Design of Liquid-Liquid Separation Hydrocyclones using Parabolic and Hyperbolic Swirl Chambers for Efficiency Enhancement

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Abstract

The wall profile of the swirl chamber greatly impacts the internal flow structures and separation efficiency of a liquid-liquid separation hydrocyclone. The objective of this study is to examine the effects of parabolic and hyperbolic wall profiles of hydrocyclone swirl chamber on the internal flow structures and separation efficiency based on the numerical simulations. The internal flow structures observed for the different wall profiles of swirl chamber motivates the redesign of hydrocyclone geometry to achieve enhanced separation efficiency. Results show that, for a dilute system (oil concentration less than 1%), the hyperbolic and parabolic swirl chambers without a tail pipe yield, respectively, 16.5% and 25% higher separation efficiency for a droplet size of 30 μm when compared with a conical swirl chamber without tail pipe. However, the hyperbolic swirl chamber has a greater potential for the reduction of effective length of hydrocyclone with maintaining high separation efficiency. In addition, a hydrocyclone with truncated hyperbolic swirl chamber and tail pipe provides very long reverse flow core and yields 17% and 33% higher efficiency than that of full hyperbolic and conical swirl chambers without tail pipe, respectively.

Keywords: Hydrocyclone; liquid-liquid separation; separation efficiency; parabolic swirl chamber; hyperbolic swirl chamber.

1. Introduction

The liquid-liquid separation hydrocyclones are widely used in various industries for removing dispersed oil (oil concentration less than 1%) from produced water, oil spill and oily wastewater (Motin, 2015a, 2015b; Liu et al., 2015; Motin et al. 2014a, 2014b, 2013). Hydrocyclones separate two phases of dissimilar densities based on the centrifugal force differential between the two phases created by strong swirling motion (Motin, 2015a, 2015b; Liu et al., 2015; Motin et al. 2014a, 2014b, 2013; Bradley, 1965; Svarovsky and Thew, 1992; Meldrum, 1988; Colman, 1981; Bai et al. 2009; Petty and Parks, 2011; Slack et al. 2000). The geometry of a hydrocyclone typically consists of an inlet chamber, a swirl chamber, a vortex finder, and a tail pipe (Fig. 1). The swirling motion developed by a tangential feed into the inlet chamber creates an axial pressure gradient at the core, which thereby provides a driving force for a reverse flow core to be developed (Motin, 2015a; Motin et al. 2014a; Bradley, 1965; Svarovsky and Thew, 1992; Meldrum, 1988; Colman, 1981). The reverse flow core moves the oil rich core to the overflow orifice. Thus the overflow is a reject stream (highly concentrated mixture) and the underflow is a cleaner primary phase (Fig. 1a).

One of the major challenges in the liquid-liquid separation hydrocyclone is maintaining a long reverse flow core for a wide range of feed flow rate (Motin, 2015a). Recent studies performed on single cone hydrocyclones demonstrate that the conventional design (conical swirl chamber) is lacking in maintaining a long reverse flow core, and possesses a finite turndown ratio (Motin, 2015a, Meldrum, 1988; Bennett and Williams, 2004; Motin et al. 2014c). A high cone angle generates severe pressure drop along the axis and breakdowns the reverse flow core (Motin, 2015a; Motin et al. 2014d). A small cone angle produces the necessary stable vortex (Thew, 2000) and a long reverse flow core (Motin, 2015a; Motin et al. 2014d), but it reduces centrifugal force differential between two phases and yields a hydrocyclone to be very long (Motin, 2015a; Motin et al. 2014d). In order to improve turndown ratio, stability of reverse flow core and separation efficiency, researchers have developed a number of new hydrocyclones such as single cone (Fig. 1c), dual cone (Fig. 1b), and mini hydrocyclones with various shapes and angles of conical swirl

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chamber (Thew, 2000; Young et al. 1994; Belaidi and Thew, 2003; Colman and Thew, 1988; Kharoua et al. 2010a; Lv et al. 2015; Zhu et al. 2012).

Young et al. (1994) and Saidi et al. (2013) observed in a single cone hydrocyclone that a cone angle of 6 degree provides a better separation over a wide range of inlet conditions. However, Liu et al. (2014) found that a 10 degree cone provides a better separation than a 6 degree cone. For a dual cone hydrocyclone, a cone angle between 1.5 and 2.0 degree provide good separation and maintain a stable vortex core (Thew, 2000; Colman and Thew, 1988). Motin et al. (2015a and 2014d) observed that the length of reverse flow core and separation efficiency increases with a decrease in the cone angle. Noroozi and Hashemabadi (Noroozi and Hashemabadi, 2011) replaced the cylindrical inlet and reducing chambers of a dual cone hydrocyclone by exponential and conical profiles and investigated its effects on the separation efficiency. They observed that the conical and the exponential inlet chambers improve the separation efficiency by approximately 4.5% and 8%, respectively. However, the conical and exponential swirl chamber breakdowns reverse flow core at high Reynolds numbers. The wall profile of swirl chamber affects the internal hydrodynamics and separation performance significantly (Motin et al. 2014a). A detail analysis of the effects of swirl chamber wall profiles on the internal flow structures and separation performance are crucial.

