Abstract

In his research, a modified Dynamic Causal Diagram (DCD) based process operating performance optimality assessment and non-optimal cause identification method is proposed to handle the online assessment of the complicated chemical processes, where certainties and uncertainties coexist. In addition, the multi-source information usage problem of the traditional DCD is addressed. In the online application, a newly developed scheme is developed, which distinguishes stable performance grade and grades switch transition. For the non-optimal performances, a contribution based root cause identification method is developed, on the framework of the constructed DCD. Finally, the operating performance of a gold hydrometallurgy process is assessed by both the traditional DCD and the modified DCD. The superiority on accuracy and robustness of the modified DCD is demonstrated, and the responsible events for non-optimality is successfully isolated by the proposed contribution based non-optimal cause identification technique.

Keywords Process operating performance assessment; Dynamic Causality Diagram (DCD); Uncertainties.

1. Introduction

With the rapid development of industrial production technology in the past several decades, the demand for quality and benefits of products increases accordingly. Optimal operating performance is helpful to improve economic benefit, manage production, avoid environmental pollution, and ensure efficient production. On condition of safety, Process Operating Performance Optimality Assessment (POPOA)

further assesses the optimal degree of process operating status and gives guidance on non-optimality causes. Compared to the traditional process monitoring which portions the process into normal and abnormal situations, POPOA further divides the normal situation into several grades, such as optimal, medium and poor, under synthetically consideration of the quality, raw material costs, and energy consumption etc. Therefore, POPOA has become a key issue that has gained tremendous attention from both academia and industry recently. In general, process operating performance finally reflects on Comprehensive Economic Benefit (CEB), which is a balance between the quality and cost. It is reasonable to take CEB as comprehensive assessing index for POPOA. According to the state of CEB, operating performance is classified into multiple grades, such as optimal, good, medium, not bad, and poor grades. The CEB can be viewed as being the higher the better. Factories pursue the optimal operating performance grade during production, to achieve the highest CEB.

Multivariate statistical methods based POPOA techniques are intensively investigated due to the development of the measurement techniques (Liu et al, 2014, 2015a, 2015b, 2016). The multivariate statistical methods extract characteristic information that is related to the process operating performance, through process operation data. Reliable measurement of process data is essential to this category of approaches. It is challenging to perform POPOA based on the multivariate statistical methods under a large amount of uncertainties.

In an actual industrial production process, coexistence of certainties and uncertainties is common. In the following content, the variables that are measured

with high Signal to Noise Ratio (SNR) are called certain variables. Meanwhile, the variables that are measured with low SNR or judged through subjective interpretation like semantic descriptions are called uncertain variables. To handle such uncertainties, some qualitative approaches are employed, such as process hazards analysis (Srinivasan and Venkatasubramanian, 1998), hazards and operability analysis (Paul, 2003), qualitative simulation (Wang et al, 1996), rough set (Slowinski and Zopounidis, 1995; Liu and Zhu, 2006), and fuzzy theory (Liu et al, 2013) etc. Hard partition method is adopted to performance grade division by a majority of the conventional qualitative approaches, which results in information loss between different performance grades.

Another kind of methods to tackle uncertainties is based on the probabilistic framework, such as Belief Network (BN) (Pearl, 1986) and Markov Network (Pearl, 2014) etc. Among them, BN which is developed by Professor Pearl, is the most widely used one. An improving version of BN, called Dynamic Causality Diagram (DCD), is proposed by Professor Zhang (Zhang, 1994). DCD is a well-established mathematical tool. The traditional DCD has the following properties. 1) The Boolean set algorithm is applied in DCD. 2) Causal relations are intuitively displayed. 3) The causal intensity between the parent and child nodes is represented with the linkage event variable. 4) Causal loops are allowed. 5) Logic gates are utilized and complex variable relations are easily and clearly described. 6) Each variable is classified into a set of mutually exclusive and complete states, and each state corresponds to an event. 7) The root and linkage events are independent. However, DCD has two essential

problems as illustrated below. 1) All the event variables are discrete into multiple states through hard partition, which leads to information loss or even model failure. 2) DCD cannot take full use of both measurable and reasoning information, which can be regarded as an information fusion problem and will be investigated in detail in the later section.

Information fusion problem has been deeply researched with the increase in the amount of available information sources. It aims at combining multi-source information to form consistent explanation or description. A large number of the researches have been reported to data fusion technologies on Bayesian theory (Fasbender et al, 2008) and Dempster-Shafer Theory of Evidence (DST) (Mu et al, 2008; Xu and Liu, 2005). Bayesian theory has strict axiomatic basis and mathematical properties. Only moderate computing time is required for Bayesian theory, whereas, prior knowledge is necessary. DST is developed as a method of qualitative reasoning under uncertainties. Belief and plausibility functions are defined to describe a random interval. Therefore, the uncertain and unknown information is expressed without prior knowledge.

This research mainly concentrates on POPOA problems with multi-source information, where certainties and uncertainties coexist. To address this problem, a modified reasoning approach based on DCD for POPOA is developed in this study.

The main contributions of this research are as follows:

- The traditional DCD is extended to solve its essential problems that it cannot combine multi-source information. When a node can be measured online as

well as derived through its father nodes, the modified DCD can make full use of both information sources.

- The certain and uncertain variables are handled separately to preserve useful information as much as possible. Hard partition of the states is avoided for certain variables in this article.
- A novel online assessing strategy is developed to distinguish the stable performance grade and performance grade shifting transition. A contribution based non-optimal root cause identification approach is further proposed to identify the reason for the non-optimal performance.
- The proposed approach is successfully applied to a gold hydrometallurgy process. Compared to the assessment result of the traditional DCD, the superiority of the novel approach is illustrated.

The remainder of this article is organized as below. Primary techniques to be utilized in this research are briefly revisited in Section 2. In Section 3, the novel reasoning method of DCD named Fuzzy DCD based on DST (FDbD) is proposed to characterize complex process with coexistence of certainties and uncertainties. The novel POPOA scheme on basis of FDbD and the corresponding non-optimal cause identification is established in Section 4. The proposed POPOA approach is applied to a gold hydrometallurgical process to demonstrate its validity and effectiveness in Section 5. Finally, the conclusions and future work are outlined in Section 6.

2. Preview of primary techniques

2.1. Background of this research

This research is aimed at implementing POPOA with multi-source information under coexistence of certainties and uncertainties. Certainties indicate the information that is deterministic or achieved with high SNR. Uncertainties mainly include the following two kinds: 1) the measurement uncertainty with low SNR; 2) subjective interpretation or estimation of the variable. In actual production processes, certain and uncertain information are usually mixed together. Such coexistence renders the traditional POPOA methods inappropriate.

For uncertain variables, online information may come from different information sources, like online measurements from different sensors, inference through the effecting variables, and so on. It is hard to judge if one single information source reflects the variable's state truthfully, because any one of the information source is uncertain. Determining the performance of an uncertain plant within a single aspect is incomplete. It is better to fuse multi-source information to form a relatively reliable result.

