratios such as D_{10}/D_{75} and D_{20}/D_{80} . Using Sample S16 as an example, D_{10}/D_{60} from thin section of S16 is 0.299 (Figure 8A), which gives the closest value to the mean pore throat/pore body ratio (0.3) from 3D pore network analysis in Section 4.2. With D_{10}/D_{60} as the value of ρ , constriction factor can be calculated based on Equation 6b. Therefore, with the constriction factor and hydraulic tortuosity obtained through thin-section image analysis and with K_{2D} from thin section permeability simulation, K_{3D} can be estimated with Equation 6a.

Table 2 includes the results of predicted permeability for all the 24 samples. Figure 9 shows the comparison of the predicted and measured permeability. The solid line is the 1:1 line. The two dash lines indicate errors of a factor of five in either direction. Among the total of 24 estimations, 22 estimated values are within a factor of ± 5 from the measured values, which accounts for about 92% of the total estimated permeabilities, and about 44% are within a factor of ± 2 . Two data points are out of the range of the factor of ± 5 . Specifically, permeability was overestimated by a factor of 62 (estimated K /measured K) for sample L4, and by a factor of 32 for sample R82.

There are several potential sources for such high error. First, as mentioned earlier, representativeness of the thin section relative to the 1-inch plug that is used for laboratory measurement is the key for good estimation. In other words, representativeness is also the large source of error if the representativeness is problematic (Lucia et al., 2001). Figure 10A shows the thin section of sample L4. Although the vuggy pores are carefully identified and excluded from

the effective pores and porosity as shown in the result of pore segmentation (Figure 10B), the permeability is still overestimated by a factor of 62, the largest error among the 24 samples. Other sources of error may contribute, as will be discussed in more detail later; however, the ultra-high error is more likely caused by the representativeness of the thin section. It is possible that the thin section obtained for this sample is not representative for the plug sample that was used in laboratory measurement in the aspect of pore size and pore size distribution. Specifically, the thin section may have extra larger pores than the sample used for laboratory measurement. The D10/D60 from the thin section is 0.33. This may overestimate the real pore throat/pore body ratio. However, it is difficult to determine the representativeness independently with only one thin section image; therefore, careful examination of the thin section before selecting a representative thin section image is critical. When one representative thin section image cannot be selected, a few typical thin section images should be used, and the average permeability should be used as a better estimate for such sample. Thin sections should always be obtained from the same sample that is used for laboratory measurement or from a location as close as possible to minimize the representativeness-related error.

The second source of high error is the determination of effective pores. As mentioned earlier, effective pores refer to interparticle pores that are well connected in carbonates. Figure 10C shows the thin section for sample R82. The predicted permeability based on this thin section is 3.2 mD, which is 32 times larger than the measured value of 0.1 mD. Another thin section was obtained for this sample (not shown). Distribution of the grains and pores in the two thin sections are basically similar, which indicates the good representativeness of the thin sections. Therefore, a more likely cause for such high error factor of 32 is that the pores identified as interparticle pores (red areas in Figure 10D) are not dominating the flow; instead, they are isolated by the

cement that make them poorly-connected and thus act as “vuggy” pores. The high degree of cementation visible in the thin section may have cut pores off from each other. Fluorescence microscopy revealed the presence of abundant micropores existing mainly within concentric ooids and well-rounded peloids (image not shown). Therefore, we interpret the pore network of this sample is dominated by the intraparticle micropores and to a much lesser extent, by scattered and poorly-connected, cemented interparticle pores. Therefore, estimation based on the interparticle pores for this sample did not generate correct permeability. For such type of samples with high degree of cementation, MICP pore throat size distribution should be measured to help to determine the dominating pore throat size. If the dominating pore throat size is much smaller than that of the observed interparticle pores in thin section, this method will overestimate the permeability and thus is not reliable.

Sample SN51 has the smallest measured permeability (0.12 mD) among the 24 samples. The mean diameter for sample SN51 is approximately 40 μm , and the smallest pores resolvable for SN51 are approximately 10 μm , which are regarded as the size of pore throats. Further enhancement of magnification of the thin section does not improve the ability significantly to resolve pores smaller than 10 μm . For samples with dominant pore throat size less than 10 μm , thin section images (under optical microscopy) is not the best method for pore characterization and the method based on thin section will not generate reliable permeability estimation. For this reason, the method in this paper will not work for mud-dominated carbonates, for which it is the micropores, generally <10 μm , that dominate in flow. Instead, scanning electron microscopy (SEM) images on polished thin sections or ion-milled samples should be used for such samples. Future work is needed for samples with permeability smaller than 0.1 mD or with dominant pore throats smaller than 10 μm using SEM images.

