A modified Lou-Huh model for characterization of ductile fracture of DP590 sheet

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Abstract

In this paper, a modified ductile fracture criterion was established to predict the fracture behavior of DP590 sheet based on Lou-Huh model. Considering the change of loading path during the plastic forming process, the C-H model was introduced to interpret the whole loading history. According to C-H model, the plastic loading process is divided into three stages including homogeneous deformation, deformation localization and localized instability. In addition, the variations of stress triaxiality and Lode parameter were tracked by the corresponding stress ratios under three tensile deformation patterns including uniaxial tension, plane strain tension and equi-biaxial tension. Based on the optimization algorithm, the material constants of the original and modified Lou-Huh fracture criteria were determined and their theoretical fracture limit strains were compared with the experimental one. Furthermore, the morphology of fracture surfaces was also taken to characterize the ductile fracture behavior of DP590 sheet. Results showed that the modified Lou-Huh model is more suitable for the prediction of ductile fracture of DP590 sheet, and the growth and shear coalescence of microvoids are correlated with stress triaxiality and Lode parameter, which change with the development of plastic deformation.

Keywords: High strength steel, Fracture limit, C-H model, Ductile fracture criterion, Loading path.

1. Introduction

With the advancement of automobile industry, especially the trend towards energy saving, environment protection and enhanced crash performance for the vehicles, the ubiquitous use of advanced high strength steel (AHSS) is facilitated due to its high strength and good formability. However, compared with other conventional steel sheets, the application of AHSS is limited by its complex fracture behavior[1]. The sheet metal formability is usually assessed via the forming limit diagram (FLD), which is an important performance indicator used to predict localized necking in the field of sheet forming. The FLD can be determined by corresponding plastic instability theory based on the onset of through-thickness necking of sheet. Because the fracture of AHSS often occurs without obvious necking phenomenon, the conventional plastic instability theories are no longer suitable for the failure prediction. Thus many researchers [2-4] employed the fracture limit diagram (FFLD) to describe the deformation achievable of sheet metal controlled by fracture, which can be obtained by the ductile fracture criteria.

In the process of sheet metal forming, the plastic deformation stage which occurs after through-thickness necking should be considered in the study of ductile damage and fracture. In addition, the fracture phenomenon of some materials may take place prior to failure caused by localized necking. To consider the above two cases, a suitable ductile fracture criterion (DFC) is vital for the precision prediction of the failure pattern of AHSS sheet. From the micro-level viewpoint, ductile fracture of metal is the combined results of nucleation, growth and coalescence of microscopic voids[5]. On the one hand, some damage models coupled with the constitutive equation are used to describe both plasticity and ductile fracture of porous materials based on the damage accumulation mechanism. For example, the GTN model was proposed and developed by Gurson[6], Tvergaard and Needleman[7], which is a porous plasticity model in terms of the micromechanical analysis of a thick spherical shell subjected to hydrostatic pressure. Recently, the inaccuracy of GTN model for the shear-dominated stress state has been validated by Xue[8], Nielsen and

Tvergaard[9]. Another coupled ductile fracture criterion is dependent on the continuum damage mechanics (CDM), which is proposed by Lemaitre[10]. On the other hand, phenomenological ductile fracture criteria are uncoupled with the plasticity formulation, which are particularly popular in engineering practice because of their simplicity and less material constants to be calibrated. The fracture is assumed to occur when the material critical damage value is reached. At present, the uncoupled fracture criteria are also used to predict the ductile fracture and deformation behavior in progressive microforming[11].

The early work of phenomenological ductile fracture criteria focused on the effect of stress triaxiality, which is expressed as the ratio of the hydrostatic stress to von Mises equivalent stress, on the ductile fracture of the used material. Many investigators found that the material ductility usually decreases with the increase of stress triaxiality. The work of Hancock and Mackenzie revealed that the ductile fracture clearly depends on the triaxiality of stress state via performing a series of tensile tests[12]. Johnson and Cook carried out a great deal of experimental studies on various materials and introduced a cumulative-damage fracture model, which works well over the positive range of the stress triaxiality values[13]. Mirza et al. [14] implemented the notched tensile tests and numerical simulations on three different materials, and discovered that the ductilities of all the three materials are strongly rooted in the level of stress triaxiality. Subsequently, several recent researches indicated that Lode parameter, which is associated with the third invariant of deviator stress tensor, also plays a crucial role in ductile damage and fracture, especially in low stress triaxiality scenario. Bai and Wierzbicki[15] modified the Mohr-Coulomb model and defined the fracture strain as the function of stress triaxiality and Lode parameter in plane stress condition. The modified criterion could predict all the experimental points well. According to the investigations of Danas [16] and Brünig[17], the stress state obviously affects the fracture behavior and damage mechanism of the used material. Like the stress triaxiality, Lode parameter is also an important factor to control the initiation and evolution of ductile damage and fracture. Based on the interactive effect of stress triaxiality and Lode parameter, Lou et al. [18] proposed a micro-mechanism-motivated macroscopic

ductile fracture criterion considering the combination of stress triaxiality and Lode parameter to describe the ductile fracture behavior of sheet metal.

