Twisting about Constrained Line of Parallel Purlins Inter Braced by Sagrods Under Wind Suctions

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Abstract: Simply supported purlins inter braced by one line of sagrods in a parallel purlin system under wind suction are analyzed. The sagrods are assumed to be zigzag placed with 30 ~ 50mm offset in adjacent spacing. The effect of this offset and the local deformation on the effectiveness of sagrod is analyzed. A purlin is divided into 3 segments by offsetted sagrods, and the general solution is found for such a purlin under wind suction. The solution is then extended to a parallel purlin system to determine the deformations of all purlins in the system. Examples are given to reveal the bracing effect of sagrods to purlins away from the sagrod anchor spacing. It is found that the sagrod bracing effect drops quickly as the number of spacings to the sagrod anchor spacing increases. The deformation of the last purlin in the 10 purlin system is close to a purlin without sagrod if the purlin thickness is 1.5mm. FE analysis is carried out to verify the analytic results and excellent agreement is found. Simplified equations are proposed to find the tensile forces in sagrods, purlin deformations and internal forces easily. Practical meaning of this study may be formulated as follows: (1) the distance of two diagonally braced spacing, as shown in Fig. 1, should be smaller if the purlin thickness is small; (2) it is welcome to take measures to reduce the local deformation of purlin web at the sagrod points.

Keywords: purlin; line of parallel purlins; C-section; sagrod; offset

1. Introduction

The Chinese technical code of cold-formed thin-wall steel structures (GB 50018-2002) requires that purlins with a span of more than 4m shall be braced by sagrods. Sagrods are used to connect parallel purlins and transfer the bracing effect of the first diagonal brace to the remaining purlins, as shown in Fig. 1. Deformations take place in parallel purlins together under wind suction.

In current design practice the sagrods are assumed to be able to provide full bracing to purlins, i.e., in the moment analysis, the purlins are assumed to be a continuous beam with intermediate rigid support at the cross-section of sagrods. But Dimos Polyzois [1] have found that the sagrods are unable to prohibit the lateral movement of the purlin at the sagrod cross-section. The thickness of the purlin web is usually 1.5mm to 2.5mm, and are easy to be locally deformed under the action of sagrod tension [2 – 6], see Fig.3.

We noted that the sagrods in the adjacent spaces have offsets, tensile forces in sagrods will produce distortional deformation in purlin webs, leading to remarkable reduction in the effective stiffness of the sagrods. Based on the study of twisting of simply supported purlin with one sagrod under wind suction with roof sheeting connected to the top flange of the purlin by self-drilled screws in Ref [2], the deformations and stresses of parallel purlins in system under wind suction are investigated in this paper.

2. Twisting about line of parallel purlins inter braced by offsetted sagrods

The sagrods connected to one purlin are arranged with longitudinal offset (see Fig. 2a), the purlin is divided into 3 segments (see Fig. 2b). This section is focused on the deformation of such a purlin acted by wind suction and the sagrod tensions T_1 and T_2 , the coordinate system is established as Fig. 2c.

2.1. General solution to lateral displacement of the shear center of cross-section in single purlin

Considering the influence of offset in sagrods arrangement (the offset distance of two sagrods is d , usually adopted as 30mm to 50mm), the model is illustrated in Fig. 2a, 2b and 2c. The expression of total potential of the purlin can be given by

$$\Pi = \frac{1}{2} \int_0^L (EI_y u''^2 + GJ \theta'^2 + EI_\omega \theta''^2) dz + T_1 u_{b1} - T_2 u_{b2} - \int_0^L m_z \theta dz \quad (1)$$

in which L is the purlin span; u is the displacement of shear center of cross-section in the x direction and θ is the twist of cross-section around z axis; E and G are respectively Young's and shear modulus of material; I_y and I_ω are respectively the second moment of area about y axis and the warping inertia moment of cross-section; J is the free torsion constant of the cross-section; T_1, T_2 are the tensile forces in sagrods connected to the purlin respectively; u_{b1}, u_{b2} are respectively the lateral displacements of the purlin web at the sagrod points in the x direction. The distributed torque around z axis in top flange is

$$m_z = q_y e_y \quad (2)$$

Since the top flange of the C-section purlin is horizontally restrained, the relation between the displacement u and the twist θ can be expressed as

$$u = 0.5h\theta, \quad u_{b1} = e_0 \theta_{L1}, \quad u_{b2} = e_0 \theta_{L2} \quad (3a, b, c)$$

in which θ_{L1}, θ_{L2} are respectively the rotations at the sagrod cross-sections.