To address the above issues, a novel approach is proposed in this research, and the basic techniques are briefly introduced in this section.

2.2. Review of DCD

The uncertainty is a crucial issue to be addressed in the actual POPOA. The probabilistic framework based method is a most widely used technique to handle uncertain information. To avoid the determination of the joint probability which requires large amounts of computation, DCD emerges.

Developed from BN, DCD is more suitable for complicated processes which

lacks prior knowledge. The parameters are relatively easier to be learned, and intricate logics are illustrated easily. The DCD can be either based on events or event variables. The term ‘variable’ indicates an event variable that consists of a group of mutually exclusive and complementary events. Each event represents an instance or a state of a variable. The variable based DCD is introduced then. The event based DCD is a special case of the variable based DCD. As shown in Fig. 1, the symbols are explained below (Zhang, 1994). The beginning of a DCD is the root node, and the ending of a DCD is the leaf node. A direct cause of a node is its father node, and a direct influence of a node is its child node.

Box: a root event variable that does not have any parent nodes.

Circle: a node event variable that has at least one parent node.

Gate: a logic operator that switches a series of inputs into a single output, e.g.

AND, OR, etc.

Directed arc: a linkage event variable that describes the causal intensity between the parent and child nodes.

Fig. 1 - Example 1 of DCD.

Fig. 2 - Example 2 of DCD.

The arcs pointing to one node without any gates are in an OR relationship. All the root event variables and linkage event variables are independent in DCD. Their

probabilities are supposed to be known, which can be easily obtained from the subjective belief or historic data analysis.

The letters used in DCD theory are explained as follows (Zhang, 1994).

B_i is a root event variable indexed by i .

X_i is a node event variable indexed by i .

G_i is a gate event variable indexed by i .

V_i is any one of B_i , X_i and G_i , and $V_i^{k_i}$ is the k_i -th event of V_i .

P_{ij} is a linkage event variable from V_i to V_j , $P_{i,j}^{k_i,k_j}$ is the linkage event of $V_i^{k_i}$ causing $V_j^{k_j}$, and $P_{i,j}^{k_i,k_j}$ can be shortened as $P_{ij}^{k_i,k_j}$ if there is no ambiguity.

An arc without a letter on it implies that the father node and the child node have the same number of states, and the probability of the linkage event variable is a unit matrix.

Assume that V_i has K_i states, V_j has K_j states, and V_i is a parent node of V_j . Then

$$\begin{aligned} V_i &= [V_i^1, V_i^2, \dots, V_i^{K_i}]^T & V_j &= [V_j^1, V_j^2, \dots, V_j^{K_j}]^T \\ P_{ij} &= \begin{bmatrix} P_{ij}^{11} & P_{ij}^{21} & \dots & P_{ij}^{K_i 1} \\ P_{ij}^{12} & P_{ij}^{22} & \dots & P_{ij}^{K_i 2} \\ \vdots & \vdots & \ddots & \vdots \\ P_{ij}^{1K_j} & P_{ij}^{2K_j} & \dots & P_{ij}^{K_i K_j} \end{bmatrix}_{K_j \times K_i} \end{aligned} \quad (1)$$

And the following equations hold

$$\sum_{k=1}^{K_i} \Pr\{V_i^{k_i}\} = 1 \quad \sum_{k=1}^{K_j} \Pr\{V_j^{k_j}\} = 1 \quad \sum_{k_j=1}^{K_j} \Pr\{P_{ij}^{k_i,k_j}\} = 1 \quad (2)$$

where $\Pr\{\cdot\}$ is the probability of $\cdot$.

During the probabilistic reasoning under the framework of DCD, Cut Set (CS)

needs to be outlined at first. A node with m parent nodes is decomposed into m CSs, where a CS contains a group of variables with AND relation, and the CSs are in OR relation. The construction of CSs is similar to the fault-tree techniques (Barlow, 1973; Henley and Kumamoto, 1981) except that the linkage event variables are ANDed with their inputs. If CSs of a node consist of its parent nodes and corresponding linkage event variables, the CSs are called level-1 CSs, written as CSs-1. If CSs have no node event variables, i.e. they only include the root and linkage event variables, such CSs are called final level CSs, denoted by CSs-f. To obtain CSs-f, the neighbor node event variables in CSs-1 are replaced with their CSs until they are all root and linkage event variables.

If causal loops occur, Rule 1 is utilized (Zhang, 1994). Rule 1 demonstrates that a node event variable cannot be the ancestor of itself. Hence, when a causal loop occurs, break down the loop at the repeating node, and treat its descendant nodes as null sets (Zhang, 1994). When such a causality loop is encountered, the descendant variable that is the same as the head variable of the loop is treated as a null set vector $\mathbf{0} = [0\ 0 \cdots 0]^T$. It implies that when X_i is encountered as a descendant in expanding X_i , the descendant X_i is treated as $\mathbf{0}$. Then the term including the descendant X_i in expanding X_i equals $\mathbf{0}$ and is deleted.

The DCD in Fig. 1 is taken as an example to explain the rule to deal with causal loops. The cut sets of each node event variables are present as below.

CSs-1:

$$\begin{aligned} X_5 &= P_{45} B_1 \cup P_{65} X_6 \\ X_6 &= P_{56} X_5 \end{aligned}$$

CSs-f:

$$\begin{aligned} X_5 &= P_{45} B_1 \cup P_{65} (P_{56} X_5) = P_{45} B_1 \cup P_{65} (P_{56} \mathbf{0}) = P_{45} B_1 \\ X_6 &= P_{56} X_5 = P_{56} (P_{45} B_1 \cup P_{65} X_6) = P_{56} (P_{45} B_1 \cup P_{65} \mathbf{0}) = P_{56} P_{45} B_1 \end{aligned}$$

CSs-f of a node X are written as

$$\mathbf{X} = \bigcup_{l=1}^L \mathbf{C}_l \quad \text{and} \quad \mathbf{C}_l = \bigcap_{s=1}^{S_l} \mathbf{V}_{ls} \quad (3)$$

where $\mathbf{C}_l$ is the l -th CS of $\mathbf{X}$, $\mathbf{V}_{ls}$ is the s -th element in $\mathbf{C}_l$, L is the number of $\mathbf{C}_l$, S_l is the number of $\mathbf{V}_{ls}$, $\bigcup$ is the OR logical gate, $\bigcap$ is the AND logical gate, and $\bigcap$ can be omitted.

Furthermore, during the computation, the following laws hold.

- 1) $V_i^{k_i} V_i^{k_i} = V_i^{k_i}$
- 2) $V_i^1 V_i^2 \dots V_i^{K_i} = 0$
- 3) $V_i^{k_i} \cup V_i^{k_i} V_j^{k_j} = V_i^{k_i}$
- 4) $V_i^{k_i} + V_i^{k_i} V_j^{k_j} = V_i^{k_i}$

where $V_i^{k_i}$ is the k_i -th state of $\mathbf{V}_i$, and K_i is the number of $\mathbf{V}_i$'s states.