This paper examines the effects of swirl chamber wall profiles on the internal flow structures and the separation efficiency of a liquid-liquid separation hydrocyclone based on the computational fluid dynamics simulations. The commercial software package ANSYS/Fluent-14.5 was applied for the numerical simulations. The hydrocyclone is projected to be used for produced water treatment and oil-spill cleanup where the oil-water mixture is diluted (oil concentration is typically less than 1%). The conical swirl chamber of a conventional single cone hydrocyclone is tailored by using parabolic and hyperbolic wall profiles to enhance length of reverse flow core and separation efficiency. Moreover, an efficient, simplified and commercially viable design of a liquid-liquid separation hydrocyclone is developed based on the hyperbolic swirl chamber.

Figure 1: (a) Schematic of a hydrocyclone illustrating internal flow structures, and schematics of (b) dual cone and (c) single cone hydrocyclones. Drawings are not in appropriate scale.

2. Swirl chamber designs

The swirl chamber of a single cone hydrocyclone (see Fig. 1c) is tailored using parabolic and hyperbolic wall profiles. The schematics of three different swirl chamber designs are presented in Fig. 2. Design-A and Design-B have parabolic profiles and Design-C has hyperbolic profile. Design-A and Design-B have asymptotic profiles at the bottom and the top end of the swirl chamber. The swirl chamber for the Design-C (hyperbolic swirl chamber) is asymptotic at both ends. Two design cases are considered: Case-1 has a cylindrical tail pipe and Case-2 has no tail pipe. The swirl chamber for the Case-2 is about 2.5 times longer than that of the Case-1. The dimensions of the hydrocyclones for the two design cases are presented in Table 1.

Table 1: Geometric groups and dimensions of hydrocyclones illustrated in Fig. 2

Design cases	Swirl Chamber	D (mm)	Do/D	Du/D	D _F /D	Li/D	Ls/D	Lv/D	Lt/D
Case-1	Short swirl chamber with tail pipe	26	0.07	0.23	0.275	1	7.37	0.1	9.71
Case-2	Long swirl chamber without tail pipe	26	0.07	0.23	0.275	1	19.23	0.1	0

$$D_F = 2ab/(a + b)$$

(a) Conventional: Conical

$$r = \begin{cases} \frac{D}{2} & \text{for } z < L_i \\ \frac{D}{2} - \left(\frac{D - D_u}{2L_s} \right) (z - L_i) & \text{for } L_i \leq z \leq (L_i + L_s) \\ \frac{D_u}{2} & \text{for } z > (L_i + L_s) \end{cases}$$

(b) Design-A: Parabolic

$$r = \begin{cases} \frac{D}{2} & \text{for } z < L_i \\ \frac{D}{2} + \left(\frac{D - D_u}{2L_s^2} \right) [(z - L_i)^2 - 2L_s(z - L_i)] & \text{for } L_i \leq z \leq (L_i + L_s) \\ \frac{D_u}{2} & \text{for } z > (L_i + L_s) \end{cases}$$

(c) Design-B: Parabolic

$$r = \begin{cases} \frac{D}{2} & \text{for } z < L_i \\ \frac{D}{2} - \left(\frac{D - D_u}{2} \right) \sqrt{\frac{z - L_i}{L_s}} & \text{for } L_i \leq z \leq (L_i + L_s) \\ \frac{D_u}{2} & \text{for } z > (L_i + L_s) \end{cases}$$

(d) Design-C: Hyperbolic

$$r = \begin{cases} \frac{D}{2} & \text{for } z < L_i \\ \frac{D}{2} - \left(\frac{D - D_u}{2} \right) \sqrt{1 - \left(\frac{L_i + L_s - z}{L_s} \right)^2} & \text{for } L_i \leq z \leq (L_i + L_s) \\ \frac{D_u}{2} & \text{for } z > (L_i + L_s) \end{cases}$$

Figure 2: Schematic of four different swirl chambers; (a) Conical swirl chamber, (b) Design-A (parabolic swirl chamber with asymptote at the tail end), (c) Design-B (parabolic swirl chamber with asymptote at the leading end), and (d) Design-C (hyperbolic swirl chamber).

2. Numerical approaches for hydrodynamic modeling

An Eulerian-Lagrangian approach is utilized for modeling the motion of continuous (water) and dispersed (oil) phases. The typical oil concentration of the produced water is less than 1% (Motin, 2015a). The Eulerian-Lagrangian approach is appropriate for this dilute system (Motin, 2015a; Motin et al. 2014a; Kharoua et al. 2010a). This approach has been used successfully in a variety of settings (Motin, 2015a; Motin et al. 2015b, 2015c, 2014a, 2014b, 2014d, 2013; Saidi et al. 2013, 2012a, 2012b; Noroozi and Hashemabadi, 2011, 2009; Ji et al. 2016). The RANS- equation (Eq. (2)) for the flow of a constant property Newtonian fluid is solved to calculate mean velocity and pressure fields of continuous phase (Motin, 2015a; Pope, 2000).

$$\nabla \cdot \langle \underline{U} \rangle = 0 \quad (2a)$$

$$\rho \frac{\partial \langle \underline{U} \rangle}{\partial t} + \rho \langle \underline{U} \rangle \cdot \nabla \langle \underline{U} \rangle = -\nabla \langle p \rangle + \mu \nabla^2 \langle \underline{U} \rangle - \rho \nabla \cdot \langle \underline{uu} \rangle \quad (2b)$$