Substituting Eqs. (3a)–(3c) into Eq. (1) leads to

$$\Pi = \frac{1}{2} \int_0^L (EI'_\omega \theta''^2 + GJ \theta'^2) dz + T_1 e_0 \theta_{L1} - T_2 e_0 \theta_{L2} - \int_0^L m_z \theta dz \quad (4)$$

where

$$EI'_\omega = 0.25h^2 EI_y + EI_\omega \quad (5)$$

The variation of the total potential Eq. (4), after partial integration, is

$$\delta \Pi = EI'_\omega \theta'' \delta \theta' \Big|_0^L + (GJ \theta' - EI'_\omega \theta''') \delta \theta \Big|_0^L - \int_0^L (GJ \theta'' - EI'_\omega \theta^{(4)} + m_z) \delta \theta dz + T_1 e_0 \delta \theta_{L1} - T_2 e_0 \delta \theta_{L2} = 0$$

The following equilibrium equation can be obtained:

$$\theta^{(4)} - \lambda^2 \theta'' = \frac{m_z}{EI'_\omega} \quad (6)$$

where

$$\lambda = \sqrt{\frac{GJ}{EI'_\omega}} \quad (7)$$

Considering the boundary and continuity conditions:

(1) In simply restrained sections, as well as where $z = 0$ and $z = L$:

$$\theta = 0, \quad \theta'' = 0 \quad (8a, b)$$

(2) In the sagrod cross-sections, as well as where $z = L_1$ and $z = L_2$:

$$\theta_{L1-} = \theta_{L1+}, \quad \theta_{L2-} = \theta_{L2+} \quad (9a, b)$$

$$\theta'_{L1-} = \theta'_{L1+}, \quad \theta'_{L2-} = \theta'_{L2+} \quad (9c, d)$$

$$\theta''_{L1-} = \theta''_{L1+}, \quad \theta''_{L2-} = \theta''_{L2+} \quad (9e, f)$$

$$(GJ \theta' - EI'_\omega \theta''')_{L1-} - (GJ \theta' - EI'_\omega \theta''')_{L1+} + T_1 e_0 = 0 \quad (9g)$$

$$(GJ \theta' - EI'_\omega \theta''')_{L2-} - (GJ \theta' - EI'_\omega \theta''')_{L2+} - T_2 e_0 = 0 \quad (9h)$$

in which the subscripts $L1 -$ and $L2 -$ represent the left sides (where the z coordinate is smaller) of the sagrod cross-sections (where $z = L_1$ and $z = L_2$), and the subscripts $L1 +$ and $L2 +$ represent the right side of the sagrod

cross-sections. Considering Eqs. (7), (9c) and (9d), Eq. (9g) and (9h) can be simplified as

$$\theta_{L1-}''' - \theta_{L1+}''' = \frac{T_1 e_0}{EI_{\omega}'}, \quad \theta_{L2-}''' - \theta_{L2+}''' = -\frac{T_2 e_0}{EI_{\omega}'} \quad (10a, b)$$

The solutions of Eq. (6) are:

$$(1) 0 \leq z \leq L_1 : \theta_{L1-} = C_1 + C_2 z + C_3 \sinh \lambda z + C_4 \cosh \lambda z - \frac{m_z}{2GJ} z^2 \quad (11a)$$

$$(2) L_1 \leq z \leq L_2 : \theta_{L1+} = C_5 + C_6 z + C_7 \sinh \lambda z + C_8 \cosh \lambda z - \frac{m_z}{2GJ} z^2 \quad (11b)$$

$$(3) L_2 \leq z \leq L : \theta_{L2+} = C_9 + C_{10} z + C_{11} \sinh \lambda z + C_{12} \cosh \lambda z - \frac{m_z}{2GJ} z^2 \quad (11c)$$

Substituting the boundary and continuity conditions (Eqs. (8)–(10)) into Eqs. (11a)–(11c), the constants $C_1 - C_{12}$ can be determined as

$$C_1 = -\frac{m_z}{GJ \lambda^2} \quad (12a)$$

$$C_2 = \frac{m_z L}{2GJ} + \frac{T_2 L_1 - T_1 L_2}{GJ L} e_0 \quad (12b)$$

$$C_3 = -\frac{m_z}{GJ \lambda^2} \tanh \frac{\lambda L}{2} + \frac{T_1 \sinh \lambda L_2 - T_2 \sinh \lambda L_1}{GJ \lambda \sinh \lambda L} e_0 \quad (12c)$$

$$C_4 = \frac{m_z}{GJ \lambda^2} \quad (12d)$$

$$C_5 = -\frac{m_z}{GJ \lambda^2} - \frac{T_1 L_1 e_0}{GJ} \quad (12e)$$

$$C_6 = \frac{m_z L}{2GJ} + \frac{T_2 L_1 + T_1 L_1}{GJ L} e_0 \quad (12f)$$

$$C_7 = -\frac{m_z}{GJ \lambda^2} \tanh \frac{\lambda L}{2} - \frac{e_0 \sinh \lambda L_1}{\lambda GJ} \left(\frac{T_1}{\tanh \lambda L} + \frac{T_2}{\sinh \lambda L} \right) \quad (12g)$$

$$C_8 = \frac{m_z}{\lambda^2 GJ} + \frac{T_1 e_0}{\lambda GJ} \sinh \lambda L_1 \quad (12h)$$

$$C_9 = -\frac{m_z}{GJ \lambda^2} + \frac{T_2 L_2 - T_1 L_1}{GJ} e_0 \quad (12i)$$

$$C_{10} = \frac{m_z L}{2GJ} + \frac{T_1 L_1 - T_2 L_2}{GJ L} e_0 \quad (12j)$$

$$C_{11} = -\frac{m_z}{GJ \lambda^2} \tanh \frac{\lambda L}{2} + \frac{T_2 \sinh \lambda L_2 - T_1 \sinh \lambda L_1}{GJ \lambda \tanh \lambda L} e_0 \quad (12k)$$

$$C_{12} = \frac{m_z}{GJ \lambda^2} + \frac{T_1 \sinh \lambda L_1 - T_2 \sinh \lambda L_2}{GJ \lambda} e_0 \quad (12l)$$