The representativeness and correct determination of effective pores are the two most important sources of error that will significantly influence the permeability estimation. Samples L4 and R82, with each of these two conditions unsatisfied, produced the largest errors of estimation among the 24 samples. The errors for the other samples are within a factor of ± 5 . This error can be considered as the systematic error of the methodology proposed in this study.

The systematic error is mainly related to the determination of effective porosity, D_{10}/D_{60} , and the m value. For samples in which cementation is not significant and interparticle pores dominate the flow, determination of effective porosity is also a source of systematic error. There are two possible sources of error in effective porosity determination. The first is related to the determination of interparticle vs. vuggy pores. As discussed earlier, vuggy pores include moldic pores, intragrain micropores, and pores that are substantially larger than the particle size. Determination of vuggy pores is evident sometimes, while sometimes it is subjective and thus there is uncertainty. Careful and correct interpretation of the pores is important to minimize the uncertainty. This uncertainty, on the other hand, is minor for grain-dominated carbonates. The second source of error in effective porosity determination is related to pore size. With images of thin sections obtained with optical microscopy, pores smaller than $\sim 10 \mu\text{m}$ is difficult to resolve. For samples with dominant pore system in which pore diameter is 50-100 μm or larger, smaller than 10 μm pores are either intraparticle pores or small interparticle pores, both of which do not significantly contribute to permeability. Ignoring such small pores thus will not produce large errors. However, for samples with smaller dominating pores ($< 50 \mu\text{m}$), the inability to resolve pores smaller than 10 μm that are part of the dominant pore network will lead to underestimated permeability. The extent of underestimation on permeability is proportional to the extent of underestimating of effective porosity according to Equations 6a and 8c. However, quantifying

the actual effective porosity is not straightforward – the laboratory measurement generally produces total porosity that includes vuggy pores. Measured permeability and total porosity relationship was plotted in Figure 11 (orange data points). A weak correlation between measured permeability and total porosity can be observed, with the low R^2 value of 0.40. In contrast, a better correlation is shown for the relationship between the measured permeability and thin-section-based effective porosity, with the R^2 value of 0.67 (green data points in Figure 11). This indicates that the effective porosity is more relevant to permeability than the total porosity. It also suggests that the effective porosity determined from thin section image analysis for the samples studied here can be considered as reasonable. It is worth noting that even with effective porosity, the R^2 value (0.67) of the fitting is still not high, indicating the importance of other pore parameters such as pore size and pore or pore throat size distribution.

The other source of error is the determination of D10/D60. Not incorporating the smaller pores into the estimate will generate larger D10. Although it will result in larger D60 as well, the increase of D60 is generally smaller than that for D10. This is caused by the shape of the cumulative pore fraction. Slope of the cumulative fraction is commonly steeper at small pore sizes than at large pore sizes (see Figure 8 as examples). With sample S41 as an example, D60 for S41 is 55 μm , D50 is 40 μm , and D10 is 20 μm . If the interparticle pores smaller than 10 μm account for 10% of the actual effective porosity, then the actual D10 is <10 μm , and the actual D60 is the observed D50, which is 40 μm . The observed D10/D60 is 0.36, whereas the actual D10/D60 is smaller than 0.25. Therefore, D10/D60 is overestimated without taking the small interparticle pores into account. Consequently, the constriction fraction f will be overestimated, and the permeability will be overestimated according to Equation 6a. On the other hand, even if the D10/D60 is overestimated, it is not necessary that the actual pore throat/pore body ratio is

overestimated because the equality of D_{10}/D_{60} and pore throat/pore body ratio is only verified for one sample (S16). Using the thin-section-based D_{10}/D_{60} may either overestimate the actual pore throat/pore body ratio and thus overestimate the permeability, or underestimate the actual pore throat/pore body ratio and permeability. Nevertheless, the D_{10}/D_{60} values obtained from the thin section analysis have a range of 0.2-0.5, which are reasonable values for pore throat/pore body ratio. To evaluate the worst case scenario, if the sample with D_{10}/D_{60} of 0.2 has the actual mean pore throat/pore body ratio of 0.1, according to Equations 6a, b, the overestimation factor (predicted K /actual K) is 7.4. If the sample with D_{10}/D_{60} of 0.5 has the actual mean pore throat/pore body ratio of 0.8, the underestimation factor is 0.6.