During the online reasoning, if I_E pieces of evidence are observed, intersection of the evidence is obtained

$$E = \bigcap_{i=1}^{I_E} V_i^{k_i} \quad (4)$$

Suppose that $\mathbf{H}$ is the event variable which is concerned, and the probability of occurrence of its k -th state H^k is

$$\Pr\{H^k | E\} = \frac{\Pr\{H^k E\}}{\Pr\{E\}} \quad (5)$$

And then normalization is performed

$$\Pr\{H^k | E\} = \frac{\Pr\{H^k | E\}}{\sum_{k'=1}^K \Pr\{H^{k'} | E\}} \quad (6)$$

Hence, $\Pr\{H^k | E\}$ is the final probability of H^k given achieved evidence.

However, in terms of E and $H^k E$, Disjoint Cut Sets (DCSSs-f) based on CSs-f

requires to be done. DCSs-f of a node event $X_i^{k_i}$ is

$$\begin{aligned} X_i^{k_i} &= C_1 + C_2 \bar{C}_1 + C_3 \bar{C}_1 \bar{C}_2 + \dots + C_L \bar{C}_1 \bar{C}_2 \dots \bar{C}_{L-1} \\ \bar{C}_l &= \bar{V}_{l1} + \bar{V}_{l2} V_{l1} + \bar{V}_{l3} V_{l1} V_{l2} + \dots + \bar{V}_{lS_l} V_{l1} V_{l2} \dots V_{l(S_l-1)} \end{aligned} \quad (7)$$

where C_l is a CS and V_{ls} is an event that is included in C_l .

A simple DCD in Fig. 2 is taken as an example to show online reasoning procedures of DCD. Let $\mathbf{B}_1 = [B_1^1, B_1^2]^T$, $\mathbf{B}_2 = [B_2^1, B_2^2]^T$, $\mathbf{X}_3 = [X_3^1, X_3^2]^T$,

$$\mathbf{P}_{34} = \begin{pmatrix} P_{34}^{11} & P_{34}^{21} \\ P_{34}^{12} & P_{34}^{22} \end{pmatrix}, \quad \mathbf{P}_{24} = \begin{pmatrix} P_{24}^{11} & P_{24}^{21} \\ P_{24}^{12} & P_{24}^{22} \end{pmatrix} \text{ and } \mathbf{X}_4 = [X_4^1, X_4^2]^T.$$

CSs-1:

$$\begin{aligned} \mathbf{X}_3 &= \mathbf{B}_1 \\ \mathbf{X}_4 &= \mathbf{P}_{34} \mathbf{X}_3 \cup \mathbf{P}_{24} \mathbf{B}_2 \end{aligned} \quad (8)$$

CSs-f:

$$\begin{aligned} \mathbf{X}_3 &= \mathbf{B}_1 = \begin{pmatrix} B_1^1 \\ B_1^2 \end{pmatrix} \\ \mathbf{X}_4 &= \mathbf{P}_{34} \mathbf{B}_1 \cup \mathbf{P}_{24} \mathbf{B}_2 = \begin{pmatrix} P_{34}^{11} & P_{34}^{21} \\ P_{34}^{12} & P_{34}^{22} \end{pmatrix} \begin{pmatrix} B_1^1 \\ B_1^2 \end{pmatrix} \cup \begin{pmatrix} P_{24}^{11} & P_{24}^{21} \\ P_{24}^{12} & P_{24}^{22} \end{pmatrix} \begin{pmatrix} B_2^1 \\ B_2^2 \end{pmatrix} \end{aligned} \quad (9)$$

DCSs-f:

$$\begin{aligned} \mathbf{X}_3 &= \begin{pmatrix} B_1^1 \\ B_1^2 \end{pmatrix} \\ \mathbf{X}_4 &= \begin{pmatrix} P_{34}^{11} B_1^1 + P_{34}^{21} B_1^2 + P_{34}^{12} B_1^1 P_{24}^{11} B_2^1 + P_{34}^{12} B_1^1 P_{24}^{21} B_2^2 + P_{34}^{22} B_1^2 P_{24}^{11} B_2^1 + P_{34}^{22} B_1^2 P_{24}^{21} B_2^2 \\ P_{34}^{12} B_1^1 + P_{34}^{22} B_1^2 + P_{34}^{11} B_1^1 P_{24}^{12} B_2^1 + P_{34}^{11} B_1^1 P_{24}^{22} B_2^2 + P_{34}^{21} B_1^2 P_{24}^{12} B_2^1 + P_{34}^{21} B_1^2 P_{24}^{22} B_2^2 \end{pmatrix} \end{aligned} \quad (10)$$

Let X_4 be the event variable which is concerned. Assume that B_1^1 and B_2^1 are

observed. Then

$$\begin{aligned}
E &= B_1^1 B_2^1 \\
H^1 E &= X_4^1 B_1^1 B_2^1 \\
&= (P_{34}^{11} B_1^1 + P_{34}^{21} B_1^2 + P_{34}^{12} B_1^1 P_{24}^{11} B_2^1 + P_{34}^{12} B_1^1 P_{24}^{21} B_2^2 + P_{34}^{22} B_1^2 P_{24}^{11} B_2^1 + P_{34}^{22} B_1^2 P_{24}^{21} B_2^2) B_1^1 B_2^1 \\
&= P_{34}^{11} B_1^1 B_2^1 + P_{34}^{12} B_1^1 P_{24}^{11} B_2^1 \\
H^2 E &= X_3^2 B_1^1 B_2^1 \\
&= (P_{34}^{12} B_1^1 + P_{34}^{22} B_1^2 + P_{34}^{11} B_1^1 P_{24}^{12} B_2^1 + P_{34}^{11} B_1^1 P_{24}^{22} B_2^2 + P_{34}^{21} B_1^2 P_{24}^{12} B_2^1 + P_{34}^{21} B_1^2 P_{24}^{22} B_2^2) B_1^1 B_2^1 \\
&= P_{34}^{12} B_1^1 B_2^1 + P_{34}^{11} B_1^1 P_{24}^{12} B_2^1
\end{aligned} \tag{11}$$

Hence

$$\begin{aligned}
\Pr\{E\} &= \Pr\{B_1^1\} \Pr\{B_2^1\} \\
\Pr\{H^1 E\} &= \Pr\{P_{34}^{11}\} \Pr\{B_1^1\} \Pr\{B_2^1\} + \Pr\{P_{34}^{12}\} \Pr\{B_1^1\} \Pr\{P_{24}^{11}\} \Pr\{B_2^1\} \\
\Pr\{H^2 E\} &= \Pr\{P_{34}^{12}\} \Pr\{B_1^1\} \Pr\{B_2^1\} + \Pr\{P_{34}^{11}\} \Pr\{B_1^1\} \Pr\{P_{24}^{12}\} \Pr\{B_2^1\}
\end{aligned} \tag{12}$$

and

$$\begin{aligned}
\Pr\{H^1 | E\} &= \frac{\Pr\{H^1 E\}}{\Pr\{E\}} \\
\Pr\{H^2 | E\} &= \frac{\Pr\{H^2 E\}}{\Pr\{E\}}
\end{aligned} \tag{13}$$

