Two-level optimization for a new family of cold-formed steel lipped channel sections against local and distortional buckling

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Abstract

The objective of this paper is to provide the development of a new algorithm that produces a family of minimal weight cold-formed steel lipped channels with the smallest number of individual cross-sections (family size) that are still capable of covering the current engineering design demands as commonly found in light steel framing. Current research in cold-formed steel optimization has largely sought optimal cross-sections for single members under a single applied action. In this paper, the optimization effort is extended to a broad set of axial (P) and bending (M) demands. A two-level optimization framework is proposed: level one focuses on member optimization of the P-M demand space as derived from current commercially available lipped channel sections in the United States; while level two focuses on the selection of a new family of optimal lipped channel sections that have the same efficiency in covering the design space, but utilize a minimal family size. As the focus of the effort is on the two-level optimization the cross-section optimization is simplified in this case to lipped channels against local and

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distortional buckling only; however the adopted algorithms are readily extensible. The results show that a new family of shapes with only 12 sections can achieve the same or better performance as the 108 sections commercially available in the United States. The developed family of shapes provides a direct demonstration of the potential for optimization to improve cold-formed steel manufacturing. In the future, the first-level member optimization will include new cross-section shapes, e.g., sigma sections, as well as global buckling, and will be performed along with the second family-level optimization to demonstrate additional potential benefits.

Keywords: member optimization; family optimization; cold-formed steel; particle swarm optimization; genetic algorithms

Introduction

Cold-formed steel members, both as load bearing and non-load bearing, have been widely used in construction of low and mid-rise buildings worldwide. The merits of cold-formed steel members include high strength-to-weight ratio, high recycled content, ease of construction, and also the ease and low cost of manufacture. Cold-formed steel sections are manufactured by bending sheet steel into useful shapes at room temperature, typically using a roll former. Since the sheet steel material is relatively thin the energy required to shape the sections is minimal and the possibility for creating different shapes quite broad. Commonly used shapes for different applications are summarized in [1]. For cold-formed steel framing in the United States manufacturing associations provide catalogs of standard shapes, such as that provided by the Steel Framing Industry Association (SFIA) [2].

Due to the ease of manufacturing, cold-formed steel has a potential advantage in maximizing material efficiency through selection of cross-sectional shape. While sections in current

production by industry hold certain advantages, they may not be the most efficient. Hence, a significant body of research has developed seeking new and/or more efficient cross sections using optimization algorithms. Initial efforts on the optimization of cold-formed steel members focused on optimizing the strength based on rules provided in specifications such as AISI [3], BS5950 [4], or Eurocode [5]. In particular, all of these specifications employ the effective width method for strength determination. This design method affords a limited degree of generality and for best accuracy the basic cross-section typology (shape) must be pre-determined. Optimization efforts have included hat sections [6–8] channel sections [9–13], and purlins [14] and both gradient-based and stochastic search algorithms have been employed.

More recently, with the adoption of new strength prediction methods such as [15], which embed numerical elastic buckling predictions (e.g., [16]) capable of evaluating arbitrary sections, cold-formed steel optimization has expanded. First, shape optimization of a cold-formed steel column with fixed perimeter width and arbitrary shape was completed [17]. This was followed by generalization of the optimization across different member lengths and refinement of the stochastic search optimization scheme [18], and finally incorporation of realistic manufacturing and end use constraints [19]. Efforts along these lines have continued by multiple researchers [20–22]. The optimization results indicate that significant strength improvements over conventionally provided sections are possible, even using current manufacturing methods and design rules.

However, cold-formed steel manufacturers do not produce individual shapes for single applied actions (isolated bending, isolated compression, etc.). Rather, they produce a family of shapes that engineers specify to complete their designs under a variety of applied actions. Thus, a key remaining problem is the optimization of families of cold-formed steel shapes under combined

actions. Specifically, we seek the smallest number of cold-formed steel sections that can provide, over a range of applied axial and bending actions, a solution using equal to or less material than optimally selected members from currently available industry sections. The solution scheme adopted is capable of handling arbitrary cold-formed steel sections against any buckling behaviors, but is implemented here for lipped channels, the most commonly used section in industry, against specifically local and distortional buckling.