Moreover, there is uncertainty in m value in Equation 8c. The m value of 2.0 is used for estimation; however, as discussed earlier in Section 4.2, a universal value of 2.0 may not be the case for different carbonate rocks. Based on Equation 8c, higher m value will generate larger tortuosity and thus lower permeability, lower m value will lead to higher permeability. This possible uncertainty of m value will add the uncertainty to the estimation. Nevertheless, using m value of 2.0 generated fairly reasonable estimations for the 24 carbonate samples. This can be caused by the use of effective porosity instead of the total porosity in the calculation of tortuosity (Equation 8c). Use of effective porosity essentially has the effect of simplifying the complex pore structure of the carbonates and extracting the dominating pore network. According to discussions in Section 4.2, when each pore plays equal important role in connection and no dead-end pores, the m value approaches to 2.0. Laboratory measurement of formation factor may provide a way for more accurate determination of m ; if doing so, it is worth to note that effective porosity, instead of total porosity, should be used to calculate m in order to apply the method in this study.

Overall, the combining effect of underestimated effective porosity, over- or underestimated pore throat/pore body ratio, and the uncertainty with m resulted in either over- or underestimation of the permeability, with an error factor of ± 5 as shown in Figure 9. The error factor of ± 5 is considered as reasonable for the estimation based on 2D thin section by considering the sources of error for the carbonate samples.

6. Conclusions

The relationship between 2D and 3D permeability is evaluated mathematically based on Kozeny-Carman Equation and a constriction factor. The K_{2D}/K_{3D} ratio is proportional to tortuosity square and inversely to constriction factor. This relationship was examined and validated with a grainstone sample, for which both K_{2D} and K_{3D} were obtained through permeability simulation based on the micro-CT images. The mathematical expression of K_{2D}/K_{3D} provides a semi-theoretical basis for the estimation of permeability based on 2D image analysis and modeling using thin sections.

The results of estimated permeability for the 24 grain-dominated carbonate samples are summarized in Table 2 and Figure 9. Among the 24 samples, 22 of them (92%) has predicted permeability within a factor of ± 5 of the measured value, 44% within a factor of ± 2 . Representativeness of the thin sections and correct determination of the effective pores are the two prerequisites for reliable permeability estimation. For samples with high degree of cementation, interparticle pores may not dominate in flow and thus should not be considered as effective pores. For samples in which interparticle pores dominate in flow, determination of effective porosity is also a source of systematic error. Determination of pore throat/pore body

ratio and using an m value of 2.0 are the other two sources of systematic error. Overall, the method developed in this study works reasonably well for grain-dominated carbonates, with the systematic error of the factor of ± 5 . This method is built on a relatively easy and fast 2D modeling. In the same time, the 2D modeling is based on thin section images, which inherently include the information of the complex pore geometry and pore size distribution of the carbonate samples. Therefore, the method developed in this study provides a quick yet reliable means to estimate permeability for grain-dominated carbonate samples that still have large differences in rock texture and fabric.

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References

624 Al-Bazzaz, W.H., Y.W. Al-Mehanna, 2007. Porosity, permeability, and MHR calculations using
625 SEM and thin-section images for characterizing complex Maaddud-Burgan carbonate reservoir.
626 SPE 110730.

627 Al-Kharusi, A.S., M.J. Blunt, 2008. Multiphase flow predictions from carbonate pore space
628 images using extracted network models. *Water Resources Research*, 44, W06S01, doi:
629 10.1029/2006WR005695.

630 Anselmetti, F.S., S. Luthi, and G.P. Eberli, 1998. Quantitative Characterization of Carbonate
631 Pore Systems by Digital Image Analysis. *AAPG Bulletin*, 82, 1815–1836.

632 Archie, G.E., 1942. The electrical resistivity log as an aid in determining some reservoir
633 characteristics. *Petroleum Transactions of AIME* 146: 54–62. doi:10.2118/942054.

634 Ben Clennell, 1997. *Tortuosity: a guide through the maze*. Geological Society, London, Special
635 Publications, 122: 299-344.

636 Berryman and Blair, 1986. Use of digital image analysis to estimate fluid permeability of porous
637 materials I. Application of two-point correlation functions. *J. Appl. Phys.* 60, 1930-1938.