The determination of material constants of ductile fracture criterion depends on the corresponding magnitudes of stress triaxiality and Lode parameter, which are usually obtained by a combined experimental-numerical method. Currently, the average values, initial values or final values of the two parameters during the plastic deformation process up to the onset of ductile fracture were adopted by many scholars [2, 19, 20]. However, the fracture initiation is an evolutionary process, the stress state changes dramatically since all the prior loading paths become plane strain state ($d\varepsilon_2=0$) after the occurrence of diffuse necking [21]. Based on Marciniak-Kuczynski (M-K) model [22], Ding [23] and Friedman [24] demonstrated that the loading path in the groove steadily develops from a certain moment to the state of plane strain. Qian [25] and Li [26] proved that stress triaxiality and Lode parameter not only change with the equivalent plastic strain but also have a close relationship with the distribution of plastic strain. In the study of Park [4], the effect of the change of loading path was taken into account in the calibration of material constants of ductile fracture criterion. Due to the dependence of stress triaxiality and Lode parameter on the loading history, the construction of fracture limit diagram based on the invariable parameters was thus inappropriate and inaccurate.

In this research, the fracture behavior of DP590 sheet was characterized, and a modified Lou-Huh model was proposed to predict its fracture limit strains considering the dependence of stress triaxiality and Lode parameter on the loading history. The C-H model [27] was applied to describe the variation of stress ratio for the three typical loading cases including uniaxial tension, plane strain tension and equi-biaxial tension. Combining with the optimization algorithm, the material constants of original and modified Lou-Huh model were calculated. The corresponding fracture locus of DP590 sheet were constructed and compared with the experimental one. Meanwhile, the characterization of ductile fracture of DP590 sheet was investigated and discussed via SEM images of fracture surfaces.

2. Material and methodology

The material adopted in the research was dual-phase steel DP590 with thickness of 1.5 mm, which was produced by BaoSteel Inc. The material has the typical DP structure consisting of ferrite and martensite islands. The chemical composition of DP590 sheet (weight percent) were summarized in **Table 1**. To get the mechanical properties of DP590 sheet, the uniaxial tensile tests were performed on the Instron tensile test machine. The material properties were listed in **Table 2**. **Fig. 1** showed the true stress-strain curves from tensile tests along 0°, 45° and 90° to the rolling direction. From **Fig. 1**, it could be observed that DP590 sheet exhibits slight difference of mechanical properties in three testing directions. For simplicity, the planar isotropy of DP590 sheet is assumed, the average normal anisotropic coefficient r -value equals to 0.922.

The research procedure was summarized in **Fig. 2**. The Lou-Huh ductile fracture criterion was modified by incorporating Hill'48 yield function and C-H model which can describe the variation of loading path. Based on three typical tensile tests and optimization algorithm, the material constants in the original and modified Lou-Huh ductile fracture criteria were calculated and their corresponding FFLD were compared with the experimental result.

2.1 C-H model

2.1.1 General description

The loading path instability theory was originally proposed by Chen and Hu, called C-H model. Unlike M-K model which considers that the localized instability is caused by the surface defects of sheet metal, C-H model argues that the occurrence of localized instability of sheet metal is a gradual process and the evolution of internal damage promotes the transformation of loading path from stress states between uniaxial tension and equi-biaxial tension to plane strain state. According to C-H model, plastic deformation process is decomposed into three phases as schematically shown in **Fig. 3**.

Phase I: Homogeneous deformation up to load instability (*ab* Section);

Phase II: Deformation localization with the transformation of loading path under constant load (*bc* Section);

Phase III: Localized instability or macroscopic crack appearing with the sudden drop of load.

The ability of this model to predict FFLD has been discussed by several researchers. For example, Chen and Hu [28] employed C-H model to calculate the fracture limit strain of aluminum alloy based on Hosford yield criterion. According to Xie's research [29], the fracture limit diagram determined by C-H model was compared with that predicted by M-K model, Swift and Hill instability theories. Based on the above studies, it was found that the deviation between the theoretical and experimental FFLDs near plane strain state can not be neglected, which may be attributed to the assumption of the condition of localized instability. In the original C-H model, the occurrence of localized instability ($d\epsilon_2=0$) is taken as a fracture criterion, and the diffuse and localized instabilities are considered to appear simultaneously under plane strain state.