The paper begins by exploring the performance of an existing family of cold-formed steel shapes: the 108 structural lipped channels supplied by SFIA. These sections are investigated in terms of their capacity under axial load and major-axis bending to establish the practical range of demands to be studied. In addition, for baseline comparisons the least weight SFIA section for any combination of axial and bending demand is established. The next section of the paper focuses on the first-level optimization: application of particle swarm optimization to determine the least weight lipped channel section for the same combinations of axial and bending demand. Given a fixed typology and minimal manufacturing constraints this first-level optimization establishes the least weight solution for every combined demand. The developed solution is studied and reveals specific trends about optimal cold-formed steel sections. Finally, the second-level optimization is performed using genetic algorithms, whereby the smallest number of optimal sections that has an equal to or improved performance to the SFIA sections is determined. Different measures of performance (fitness) are explored. The paper finishes with a discussion of limitations, and potential improvements and future research possibilities, followed by the conclusions and acknowledgements.

Current P-M Space of U.S. CFS Structural Sections

U.S. CFS Structural Sections

For framing, the lipped channel (Figure 1) is the dominant cold-formed steel structural member in the United States and is employed for both wall studs and floor joists. Manufacturer consortiums such as the Steel Framing Industry Association (SFIA) [2], supply a broad range of structural (intended for load beading application) sections from 2.5 in. (63.5 mm) to 16 in. (406.4 mm) deep and 33 mils (0.84 mm) to 118 mils (3.00 mm) thick as summarized in Table 1. In total, SFIA produces 108 different sections across 11 different depths and 6 different thicknesses for structural use.

Strength prediction methodology

To determine the axial and major-axis bending capacity of the SFIA lipped channel sections the Direct Strength Method in the North American Specification for Cold-formed Steel Structural Members: AISI-S100 [3] is employed. This method requires that the elastic buckling loads in compression for local ($P_{cr\ell}$), distortional (P_{crd}), and global buckling (P_{cre}), and for major-axis bending ($M_{cr\ell}$, M_{crd} , M_{cre}) be calculated for every cross-section. The method explicitly recognizes the decreased capacity that is inherent in thin-walled members through consideration of these elastic buckling loads or moments in design formulae that combine the elastic buckling expressions with appropriate yield load (P_y) or moment (M_y) to produce axial (P_n) or bending (M_n) capacity.

For the capacity calculations performed here on the SFIA lipped channel sections the following additional assumptions are employed. For elastic buckling the material of the steel is assumed to be linear elastic, isotropic, with Young's modulus $E=29500$ ksi (203400 MPa) and Poisson's

ratio $\nu=0.3$. Consistent with final applications where members are sheathed and/or discretely braced all sections are assumed to be globally-braced; i.e., global buckling is removed ($P_{cre} \rightarrow \infty$, $M_{cre} \rightarrow \infty$). Local and distortional cross-section buckling loads (or moments) are determined using the semi-analytical finite strip method under simply-supported end boundary conditions as implemented in CUFSM [16]. Corners are not included in the finite strip models of the cross-sections. To address cross-sections where distinct elastic buckling minima do not exist in the finite strip signature curve the two-step procedure for identifying unique minimum is employed [23]. For strength, a material yield stress of $F_y=50$ ksi (345 MPa) is used throughout. The inelastic bending provisions of AISI-S100 are ignored, thus the maximum possible moment is the moment at first yield, M_y . Inclusion of the potential reduction for local or distortional buckling is critical in the method: the nominal axial capacity (P_n) of the 108 SFIA lipped channel sections is found to vary between 28% and 97% of the squash load (P_y).

Nominal P-M capacity space

If the developed P_n and M_n capacity for each SFIA lipped channel section is combined with a linear beam-column interaction expression, such as:

$$\frac{P_r^*}{P_n} + \frac{M_r^*}{M_n} \leq 1.0 \quad (1)$$

where P_r^* and M_r^* are the required axial and bending demand, then the nominal beam-column interaction space for all SFIA lipped channel sections may be generated as per Figure 2. Note, AISI-S100 employs a linear interaction expression for beam-columns, here (Eq. 1) provided without resistance factors or amplification for required second-order moments. Figure 2 provides the traditional view of member strength, where P_n and M_n form the x-axis and y-axis anchor points and any demand combination P_r^* and M_r^* that fall below the Eq. 1 line are satisfied, and

those above it indicate the section is inadequate. Figure 2 indicates the range of demands that are covered by sections currently available in the United States. The maximum nominal axial strength of all SFIA stud sections is 78.3 kips (348.3 kN) and the maximum nominal bending strength is 541.7 kip-in. (61.2 kN-m).