The unclosed kinematic Reynolds momentum flux, $\langle \underline{uu} \rangle$, is closed using the Reynolds Stress Model (RSM) (see Motin (2015a), Launder (1989), Lien and Leschziner (1978), Gibsior and Launder (1978), Fu et al. (1987) for a review of the model). The Fluent 6.0 supported by ANSYS 14.5 commercial software package is used to solve the RSM. The RSM supported by ANSYS/Fluent commercial software is used successfully to study the internal flow structures and the separation of oil and water in a hydrocyclone in many previous researches (Motin, 2015a; Motin et al. 2015b, 2014a, 2014b, 2014d, 2013, Noroozi and Hashemabadi, 2011; Kharoua et al. 2010b; Huag, 2005). The pressure strain term in the unclosed transport equation of Reynolds stress (Motin et al. 2014a, 2013) is modeled using a Linear Pressure Strain Model (LPSM) available in Ansys/Fluent. The redistribution of normal component of Reynolds stress near the wall is obtained by enabling the wall reflection effect in the LPSM. The wall boundary conditions for the Reynolds stresses are obtained by solving the transport equation of turbulent kinetic energy. A scalable wall function has been used for near wall treatment since it produces consistent results for grids with arbitrary refinement (Motin et al. 2014a). The trajectories of dispersed droplets denoted as $\chi_D(t)$ are predicted by solving the kinematic equation (Eq. (3)) of droplet motion, which is written in a Lagrangian frame of reference as (Motin, 2015a; Motin et al. 2014a)

$$\chi_D(t + \Delta t) = \chi_D(t) + \int_t^{t+\Delta t} \underline{U}_D(t') dt' \quad (3)$$

$$\text{where } \underline{U}_D(t + \Delta t) = \underline{U}_D(t) + \frac{\rho_c}{\rho_D} \int_t^{t+\Delta t} \left[\frac{\underline{U}_{drift}(t')}{\tau_D} + \sum_{n=1}^N \underline{a}_n(t') \right] dt'$$

A pressure-based segregated solver available with Fluent 6.0 supported by ANSYS 14.5 is applied for solving the RANS-equation (Eq. (2)). The coupling between pressure and velocity is achieved using Pressure Implicit with Splitting of Operator (PISO). The time step used for the simulation changes from 0.0001 to 0.0005s. Pressure interpolation is performed using the PREssure STaggering Option (PRESTO) scheme. The PRESTO scheme provides a better estimation for high speed swirling flows, and flows in strongly curved domain. The momentum equations are discretized using Quadratic Upstream Interpolation for Convective Kinetics (QUICK) scheme since it provides a better accuracy and is suitable for a hexahedral mesh. A progressive treatment approach is used for the application of turbulence models and numerical discretization schemes to get a converged statistically steady-state solution of the complex flow field (see Motin (2015a), Motin et al. (2014a) for details of the approach). Converged results were achieved by setting under-relaxation factors for pressure, momentum, turbulent kinetic energy, turbulent dissipation rate, and Reynolds stresses to 0.3, 0.3, 0.8, 0.8, and 0.5 respectively. At every time step the residual levels for continuity, momentum and all the stresses is less than 10^{-4} . Normalized residuals in the range of 10^{-3} typically indicate a practically-converged solution for the RSM. A tighter convergence criterion (in the range of 10^{-4}), however, is maintained to ensure accuracy.

The Lagrangian tracking of dispersed droplets in the calculated flow field are solved by using Discrete Phase Model supported by ANSYS/Fluent software package. A 5th order Runge-Kutta scheme is used for the integration of the equations of dispersed phase motion (Eq. (3)). The effects of pressure gradient and virtual mass force on the acceleration of dispersed droplets (represented by an in Eq. (3)) are considered. The dispersed phase droplets are assumed to be spherical and the coefficient of drag force C_D is calculated by applying a spherical drag law. In the Lagrangian tracking method, for each droplet size, about 2,500 uniformly distributed mono sized droplets of specific gravity 0.85 mimicking the crude oil are injected from the inlet surfaces (see Motin (2015a), Motin et al. (2014a and 2014b) for detail). A one way coupling has been used to implement effect of continuous phase turbulence on the trajectories of dispersed droplets. The dispersion of droplets due to turbulence in the continuous phase is predicted based on the stochastic tracking model. For a review of the stochastic tracking approach using the Lagrangian method, see Motin (2015a). The accuracy of the solution is controlled by specifying a tolerance of 10^{-6} for the numerical integration. The maximum number of time steps was set as 5×10^5 , which is sufficient to ensure that there were no droplets with incomplete trajectories in the flow domain. The step length factor λ is equal to 5, which means that there are five integration steps for each grid cell.

3. Boundary conditions

A plug flow with average velocity of 4.167 m/s, which gives a volumetric flow rate into the hydrocyclone about 1.2 m³/hr, is specified at the inlets. The average gauge pressure at the overflow and underflow outlets is set to zero and 120 kPa, respectively. The turbulence intensity implemented at the inlets and for backflows (if any) at the overflow and underflow outlets is 5%. The turbulent kinetic energy and viscosity ratio calculated based on the turbulent intensity and the hydraulic diameter. The Reynolds stresses at the inlets are specified by turbulent intensity (I) as $\langle \underline{u}_F \underline{u}_F \rangle = e_i e_j = (U_F I)^2$ for $i = j$ and $\langle \underline{u}_F \underline{u}_F \rangle = e_i e_j = 0$ for $i \neq j$. The solid wall boundaries are set as no-slip condition.