The lateral displacements at the connecting points of the sagrods on the purlin web (where $z = L_1$ and $z = L_2$) are expressed by

$$u_{b1} = g_1 T_1 + g_2 T_2 + g_3 m_z \quad (13a)$$

$$u_{b2} = -g_1 T_2 - g_2 T_1 + g_3 m_z \quad (13b)$$

where

$$g_1 = \frac{e_0^2}{GJ} \frac{L \cosh \lambda L - L \cosh \lambda d - 2\lambda L_1 L_2 \sinh \lambda L}{2\lambda L \sinh \lambda L} \quad (14a)$$

$$g_2 = \frac{e_0^2}{GJ} \frac{\lambda L_1^2 \sinh \lambda L - L \sinh^2 \lambda L_1}{\lambda L \sinh \lambda L} \quad (14b)$$

$$g_3 = \frac{e_0}{GJ} \frac{2 \sinh \lambda L_1 + 2 \sinh \lambda L_2 + (\lambda^2 L_1 L_2 - 2) \sinh \lambda L}{2\lambda^2 \sinh \lambda L} \quad (14c)$$

Since the offset distance of two sagrods connected to the same purlin is much less than the purlin span ($d \ll L$), which means $L_1 \approx L/2$, $L_2 \approx L/2$, the following relationship can be obtained in approximate solution:

$$\frac{g_1}{f_3} \approx 1, \quad \frac{g_2}{f_3} \approx -1, \quad \frac{g_3}{f_4} \approx 1 \quad (15a, b, c)$$

in which f_3 and f_4 are determined by substituting $d = 0$ into Eqs. (14a)–(14c):

$$f_3 = \frac{e_0^2}{2\lambda GJ} (\tanh 0.5\lambda L - 0.5\lambda L) \quad (16a)$$

$$f_4 = \frac{e_0}{GJ\lambda^2} \left(\frac{1}{\cosh 0.5\lambda L} - 1 + \frac{1}{8} \lambda^2 L^2 \right) \quad (16b)$$

The lateral displacement and the twist of the section if the sagrods were connected to purlin web without offset (i.e. $d = 0$) can be expressed by

$$u_L = f_3 F + f_4 m_z \quad (17a)$$

$$\theta_L = f_1 F + f_2 m_z \quad (17b)$$

where

$$F = T_1 - T_2, \quad f_1 = f_3 / e_0, \quad f_2 = f_4 / e_0 \quad (18a, b, c)$$

The twisting of the purlin along span can be approximated by

$$\theta = \frac{e_0}{2GJ} \left(\frac{\sinh \lambda z}{\lambda \cosh \lambda l} - z \right) F + \frac{1}{GJ\lambda^2} \left(-1 + \lambda^2 l z - \frac{\sinh \lambda l \sinh \lambda z}{\cosh \lambda l} + \cosh \lambda z - \frac{\lambda^2 z^2}{2} \right) m_z \quad (19)$$

2.2. Effective stiffness of sagrods

2.2.1. Tensile stiffness of sagrods

Supposing that the sectional area and the length of sagrods are respectively A_b and l_b , the axial stiffness of the sagrods is

$$k_b = \frac{EA_b}{l_b} = \frac{1}{f_b} \quad (20)$$

where f_b is the flexibility corresponding to the axial stiffness.

The tensile force in the sagrod produce local bending deformation of the purlin web, as shown in Fig. 3. The stiffness corresponding to this local deformation is approximated by

$$k_w = \frac{13Et^3}{h^2(1 - \cos 2\pi\xi)^{0.65(\xi^2 - \xi + 1.25)}} \left(1 - 25 \frac{t^{1.5}}{h^{1.5}} \right) = \frac{1}{f_w} \quad (21)$$

where f_w is the flexibility corresponding to k_w ; h is the distance between the top and bottom flanges of the cross-section; t is the thickness of the purlin section; ξ is the normalized distance between the sagrod location and the bottom flange given by

$$\xi = 1 - \frac{e_0}{h} \quad (22)$$

where e_0 is the distance between the middle thickness of the top flange of cross-section and the sagrod location (shown in Fig. 1c).

The local deformation of purlin web result in a considerable reduction in effectiveness of lateral restraint to purlin by sagrod. Two kinds of deformations (sagrod elongation and web local deformation) occur and must be considered simultaneously, the effective stiffness of the sagrod is thus reduced to

$$k_{LT} = \frac{k_w k_b}{k_b + k_w} \quad (23)$$

Because k_w is far less than k_b , k_{LT} is less than but close to k_w , and k_{LT} is (1% ~ 17%) k_b , dependent on the thickness and the sagrod location on the web.