Subsequently normalization is performed

$$\begin{aligned}
\Pr\{H^1 | E\} &= \frac{\Pr\{H^1 | E\}}{\Pr\{H^1 | E\} + \Pr\{H^2 | E\}} \\
\Pr\{H^2 | E\} &= 1 - \Pr\{H^1 | E\}
\end{aligned} \tag{14}$$

$$\text{If } \Pr\{B_i^{k_i}\} = 0.5 \quad , \quad \Pr\{\mathbf{P}_{34}\} = \begin{pmatrix} \Pr\{P_{34}^{11}\} & \Pr\{P_{34}^{21}\} \\ \Pr\{P_{34}^{12}\} & \Pr\{P_{34}^{22}\} \end{pmatrix} = \begin{pmatrix} 0.9 & 0.15 \\ 0.1 & 0.85 \end{pmatrix} \quad , \quad \text{and}$$

$$\Pr\{\mathbf{P}_{24}\} = \begin{pmatrix} \Pr\{P_{24}^{11}\} & \Pr\{P_{24}^{21}\} \\ \Pr\{P_{24}^{12}\} & \Pr\{P_{24}^{22}\} \end{pmatrix} = \begin{pmatrix} 0.95 & 0.3 \\ 0.05 & 0.7 \end{pmatrix} \quad , \quad \text{then} \quad \Pr\{H^1 | E\} = 0.87 \quad \text{and}$$

$$\Pr\{H^2 | E\} = 0.13 .$$

2.3. Dempster-Shafer Theory of Evidence (DST)

Dempster-Shafer Theory of Evidence is first proposed by Dempster (Dempster,

1967) and then enhanced by Shafer (Shafer, 1976). Compared to the traditional Bayesian theory based information fusion technique, DST avoids determination of prior probabilities.

A frame of discernment Θ is a set of mutually exclusive and exhaustive propositional hypotheses (Ali et al, 2012). Let 2^Θ be the powerset of Θ . A basic belief assignment (bba) is

$$m: 2^\Theta \rightarrow [0,1] \quad (15)$$

m is also called mass function that assigns masses of belief to subsets of 2^Θ . The following two conditions hold:

$$\begin{aligned} m(\emptyset) &= 0 \\ \sum_{A \subseteq \Theta} m(A) &= 1 \end{aligned} \quad (16)$$

where $\emptyset$ is an empty set and A is a subset of Θ .

Given a set of bba m_q ($q=1,2,\dots,Q$) where Q is the number of information sources, combination rule of DST is

$$\begin{aligned} m(B) &= \frac{\sum_{A_1 \cap \dots \cap A_Q = B} m_1(A_1)m_2(A_2)\dots m_Q(A_Q)}{1 - \sum_{A_1 \cap \dots \cap A_Q = \emptyset} m_1(A_1)m_2(A_2)\dots m_Q(A_Q)} \\ (A_q &\subseteq \Theta; m(\emptyset) = 0) \end{aligned} \quad (17)$$

where $K = \sum_{A_1 \cap \dots \cap A_Q = \emptyset} m_1(A_1)m_2(A_2)\dots m_Q(A_Q)$ is the conflict between different information sources, and $m(B)$ is comprehensive bba of event B that combines I information sources.

3. Fuzzy DCD based on DST

3.1. Motivation of improving DCD

DCD deals with uncertainties in a probabilistic way. It has been widely used in actual production where uncertainties exist. Coexistence of certainties and uncertainties is common in real production. In this study, the certain information is obtained online with high SNR. Conversely, the uncertain information is measured with low SNR, expressed through semantic descriptions, or evaluated subjectively. It indicates that the measurements of the certain variables are reliable, while those of the uncertain variables are unreliable. Uncertain variables may be achieved by various sources, such as inference by the father nodes, experts' evaluation, rough measurements and so on. A single information source is not reliable enough. The traditional DCD has a problem of taking full use of all the information sources. It is necessary to fuse the valuable information sources to obtain a relatively reliable result. Besides, the traditional DCD has another problem of hard partition. It classifies every variable into multiple exclusive states by hard partition, hence useful information and precision during discretization procedure are lost.

To solve the valuable information usage problem, DST is added into the traditional DCD. To reduce information loss during hard partition, Fuzzy membership is used for the certain variable states classification. Fuzzy DCD based on DST (FDbD) will be proposed in this section.

3.2. Data processing

In the traditional DCD, both certain and uncertain variables are discrete into several states through hard partition. Discretion of certain variables inevitably results

in information loss. Hence, the traditional DCD cannot ensure precision or even makes wrong reasoning result. To reduce the information loss from hard partition, certain and uncertain variables are handled separately for further reasoning in this research.

Obviously, the value/state of a certain variable is accurately known. Therefore, certain nodes are root nodes without any father nodes. Compared to the traditional DCD, the proposed FDbD adds certain root event variable to the DCD, lettered by $\tilde{\mathbf{B}}_i$, as illustrated in Fig. 3. For a certain variable, discretion is performed through membership function. Instead of traditional hard partition, the state of a certain variable is determined with a membership degree between 0 and 1 in the proposed FDbD. For an uncertain variable, similar to the traditional DCD, the variable is discrete into a series of mutually independent and complementary events by hard partition.

A linkage event variable that represents V_i causing V_j , is written as $\mathbf{P}_{ij}$. Let $\mathbf{V}_i = [V_i^1, V_i^2, \dots, V_i^{K_i}]^T$ and $\mathbf{V}_j = [V_j^1, V_j^2, \dots, V_j^{K_j}]^T$, where $V_i^{k_i}$ ($V_j^{k_j}$) is the k_i (k_j)-th state of V_i (V_j), K_i (K_j) is the state number of V_i (V_j), and V_i is a direct cause of V_j . Therefore, $\mathbf{P}_{ij}$ is a $K_j \times K_i$ dimensional matrix, where $P_{ij}^{k_i k_j}$ is the causal intensity of $V_i^{k_i}$ causing $V_j^{k_j}$, and $k_j = 1, 2, \dots, K_j$.

3.3. Offline parameter determination

The construction of DCD requires process knowledge, which can be obtained through experts' experiences. Then offline parameter determination is focused on, where the parameter indicates $\Pr\{\mathbf{P}_{ij}\}$. Actually $\Pr\{\mathbf{B}_i\}$ and $\Pr\{\tilde{\mathbf{B}}_i\}$ are changing

with real-time information, while $\Pr\{\mathbf{P}_{ij}\}$ is not. $\Pr\{\mathbf{P}_{ij}\}$ can be determined through either experts' experiences or data statistics. The former is subjective. The latter requires a large amount of data.