4. Computational grid

A structured hexahedral O-grid mesh of about 900,000 cells with orthogonal quality of 0.35 is generated for the numerical simulations. A finer mesh is generated at the core and near the wall regions to accurately capture core profile and near wall flow profile (see Appendix-A, Fig. A-1). The mesh density near the wall region satisfies the criterion of dimensionless wall distance for the selected scalable wall function (SWF) (see section 3.3 of Motin (2015a) for the details about the SWF and y^+ value selected for this simulation). This mesh size is selected based on the grid independence study (GIS) (Celik, 2008) (see Motin (2015a) for the detail analysis of GIS). The grid independent study is performed by calculating approximate relative error, E_a , extrapolated relative error, E_{ext} , and grid convergence index, GCI, for the normalized axial velocity, tangential velocity and static pressure (see Appendix-C of Motin (2015a) for the definitions). The E_{ext} and GCI are calculated at a location where differences in the velocity components and pressure are the maximum. A summary of the E_a , E_{ext} and GCI are given in Table 2. The GCI also calculated for the grade separation efficiency for the droplet sizes varies from 20 microns to 100 microns. The maximum GCI is found for the 20 microns, which is less than 10%. The GCI drastically decreases with increasing droplet sizes, which is less than 1% for droplet size of 100 microns. The grid independent study indicates that the mesh with about 900,000 cells is sufficiently fine for an accurate numerical solution. The effect of mesh size on the tangential velocity, axial velocity, static pressure and reduced grade separation efficiency is shown in Appendix-A.

Table 2: Effect of mesh refinement on the predicted properties

Parameters	Ro	PDR	$\langle U_\theta \rangle / \langle U_F \rangle$	$\langle U_z \rangle / \langle U_F \rangle$	$\langle P \rangle / \langle P_F \rangle$
Approximate relative error Ea (%)	0.97	0.37	0.46	0.97	5.7
Extrapolated relative error Ea _{ext} (%)	0.15	0.092	0.013	0.11	2.1
Grid Convergence Index GCI (%)	0.19	0.12	0.017	0.14	2.3

5. Validation of numerical results

The tangential velocity profile, which is obtained using the simulation approaches presented in the previous sections, of a 35 mm hydrocyclone is qualitatively compared with experimental and numerical investigations performed on the same hydrocyclone by Bai et al. (2009) and Saidi et al. (2013), respectively. There is no experimental or numerical investigation of hydrodynamics on a 26 mm single cone hydrocyclone found in the literature. Figure 3 shows that the tangential velocity profile obtained in the present work by solving the RANS equations with the RSM is qualitatively and quantitatively more closer to the experimental value when compared with the LES result. Near the wall region, the tangential velocity obtained by LES approach shows a larger gradient. However, the tangential velocity calculated by the RANS simulation provides a trend similar to the experimental data. The reduced grade separation efficiency, $G_R(d_D)$, of a 35 mm hydrocyclone is also qualitatively compared with a numerical predictions obtained for the same hydrocyclone by Saidi et al. (2013), which is shown in Fig. 4. The $G_R(d_D)$ calculated in the present study is both qualitatively and quantitatively similar to that obtained by these authors (Saidi et al. 2013).

Figure 3: Qualitative comparison of tangential velocity calculated using the RANS simulation on a 35 mm single cone hydrocyclone with that obtained from previous studies of a similar hydrocyclone based on the LES approach and experimental investigation. Velocity profile is taken at $z/D = 2.29$.

Figure 4: Qualitative comparison of reduced grade separation efficiency, $G_R(d_D)$, calculated using the RANS simulation on a 35 mm single cone hydrocyclone with that obtained earlier for a similar hydrocyclone using the LES approach.

6. Internal flow structures for the design Case-1 (Table 1)

The flow structures within hydrocyclones with the three designs of swirl chamber (Fig. 2b-2d) having dimensions presented in Table 1 (Case-1) are compared with that of a hydrocyclone with conical swirl chamber (Fig. 2a). The conical swirl chamber yields less normalized tangential velocity, $\langle U_\theta \rangle / \langle U_F \rangle$, when compared with the parabolic and hyperbolic swirl chambers (Fig. 5). The Design-C yields the greatest $\langle U_\theta \rangle / \langle U_F \rangle$ at $z/D = 1$ because of the largest wall curvature (see Fig. 2). However, the location of peak core pressure, $\langle P_C \rangle / \langle P_F \rangle$, for the Design-C is closer to the overflow orifice when compared with other designs (Fig. 6). The conical swirl chamber yields the location of peak $\langle P_C \rangle / \langle P_F \rangle$ to the farther downstream. It indicates that the new swirl chambers (Design-A, Design-B and Design-C with design Case-1) do not increase the length of reverse flow core. However, the conical chamber reduces the $\langle P_C \rangle / \langle P_F \rangle$ significantly at the end of the chamber ($z/D \approx 8.37$). A high negative value of $\langle P_C \rangle / \langle P_F \rangle$ near the end of swirl chamber can initiate cavitation. The Design-C yields a significant improvement in regards to the minimization of negative $\langle P_C \rangle / \langle P_F \rangle$ near the end of swirl chamber. A hydrodynamic reason for the catastrophic reduction of the $\langle P_C \rangle / \langle P_F \rangle$ at $z/D \approx 8.3$ can be explained using a chart of radial pressure differential, $\langle P_W - P_C \rangle / \langle P_F \rangle$, which is shown in Fig. 7. The $\langle P_W - P_C \rangle / \langle P_F \rangle$ at $z/D = 1$ for the conical swirl chamber is less than that for the other designs. The conical chamber accelerates swirling motion, which thereby increases the $\langle P_W - P_C \rangle / \langle P_F \rangle$ significantly. Among the three designs, the Design-C provides the least $\langle P_W - P_C \rangle / \langle P_F \rangle$, which thereby supports the smallest drop of $\langle P_C \rangle / \langle P_F \rangle$ at $z/D \approx 8.37$ as shown in Fig. 4.