Utilizing Eq. (17b), the twist of the sagrod cross-section with sagrod connected in the case of single purlin with one sagrod can be obtained by

$$\theta_L = \frac{f_2}{1 - f_1 k_{LT} e_0} m_z \quad (24)$$

And the tensile force in the sagrod is

$$F_{LT} = k_{LT} e_0 \theta_L = \frac{f_4}{f_w + f_b - f_3} m_z \quad (25)$$

2.2.2. The local flexural stiffness of purlins

Fig. 4 shows a kind of deformation occurring as a result of the forces in offsetted sagrods. The two tensile forces in the offsetted sagrods constitute a local bending moment. The aim of this section is to determine the local flexural stiffness of resisting this local bending moment by observing the local deformation caused by this moment in FE analysis.

The purlins are modeled using the elastic thin shell element SHELL63 of ANSYS in FE analysis. The lengths of all purlins are 5m, which are longitudinally divided into 1000 meshes. Two unit concentrated forces are applied oppositely perpendicular to the purlin web with a distance of d ; the positions of the sagrod cross-sections are symmetric about the midspan of the purlin; the sagrod positions on the cross-section are $e_0 = 0.5h$. The horizontal and vertical displacements (in x and y directions, see Fig. 2c) of the middle width of the top flange are restrained at intervals of 250mm in the longitudinal direction, to simulate the lateral braces from the roof sheeting. To prohibit the twisting of the cross section, the x -displacement of the bottom web-to-flange juncture is also restrained. The model in FE analysis is shown in Fig. 4.

In Fig. 4a, denote the displacements at the loaded points by δ_M . In Fig. 3b, there is only one sagrod, we exert also a unit force at the sagrod point, and denote the displacement at the loaded point by δ_0 . δ_M is related to δ_0 by the following equation

$$\delta_0 = 2\chi\delta_M \quad (26a)$$

Where χ is a parameter to be determined by FE analysis. δ_0 is the reciprocal of k_w :

$$k_w = \frac{1}{\delta_0} \quad (27a)$$

And the local bending stiffness k_θ of resisting the deformation in Fig. 4 is

$$k_\theta = \frac{M}{\theta} = \frac{1 \times d}{2\delta_M / d} = \frac{d^2}{2\delta_M} = \frac{2\chi d^2}{2\delta_0} = \chi d^2 k_w \quad (27b)$$

in which M is the local bending moment by the two opposite unit concentrated loads; θ is the angle between the line connecting two loaded points with displacements and the longitudinal axis; χ is a coefficient for

approximation.

In FE analysis, 30 typical cold-formed steel sections are adopted where the thickness and height of cross sections are 1.5 ~ 3.5mm and 140 ~ 250mm, respectively. The offset distance d is 30 ~ 50mm at intervals of 5mm. Utilizing the FE results of displacement δ_M , considering the approximate solution of k_w in Eq. (21), an almost accurate approximation of coefficient χ can be obtained as

$$\chi_{\xi=0.5} = 0.158\rho^{-1.6} + 0.4 \quad (28a)$$

where $\rho = d/h$. By varying the distance e_0 ($\xi = 0.375, 0.25$ and 0.125), the following equations for χ are obtained:

$$\xi = 0.375: \chi = 0.147\rho^{-1.6} + 0.43 \quad (28b)$$

$$\xi = 0.25: \chi = 0.114\rho^{-1.6} + 0.519 \quad (28c)$$

$$\xi = 0.125: \chi = 0.06\rho^{-1.6} + 0.642 \quad (28d)$$

Based on Eqs. (28a)–(28d), the local bending stiffness can be proposed as

$$k_\theta = k_w h^2 \left[(1.86\xi^2 - 1.86\xi + 0.84)\rho^2 - (0.7\xi^2 - 0.7\xi + 0.017)\rho^{0.4} \right] \quad (29a)$$

The flexibility corresponding to this local bending flexural stiffness is

$$f_\theta = \delta_{M=d} = \frac{d^2}{2k_\theta} = \frac{0.5f_w}{1.86\xi^2 - 1.86\xi + 0.84 - (0.7\xi^2 - 0.7\xi + 0.017)\rho^{-1.6}} \quad (29b)$$

Comparisons of Eq. (29a) and the FE results with various values of ξ are shown in Fig. 5. It can be observed that Eq. (29a) has excellent accuracy.

2.3. Twisting about line of parallel purlins

2.3.1. Theoretical solution

In the parallel purlins shown in Fig. 6, u_{iL1} and u_{iL2} are the x -displacements at the sagrod locations in purlin webs (i.e. $z = L_1$ and $z = L_2$) respectively, with local bending deformations included (the subscript L means local bending included); u_{i1} and u_{i2} are respectively the x -displacements at the same locations with rigid profile assumption; θ_{i1} and θ_{i2} are respectively the twists of the sections connected with sagrods. The relation between displacements and twists are

$$u_{i1} = e_0\theta_{i1}, \quad u_{i2} = e_0\theta_{i2} \quad (30a, b)$$

It should be noticed that $u_{iL1} \neq u_{i1}$, $u_{iL2} \neq u_{i2}$. The tensile forces in sagrods are computed based on the locations after local deformation. The local web bending deformation is $u_{i1} - u_{iL1}$, $u_{i2} - u_{iL2}$ (see Fig. 3c).