To precisely describe the plastic deformation behavior and ductile fracture characteristic of DP590 sheet under various stress states, C-H model was adopted to reflect the change of loading path before fracture in the research.

2.1.2 Plasticity relationship

Due to the fact that the employed yield function has a significant impact on the prediction of the forming behavior of DP590 sheet, the biaxial tensile tests and cruciform specimens [30] were adopted to obtain the plastic work contour of DP590 sheet, which is considered as yield locus. The experimental yield locus of DP590 sheet and the theoretical yield loci based on Hill'48, Hosford, Barlat'89, Yld2000-2d yield criteria are shown in **Fig. 4**. It is found that Barlat'89 yield criterion has the closest agreement with the experimental one. Although its accuracy for describing yield locus is slightly lower than Barlat'89 and Yld2000-2d yield functions, Hill'48 yield criterion was employed in this study for the purpose of simplifying calculation.

Under plane stress state ($\sigma_3=0$), Hill'48 yield function and its corresponding equivalent plastic strain could be written as Eqs. (1) and (2).

$$\sigma_1^2 - \frac{2r}{1+r}\sigma_1\sigma_2 + \sigma_2^2 = \sigma_e^2 \quad (1)$$

$$\varepsilon_e = \frac{1+r}{\sqrt{1+2r}}(\varepsilon_1^2 + \frac{2r}{1+r}\varepsilon_1\varepsilon_2 + \varepsilon_2^2)^{1/2} \quad (2)$$

where σ_e represents the yield stress; r is the average normal anisotropic coefficient.

According to Eq.(1), the following equation is obtained,

$$\varphi = \frac{\sigma_e}{\sigma_1} = \sqrt{1 - \frac{2r}{1+r}\alpha + \alpha^2} \quad (3)$$

where α is the loading path defined as the ratio of the minor principal stress σ_2 to the major principal stress σ_1 under the plane stress condition.

Based on the hypothesis of the associated flow rule, the relationship between the plastic strain increments is described as,

$$\frac{d\varepsilon_1}{\varphi_1} = \frac{d\varepsilon_2}{\varphi_2} = \frac{d\varepsilon_3}{\varphi_3} = \frac{d\varepsilon_e}{\varphi} \quad (4)$$

where $\varphi_1 = 1 - \frac{r}{1+r}\alpha$; $\varphi_2 = \alpha - \frac{r}{1+r}$; $\varphi_3 = -(\varphi_1 + \varphi_2) = -\frac{1+\alpha}{1+r}$.

2.1.3 Phase I: Homogeneous deformation zone

During this stage, homogeneous deformation is assumed until the load instability occurs. With the proceeding of deformation, the loading path remains linear. Therefore, the strain of this stage can be calculated by the total strain theory of plasticity. The load instability criterion in the direction of the major principal stress is described by Considere-type criterion [31], as the load acting on the sheet cross section attains its maximum ($dP_1=0$), that is formulated as:

$$\frac{d\sigma_1}{\sigma_1} = d\varepsilon_1 \quad (5)$$

Considering the power exponent hardening law,

$$\sigma_e = k\varepsilon_e^n \quad (6)$$

where k and n are the strength coefficient and strain hardening exponent, respectively.

The following equation is achieved by combining Eqs. (5) with (6),

$$d\sigma_e = kn\varepsilon_e^{n-1}d\varepsilon_e = \sigma_e \frac{n}{\varepsilon_e} d\varepsilon_e \quad (7)$$

During the steady loading stage, the material deformation resistance is equal to the stress increment:

$$d\sigma_e = \beta_1 d\sigma_1 + \beta_2 d\sigma_2 \quad (8)$$

$$\text{where } \beta_1 = \frac{\partial \sigma_e}{\partial \sigma_1} = \frac{\varphi_1}{\varphi}, \quad \beta_2 = \frac{\partial \sigma_e}{\partial \sigma_2} = \frac{\varphi_2}{\varphi}.$$

Based on the assumption of homogeneous deformation, the following relationship is obtained:

$$\frac{d\sigma_2}{d\sigma_1} = \frac{\sigma_2}{\sigma_1} = \alpha \quad (9)$$

In terms of the associated flow rule and the total strain theory of plasticity, the ratios of plastic strains are expressed as:

$$\begin{aligned} \frac{\varepsilon_e}{\varepsilon_1} &= \frac{d\varepsilon_e}{d\varepsilon_1} = \frac{\varphi}{\varphi_1} \\ \frac{\varepsilon_2}{\varepsilon_1} &= \frac{d\varepsilon_2}{d\varepsilon_1} = \frac{\varphi_2}{\varphi_1} \end{aligned} \quad (10)$$