$P_{ij}^{k_i k_j}$ is an instance of $\mathbf{P}_{ij}$. After sufficient data are collected, the number of samples is counted then. Suppose that $N_i^{k_i}$ samples are at the k_i -th state of $\mathbf{V}_i$, among which $N_j^{k_j}$ samples are at the k_j -th state of $\mathbf{V}_j$. Hence the probability of $P_{ij}^{k_i k_j}$ can be achieved as

$$\Pr\{P_{ij}^{k_i k_j}\} = \frac{N_j^{k_j}}{N_i^{k_i}} (k_i = 1, 2, \dots, K_i; k_j = 1, 2, \dots, K_j) \quad (18)$$

where K_i and K_j are respectively the state number of $\mathbf{V}_i$ and $\mathbf{V}_j$.

3.4. Online information acquirement

In this subsection, the online probabilities are determined.

3.4.1. The probability of a certain root event variable $\Pr\{\tilde{\mathbf{B}}_i\}$

$\Pr\{\tilde{\mathbf{B}}_i\} = \Pr\{[\tilde{B}_i^1, \tilde{B}_i^2, \dots, \tilde{B}_i^K]^T\} = [\Pr\{\tilde{B}_i^1\}, \Pr\{\tilde{B}_i^2\}, \dots, \Pr\{\tilde{B}_i^K\}]^T$ is the probability of each state of $\tilde{\mathbf{B}}_i$, where K is the number of states for $\tilde{\mathbf{B}}_i$. For a certain root event variable, the measurement of $\tilde{\mathbf{B}}_i$ is relatively precise. The traditional DCD classifies $\tilde{\mathbf{B}}_i$ into a unique state through critical values, namely hard partition. Such classification inevitably leads to loss of information. Fuzzy membership function is employed to take the place of traditional $\Pr\{\tilde{B}_i^k\}$.

Fuzzy membership function is a concept in Fuzzy theory that is put forward by Zadeh¹⁰. Fuzzy membership $\mu_C(x)$ describes belonging degree of an element x to a set C . x may partially belongs to C . When the measurement of $\tilde{\mathbf{B}}_i$ is obtained,

$\mu_{\tilde{B}_i^k}$ ($k=1,2,\dots,K$) is computed. The form of $\mu_{\tilde{B}_i^k}$ is determined by the data characteristics of variable $\tilde{B}_i$. Among all the membership functions, triangular and trapezoidal ones are the most widely used. Through membership function, hard classification is avoided. Hence, $\mu_{\tilde{B}_i^k}$ is substituted for $\Pr\{\tilde{B}_i^k\}$.

3.4.2. The probability of an uncertain event variable $\Pr\{V_i\}$

In actual processes, limited to the drawback of measurement techniques and production circumstances, there are variables that cannot be precisely achieved. Such variables are called the uncertain variables. $\Pr\{V_i\} = \Pr\{[V_i^1, V_i^2, \dots, V_i^K]^T\} = [\Pr\{V_i^1\}, \Pr\{V_i^2\}, \dots, \Pr\{V_i^K\}]^T$ is the probability of each state of an uncertain variable V_i , where V_i represents B_i or X_i , and K is the number of states for V_i . V_i may have multiple information sources for its online state determination. The information sources are denoted as $E_{i,1}, E_{i,2}, \dots, E_{i,Q}$, where Q is the number of information sources.

In this paper, DST is employed to estimate V_i 's state when it has more than one information sources. Then the frame of discernment is expressed as $\Theta = \{V_i^1, V_i^2, \dots, V_i^K\}$. The degree of $E_{i,q}$ that supports V_i in its k -th state is denoted by the mass function $m_q(V_i^k)$ ($q=1,2,\dots,Q$). If there are sufficient historic data, $m_q(V_i^k)$ is determined through statistical analysis. Otherwise, $m_q(V_i^k)$ is determined by experts' knowledge, which has strong subjectivity. However, there is no strict guidance for choosing $m_q(V_i^k)$.

After $m_q(V_i^k)$ is achieved, DST is adopted to fuse multi-source information (Dempster, 1967; Shafer, 1976; Ali et al, 2012)

$$\begin{aligned}
m(V_i^k) &= \frac{\sum_{A_1 \cap \dots \cap A_Q = V_i^k} m_1(A_1)m_2(A_2)\dots m_Q(A_Q)}{1 - \sum_{A_1 \cap \dots \cap A_Q = \emptyset} m_1(A_1)m_2(A_2)\dots m_Q(A_Q)} \\
&= \frac{m_1(V_i^k)m_2(V_i^k)\dots m_Q(V_i^k)}{1 - \sum_{A_1 \cap \dots \cap A_Q = \emptyset} m_1(A_1)m_2(A_2)\dots m_Q(A_Q)} \\
&\quad (A_q \subseteq \Theta; m(\emptyset) = 0)
\end{aligned} \tag{19}$$

where $\sum_{k=1}^K m_q(V_i^k) = 1$, i.e., the bba of subsets of Θ all equal to zero except for $V_i^1, V_i^2, \dots, V_i^K$. Hence, the belief of each state of V_i is acquired, and the state of V_i is then judged through comparison of $m(V_i^k)$ for $k = 1, 2, \dots, K$.

This research mainly concentrates on two kinds of information sources. One is the measurement that is measured or estimated online. The other one is derived through the father nodes. It is to be noted that some uncertain event variables may have only one information source, and then DST is not necessary.

The probability of $\Pr\{\mathbf{P}_{ij}\}$ does not change online. $\Pr\{\mathbf{P}_{ij}\}$ uses the value obtained offline.

3.5. Online reasoning based on the proposed FD bD

The FD bD based online reasoning starts from the root nodes, and ends at the leaf nodes. The probabilities are calculated hierarchically. The state of a node event variable can be deduced by its father nodes. Suppose that the father nodes are mutually independent. It indicates that the probabilities of the root nodes need to be computed at first. Subsequently, the probabilities of the node event variables whose father nodes' states have been achieved are then computed through their father nodes. Similarly, the probabilities of the rest nodes are calculated.

Procedures of online reasoning is summarized below.

1) Compute the probabilities of each state of root variables. If the variable is certain, the probability is calculated as mentioned in Section 3.4.1. If the variable is uncertain, the probability is respectively calculated from each information source and then fused by DST as mentioned in Section 3.4.2.

2) Calculate the probabilities of the rest node event variables hierarchically from the root nodes to the leaf nodes. The node event variables are uncertain. The probability is separately calculated from each information source at first and then fused by DST as mentioned in Section 3.4.2. Each computation only involves the father and the child nodes, hence the calculation amount decreases largely.

3) When the probabilities of all the leaf event node variables' are obtained, the algorithm stops.