The relationship of tensile forces in sagrods, tensile stiffness of sagrods and axial deformation can be obtained by following analysis.

(1) In the first purlin of the parallel purlin system:

In the first purlin (the purlin connected with the sagrod whose other end is completely restrained), considering the tensile stiffness of sagrod itself, the following expressions can be obtained:

$$T_1 = k_b u_{iL1} \quad (31a)$$

$$T_2 = k_b (u_{iL2} - u_{iL1}) \quad (31b)$$

In the first purlin, the local deformation $u_{i1} - u_{iL1}$ at $z = L_1$ is equivalently caused by a tensile force equal to $T_1 - T_2$ applied at the location and a bending moment equal to $d \times T_2$ applied around the location, as shown in Fig. 7b. The local flexibilities corresponding to the two types of forces in Fig. 7b are f_w and f_θ respectively, so one has

$$f_w(T_1 - T_2) + f_\theta T_2 = u_{i1} - u_{iL1} \quad (31c)$$

Similarly, referring to Fig. 7(c), one has

$$f_w(T_1 - T_2) - f_\theta T_1 = u_{12} - u_{1L2} \quad (31d)$$

Considering Eqs. (13a) and (13b), we have

$$u_{11} = g_1 T_1 + g_2 T_2 + g_3 m_z \quad (31e)$$

$$u_{12} = -g_2 T_1 - g_1 T_2 + g_3 m_z \quad (31f)$$

(2) In the purlin with even number of i :

Following the similar procedure, considering Eqs. (13a) and (13b), we can write out

$$T_i = k_b(u_{iL2} - u_{i-1L2}) \quad (32a)$$

$$f_w(T_i - T_{i+1}) - f_\theta T_i = u_{i1} - u_{iL1} \quad (32b)$$

$$f_w(T_i - T_{i+1}) + f_\theta T_{i+1} = u_{i2} - u_{iL2} \quad (32c)$$

$$u_{i1} = g_1(-T_{i+1}) + g_2(-T_i) + g_3 m_z \quad (32d)$$

$$u_{i2} = g_2 T_{i+1} + g_1 T_i + g_3 m_z \quad (32e)$$

In the purlin with odd number of i :

$$T_i = k_b(u_{iL1} - u_{i-1L1}) \quad (33a)$$

$$f_w(T_i - T_{i+1}) + f_\theta T_{i+1} = u_{i1} - u_{iL1} \quad (33b)$$

$$f_w(T_i - T_{i+1}) - f_\theta T_i = u_{i2} - u_{iL2} \quad (33c)$$

$$u_{i1} = g_1 T_i + g_2 T_{i+1} + g_3 m_z \quad (33d)$$

$$u_{i2} = -g_2 T_i - g_1 T_{i+1} + g_3 m_z \quad (33e)$$

(3) In the purlin with even n :

$$T_n = k_b(u_{nL2} - u_{n-1L2}) \quad (34a)$$

$$f_w T_n = u_{n2} - u_{nL2} \quad (34b)$$

$$u_{n2} = g_5(-T_n) + g_3 m_z \quad (34c)$$

In the purlin with odd n :

$$T_n = k_b(u_{nL1} - u_{n-1L1}) \quad (35a)$$

$$f_w T_n = u_{n1} - u_{nL1} \quad (35b)$$

$$u_{n1} = g_1 T_n + g_3 m_z \quad (35c)$$

By eliminating the terms of displacements, a set of simultaneous equations combining Eqs. (31)–(35) (compatible with arbitrary value of i or n) can be obtained as

$$\beta T_1 - T_2 = g'_3 m_z \quad (36a)$$

$$-T_{i-1} + \alpha T_i - T_{i+1} = 0 \quad (36b)$$

$$-T_{n-1} + \alpha T_n = 0 \quad (36c)$$

where (considering $\frac{g_1}{f_3} \approx 1$, $\frac{g_2}{f_3} \approx -1$, $\frac{g_3}{f_4} \approx 1$)

$$\alpha = -\frac{2f_w + f_b - 2g_1}{f_\theta - f_w - g_2} \approx 2 + \gamma \quad (37a)$$

$$\beta = -\frac{f_w + f_b - g_1}{f_\theta - f_w - g_2} \approx 1 + \frac{f_b + f_\theta}{f_w - (f_\theta + f_3)} \quad (37b)$$

$$g'_3 = -\frac{g_3}{f_\theta - f_w - g_2} \approx \frac{f_4}{f_w - (f_\theta + f_3)} \quad (37c)$$

$$\gamma = \frac{f_b + 2f_\theta}{f_w - (f_\theta + f_3)} \quad (37d)$$

Equations (36a, b and c) are put in matrix form as

$$\begin{bmatrix} \beta & -1 & 0 & 0 & \cdots & 0 \\ -1 & \alpha & -1 & 0 & \cdots & 0 \\ 0 & -1 & \alpha & -1 & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & -1 & \alpha & -1 \\ 0 & \cdots & 0 & 0 & -1 & \alpha \end{bmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ \vdots \\ T_{n-1} \\ T_n \end{pmatrix} = \begin{pmatrix} g'_3 m_z \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad (38)$$

Compared with the special case of $d = 0$ (i.e. $f_\theta = 0$), the presence of offset distance d between adjacent sagrods makes the sagrod flexibility increase to $f_b + 2f_\theta$, whereas the flexibility of the first sagrod increases to $f_b + f_\theta$ since it is connected to the purlin web at one end only, and the flexibility coefficient f_3 increases to $f_3 + f_\theta$.