Substituting Eqs. (3), (5), (7), (9) and (10) into Eq.(8), the strains of Phase I can be deduced as follows:

$$\begin{aligned} \varepsilon_e &= \frac{\varphi}{\varphi_1} n \\ \varepsilon_1 &= n \\ \varepsilon_2 &= \frac{\varphi_2}{\varphi_1} n \end{aligned} \quad (11)$$

2.1.4 Phase II: Deformation localization zone

After reaching the maximum load, the load in the direction of the major principal stress of Phase II still remains constant. It indicates that Eq.(5) not only characterizes the end of Phase I, but also keeps valid throughout Phase II. Due to the deformation restriction and discontinuity caused by the evolution of damage, the loading path gradually drifts to the plane strain state during Phase II. Consequently, the strains are composed of two parts. The first part is the strain of Phase I and the second one can be calculated by numerical iterative algorithm based on elastic-plastic incremental strain theory. Furthermore, the hypothesis of load constancy during Phase II has been

verified by the experiment performed by Chamanfar and Mahmudi[32]. Combining Eqs. (5), (7) with (8), the following relationship is obtained,

$$d\sigma_2 = \frac{1}{\beta_2} \left(\frac{n}{\varepsilon_e} \sigma_e d\varepsilon_e - \beta_1 \sigma_1 d\varepsilon_1 \right) \quad (12)$$

Thus,

$$\begin{cases} d\sigma_2 = \left(\frac{n}{\varepsilon_e} \varphi - \beta_1 \frac{\varphi_1}{\varphi} \right) \frac{\sigma_1 d\varepsilon_e}{\beta_2} \\ d\sigma_1 = \sigma_1 d\varepsilon_1 = \frac{\varphi_1}{\varphi} \sigma_1 d\varepsilon_e \end{cases} \quad (13)$$

To describe the change of loading path, Eq. (13) is integrated [27]. Considering the simplification of numerical calculation, the equation is rewritten as:

$$\begin{cases} \sigma'_1 = \sigma_1 / \sigma_{1d} \\ \sigma'_2 = \sigma_2 / \sigma_{1d} \end{cases} \quad (14)$$

where σ_{1d} and σ_{2d} are in-plane principal stresses at the starting point of Phase II. Apparently, for the initial state of Phase II:

$$\begin{cases} \sigma'_1 = 1 \\ \sigma'_2 = \alpha_d \end{cases} \quad (15)$$

Rearranging Eq. (13), the following equation is obtained,

$$\begin{cases} d\sigma'_2 = \left(\frac{n}{\varepsilon_e} \varphi - \beta_1 \frac{\varphi_1}{\varphi} \right) \frac{\sigma'_1 d\varepsilon_e}{\beta_2} \\ d\sigma'_1 = \sigma'_1 d\varepsilon_1 = \frac{\varphi_1}{\varphi} \sigma'_1 d\varepsilon_e \end{cases} \quad (16)$$

Therefore, the loading path of the whole Phase II can be calculated by Eqs. (15) and (16). Assuming the iteration step-size ($d\varepsilon_e$) is 0.0001, the following results can be deduced:

$$\left\{ \begin{array}{l}
\varepsilon_e = \varepsilon_{ed} + \sum_{i=1}^{N-1} d\varepsilon_e \\
\sigma_1' = 1 + \sum_{i=1}^{N-1} d\sigma_1' \\
\sigma_2' = \alpha_d + \sum_{i=1}^{N-1} d\sigma_2' \\
\alpha = \frac{\sigma_2'}{\sigma_1'} \\
d\varepsilon_1 = \frac{\varphi_1}{\varphi} d\varepsilon_e \\
d\varepsilon_2 = \frac{\varphi_2}{\varphi} d\varepsilon_e \\
\rho = \frac{d\varepsilon_2}{d\varepsilon_1} \\
\varepsilon_1 = \varepsilon_{1d} + \sum_{i=1}^{N-1} d\varepsilon_1 \\
\varepsilon_2 = \varepsilon_{2d} + \sum_{i=1}^{N-1} d\varepsilon_2
\end{array} \right. \quad (17)$$

The deformation mode of Phase III is described by localized instability or macroscopic crack appearing with a precipitous drop of loading force. The realization of the plane strain state ($d\varepsilon_2=0$) is taken as the condition of the occurrence of localized instability in the original C-H model. However, previous research has shown that the error of the theoretical prediction via C-H model is especially large under plane strain state. Therefore, Phase III describing the onset of ductile fracture in forming process is identified by the modified Lou-Huh ductile fracture criterion in this research.