638 Cancelliere, A., C. Chang, E. Foti, D.H. Rothman, S. Succi, 1990. The permeability of a random
639 medium: comparison of simulation with theory. *Physics of Fluids*, A2, 2085-2087.

640 Carman, P.C., 1956. *Flow of gases through porous media*. Butterworth, London.

641 Coskun, S.B., and N.C. Wardlaw, 1993. Estimation of permeability from image analysis of
642 reservoir sandstones. *Journal of Petroleum Science and Engineering*, 10, 1-16.

643 Dashtian, H., Y. Yang, and M. Sahimi, 2015. Nonuniversality of the Archie exponent due to
 644 multifractality of resistivity well logs. *Geophysical Research Letters*, doi:
 645 10.1002/2015GL066400.

646 De Boever, E., C. Varloteaux, F. Nader, A. Foubert, S. Bekri, S. Youssef, E. Rosenberg, 2012.
 647 Quantification and Prediction of the 3D Pore Network Evolution in Carbonate Reservoir Rocks.
 648 *Oil & Gas Science and Technology*, 67, 161-178.

649 Dullien, F.A.L., 1992. *Porous media: Fluid transport and pore structure*. Second edition.
 650 Academic Press, San Diego.

651 Focke, J. W., and Munn, D., 1987. Cementation exponents in middle-eastern carbonate
 652 reservoirs: *SPE Formation Evaluation*, 2, 155–167.

653 Jurgawczynski, M., R.W. Zimmerman, X.D. Jing, 2004. Estimating the permeability of
 654 carbonate rocks using image analysis and effective medium theory. SCA2004-37. Society of
 655 Core Analysts held in Abu Dhabi, UAE, 5-9 October, 2004.

656 Keehm, Y, T. Mukerji, A. Nur, 2004. Permeability prediction from thin sections: 3D
 657 reconstruction and Lattice-Boltzmann flow simulation. *Geophysical Research Letters*, 31,
 658 L04606, doi: 10.1029/2003GL018761.

659 Kirkpatrick, S., 1973. Percolation and conduction, *Rev. Mod. Phys.* 45, 574–588.

660 Knackstedt, M.A., R.M. Sok, A.P. Sheppard, S.J. Latham, M. Madadi, T. Varslot, C.H. Arns, G.
 661 Bachle, G. Eberli, 2008. Probing pore systems in carbonates: correlations to petrophysical
 662 properties. SPWLA 49th Annual Logging Symposium, May 25-28, 2008.

663 Lindquist, W.B., 1999. *3DMA General Users Manual*. New York, Stony Brook AMS Preprints.

664 Lock, P.A., X. Jing, R.W. Zimmerman, E.M. Schlueter, 2002. Predicting the permeability of
 665 sandstone from image analysis of pore structure. *Journal of Applied Physics*, 92, 6311, doi:
 666 10.1063/1.1516271.

667 Lucia, F.J., 1983, Petrophysical parameters estimated from visual descriptions of carbonate
 668 rocks: A field classification of carbonate pore space: *Journal of Petroleum Technology*, 35, 629–
 669 637.

670 Lucia, F. J., 1995, Rock-fabric/petrophysical classification of carbonate pore space for reservoir
 671 characterization: *American Association of Petroleum Geologists Bulletin*, v. 79, no. 9, p. 1275–
 672 1300.

673 Lucia, F. J., Jennings, J. W., Jr., Rahnis, M. A., and Meyer, F. O., 2001, Permeability and rock
 674 fabric from wireline logs, Arab-D reservoir, Ghawar field, Saudi Arabia: *GeoArabia*, v. 6, no. 4,
 675 p. 619–646.

676 Madden, T.R., 1976. Random networks and mixing laws. *Geophysics*, 41, 1104-1125.

677 Mostaghimi, P., M.J. Blunt, B. Bijeljic, 2012. Computations of absolute permeability on micro-
 678 CT images. *Math. Geosci.*, doi: 10.1007/s11004-012-9431-4.

679 Peng, S., F. Marone, S. Dultz, 2014. Resolution effect in X-ray microcomputed tomography
 680 imaging and small pore's contribution to permeability for a Berea sandstone. *Journal of*
 681 *Hydrology*, 510, 403-411.