The traditional beam-column interaction diagram may be collapsed to a more convenient point representation in the P-M space for studying optimal cross-sections within this strength range. The anchor points, or isolated P_n and M_n capacities, are used as unique coordinates for each member in this space. The resulting representation in the P-M space, Figure 3, provides a single point representation of each cross-section and creates a less cluttered view of how the SFIA sections cover the P-M space. The role of thickness in determining capacity is highlighted through the choice of markers. In addition, members that are more efficient as beams or columns are easily identified based on their deviation from the norm with beam efficient members being closer to the y-axis and column efficient members closer to the x-axis.

Optimal Industry Available (SFIA) Lipped Channel Sections

This section determines the optimal (least weight) SFIA lipped channel section across the full range of current P_n and M_n capacities. First, the P-M search space is discretized. Next, the minimal weight section is found for each search point. Finally, the resulting optimal sections are presented and discussed.

Regularization and discretization of P-M space

To explore optimal solutions the P-M space is discretized into a 55x16 search grid spaced at 5 kip (22.24 kN) axial demand and 10 kip-in. (1.13 kN-m) bending demand as illustrated in Figure 4. Optimal solutions are explored at every intersection. Every grid point represents a potential

required demand for the isolated actions, P_r and M_r , for a given cross-section. Note P_r and M_r differ from P_r^* and M_r^* of Eq. 1, in that P_r and M_r are the required anchor points for the section i.e. in isolation of the other actions – consistent with the transformation from Figure 2 to Figure 3 as discussed in the previous section. The search space encompasses the complete SFIA P-M space, as Figure 4 indicates, and explores in detail sections that are potentially more optimized for bending or compression alone – i.e. the upper left of the P-M space explores sections which have high capacity in bending but may have low compression capacity and the lower right explores sections which have high capacity in compression, but may have lower bending capacity.

Lightest SFIA lipped channel shapes for P-M requirements

As a basis of comparison for potential optimal solutions, the least weight (minimum area) optimal SFIA section that is available at every grid point in the P-M space is determined and provided in Figure 5. Of the 108 potential sections 72 are found to have at least one unique (P_r , M_r) demand pair where they provide an optimal solution (these members are called out in Figure 4). In Figure 5 each optimal solution is indicated with a unique marker and color and designated using the section nomenclature of SFIA. For example the lightest SFIA section that can provide an isolated axial demand of at least 20 kip (89 kN) and bending demand of 100 kip-in (11.3 kN-m) is the 800S300-68. (Note, the SFIA section nomenclature as defined in the figure is H×100 S B×100 – t×1000 with all units in inches.)

As provided in Figure 6, three SFIA sections are selected to graphically depict the variation in optimal sections across different demands, and are used in later comparisons. The Figure 6 sections are optimal SFIA solutions for (a) high axial demand (with no bending requirement), (b) high bending demand (with no axial requirement), and (c) a relatively high axial and bending

demand. Overall, the optimal solutions generally follow the same thickness trends as the underlying family of sections. The greatest variation of optimal solutions occurs for the lighter demands: less than 30 kips (133 kN) axial and 150 kip-in (16.9 kN-m) bending, where the greatest number of potential SFIA sections also exist.

Level One: Member Optimization for Lipped Channels in the P-M space

For every search point in the selected P-M space (Figure 4) there exists an optimal (least weight) lipped channel that can meet the required P_r and M_r demands. First, this section introduces the Particle Swarm Optimization scheme that is used for the optimization. Next, this section provides the specific objective function and constraints that are employed for the lipped channel optimization. Finally, the results are presented, compared with the optimal industry shapes, and discussed.