2.2 The modified Lou-Huh ductile fracture criterion

2.2.1 Application of Hill'48 yield function

To consider the influence of material normal anisotropy on the ductile fracture behavior of DP590 sheet, von Mises yield function is replaced by Hill'48 yield function, which is applied to combine with Lou-Huh ductile fracture criterion. The Lou-Huh model is constructed based on the mechanism of voids nucleation, growth and shear coalescence as expressed in Eq.(18).

$$\left(\frac{2\tau_{\max}}{\sigma_e} \right)^{c_1} \left(\frac{\langle 1+3\eta \rangle}{2} \right)^{c_2} \varepsilon_e^f = C_3 \quad \langle x \rangle = \begin{cases} x & \text{when } x \geq 0 \\ 0 & \text{when } x < 0 \end{cases} \quad (18)$$

where $\tau_{\max}$ is the maximum shear stress, ε_e^f is the equivalent plastic strain at the onset of the final fracture, η is the stress triaxiality, and C_1, C_2, C_3 are the material constants of the ductile fracture criterion, respectively.

Since C-H model is only applicable to the stress states between uniaxial tension and equi-biaxial tension, the following relationships are satisfied,

$$\sigma_1 \geq \sigma_2 > \sigma_3 = 0 \quad (19)$$

$$1/3 \leq \eta \leq 2/3 \quad (20)$$

Under the plane stress state, $\tau_{\max}, \mu_\sigma$ and η are reformulated as,

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_1}{2} \quad (21)$$

$$\mu_\sigma = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3} = \frac{2\sigma_2 - \sigma_1}{\sigma_1} = 2\alpha - 1 \quad (22)$$

$$\eta = \frac{\sigma_m}{\sigma_e} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3\sqrt{\sigma_1^2 - \frac{2r}{1+r}\sigma_1\sigma_2 + \sigma_2^2}} = \frac{1+\alpha}{3\sqrt{1 - \frac{2r}{1+r}\alpha + \alpha^2}} \quad (23)$$

where μ_σ denotes Lode parameter.

Combining Eqs.(21),(22) with Eq.(1), the following equation is gained,

$$\frac{2\tau_{\max}}{\sigma_e} = \frac{\sigma_1}{\sigma_e} = \frac{2}{\sqrt{\mu_\sigma^2 + 3 + (4 - \frac{8r}{1+r})\alpha}} \quad (24)$$

Furthermore, to consider the change of loading path during the whole plastic deformation process including homogeneous deformation zone and deformation localization zone, the modified Lou-Huh ductile fracture criterion is expressed as an integral form:

$$\frac{1}{C_3} \int_0^{\bar{\varepsilon}_f} \left(\frac{2}{\sqrt{\mu_\sigma^2 + 3 + (4 - \frac{8r}{1+r})\alpha}} \right)^{C_1} \left(\frac{1+3\eta}{2} \right)^{C_2} d\bar{\varepsilon} = D \quad (25)$$

where the average normal anisotropic coefficient r equals to 1, Eq. (25) can be simplified to the original form of Lou-Huh ductile fracture criterion with

$$\frac{2\tau_{\max}}{\sigma_e} = \frac{\sigma_1}{\sigma_e} = \frac{2}{\sqrt{\mu_\sigma^2 + 3}} [18].$$

Many investigators took stress triaxiality and Lode parameter as input data to calculate the material constants in the ductile fracture criterion. As η and μ_σ vary with the

advancement of plastic deformation, it is inappropriate to adopt the mean values, initial values or final values during the plastic forming process. According to the abovementioned analysis, the variation of stress ratio, which is obtained based on C-H model, can reflect the change of stress triaxiality and Lode parameter. Consequently, the modified Lou-Huh ductile fracture criterion combining with the variation of stress ratio is considered to characterize the effect of the whole loading history on the fracture behavior of DP590 sheet.

2.2.2 Calibration of material constants

The calculation accuracy of materials constants C in ductile fracture criterion (DFC) directly affects the precision of predicted result, the accurate determination of material constants C in DFC is thus critical. Considering there are three material constants in the Lou-Huh criterion: C_1 , C_2 , C_3 , which can be calculated by at least three experimental data. Hence, three typical tensile tests including uniaxial tension, plane strain tension and equi-biaxial tension were conducted to obtain the equivalent plastic strains at the onset of the final fracture, which are used as the termination conditions of iterative algorithm of C-H model. Since the loading paths of three typical tensile tests are changed during the forming process, the whole loading history described by the evolution of stress ratio with equivalent plastic strain should be considered in the determination of materials constants C . Based on the obtained loading history, the material constants in the modified Lou-Huh ductile fracture criterion can be calibrated by using the optimization algorithm as summarized in Eq. (26).

$$[C_1, C_2, C_3] = \min_{C_1, C_2, C_3} \delta_j \quad \text{where } \delta_j = |1 - D_j| \quad (26)$$

Here, the subscripts, $j = 1, 2$ and 3 , represent the three typical loading cases. The deviation δ denotes the discrepancy between the calculated damage value and the unity. The damage index D can be numerically calculated based on Eq. (25), which will reach unity at the onset of the final fracture. Therefore, the purpose of the optimization algorithm is to make the calculated damage index D_j in all cases reach unity simultaneously at the time of which the fracture initiates.