Fig. 3 - A simple example for FDbD

The simple example in Fig. 3 is brought to illustrate the proposed reasoning approach. In Fig. 3, $\tilde{B}_1$ is certain, while X_2 and X_3 are uncertain. $\tilde{B}_1$ and X_2 are measurable online. Each variable has two states. The states of X_2 can be determined through two information sources, namely measurement and reasoning. The status of X_3 is to be evaluated. Therefore, $\Pr\{\tilde{B}_1^k\}$ is computed at first, and $\Pr\{\tilde{B}_1^k\} = \mu_{\tilde{B}_1^k}$ for $k = 1, 2$. Then $\Pr\{X_2^k\} (k = 1, 2)$ is determined. The measurement result of X_2 is denoted as $m_1(X_2^k)$, and the reasoning result of X_2 is computed as $m_2(X_2^k)$, where $m_2(X_2^k)$ is computed as

$$\begin{aligned}
\mathbf{X}_2 &= \mathbf{P}_{12} \tilde{\mathbf{B}}_1 = \begin{bmatrix} P_{12}^{11} & P_{12}^{21} \\ P_{12}^{12} & P_{12}^{22} \end{bmatrix} \begin{bmatrix} \tilde{B}_1^1 \\ \tilde{B}_1^2 \end{bmatrix} = \begin{bmatrix} P_{12}^{11} \tilde{B}_1^1 + P_{12}^{21} \tilde{B}_1^2 \\ P_{12}^{12} \tilde{B}_1^1 + P_{12}^{22} \tilde{B}_1^2 \end{bmatrix} \\
\Pr \left\{ \begin{bmatrix} X_2^1 \\ X_2^2 \end{bmatrix} \right\} &= \Pr \left\{ \begin{bmatrix} P_{12}^{11} \tilde{B}_1^1 + P_{12}^{21} \tilde{B}_1^2 \\ P_{12}^{12} \tilde{B}_1^1 + P_{12}^{22} \tilde{B}_1^2 \end{bmatrix} \right\} \\
m_2(X_2^1) &= \Pr \{ P_{12}^{11} \tilde{B}_1^1 + P_{12}^{21} \tilde{B}_1^2 \} \\
m_2(X_2^2) &= \Pr \{ P_{12}^{12} \tilde{B}_1^1 + P_{12}^{22} \tilde{B}_1^2 \}
\end{aligned}$$

After $m_1(X_2^k)$ and $m_2(X_2^k)$ are achieved, the two information sources are fused as equation (19). Therefore

$$\begin{aligned}
m(X_2^1) &= \frac{m_1(X_2^1)m_2(X_2^1)}{1 - [m_1(X_2^1)m_2(X_2^2) + m_1(X_2^2)m_2(X_2^1)]} \\
m(X_2^2) &= \frac{m_1(X_2^2)m_2(X_2^2)}{1 - [m_1(X_2^1)m_2(X_2^2) + m_1(X_2^2)m_2(X_2^1)]} \\
\Pr \{ \mathbf{X}_2 \} &= [m(X_2^1), m(X_2^2)]^T
\end{aligned}$$

$\mathbf{X}_3$ is unmeasurable, so $\mathbf{X}_3$ is evaluated through reasoning with its father nodes.

The reasoning procedure of $\mathbf{X}_3$ is

$$\begin{aligned}
\mathbf{X}_3 &= \mathbf{P}_{23} \mathbf{X}_2 = \begin{bmatrix} P_{23}^{11} & P_{23}^{21} \\ P_{23}^{12} & P_{23}^{22} \end{bmatrix} \begin{bmatrix} X_2^1 \\ X_2^2 \end{bmatrix} = \begin{bmatrix} P_{23}^{11} X_2^1 + P_{23}^{21} X_2^2 \\ P_{23}^{12} X_2^1 + P_{23}^{22} X_2^2 \end{bmatrix} \\
\Pr(X_3^1) &= \Pr \{ P_{23}^{11} X_2^1 + P_{23}^{21} X_2^2 \} \\
\Pr(X_3^2) &= \Pr \{ P_{23}^{12} X_2^1 + P_{23}^{22} X_2^2 \}
\end{aligned}$$

where the probability of $\mathbf{X}_2$ is $\Pr \{ \mathbf{X}_2 \} = [m(X_2^1), m(X_2^2)]^T$, as computed above.

Finally, the state of $\mathbf{X}_3$ is determined.

4. FDbD based process operating performance optimality assessment and cause identification for non-optimality

When the proposed FDbD is explored in operating performance optimality assessment, it is assumed that the causal diagram covers all the possible steady

operating performances and different performance grades have adequate difference.

4.1. FDbD based process operating performance optimality assessment

Let $\mathbf{H}$ be the final assessing index, namely the unique leaf node event variable in FDbD for POPOA. Suppose that $\mathbf{H}$ has N states, corresponding to N performance grades. After online data $\mathbf{x}_k$ is obtained at time k , $P(H^1 | \mathbf{x}_k), P(H^2 | \mathbf{x}_k), \dots, P(H^N | \mathbf{x}_k)$ are computed through FDbD, where H^n is the n -th operating performance grade, H^1 denotes the optimal performance, H^N represents the worst performance, and $\sum_{n=1}^N P(H^n | \mathbf{x}_k) = 1$.

The schematic diagram of POPOA grade determination is shown in Fig. 4. Suppose that the initial performance grade is known. An assessing threshold α ($\alpha > 0.5$) is defined to judge whether it is a stable or transition performance grade. To avoid external interference and achieve a reliable assessment result, a M -length time sliding window is utilized, which serves as the basic analysis unit during online assessment. When $P(H^n | \mathbf{x}_{k'})$ ($k' = k, k-1, \dots, k-M+1; n = 1, 2, \dots, N$) are computed, operating performance grade is determined, under some parameters.

Fig. 4 - Schematic diagram of POPOA grade determination

The operating performance grade assessment can result in the following three cases.

Case 3: Otherwise, keep original performance grade assessment unchanged and go on assessing.

The detailed flow chart is shown in Fig. 5.

4.2. Cause identification for the non-optimal performance

When the process is operating at a non-optimal grade, cause identification needs to be done. Valid cause identification searches responsible variables for the non-optimality, therefore the operators can adjust production according to the identification guidance to bring the performance back to the optimal grade.

Inspired by the variable contribution plot methods, the assessing index $P(H^n | \mathbf{x}_k)$ ($n=1,2,\dots,N$) can also be decomposed as the summation of the event contributions. Non-optimal performances results in $P(H^{n_{\text{non-opt}}} | \mathbf{x}_k)$ over its general level under definite production conditions, where $P(H^{n_{\text{non-opt}}} | \mathbf{x}_k)$ is the probability of the current non-optimal performance grade, namely the $n_{\text{non-opt}}$ -th grade. The events whose contributions exceed corresponding limits at the optimal performance grade, take responsibilities for the non-optimality.

Due to the hierarchical structure of DCD, cause identification is performed backwardly from the leaf node to the root nodes. First of all, the responsible father nodes for the non-optimal assessing index $\mathbf{H}$ are figured out. Then similarly, the direct cause for each non-optimal event whose contribution exceeds its limit within optimal performance grade is searched, until all the responsible events are root events. Then these root events are causes for the non-optimality. This is an inverse process of online reasoning as mentioned in Section 3.5. The detailed procedure of cause

identification is illustrated below.