Figure 5: Effect of swirl chamber designs on the normalized tangential velocity, $\langle U_\theta \rangle / \langle U_F \rangle$, profile at $z/D = 1$ for the $Re_F = 23835$. Design-C (hyperbolic swirl chamber) provides a better centrifugal acceleration, U_θ^2/r , which may enhance the migration rate of dispersed droplets toward the center.

Figure 6: Effect of swirl chamber geometry on the center pressure, $\langle P_C \rangle / \langle P_F \rangle$, profiles. A lower variation of $\langle P_C \rangle / \langle P_F \rangle$ along z/D for the Design-C indicates less momentum loss in the hydrocyclone chambers.

Figure 7: The bar chart illustrating normalized radial pressure differential, $\langle P_w - P_c \rangle / \langle P_F \rangle$, between the wall and center. Design-C provides the minimum $\langle P_w - P_c \rangle / \langle P_F \rangle$ at $z/D = 8.37$, which thereby supports a less drop in the $\langle P_c \rangle / \langle P_F \rangle$ as shown in Fig. 4.

7. Internal flow structures for the design case-2 (Table-1)

The operating conditions for the hydrocyclones having long swirl chamber without tail pipe (see Fig. 2b-2d and Case-2 in Table-1) are presented in Table 3. For the same inlet conditions, the overflow ratio, $R_O = Q_O / Q_F$, for the Design-C is less than that for the Design-A and the Design-B. A smaller R_O enhances the effectiveness of separation (see Eq. (2.7) in Motin (2015a)). However, the pressure drop ratio, $\Delta P^* = (P_O - P_F) / (P_U - P_F)$, for the Design-B is higher than that of the other two designs, which thereby yields a smaller underflow Euler number, E_{UU} . It indicates that the energy loss in the Design-B is less than that of the other two designs.

Table 3: Operating conditions simulated for the three designs of swirl chamber without tail pipe (see Fig. 2b-2d and Case-2 in Table-1)

Design	Re_F	P_F (Pa)	ΔP^*	R_O	E_{uo}	E_{uu}
Design-A	23,835	308,107	1.64	0.091	17.7	10.8
Design-B	23,835	293,516	1.7	0.089	16.9	10.0
Design-C	23,835	362,470	1.5	0.077	20.9	14.0

The effects of the swirl chamber designs on the normalized tangential velocity, $\langle U_\theta \rangle / \langle U_F \rangle$, normalized axial velocity, $\langle U_z \rangle / \langle U_F \rangle$, normalized radial velocity, $\langle U_r \rangle / \langle U_F \rangle$, and normalized static pressure, $\langle P \rangle / \langle P_F \rangle$, at $z/D = 1$ are shown in Fig. 8. The peak $\langle U_\theta \rangle / \langle U_F \rangle$ for the Design-C is much higher than that for both the Design-A and Design-B (Fig. 8a). A higher longitudinal wall curvature of the Design-C (see Fig. 2) increases the U_θ^2 / r , which thereby yields a greater $\langle U_\theta \rangle / \langle U_F \rangle$ at the interface between inner and outer vortex ($r/R \approx 0.1$). The locations of peak $\langle U_\theta \rangle / \langle U_F \rangle$ for all the designs are almost

the same, which supports an equal radius of reverse flow core (represented by r_c in Fig. 8b). However, no considerable difference appears in the $\langle U_z \rangle / \langle U_F \rangle$ except at the center. Near the wall ($r/R \approx 1$), the $\langle U_r \rangle / \langle U_F \rangle$ for all the designs is negative because of an inward curvature of swirl chambers (Fig. 8c). The Design-C provides a greater radially inward velocity near the wall due to a higher longitudinal wall curvature. Near the core ($r/R \approx 0.05$), the negative value in the $\langle U_r \rangle / \langle U_F \rangle$ indicates a local radial inflow to the reverse flow core. The Design-B yields less local radial inflow to the reverse flow core at $z/D = 1$. This less radial inflow reduces short circuit flows at the inlet chamber, which thereby increases residence time of reverse flow streams.

Figure 8: Effects of the swirl chamber designs on the (a) normalized tangential velocity, $\langle U_\theta \rangle / \langle U_F \rangle$, (b) normalized axial velocity, $\langle U_z \rangle / \langle U_F \rangle$, (c) normalized radial velocity, $\langle U_r \rangle / \langle U_F \rangle$, and (d) normalized static pressure, $\langle P \rangle / \langle P_F \rangle$ at $z/D = 1$. The Design-C yields a higher centrifugal acceleration, U_θ^2/r and less $\langle P \rangle / \langle P_F \rangle$ near the center.

Figure 9: Effects of the swirl chamber geometries on the normalized static pressure on the wall, $\langle P_w \rangle / \langle P_F \rangle$, and the normalized pressure differential between wall and center, $\langle P_w - P_c \rangle / \langle P_F \rangle$.