3. Experimental procedures

3.1 Determination of stress-strain curve

Since sheet metal undergoes large plastic deformation before the fracture initiates, it is necessary to determine the whole stress-strain curve. The stress-strain curve from uniaxial tensile test can only be determined before the onset of diffuse instability, the stress-strain relationship beyond the uniform elongation of uniaxial tensile test is usually gained by extrapolating the curve based on the selected hardening law. However, enormous deviation in prediction of flow behavior of the material is forecasted when a unreasonable hardening function is employed [33]. To minimize this deviation, the hydraulic bulge tests are adopted to study the flow behavior of dual-phase steel sheet because the deformation uniformity of sheet metal from the hydraulic bulge test far exceeds that from uniaxial tension test. In the hydraulic bulging test, the stress state of equi-biaxial stretching can be achieved at the vertex position of sheet. Since the shape of sheet near the vertex position is considered to be spherical, its curvature radius R is written as Eq. (27), which is shown in Fig. 5.

$$R = \frac{b^2 + h^2}{2h} \quad (27)$$

where b is the length of curvature meter spacing, and h is the dome height.

In addition, circular grids with diameter of 2.5 mm were electroplated on the surface of sheet to record local plastic strains ε_1 and ε_2 near the vertex position of bulging-sheet. Then normal strain ε_3 is calculated according to the principle of volume constancy. With the liquid chamber pressure p , the curvature radius R and initial thickness t_0 of sheet, the biaxial stresses σ_1 and σ_2 at the vertex position of bulging-sheet are deduced as:

$$\sigma_1 = \sigma_2 = \frac{Rp}{2t} \quad (28)$$

$$t = t_0 \exp(-\varepsilon_3) \quad (29)$$

To get as many experimental data under equi-biaxial tensile stress state as possible,

the hydraulic bulging tests were conducted under different liquid chamber pressure p until the sheet failure occurs. Considering the true stress-strain points from the hydraulic bulge tests are not accurate at low bulge curvatures, the uniaxial tensile test was adopted to describe the deformation behavior of DP590 sheet at small plastic deformation condition [34]. **Fig. 6** shows the data obtained from uniaxial tensile test up to the uniform elongation and the achieved equivalent stress-strain data based on Hill'48 yield criterion from the hydraulic bulging tests. These data points were fitted by Hollomon function, as shown in Eq. (6). It was found from **Fig. 6** that a satisfactory fitting result was acquired with hardening parameters of $k=1135\text{MPa}$ and $n=0.245$.

3.2 Determination of the fracture limit strain

To investigate the fracture limit of DP590 sheet, the Nakajima tests were conducted. **Fig. 7** depicts the schematic illustration of the assembly of Nakajima test. Experimental limit strains under all possible stress states from the uniaxial tension to the equi-biaxial tension were obtained via using different specimen shapes and lubrication conditions. The shape and geometric dimension of the specimens are also presented in **Fig. 7**. The widths of arc-shaped specimens are selected as 20, 40, 60, 90, 120, 140, 160 and 180mm with the same length of 180mm. Before the forming process, circular grids with diameter of 2.5mm were electro-etched on the surface of sample. During the experiments, a constant blank-holder force of 300kN was applied on the sheet metal, and the specimens were deformed until fracture with a constant punch velocity of 30mm/min. Meanwhile, the circular grids on the surface of specimens were distorted into elliptic grids.

Fig. 8 illustrates the deformed specimens after Nakajima tests. It is noted that the ductile fracture of DP590 sheet takes place with moderate necking phenomenon. In this paper, to investigate the ductile fracture behavior of DP590 sheet, the fracture limit strains in the fracture zone that are obtained by extrapolating the measured strains in the edge of the fracture section are adopted. According to the investigations of Lou [2] and Ma [35], the FFLD measured by the optical measuring system is utilized to approximate the equivalent plastic strains to fracture under the three typical loading

cases, i.e., uniaxial tension, plane strain tension and equi-biaxial tension. In addition, it has been demonstrated by Ma et al. [36] that the loading paths prior to fracture of Nakajima tests with the arc-shaped specimen widths of 20, 90 and 180mm approach to those from the uniaxial tension, plane strain tension and equi-biaxial tension stress states. Thus, the specimen dimensions of three calibration tests are selected as the arc-shaped widths of 20, 90 and 180 mm with the same length of 180 mm. In order to ensure the accuracy of material constants, each calibration tests are tested two times with the same punch velocity of 30 mm/min. Here, the measured fracture limit strains and corresponding equivalent plastic strains are summarized in **Table 3**.