Optimal lipped channel dimension: Particle Swarm Optimization background

Given that a lipped channel is defined by variables H , B , D , and t , and fixed material properties E , and F_y , a minimal weight lipped channel solution is sought for all studied P_r and M_r in the P-M space. The optimization is performed using Particle Swarm Optimization (PSO) a population-based stochastic search method for continuous nonlinear functions. Given that this is the first application of PSO in this context a brief review of PSO is provided followed by the objective function formulation, the incorporation of constraints, and the final results.

The PSO algorithm was first proposed by Kennedy and Eberhart [24]. The method is inspired through simulation of a simplified social model such as bird flocking or fish schooling in particular [25]. The methodology of PSO is to optimize a problem by having a population of

candidate solutions, known as particles, and moving these particles around in the search-space according to simple mathematical formulae over the particle's position and velocity [24]. Specifically, the movement of each particle is influenced by its local best-known position and is also guided toward the best-known positions in the search-space that is shared among the swarm and is updated as better positions are found by other particles. Thus, the swarm is expected to move toward the best solutions.

There are a number of advantages of PSO with respect to other optimization algorithms. The algorithm is robust and well suited to handle non-linear, non-convex design spaces. Compared to other design optimization algorithms, PSO is more efficient and leads to the same or better quality of results with fewer number of function evaluations [26]. In addition, PSO is easy to implement making it very attractive. Applications of PSO in structures can be found in the area of structural shape optimization [27], [28].

A general optimal design problem is formulated as finding the minimum of the objective function f such that

$$\begin{aligned} & \min f(x) \\ \text{s.t.: } & g_j(x) \leq 0, j=1,2,\dots,m \end{aligned} \quad (2)$$

where, x is a column vectors of the design variables, g_j , representing the general inequality constraints, are the scalar functions of the design variables x .

Consider a swarm of n particles. For particle d (total p particles), the design variable vector x^d (i.e., positions in PSO terminology) is updated as

$$x_{k+1}^d = x_k^d + v_{k+1}^d \quad (3)$$

while the velocity v^d is updated as

$$v_{k+1}^d = wv_k^d + c_1r_1(p_k^d - x_k^d) + c_2r_2(p_k^g - x_k^d) \quad (4)$$

where, k indicates the iteration; w , c_1 and c_2 are problem dependent parameters; w is the inertia weight that is updated by fraction a if no improved solution is obtained within h_step consecutive iterations; c_1 and c_2 represent “trust” parameters indicating the confidence level the current particle has in itself and in the swarm; r_1 and r_2 represents random numbers between 0 and 1; p_k^d is the best-known position of particle d up to iteration k , and p_k^g is the best-known position in the swarm up to iteration k .

After updating the positions of each particle, one evaluates the objective function $f(x_{k+1}^d)$ using the updated design variables x_{k+1}^d and updates the best-known positions of each particle as well as the best-known positions of the swarm. The search process continues until the stopping criteria are met, which could be the maximum number of iterations (k_{max}) or when convergence of the design detected. Usually convergence is defined as when the global best-known position does not change during a large number of iterations (s_stop).

Boundary and constraint handling require additional consideration. For optimal problems in structural design, the design variables usually have specific bounds. When updating the design variables in Eq. (3), it is possible that the design variables may not be in the feasible region. For the shape optimization problem in this paper, the design variables must be positive. Any negative design variables would cause the strength evaluation to fail. Therefore, a special boundary handling technique is needed. The implemented boundary handling method is the exponential distribution method of [29]. This approach allows the particle to be brought back inside the search space, dimension-wise, in the region between its old position and the bound based on an exponential probability distribution.

When the general constraints are violated, a parameter-less adaptive penalty scheme similar to those in [27] is employed. The scheme utilizes the average of the objective function and the level

of violation of the constraints in order to define the penalty for the constraints as following:

$$f^*(x) = \begin{cases} f(x) & \text{if } x \text{ is feasible} \\ f(x) + \sum_{i=1}^m k_i g_i(x) & \text{otherwise} \end{cases} \quad (5)$$

where the penalty parameter k_i for each violated constraint is defined at each iteration as

$$k_i = |\bar{f}(x)| \frac{\bar{g}_i(x)}{\sum_{j=1}^m [\bar{g}_j(x)]^2} \quad (6)$$

where m is the number of violated constraints; $\bar{f}(x)$ is the average of the objective function in current swarm; $g_i(x)$ is the constraint value of i^{th} constraint (greater than zero when violated); $\bar{g}_i(x)$ is the violation of the i^{th} constraint averaged over the current population. A summary of the algorithm is provided in the flow chart of Figure 7.