Due to the sagrod offset, the effective axial stiffness of the offsetted sagrod is changed from $k_b = \frac{1}{f_b}$ to

$k_{b,d} = \frac{1}{f_b + 2f_\theta}$. Fig. 9 presents the reduced effective sagrod axial stiffness due to the sagrod offset, which is

$$\frac{k_{b,d}}{k_b} = \frac{f_b}{f_b + 2f_\theta}.$$

The most frequently used value of d are 30 ~ 50mm, thus $\frac{d}{h} = \frac{30}{140} \sim \frac{50}{140} = 0.214 \sim 0.357$, and

$\frac{d}{h} = \frac{30}{250} \sim \frac{50}{250} = 0.12 \sim 0.2$. From Fig. 9 it can be seen that the effective stiffness of the sagrod is reduced to a very small value.

2.3.2. Analysis of examples

In the case of only two purlins, as shown in Fig. 10, by solving Eq. (38), we obtain

$$T_1 = \frac{1}{A_2} 4k_b k_w k_\theta^2 (2k_b + k_w - 2g_1 k_b k_w) g_3 m_z \quad (39a)$$

$$T_2 = -\frac{2}{A_2} k_b^2 k_\theta k_w (k_w d^2 - 2k_\theta - 2g_2 k_\theta k_w) g_3 m_z \quad (39b)$$

in which

$$A_2 = 4k_\theta^2 (k_b + k_w)^2 + (4k_\theta - d^2 k_w + 4g_2 k_\theta k_w) d^2 k_w k_b^2 + 4(2g_1^2 - g_2^2) k_\theta^2 k_w^2 k_b^2 + 4(1 - 4g_1 k_b - 2g_2 k_b - 3g_1 k_w) k_b k_w k_\theta^2 \quad (39c)$$

Substituting Eq. (39) into Eq. (19), considering Eq. (3a), the theoretical lateral displacements along span in each purlin can be obtained. To examine the theoretical results, a typical example with FE analysis is developed in which the purlin section C250 × 75 × 20 × 2 ($h = 250\text{mm}$, $b = 75\text{mm}$, $c = 20\text{mm}$ and $t = 2\text{mm}$) is adopted; the sagrods are arranged with offset of 50mm (i.e. $d = 50\text{mm}$) and $\xi = 0.5$. In shell element modeling, the lengths of purlins are 5m, which are longitudinally divided into 2000 meshes with simple restraints at ends by restraining the displacements in the x and y directions of all nodes of the two end sections and the z -displacement of the middle height node of one end section. The x -displacement of each node at the middle width of the top flange is restrained to simulate the lateral braces from the roof sheeting. The wind suction applied is equal to 1kN/m, and the tensile stiffness of the sagrod assumed to be 1.5m in length with a rod section of 10mm in diameter can be calculated

as $k_b=10780\text{N/mm}$. The comparison of theoretical solutions and FE results (adopted as average displacements of top and bottom web-to-flange junctures in sections along span to eliminate the influence of local deformation) is shown in Fig. 11.

It can be observed in Fig. 11 that the theoretical results provide high precision, and the displacements of the same section in two purlins show significant difference, which prove that the offset of sagrods has notable influence on the deformation of purlins, reducing the effectiveness of the sagrods.

In the case of 3 purlins, the comparison of two sets of results are given in Fig. 12.

The results of 10-purlin system are presented in Fig. 13 (only purlins with $i=1, 3, 5, 10$ are shown for a clearer observation). For comparison, the displacements of a purlin without sagrod (denoted as u_0) and of a purlin with 1 sagrod in one-purlin system (denoted as u_s) are also presented in Fig.13.

Further examples are given in Fig.14 for purlin sections $C140 \times 50 \times 20 \times 2.5$ and $C250 \times 75 \times 20 \times 1.5$; $d = 30, 50\text{mm}$ and $\xi = 0.25, 0.5$.

It can be observed in Fig. 13 and Fig. 14 that

- (1) The closed theoretical solutions have excellent agreement with the FE numerical results.
- (2) The lateral displacement of a purlin is larger if it is farther away from the anchor point of the first sagrod, implying the bracing effect received by purlins weaken as the purlin is farther away from the anchor point.
- (3) The lateral displacements of the farthest purlin reach almost the values of the purlin without sagrod if the thickness of the purlin is 1.5 ($t = 1.5$), and $d = 50$, $\xi = 0.5$. That is, in such a combination of parameters, the sagrod has almost no effect on the last purlin if the total number of purlins is greater than 10.
- (4) the lateral deformation of the first purlin in a parallel purlin system is always greater than that of one purlin system.