4. Results and discussion

4.1 The evolution of stress ratio α

According to the analysis of Phases I and II of C-H model, the relationship between stress ratio and the equivalent plastic strain under three typical loading cases was worked out, as demonstrated in **Fig. 9**. It is found that the stress ratio keeps constant during Phase I. However, in Phase II, the changing tendency of stress ratio with the increase of the equivalent plastic strain is different for three loading cases. In the uniaxial tension condition, the stress ratio steadily increases with the development of plastic deformation. Whereas, the stress ratio of equi-biaxial tension gradually decreases with the increase of equivalent plastic strain. The variation of stress ratio of plane strain tension is not obvious during Phase II. The diverse variations mirror that the stress states of uniaxial tension and equi-biaxial tension do not always maintain proportional loading over the whole period, but gradually drift toward plane strain with the occurrence of load instability. But the stress state of plane strain tension remains almost constant with plastic deformation. The second order polynomial function was adopted to fit the relationship between stress ratio and the equivalent plastic strain during Phase II, as shown in **Fig. 9**.

4.2 Comparison of predicted FFLDs to experimental result

The effect of loading path on the fracture behavior is not taken into account in the process of the determination of the material constants of the original Lou-Huh ductile fracture criterion. The stress ratio of proportional loading stage is adopted for the whole loading process up to the onset of ductile fracture. Thus, the path-independent material constants in the original Lou-Huh model can be worked out via the optimization algorithm of Eq. (26) along three representative loading cases. However, based on the above analysis, the loading path does not remain linear during the deformation process. According to C-H theory, the evolution of loading path during the whole plastic deformation process is traced by stress ratio. The path-dependent material constants in the modified Lou-Huh ductile fracture criterion should be determined via the combination of the obtained stress ratio evolution equations and the optimization algorithm of Eq. (26). The material constants of the original and modified Lou-Huh models are summarized in **Table 4**. The fracture limit strains between uniaxial tensile and equi-biaxial tensile states were calculated by using these constants, but the evolution of stress ratio of other stress states must be considered in the integral process of the modified Lou-Huh model.

The theoretical FFLDs predicted by the original and modified Lou-Huh ductile fracture criteria are compared with the experimental result, as shown in **Fig. 10**. Obviously, among the two ductile fracture criteria, the theoretical fracture limit curve calculated by the modified Lou-Huh model is in best agreement with the experimental one, especially under the uniaxial tension and equi-biaxial tension states. Nevertheless, in plane strain tension condition, the fracture limit strain reckoned by the original Lou-Huh model is coincided with that calculated by the modified one. This is because the variation of stress ratio is not obvious, and the loading path almost remains constant in this scenario. Under other stress states except the plane strain state, the stress ratios vary significantly with the advancement of plastic deformation, and stress states gradually drift toward plane strain. Therefore, on the whole, the theoretical result based on the modified model is more consistent with experimental data than that

determined by the original model.

Based on the microscopic analysis of Lou-Huh model, the void growth is represented by the function of stress triaxiality, and the coalescence of voids is described by Lode parameter. Material constants in Lou-Huh model regulate the influence of nucleation, growth and coalescence of microscopic voids on ductile fracture, which are separately summarized as follows. Firstly, the material constant C_1 adjusts the effect of Lode parameter on coalescence of voids during plastic deformation. Since C_1 from the modified Lou-Huh model is larger than that from the original model, the tendency of coalescence of voids is obviously enhanced in the modified model. Secondly, the material constant C_2 accommodates the influence of stress triaxiality on growth of voids. As C_2 is slightly larger in the modified model, the effect of void growth is correspondingly increased. And lastly, the material constant C_3 just modulates the magnitude of FFLD. In original Lou-Hu model, C_3 is equal to the fracture strain of the uniaxial tensile, which is showed in **Table 3** and **Table 4**. As for C_3 in modified Lou-Hu model, it is significantly greater than that in original model for higher theoretical FFLD.