Each state of a node event variable T^k , can be expressed as the summation of a set of exclusive event combination, and each event combination only involves the events from the father nodes of T^k and corresponding linkage events. Hence, as illustrated in equation (3) and (7), T^k can be denoted as

$$\begin{aligned}
T^k &= C_1 + C_2 \bar{C}_1 + C_3 \bar{C}_1 \bar{C}_2 + \dots + C_{L_{T^k}} \bar{C}_1 \bar{C}_2 \dots \bar{C}_{L_{T^k}-1} \\
&= C_2 + C_1 \bar{C}_2 + C_3 \bar{C}_2 \bar{C}_1 + \dots + C_{L_{T^k}} \bar{C}_2 \bar{C}_1 \dots \bar{C}_{L_{T^k}-1} \\
&= \dots \\
&= C_l + \bar{C}_l
\end{aligned} \tag{20}$$

where T^k is any one of X^k or H^k , C_l is an event combination involving only one father node event of T^k , $\bar{C}_l$ is the remainder except for C_l , $l=1,2,\dots,L_{T^k}$, and L_{T^k} is the number of total state events of the father nodes. It is to be noted that the father nodes are supposed to be mutually independent. Hence,

$$\Pr\{T^k\} = \Pr\{C_l\} + \Pr\{\bar{C}_l\} \quad (l=1,2,\dots,L_{T^k}) \tag{21}$$

Let T^k be a non-optimal state. The contribution of C_l to T^k is defined as

$$I_{C_l}^{T^k} = \frac{\Pr\{C_l\}}{\Pr\{T^k\}} \quad (l=1,2,\dots,L_{T^k}) \tag{22}$$

The probabilities of the father node events change according to their current states, whereas, the probabilities of the linkage events do not change online. When the process is operating at a non-optimal performance, the contribution of the responsible causes for the non-optimal state T^k must exceed its limit at the optimal performance.

It is assumed that sufficient optimal data are collected. The probability density function (pdf) of contribution $I_{C_l}^{T^k}$ at the optimal performance can be determined by

Kernel Density Evaluation (KDE). Then hypothesis test is utilized online to check if C_l is responsible for the non-optimality. The following hypothesis is tested

$$H_0: C_l \text{ is not responsible for the non-optimality.}$$

Versus

$$H_1: C_l \text{ is responsible for the non-optimality.}$$

If C_l is not a cause for the non-optimality indeed, $I_{C_l}^{T^k}$ follows the distribution at the optimal performance, which is obtained offline. Hence, if $I_{C_l}^{T^k}$ follows the corresponding pdf at the optimal performance with a large probability, the null hypothesis is accepted. Otherwise, if $I_{C_l}^{T^k}$ follows the corresponding pdf at the optimal performance with a small probability, the null hypothesis is rejected, and the alternative hypothesis is accepted.

A significance level β is selected. When the process operating performance is optimal, $\Pr(I_{C_l}^{T^k} \leq \text{Lim}_{C_l}^{T^k}) = \beta$ holds, where $\text{Lim}_{C_l}^{T^k}$ is determined through the corresponding pdf and the significance level. Once the contribution $I_{C_l}^{T^k}$ exceeds the limit $\text{Lim}_{C_l}^{T^k}$, $\Pr(I_{C_l}^{T^k} > \text{Lim}_{C_l}^{T^k}) = 1 - \beta$ holds. In this case, it is obvious that C_l follows the corresponding pdf at the optimal performance with a small probability, then such event combination C_l is a candidate of cause for the non-optimality.

To be more intuitive, the contribution index is redefined as

$$I_{C_l}^{T^k} = \frac{\Pr\{C_l\} / \Pr\{T^k\} - \text{Lim}_{C_l}^{T^k}}{\text{Lim}_{C_l}^{T^k}} \quad (23)$$

If $I_{C_l}^{T^k} \leq 0$, C_l has nothing to do with the non-optimality. If $I_{C_l}^{T^k} > 0$, C_l is a cause for the non-optimality.

5. Case studies

Compared to the pyrometallurgy, the gold hydrometallurgy is environmentally friendly and high efficient. The production of gold hydrometallurgy is a rather complex chemical process. It is difficult to establish the mechanism model. Besides, limited to the measurement techniques, the measurements of gold hydrometallurgy process contain considerable uncertainties. A typical gold hydrometallurgy process consists of the cyanide leaching, filter pressing, replacement, and refining sub-processes. The cyanide leaching sub-process transforms the solid gold in the ore into liquid gold cyanide complex ion in the pulp. The filter pressing sub-process performs solid-liquid separation. Then the majority of the gold is in the liquid pulp, which is called the pregnant solution, and the rest solid slag contains only tiny amounts of gold. The pregnant solution is then transported to the replacement sub-process, where the gold is replaced by the zinc, and then the gold slime is obtained. The gold slime contains the gold and impurities and further refining process is required to purify the gold slime to achieve the gold with high-purity. Operating performance of the cyanide leaching sub-process directly affects the leaching rate and leaching cost, which are important influencing factors of CEB. The filter pressing sub-process is closely related to the cyanide leaching sub-process. In this section, the cyanide leaching process and the filter pressing sub-process from a gold hydrometallurgy are to be assessed. The POPOA efficiency based on the traditional DCD and the proposed FDbD are compared.

A large number of variables are present in the two sub-processes. CEB is taken as the final assessing index. There are two operating performance grades, i.e., the optimal grade and the non-optimal grade, corresponding to State 1 and 2 of CEB. The manipulated variables ought to be kept in the proper amount to make the operating performance optimal. The sampling time is 2.5 minutes. Sixteen variables that are closely related to the assessing index are selected as the process variables. The name, symbol and online data type of the variables' are tabulated in Table. 1. The causal diagram is established by process knowledge, as shown in Fig. 6. To illustrate the method intuitively and easily, the process variables are categorized into two states. The value of a variable that is larger than its average level, is classified into State 1, and the value of a variable that is not larger than its average level, is classified into State 2. To determine the parameters of the causal diagram for POPOA, 5000 samples are collected under each performance grade. The training data cover all the possible situations within different operating performances.

To illustrate the validity of the proposed assessing approach for processes under coexistence of certainties and uncertainties, three scenarios are designed, as briefly illustrated in Table. 2. In the first two cases, the operating performance is shifting from the optimal to non-optimal grade with corresponding reasons. In the last case, operating performance stays at the optimal performance till the end. The detailed design will be outlined below. Each case is assessed by the traditional DCD and the proposed FDbD respectively with the same assessing strategy as proposed in Section 4.1. When the process is assessed to be non-optimal, cause identification is

implemented based on the proposed approach in this article. Two hundred samples are assessed in each experiment. Taken the considerable uncertainties into consideration, the parameters are defined as $\alpha=0.6$, $\beta=0.85$, and $M=5$. The assessing and cause identification of the non-optimality results of the three experiments are respectively displayed in Fig. 7 ~ Fig. 9. In the performance grade determination chart, the grade number 1 indicates the optimal performance, the grade number 2 indicates the non-optimal performance, and the grade number 1.5 indicates a performance grade shifting transition.