682 Peng, S., J. Yang, X. Xiao, B. Loucks, S. Ruppel, T. Zhang, 2015. An integrated method for
 683 upscaling pore-network characterization and permeability estimation: Example from the
 684 Mississippian Barnett Shale. *Transport in Porous Media*. 109 (2), 359-376.

685 Schaap, M.G., and I. Lebron, 2001. Using microscope observations of thin sections to estimate
686 soil permeability with the Kozeny-Carman equation. *Journal of Hydrology*, 251, 186-201.

687 Van Brakel, J., 1975. Pore space models for transport phenomena in porous media. Review and
688 evaluation with special emphasis on capillary liquid transport. *Powder Technology*, 11, 205-236.

689 Weger, R.J., G.P. Eberli, G.T. Baechle, J.L. Massafferro, and Y-F Sun, 2009. Quantification of
690 pore structure and its effect on sonic velocity and permeability in carbonates. *AAPG Bulletin*,
691 93, 1297–1317.

692 Witt, K.-J., and J. Braus, 1983. Permeability anisotropy due to particle shape. *Journal of*
693 *Geotechnical Engineering*, 109, 1181-1187.

694 Wong, H.S., R.W. Zimmerman, N.R. Buenfeld, 2012. Estimating the permeability of cement
695 pastes and mortars using image analysis and effective medium theory. *Cement and Concrete*
696 *Research*, 42, 476-483.

697 Wu, K., A. Ryazanov, M. I. J. van Dijke, Z. Jiang, J. Ma, G. Couples, and K. Sorbie, 2008,
698 Validation of methods for multi-scale pore space reconstruction and their use in prediction of
699 flow properties in carbonate: Society of Core Analysts, Abu Dhabi, United Arab Emirates,
700 October 5–9.

701 Zakirov, T.R., A.A. Galeev, E.A. Korolev and E.O. Statsenko, 2016. Flow properties of
702 sandstone and carbonate rocks by X-ray computed tomography. *Current Science*, 110, 2142-
703 2148.

704 Zhang, S., Klimentidis, R.E., Barthelemy, P.:Micron to millimeter upscaling of shale rock
705 properties based on 3D imaging and modeling. In: International Symposium of the Society of
706 Core Analysts, 27–30 August. Aberdeen, Scotland (2012).

707 Table 1. Sample location and description.

Sample	Sample location	Sample description
S11	Lower Cretaceous Thamama Group, Abu Dhabi Fields	Peloidal grain-dominated packstone, dominant tangential grain-to-grain contacts, small interparticle pores
S14		Peloidal ooid grainstone, minor amount of pore-filling cementations
S15		Peloidal ooid grainstone, minor amount of pore-filling cementations
S16		Skeletal ooid grainstone, minor amount of pore-filling cementations
S17		Skeletal ooid grainstone, minor amount of pore-filling cementations
S18		Peloidal ooid grainstone, frequent interpenetrating grain-to-grain contacts
S20		Foraminifera peloidal grain-dominated packstone, abundant equant blocky calcite cementation
S30		Skeletal peloidal grain-dominated packstone, small interparticle pores
S38		Skeletal peloidal grain-dominated packstone, small interparticle pores, including small recrystallization pores
SN12		Coated-grain ooid grainstone, minor amount of pore-filling cementations
SN15		Skeletal ooid grainstone, minor amount of pore-filling cementations
SN21		<i>Bacinella</i> intraclastic rudstone
SN35		<i>Bacinella</i> intraclastic rudstone
SN51		Skeletal peloidal grain-dominated packstone, small interparticle pores
R20	Lower Cretaceous Sligo Ooid-Shoal Complex, South- Central Texas	Skeletal coated-grain grainstone, with blocky calcite cementation
R24		Coated-grain grainstone, isopacheous bladed calcite rim-cement is abundant
R82		Ooid grainstone, with equant blocky calcite cement crystals infilling the majority of interparticle pores
L1	Ghawar Field, Saudi Arabia	Fossil grainstone with intergrain and separate-vug pore space
L2	West Texas	Medium crystalline dolostone with small intercrystal pore space
L3	West Texas	Oomoldic grainstone with cemented intergrain pores and moldic pore space
L4	Seminole Field West Texas	Grain-dominated dolopackstone with intergrain pore space
L5	Ghawar Field Saudi Arabia	Large crystalline dolostone with large intercrystal pores
L10	Sacroc Field West Texas	Ooid grainstone with intergrain and intragrain pore space
L15	Wasson Clear Fork field, West Texas	Medium crystalline grain-dominated dolopackstone with intergrain pore space