4.3 The analysis of ductile fracture

From the relationship between stress ratio and the equivalent plastic strain, the evolution of stress triaxiality and Lode parameter can be derived by Eqs. (22) and (23), as shown in **Fig. 11**. In addition, according to the forms of Eqs. (22) and (23), the change of stress triaxiality with stress ratio is not obvious in larger stress ratio condition. However, Lode parameter varies evenly with stress ratio over the entire stress ratio range. For the uniaxial tension stress state, the η -value and μ_σ -value remain $1/3$ and -1 before load instability, respectively. Then, they increase rapidly after the load instability occurs, which apparently accelerate the growth and shear coalescence of micro-voids. Li et al. [26] also came to the same conclusion by researching the fracture mechanism of Al-6061 billet. In plane strain tension condition, the diffuse instability period becomes shorter, and the η -value and μ_σ -value remain at approximately $1/\sqrt{3}$

and 0, respectively. In addition, the plane strain state ($\mu_r=0$) facilitates shear coalescence of voids to induce lower equivalent plastic strain. Under equi-biaxial tension stress state, Lode parameter keeps constant until the occurrence of load instability, then it decreases with the equivalent plastic strain. However, the stress triaxiality decreases slightly after load instability, which is verified by Mohr et al. [20]. That means the growth of micro-voids basically remains unchanged and the coalescence of micro-voids is enhanced slightly after the load instability. In **Fig. 11**, the experimental fracture strains from Nakajima tests were compared with the predicted fracture locus of DP590 sheet. The result of comparison shows that the modified Lou-Huh model has a good prediction capability for DP590 sheet formability.

Moreover, SEM images of fracture surfaces were taken for the three typical loading specimens to characterize the ductile fracture behavior of DP590 sheet, as illustrated in **Fig. 12**. In the uniaxial tension specimen with the width of 20 mm (shown in **Fig. 12** (a)), the enlarged voids are distributed in the center region of the fracture surface, while smaller voids can be found on the periphery due to the intensified effect of shear deformation. **Fig. 12** (b) shows the plane strain tension specimen with the width of 90 mm, the enlarged voids are presented in a larger area, and the shear fracture occurs at the outer area of fracture surface. In the equi-biaxial tension specimen with the width of 180 mm, as shown in **Fig. 12** (c), the enhanced voids are located in nearly entire fracture surface area.

5. Conclusions

In this research, a new methodology to predict the onset of ductile fracture of DP590 sheet was proposed by using the modified Lou-Huh fracture criterion combined with C-H model, which is suitable to describe the change of loading paths during the deformation process in the plane stress condition. The main findings were summarized as follows:

(1) The plastic deformation process was divided into three phases according to C-H model. Phase I describes the homogeneous deformation of sheet metal with a constant stress ratio, while Phase II depicts the deformation localization of material with the

drift of loading path, which is reflected by the change of stress ratio. Phase III is predicted by the ductile fracture criterion. During plastic deformation, the stress states of uniaxial tension and equi-biaxial tension tend toward plane strain, and the stress ratio of plane strain tension is nearly invariable.

(2) Considering the effects of material anisotropy and the whole loading history, the modified Lou-Huh ductile fracture criterion was derived based on the Hill'48 yield function and the constructed relationship between stress ratio and the equivalent plastic strain. And the material constants of the modified Lou-Huh model were calibrated by using the optimization algorithm. The whole stress-strain curve was established by the true stress-strain data from uniaxial tensile test and the equivalent stress-strain data from hydraulic bulge tests. The hardening behavior beyond the uniform elongation of uniaxial tensile test was accurately described by Hollomon function.

(3) The modified Lou-Huh model which takes the change of loading path into account provided more accurate formability prediction than the original one, particularly in the uniaxial tension and equi-biaxial tension states. Furthermore, the growth and shear coalescence of micro-voids were correlated with the evolution of stress triaxiality and Lode parameter in the plastic deformation process based on the analysis of SEM images of the fracture surface.

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Table Captions and then Figures

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*Graphical Abstract

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Nominal composition (wt.%)				
C	Mn	Si	S	P
0.068	1.61	0.447	0.002	0.011

Table 2 Mechanical properties of DP590 sheet.

Material	Angle to the Rolling Direction	$\sigma_{0.2}$ /MPa	σ_b /MPa	n^*	r -value
DP590	0°	398	631	0.177	0.963
	45°	404	645	0.170	0.815
	90°	388	630	0.172	1.096

Table 3 The fracture strains of DP590 sheet under different stress states.

Fracture strain	Uniaxial tension	Plane strain tension	Equi-biaxial tension
ε_1	0.569	0.282	0.486
ε_2	-0.194	0.017	0.420
ε_e	0.576	0.331	0.889

Table 4 Material constants of different ductile fracture criteria.

	C_1	C_2	C_3
Modified Lou-Huh model	7.0864	-0.5036	0.6956
Original Lou-Huh model	6.3285	-0.7865	0.5760

Fig. 1

Fig. 2

The Lou-Huh fracture criterion considering loading path

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9(a)

Fig. 9(b)

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Fig. 10

Fig. 11

Fig. 12(a)

(a)

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(b)

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(c)