2.3.3. The coefficient of tensions in sagrods

By observing Eqs. (36a)–(36c), similar relationships of sagrods can be obtained as

$$T_{n-1} = \alpha T_n \quad (40a)$$

$$T_{n-2} = \alpha T_{n-1} - T_n = (\alpha^2 - 1)T_n \quad (40b)$$

$$T_{n-3} = \alpha T_{n-2} - T_{n-1} = (\alpha^3 - 2\alpha)T_n \quad (40c)$$

$$T_{n-4} = \alpha T_{n-3} - T_{n-2} = (\alpha^4 - 3\alpha^2 + 1)T_n \quad (40d)$$

$$T_{n-5} = \alpha T_{n-4} - T_{n-3} = (\alpha^5 - 4\alpha^3 + 3\alpha)T_n \quad (40e)$$

$$T_{n-6} = \alpha T_{n-5} - T_{n-4} = (\alpha^6 - 5\alpha^4 + 6\alpha^2 - 1)T_n \dots\dots \quad (40f)$$

It can be observed that the ratio $\alpha_i = T_{n-i+1} / T_n$ is independent of n , this is shown in Fig.15. After some induction, the ratio α_i can be expressed by

$$\alpha_i = \frac{T_{n-i+1}}{T_n} = \sum_{k=1}^{(i+1)/2} \frac{(-1)^{k-1}}{(k-1)!} \prod_{j=k}^{2k-2} (i-j) \alpha^{i-2k+1} \quad (n \text{ is odd}) \quad (41a)$$

$$\alpha_i = \frac{T_{n-i+1}}{T_n} = \sum_{k=1}^{i/2} \frac{(-1)^{k-1}}{(k-1)!} \prod_{j=k}^{2k-2} (i-j) \alpha^{i-2k+1} \quad (n \text{ is even}) \quad (41b)$$

By curve fitting, a simple approximate expression can be obtained with deviations less than 1%:

$$\alpha_i = \frac{T_{n-i+1}}{T_n} = \frac{\sinh(\gamma^{0.5025} i)}{\sinh \gamma^{0.5025}} \quad i = 1, 2, 3, \dots, n \quad (42)$$

where $\gamma = \alpha - 2$.

Comparisons of α_i by Eq. (42) and Eq. (41a, b) are shown in Fig. 16, it can be seen that the approximate expression has high precision.

Considering Eq. (36a), the tensile force in the last sagrod (i.e. T_n) can be obtained by

$$T_n = \frac{g_3 m_z}{(f_b + f_w - g_1)\alpha_n + (f_\theta - f_w - g_2)\alpha_{n-1}} \approx \frac{f_4 m_z}{(f_b + f_w - f_3)\alpha_n + (f_3 + f_\theta - f_w)\alpha_{n-1}} \quad (43a)$$

which can be simply expressed as

$$\frac{T_n}{F_{LT}} = \frac{1}{\alpha_n - \alpha_{n-1} / \beta} \quad (43b)$$

in which F_{LT} represents the tension in the sagrod when only one purlin presents, and is given by Eq. (25). And because f_3 is less than 0, but $-f_3$ is far larger than f_w, f_θ and f_b , β is greater but close to 1.0, so

$$\frac{T_n}{F_{LT}} = \frac{1}{\cosh(\gamma^{0.5025} n)} \quad (43c)$$

The tension in the first sagrod is

$$\frac{T_1}{F_{LT}} = \frac{\alpha_n}{\alpha_n - \alpha_{n-1} / \beta} \approx \frac{\tanh(\gamma^{0.5025} n)}{\sinh \gamma^{0.5025}} \approx \frac{1}{\sinh \gamma^{0.5025}} \quad \text{if } n \geq 8 \quad (44)$$

It can be seen from Eq. (42) and Fig. 16 that the increasing of the total amount of purlins (n) result in significant increment of the coefficient of tension in sagrods (α_n). Consequently, the tensile force in the last sagrod decreases rapidly according to Eq. (42), the bracing effect of the sagrod provided to the last purlin decreases significantly.

The tension in each purlin can be calculated utilizing Eqs. (42) and (43). Substituting the difference between two adjacent sagrods which is assumed as F_i (i.e. $F_i = T_i - T_{i+1}$) into Eq. (19), the twists and displacements (Eq. (3a)) of sections along span in each purlin can be obtained.

2.3.4. The out-of-plane bending moments of sections

The out-of-plane bending moments (about y axis) of sections can be expressed as

$$M_y = -EI_y u'' = -EI_y \frac{1}{2} h \theta'' \quad (44)$$

After the forces $F_i = T_i - T_{i+1}$ is obtained, Eq. (19) is used to find the twist of purlins with F replaced by F_i and then M_y is computed by Eq. (44). For verification of theoretical solutions, 36 typical examples with various sections and arrangement ways of sagrods are adopted in FE analysis (shown in Table. 1).

With 36 different conditions, the theoretical solutions M_y and FE analytic results $M_{y\text{ANSYS}}$ at midspan of the purlin with serial number of 1, 5, 10 in a 10 parallel purlins system are shown in Fig. 17.

Fig.18 shows the distribution of M_y along span, in which only the 1st, 5th and 10th purlins are shown. The purlin is C140×50×20×2.5, $\xi = 0.5$ and $d = 50\text{mm}$.