Near the wall ($r/R \approx 1$), the $\langle P \rangle / \langle P_F \rangle$ for all the swirl chamber designs is the same, which is almost equal to one (Fig. 8d). The $\langle P \rangle / \langle P_F \rangle$ decreases toward the center of the hydrocyclone and it is the minimum at the center. The radial pressure differential, $\langle P_w - P_c \rangle / \langle P_F \rangle$, for the Design-C is greater than that of the Design-B and the Design-A. Downstream of $z/D = 1$, a higher U_0^2/r due to a greater longitudinal wall curvature for the Design-C yields a larger $\langle P_w - P_c \rangle / \langle P_F \rangle$ and a less $\langle P_w \rangle / \langle P_F \rangle$, (Fig. 9). With increasing z/D , the $\langle P_w - P_c \rangle / \langle P_F \rangle$ and $\langle P_w \rangle / \langle P_F \rangle$ decreases for all the designs due to a reduction of swirling motion caused by frictional losses on the wall. However, the Design-C yields a greater $\langle P_w - P_c \rangle / \langle P_F \rangle$ up to $z/D \approx 15$ when compared with other designs. It indicates that the hyperbolic swirl chamber provides a higher swirling motion as well as develops a greater centrifugal force differential between two phases.

Figure 10a shows that the core pressure, $\langle P_c \rangle / \langle P_F \rangle$, profiles for all the designs with longer swirl chamber (see Case-2 in Table 1) are greatly improved when compared with the profiles as shown in Fig. 6. The longer swirl chamber reduces the wall curvature, which thereby makes a good balance between the acceleration and deceleration of swirling motion, respectively, by reduction of cross-sectional area and wall friction. Unlike the profiles shown in Fig. 6, there is no sharp drop and negative magnitude of $\langle P_c \rangle / \langle P_F \rangle$ in the swirl chamber. Among the three designs, the Design-C yields the smoothest pressure profile. The higher wall curvature for the Design-C yields a greater U_0^2/r , which thereby supports the lower $\langle P_c \rangle / \langle P_F \rangle$ in the entire swirl chamber. In comparison with the Design-B, the Design-A provides higher $\langle P_c \rangle / \langle P_F \rangle$ for the $z/D < 8$ due to a lower wall curvature. The $\langle P_c \rangle / \langle P_F \rangle$ profiles for the Design-A and the Design-B intersect exactly at the same location where the wall profiles (radius versus z/D chart as

shown in Fig. 10b) of these two designs intersect each other. At $z/D > 8$, the $\langle P_C \rangle / \langle P_F \rangle$ for the Design-A reduces quickly due to a higher wall curvature (smaller radius) in comparison with the Design-B. Therefore, it can be inferred that the $\langle P_C \rangle / \langle P_F \rangle$ is directly proportional to the radius of swirl chamber. The location of peak $\langle P_C \rangle / \langle P_F \rangle$ for the Design-B is observed at the $z/D \approx 7$, where it is at $z/D \approx 5.5$ for the Design-C and at $z/D \approx 6$ for the Design-A. The positive axial pressure gradient between the z/D at $dP/dz = 0$ and the overflow orifice provides the driving force for a reverse flow core to be developed. The closer the location of $dP/dz = 0$ to the overflow orifice the shorter the reverse flow core becomes. The $\langle P_C \rangle / \langle P_F \rangle$ profiles shown in Fig. 10a support the length of reverse flow cores which are presented in Fig. 11. The lengths of the reverse flow core for the Design-A and Design-C are almost the same because of their locations of $dP/dz = 0$ close to each other (see Fig. 10). The z/D at $dP/dz = 0$ for the Design-B is farther downstream, which provides a longer reverse flow core than that of other designs. However, the location of reverse flow core breakdown (see Fig. 11) is slightly different than that of the location of $dP/dz = 0$. A detail discussion on the understanding of reverse flow core breakdown could be found in Motin (2015a).

The reduced grade separation efficiency, $G_{OR} = (G_O - R_O)/(1 - R_O)$, (see Motin (2015a) for detail), curves for the three designs are shown in Fig. 12. Due to a longer reverse flow core (see Fig. 11), the Design-B supports the transport of migrated dispersed droplets to the overflow from a far downstream and yields a better G_{OR} . However, the G_{OR} for the Design-C is very close to the Design-B and much higher than that of the Design-A and the conical one. The relation between the length of reverse flow core, ℓ_c/D , and the cut size, d_{D50} , (see Motin (2015a) for definition) presented in Fig. 13 shows that the d_{D50} for the Design-B and the Design-C is almost the same and it is much less than that of the Design-A. Though the ℓ_c/D for the Design-C is much shorter than that of the Design-B, the higher U_θ^2/r for the Design-C enhances the migration rate of dispersed droplets toward the center and decreases the d_{D50} . Besides, the ℓ_c/D for the Design-A and the Design-C is almost the same but the Design-A provides significantly higher d_{D50} (less G_{OR}). Because the Design-A provides lower U_θ^2/r as well as less migration rate of dispersed droplets when compared with the Design-C. Therefore, it can be inferred that the Design-C has a better potential for a higher separation. Using the Design-C a short hydrocyclone can be made because of its ability to provide higher separation efficiency with a shorter reverse flow core.

Figure 10: Effects of the swirl chamber shape on the static pressure along the center of the hydrocyclone. Core pressure profiles (Fig. a) follow the wall curvatures (Fig. b) of the swirl chambers, i.e. a higher wall curvature yields less core pressure.

a) Design-A b) Design-B c) Design-C

Figure 11: Contours are showing the reverse flow core for the three different swirl chamber designs. The Design-A and the Design-C generates almost the same length of reverse flow core whereas the Design-B yields much longer reverse flow core.