708

709 Table 2. Porosity and permeability based on thin section and laboratory measurements.

Sample	Measured porosity (%)	Measured permeability (mD)	Thin-section-based porosity (effective porosity) (%)	Estimated permeability (mD)
S11	23.4	7.6	5.4	12.1
S14	21.7	1615.3	11.6	1774.5
S15	21.6	2117.6	16.6	4386.6
S16	20.9	1988.7	11.6	2183.2
S17	17.8	1122.0	7.5	852.6
S18	14.3	117.2	4.8	310.3
S20	12.6	2.5	3.9	11.2
S30	22.5	11.8	7.0	4.7
S38	9.5	0.4	1.2	0.5
SN12	16.9	717.0	11.1	1880.9
SN15	17.7	2620.7	9.2	1450.0
SN21	14.6	403.8	10.9	964.4
SN35	22.5	686.5	8.3	374.5
SN51	8.6	0.1	0.7	0.1
R20	16.5	10.0	8.0	49.0
R24	17.1	8.8	10.8	40.2
R82	10.9	0.1	1.7	3.2
L1	20.0	121.0	8.7	443.5
L2	16.0	30.0	5.0	13.6
L3	25.0	5.0	1.7	23.4
L4	9.0	7.3	6.3	451.7
L5	24.0	3200.0	13.0	3335.3
L10	16.1	5.5	1.0	3.4
L15	14.2	35.0	4.5	46.5

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Figure 1. Micro-CT reconstruction of sample S16 and image analysis. (A) Digital rock image of mini-plug of S16; (B) Pore segmentation. Blue shaded areas are interparticle pores, red dotted areas are intraparticle pores; (C) Interparticle and intraparticle pore

network. Yellow denotes interparticle pores, and red denotes intraparticle pores. Intraparticle pore network in the white circle was used for micropore permeability simulation. (D) Pore throat size distribution from MICP. Bimodal distribution is observed for sample S16. Right peak corresponds to interparticle pores, left peak to intraparticle pores.

Figure 2. Interparticle pore network 3D and 2D for sample S16. (A) 3D interparticle pore network; (B) Selected 2D slices along the depth having equidistance. Blue arrow denotes the direction of permeability simulation; (C) 2D porosity and permeability along the depth within the core plug. Blue data points are porosity, orange data points are permeability.

Figure 3. (A) Shortest paths; and (B) geometric tortuosity distribution for pore network of sample S16 along the vertical direction.

Figure 4. Pore throat analysis for sample S16. (A) Media axis of the pore network. The rainbow color scale represents the distance of the path to the closest grain voxels, with red denoting short distance and blue long distance. Blue transparent surface illustrates the pore surfaces; (B) Pore throat and pore body sizes; (C) Pore throat/pore body ratio distribution. Blue data points are normalized incremental pore throat/pore body ratio, and orange is cumulative pore throat/pore body ratio fraction.

Figure 5. Grainstone samples with well-defined interparticle pores and minor cementation and relatively homogeneous pore distribution. (A) and (C) are thin sections for samples S16 and S18, respectively. (B) and (D) are results of pore segmentation, with red areas being interparticle pores. Measured permeability for S16 and S18 is 2.0 D and 117.2 mD, respectively.

Figure 6. Thin section for L1 a fossil grainstone sample (A) and pore segmentation results for sample L1 (B). Red areas being interparticle pores. Measure permeability for L1 is 121.0 mD.

Figure 7. Grain-dominated packstone. (A) Thin section for sample SN51, and (B) the pore segmentation result. Red areas are interparticle pores. Measured permeability for SN51 is 0.1 mD.

Figure 8. Pore size distribution from thin section analysis for samples S16 (A) and S18 (B)

Figure 9. Comparison between predicted and measured permeability for the 24 samples. Solid line is the 1:1 line. The upper and lower dashed lines correspond to errors of a factor of 5 in either direction.

Figure 10. Thin sections for samples L4 (A) and R82 (C), and corresponding pore segmentation results (B) and (D). Measured permeability for L4 and R82 is 7.3 and 0.1 mD, respectively.

Figure 11. Permeability-and-porosity relationship with porosity being thin-section-based effective porosity (green data) and laboratory measured total porosity (orange data). Solid curves are power law equation fittings. R^2 is 0.67 and 0.40 for the green and orange data points, respectively.