It can be seen from Fig. 17 and Fig. 18 that the theoretical solutions of out-of-plane bending moments are in good agreement with the results of FE analysis, except in sections near midspan with considerable distortion. Due to reductions of sagrod stiffness, the out-of-plane bending moments in purlins away from the sagrod anchor span are all positive along span, with no negative bending moment occurs in these purlins, and the bending moments are closer to the purlin without sagrod.

3. Conclusion

The deformations and stresses of simply supported purlins with one line of sagrods in parallel purlin system

under wind suction are investigated in this paper, in which the sagrod offset 30 ~ 50mm in adjacent purlin spacing is taken into account. It is found that besides the weakening effect of local deformation of the purlin web at the sagrod point on the effective stiffness of the sagrod, the sagrod offset in adjacent purlin spacing weakens further this effective stiffness. A local flexural stiffness of purlin web (k_{θ}) due to the sagrod offset is introduced to consider the effect of sagrod offset.

A purlin is divided into 3 segments by offsetted sagrods, the general solution is found for such a purlin under wind suction. The solution is then extended to a parallel purlin system to determine the deformations of all purlins in the system.

Examples are presented to reveal the bracing effect of sagrods to purlins away from the sagrod anchor spacing. It is found that the sagrod bracing effect drops quickly as the number of spacings to the sagrod anchor spacing increases. The deformation of the last purlin in the 10-purlin system is close to a purlin without sagrod if the purlin thickness is 1.5mm.

FE analysis is carried out to verify the analytic results and excellent agreement is found.

Simplified equations are proposed to find the tension forces in sagrods and purlin deformations and internal forces easily.

Further practical meaning of this study may be formulated as follows: (1) the distance of two diagonally braced spacing, as shown in Fig.1, should be smaller if the purlin thickness is small. (2) it is welcome to take measures to reduce the local deformation of purlin web at the sagrod points.

Acknowledgement

The study carried out in this paper is financially supported by Chinese National Science Foundation Under Grant No. 51478421.

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Fig. 1. Arrangement of purlins and sagrods in line of parallel purlins

(a) Sagrods with longitudinal offset

(b) Forces applied to the purlin with offset

(c) Coordinate system

Fig. 2. Analysis diagram of arbitrary purlin in line of parallel purlins

(a) Local deformation of purlin in FE analysis

(b) Analysis diagram

(c) Purlin web with and without local deformation

Fig. 3. Local deformation of purlin web

(a) Simple modeling diagram

(b) Local flexural deformation of purlin web in FE analysis

Fig. 4. Local flexural deformation of purlin web under opposite forces with offset

Fig. 5. The approximations and the FE analytic results of the flexural stiffness

Fig. 6. Line of parallel purlins inter braced by sag rods

Fig. 7. Equivalent force condition of the first purlin (or purlin with odd serial number)

Fig. 8. Equivalent force condition of the second purlin (or purlin with even serial number)

Fig. 9. the effective axial stiffness of the offset sagrod

Fig. 10. Line of two parallel purlins

Fig. 11. Theoretical solutions and FE analytic results in line of two parallel purlins (C250×75×20×2, $d=50\text{mm}$, $\xi=0.5$)

Fig. 12. Theoretical solutions and FE analytic results in line of three parallel purlins ($C250 \times 75 \times 20 \times 2$, $d = 50\text{mm}$, $\xi = 0.5$)

Fig. 13. Theoretical solutions and FE analytic results in line of 10 parallel purlins ($C250 \times 75 \times 20 \times 2$, $d = 50\text{mm}$, $\xi = 0.5$)

(a) $C140 \times 50 \times 20 \times 2.5$

(b) $C250 \times 75 \times 20 \times 1.5$

(c) $C140 \times 50 \times 20 \times 2.5$

(d) $C250 \times 75 \times 20 \times 1.5$

(e) C140×50×20×2.5

(f) C250×75×20×1.5

(g) C140×50×20×2.5

(h) C250×75×20×1.5

Fig. 14. Theoretical solutions and FE analytic results in line of 10 parallel purlins

Fig. 15. The ratio of tensions in sagrods in line of parallel purlins with different total amount of purlins

(a)

(b)

Fig. 16. The coefficient of tensions in sagrods

Fig. 17. The theoretical and FE analytic results of M_y

Fig. 18. The distribution of the out-of-plane bending moments of sections along span

Table. 1. Sections and arrangement ways of sagrods

Num.	Sections	d	ξ	Remarks
1	250×75×20×2.5	50	0.5	Conditions Num. 10 – 18 are the same 9 sections, but with $d = 30$ and $\xi = 0.5$;
2	250×75×20×2.0			
3	250×75×20×1.5			
4	200×70×20×2.5			Conditions Num. 19 – 27 are the same 9 sections, but with $d = 50$ and $\xi = 0.25$;
5	200×70×20×2.0			
6	200×70×20×1.5			
7	140×50×20×2.5			Conditions Num. 28 – 36 are the same 9 sections, but with $d = 30$ and $\xi = 0.25$.
8	140×50×20×2.0			
9	140×50×20×1.5			