Figure 12: Grade separation efficiency, $G_{OR}(d_D)$ for the three designs. The Design-B provides about 8% more G_{OR} than that of the Design-C and the Design-A yields much lower G_{OR} , which is almost equal to the conical one.

Figure 13: Relation between the normalized length of reverse flow core, l_c/D , and the cut size, d_{D50} , for the three swirl chamber designs. A lower d_{D50} represents higher separation efficiency. Even though the l_c/D for the Design-C is much smaller than that of the Design-B, their d_{D50} is almost the same.

8. Tailoring of swirl chamber geometry based on hyperbolic wall profile

The analyses addressed in sections 6 and 7 show that the hyperbolic wall profile (Design-C) provides high separation efficiency with a short reverse flow core and has a better potential of enhanced liquid-liquid separation for a wide range of inlet conditions. This section addresses tailoring of hyperbolic swirl chamber geometry to achieve longer reverse flow core with a simplified design of hydrocyclone.

8.1. Design modification using truncated hyperbolic chamber and a tail pipe

Figure 10 shows that the peak core pressure, $dP/dz = 0$, appears at $z/D \approx 5$. At $z/D > 5$, the $\langle P_C \rangle / \langle P_F \rangle$ decreases due to a continuous reduction of the radius of swirl chamber. For avoiding the breakdown of positive gradient of $\langle P_C \rangle / \langle P_F \rangle$ at the middle of swirl chamber, it is wise not to further reduce the radius of swirl chamber after $z/D = 5$. On the basis of this concept, the long swirl chamber with the Design-C (see Fig. 2d and Case-2 in Table 1) is cut off at $z/D = 5$ and a long cylindrical tail pipe is connected. The schematic and geometric dimensions of the modified hydrocyclone are shown in Fig. 14.

Wall profile	D (mm)	Do/D	Du/D	a/D	b/D	Li/D	Ls/D	Lt/D	Lv/D
Hyperbolic (Design-C)	26	0.07	0.53	0.154	0.385	1	4	15	0.1

Figure 14: A schematic of modified hydrocyclone and its geometric dimensions. The hydrocyclone having hyperbolic swirl chamber (Design-C) and the dimensions presented in Table 1(Case-2) is cut off at $z/D = 5$ and a tail pipe is connected.

The simulations for this design of hydrocyclone are performed for the $Re_F = 23,835$. The gauge pressure at the feed, overflow and underflow is, respectively, 187,247 Pa, 0.0 Pa and 120,000 Pa. The R_O and the ΔP^* is about 0.07 and 2.8, respectively. For this modified design, the $\langle P_C \rangle / \langle P_F \rangle$ increases gradually toward the underflow orifice and maintains a positive axial pressure gradient (Fig. 15a). The $\langle P \rangle / \langle P_F \rangle$ near the wall decreases toward the underflow due to frictional losses on the wall, which thereby decreases radial pressure gradient (Fig. 15b). The contours of reverse flow core and $\langle U_\theta \rangle / \langle U_F \rangle$, and the plot of $G_{OR}(d_D)$ are shown in Fig. 16. The reverse flow core appears up to the underflow (Fig. 16b). It indicates that this class of hydrocyclone (Fig. 14) is able to transport all the migrated droplets to the overflow orifice. Figure 14a shows that the $G_{OR}(d_D)$ increases by about 17% for the droplet size of 30 microns when compared with the full hyperbolic profile (see Fig. 2d and Case-2 in Table 1). However, no

improvement in the G_{RO} appears for the droplet size of 10 microns or less. The reason is that the straight cylindrical tail pipe significantly reduces the swirling velocity (Fig. 16c). At $z/D > 10$, the $\langle U_\theta \rangle / \langle U_F \rangle$ becomes less than the feed velocity. This lower $\langle U_\theta \rangle / \langle U_F \rangle$ reduces the migration rate of small dispersed droplets toward the center. However, using the modified design (Fig. 14), it is possible to enhance the $G_{OR}(d_D)$ for the small droplets by increasing the Re_F . Because of very long reverse flow core and longer flow residence time, the modified design (Fig. 14) should provide a higher $G_{OR}(d_D)$ for small droplets at a larger Re_F , which may not be possible in the full hyperbolic swirl chamber (Fig. 2d and Case-2 in Table 1) because of shorter flow residence time and reverse flow core.

Figure 15: Normalized static pressure along the center, $\langle P_C \rangle / \langle P_F \rangle$, and a contour of normalized static pressure, $\langle P \rangle / \langle P_F \rangle$, for the $Re_F = 23,835$. The modified truncated hyperbolic swirl chamber provides positive axial pressure gradient up to the underflow.

Figure 16: The reduced grade separation efficiency, $G_{OR}(d_D)$, plot, and contours of reverse flow core and normalized tangential velocity, $\langle U_\theta \rangle / \langle U_F \rangle$, for the $Re_F = 23,835$. The truncated hyperbolic swirl chamber with tail pipe provides about 17% higher separation efficiency than that of full hyperbolic one.