Fig. 6 - DCD of the cyanide leaching process.

In Case 1, 200 samples are assessed. The first 100 samples are operating at the optimal level. Since the 100th sample, the original throughput of the ore (B_2) increases. Accordingly, the operators ought to increase valve opening of the NaCN

dosing machine (B_4), to make addition of the NaCN (X_8) at the optimal level. Nevertheless, the operators do not adjust the valve in time. Therefore, the performance turns to be non-optimal gradually since the 100th sample. The performance shifting transition ends at the 110th sample, and the non-optimal performance starts at the 111th sample. Valve opening of the NaCN dosing machine is a certain variable that is measured accurately. Theoretically, addition of NaCN has simple positive correlation with valve opening of the NaCN dosing machine, i.e. the large valve opening leads to the large amount of NaCN. However, practically, the transmission pipelines of NaCN are usually blocked due to crystallization. It implies that addition of NaCN cannot be evaluated by the valve opening of the NaCN dosing machine. The actual addition of NaCN is analyzed every 8 hours, and only the cumulative amount within the 8 hours is evaluated. And in this case, its state is correctly estimated. The whole process is assessed by the traditional DCD and the proposed FDbD respectively, as shown in Fig. 7. It appears that both approaches detect the performance transforming. However, the traditional DCD assesses the non-optimal with more delay than FDbD does. Furthermore, FDbD figures out the transition between two performance grades. When the process is assessed to be non-optimal at the 105-th sample based on FDbD, the cause identification is implemented, which indicates that the responsible variable is the valve opening of the NaCN dosing machine (B_4). The identifying result figures out that the low level of valve opening of the NaCN dosing machine results in the non-optimality. The identification matches the fact. This experiment shows that when the qualitative states

are correctly estimated, FDbD is more sensitive than the traditional DCD, and FDbD can distinguish the stable performance and the performance shifting transition.

In Case 2, 200 samples are assessed. The first 99 samples are operating at the optimal level. Since the 100th sample, the original throughput of the ore (B_2) increases. But the operators do not adjust the dosing machine valve (B_4) in time. The experiment set is the same as Case 1. But in this case, the addition amount of NaCN (X_8) is estimated to be in a wrong state. 200 samples are assessed by the traditional DCD and the proposed FDbD respectively, as shown in Fig. 8. Because the state of addition of NaCN is not evaluated correctly, the traditional DCD does not figure out performance change in this situation. In contrast, although information of addition of NaCN is not correct, FDbD fused the online measurement information and the reasoning information from valve opening of the NaCN dosing machine. Hence, FDbD accurately assesses the operating performance grade. When the process is assessed to be non-optimal at the 105-th sample based on FDbD, cause identification is implemented. Cause identification shows a similar result as Case 1. In a word, when the qualitative states are wrongly estimated, the traditional DCD may make erroneous conclusions, whereas, FDbD makes relatively accurate assessment. It implies that FDbD has strong robustness.

In Case 3, 200 samples are assessed. The 200 samples are all operating at the optimal performance. Due to a lack of maintenance for a rather long time, the measurements of dissolved oxygen content (X_{11}) contains large amounts of noise. The process is respectively assessed by the traditional DCD and the proposed FDbD,

as shown in Fig. 9. It can be observed that the performance grade assessment based on the traditional DCD makes wrong judgement for 20 samples due to measurement noise. In contrast, the result that is assessed by the proposed FDbD makes correct judgement during the whole experiment. Furthermore, the original probabilities of each performance grade is concentrated on, where the probabilities that are obtained by FDbD are relatively stable, compared to those obtained by the traditional DCD. In general, when the measurements contain a large amount of noise, compared to the traditional DCD, FDbD makes a relatively true and stable judgement.

Fig. 7 - Chart of POPOA result for Case 1.

Fig. 8 - Chart of POPOA result for Case 2.

Fig. 9 - Chart of POPOA result for Case 3.

Table 1 - Variables for assessment in the gold hydrometallurgical process.

Index	Name	Symbol	Data type
1	Average particle size of the ore	B_1	Uncertain
2	Original throughput of the ore	B_2	Uncertain
3	Water flow of the pulp conditioning	B_3	Certain
4	Valve opening of the NaCN dosing machine	B_4	Certain
5	Initial concentration of cyanide ions in the pulp	B_5	Certain
6	Air flow	B_6	Certain
7	Original gold grade in the ore	B_7	Uncertain
8	Addition of the NaCN	X_8	Uncertain
9	Pulp density	X_9	Certain
10	Pulp flow	X_{10}	Certain
11	Dissolved oxygen content	X_{11}	Uncertain
12	Cyanide ion concentration in the slag	X_{12}	Uncertain
13	Gold grade in the pulp	X_{13}	Uncertain

14	Gold grade in the slag	X_{14}	Uncertain
15	Leaching rate	X_{15}	Uncertain
16	Material cost of the leaching sub-process	X_{16}	Uncertain
17	CEB	X_{17}	Uncertain

Table 2 - Illustration of experiments design.

Case	Description
1	1-99th sample: optimal performance.
	100-110th sample: performance shifting transition.
	111-200th sample: non-optimal performnce.
	Responsible variable: Valve opening of the NaCN dosing machine (B_4)
	Brief explanation: The state of the addition of the NaCN (X_8) is correctly estimated.
2	1-99th sample: optimal performance.
	100-110th sample: performance shifting transition.
	111-200th sample: non-optimal performnce.
	Responsible variable: Valve opening of the NaCN dosing machine (B_4)
	Brief explanation: The state of the addition of the NaCN (X_8) is wrongly estimated.
3	1-200th sample: optimal performance.
	Responsible variable: Dissolved oxygen content (X_{11})

Brief explanation: Measurement of X_{11} includes large amounts of noise.

6. Conclusions

This research addresses the operating performance optimality online assessment problem for chemical processes with coexistence of certainties and uncertainties. An essential problem of the traditional DCD, namely it cannot take advantage of both online measurable and reasoning information from the father nodes, is solved. FDbD is proposed to combine multi-source information of an uncertain node. Certain and uncertain variables are tackled separately to ensure the valuable information is preserved as much as possible. The corresponding online assessing strategy is developed, which distinguishes the stable performance grade and the grades switch transitions. The non-optimal cause identification technique is proposed based on novel events contributions. The proposed FDbD based online assessment and cause identification approach is successfully applied to a gold hydrometallurgy POPOA. Compared to the traditional DCD, FDbD has superiority in accuracy and robustness. The dependence on measurement techniques and production circumstance is largely reduced. Hopefully, the novel POPOA approach could be applied to more chemical processes. However, most causal relations based methods, like the Bayesian network and DCD, have causal loop problems. The DCD method deals with the causal loops, but there still may be some information loss. The further research would be carried out to improve this issue.

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