8.2. Modified hydrocyclone with frustoconical hyperbolic swirl chamber

From the manufacturing standpoint, the hyperbolic wall profile is complex for machining. For this reason, the swirl chamber (Fig. 14) is reshaped using three frustocones of different cone angle in such a way that the equivalent wall curvature of swirl chamber almost follows the hyperbolic profile. The schematic of the modified hydrocyclone using frustocones and its geometric dimensions are presented in Fig. 17. The contours of $\langle U_\theta \rangle / \langle U_F \rangle$ and reverse flow core, and a plot for the $\langle P_C \rangle / \langle P_F \rangle$ versus z/D are presented in Fig. 18. There is no considerable variation in the contours of $\langle U_\theta \rangle / \langle U_F \rangle$ and reverse flow core appears between the frustoconical profile (Fig. 18) and the hyperbolic profile (Fig. 16). Moreover, the $\langle P \rangle / \langle P_F \rangle$ profiles for the frustoconical and hyperbolic wall profiles are almost the same (Fig. 18a). These results indicate that the reshaping of hyperbolic swirl chamber using the three frustocones with cone angles presented in Fig. 17 is appropriate. Hence, the modified hydrocyclone (Fig. 17) is suitable from both high separation efficiency and simplicity in manufacturing standpoints.

D_o/D	D_u/D	a/D	b/D	L_i/D	L_{r1}/D	L_{r2}/D	L_s/D	L_t/D	L_v/D	α_1	α_2	α_3
0.07	0.55	0.154	0.385	1	0.25	0.75	3	15.23	0.1	14°	4.5°	2°

Figure 17: Schematic of the modified hydrocyclone using frustocones and its geometric dimensions. The hyperbolic swirl chamber (Fig. 14) is tailored using three frustocones (Reducer-1, Reducer-2 and Swirl chamber).

Figure 18: Illustration of normalized center pressure, $\langle P_C \rangle / \langle P_F \rangle$, versus z/D plot, and contours of reverse flow core and normalized tangential velocity, $\langle U_\theta \rangle / \langle U_F \rangle$, for the modified hydrocyclone with three frustocones (Fig. 15) for the $Re_F = 23,835$.

9. Summary

The designs of swirl chamber studied in this work have significant impact on the internal flow structures and the separation efficiency. The hyperbolic swirl chamber provides an increased swirling motion as well as develops a larger centrifugal force differential between two phases, when compared with parabolic and conical swirl chambers. Moreover, the hyperbolic swirl chamber raises the radial pressure gradient in the flow field, which also accelerates the motion of dispersed droplets toward the center. However, for a droplet size of 30 μ m, the parabolic swirl chamber with asymptote at the leading edge (Design-B) provides 8% and 25% higher separation efficiency than that of full hyperbolic (Design-C) and conical swirl chambers, respectively. Though the length of reverse flow core for the full hyperbolic swirl chamber without tail pipe is about 34% shorter than that of the parabolic chamber of the same type, their separation efficiency are comparable. Using the hyperbolic wall profile, a short hydrocyclone can be devised because of its ability to provide higher separation efficiency with a shorter reverse flow core.

For a truncated hyperbolic swirl chamber with a tail pipe, the positive axial pressure gradient and the reverse flow core appear up to the underflow orifice. This class of hydrocyclone is able to transport all the migrated droplets to the overflow orifice because of the long reverse flow core. The truncated hyperbolic swirl chamber increases the separation efficiency by 17% and 33% for the droplet size of 30 microns when compared with the full hyperbolic and conical swirl chambers, respectively. At a feed Reynolds number of 23,835, in comparison with the full hyperbolic chamber, no considerable improvement in the separation efficiency appears for the droplet size of 10 microns or less. However, the separation efficiency for the small droplets can be improved by increasing the feed Reynolds number. In addition, the hyperbolic wall profile can be simplified by using frustocones without affecting internal flow structures and separation efficiency. Hence, the hydrocyclone with frustoconical hyperbolic swirl chamber is suitable from both high separation efficiency and simplicity in manufacturing standpoints.

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Nomenclature

<i>Symbols</i>	Description	Unit
a	Width of inlet channel	mm
b	Height of inlet channel	mm
D	Diameter of swirl chamber (nominal diameter of hydrocyclone)	mm
d_{D50}	Cut size	μm
d_D	Diameter of dispersed (oil) droplet	μm
D_o	Diameter of overflow orifice	mm
D_u	Diameter of underflow orifice	mm
G_{OR}	Overflow grade separation efficiency	-
ℓ_c	Length of reverse flow core	mm
L_i	Length of inlet chamber	mm
L_s	Length of swirl chamber	mm
L_t	Length of tail pipe	mm
P	Pressure	Pa

P_C	Static pressure at the center of hydrocyclone	Pa
P_W	Static pressure at the wall	Pa
Q	Flow rate	m^3/s
R	Radius (nominal) of hydrocyclone	mm
Re	Reynolds number	-
R_O	Overflow flow ratio	-
U	Velocity	m/s
u	Fluctuating velocity	m/s
μ	Viscosity	$Pa-s$
ρ	Density	kg/m^3

Subscripts

C	Continuous phase (water)
D	Dispersed phase (oil)
F	Feed
O	Overflow
U	Underflow
z	Longitudinal (axial) component
r	Radial component
θ	Tangential component

Operators

$\langle \rangle$	Ensemble average
—	Vector component
$\cdot$	Inner product

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Appendix-A
Effect of grid resolution on the numerical solution

Figure A-1: An O-grid structured hexahedral mesh generated by using ANSYS ICEM

(a) Tangential velocity at $z/D=8.37$

(b) Axial velocity at $z/D=8.37$

(c) Static pressure at $z/D=8.37$

(d) Reduced grade efficiency

Figure A-2: Effect of mesh refinement on the (a) normalized tangential velocity, (b) normalized axial velocity, (c) normalized static pressure at $z/D=8.37$, and (d) reduced grade separation efficiency.