Optimal lipped channel dimension: Particle Swarm Optimization implementation

The objective of the optimization is to seek a lipped channel section with minimal material (i.e., least sectional area A) for a given required strength (P_r , M_r) subject to manufacturability and usability constraints. The result is a constrained optimization problem that can be mathematically formulated as:

$$\begin{aligned} \min A &= f(H, B, D, t) \\ \text{s. t.:} \\ \text{General constraint:} \\ P_n &\geq P_r, M_n \geq M_r \\ \text{Design variable bounds:} \\ H_{min} &\ll H \ll H_{max} \\ B_{min} &\ll B \ll B_{max} \\ D_{min} &\ll D \ll (H - D_{gap})/2 \\ t_{min} &\ll t \ll t_{max} \end{aligned} \quad (7)$$

where, H_{min} is 1.5 in. (38.1 mm), B_{min} and D_{min} are 0.25 in. (6.4 mm), H_{max} and B_{max} are 30 in. (762.0 mm), D_{gap} , the distance between lips, is 1 in. (25.4 mm), t_{min} is 0.018 in. (0.46 mm) and t_{max} is 0.118 in. (3.00 mm). P_n and M_n are the nominal axial and bending strengths of the optimized shapes. The constraints are based on basic usability of the final section, including the

necessity to pass services through the web, which requires a minimum H and minimum gap between the lips, D; and the need to connect boards (sheathing) to the flanges which requires a minimum B. The maximum H and B are set high enough to allow a complete search of the parameter space. The thickness range is set equal to the full thickness range (nonstructural and structural) used by SFIA.

Strength P_n and M_n are calculated using the Direct Strength Method integrated with automated elastic buckling analysis by CUFSM as explained in the strength prediction section. The optimization is completed on the 16x55 grid using the PSO framework. The initial design for each demand (P_r , M_r) is the optimal SFIA shape, unless no optimal shape exists then the initial design is randomly selected from those closest SFIA shapes. Although PSO is a global random search algorithm, similar to genetic algorithms, mathematical proof of its convergence does exist [30], [31], and guideline for parameter selection (i.e., those problem dependent parameters, w , c_1 and c_2) are possible [31]. Based on a convergence investigation, in this paper, the size of particles is 20, initial inertia weight $w=0.7$ with fraction $a=0.95$, updating inertia weight update threshold steps $h_step=20$, maximum iteration $k_{max}=400$, $c_1=2$, $c_2=1.6$, and the stop threshold of no improvement $s_stop=80$. For each demand (P_r , M_r), a total of 2000 to 8000 objective functions will be evaluated depending on the iterations needed, with CPU times range from 40 hours to 100 hours.

Optimal lipped channel dimension: Results

The three optimal SFIA solutions of Figure 6 are re-examined here to demonstrate the difference that occurs with the less constrained PSO-based optimization results. As shown in Figure 8 the PSO-based optimal lipped channel has different section dimensions (and thickness), and in some cases, for example the high axial demand case of Figure 8a, substantially different solutions.

Convergence of the PSO scheme for the high axial demand cases is shown in Figure 9. This rate of convergence was typical across the studied sections.

It is found in every case that the SFIA optimal sections are heavier (greater area) than the PSO-based optimal lipped channel sections. A contour plot of the ratio of SFIA optimal area ($A_{SFIAopt}$) to PSO-based optimal lipped channel area (A_{LCopt}) for the complete P-M space is provided in Figure 10. The minimum area ratio is 1.04, the maximum is 2.03, and the mean is 1.14. Hence, on average, the PSO-based optimal lipped channel sections are 14% more efficient. For sections with high axial demand (but low bending demand) the PSO-based lipped channel optimal sections have the greatest efficiency gains over the SFIA optimal shapes. It is worth noting again that these solutions are optimal shapes for the minimum of local and distortional buckling strength; global buckling is assumed removed through bracing, and stiffness is not examined.

Comparison of the capacities of the SFIA sections vs. the PSO-based optimal lipped channels across the P-M space is summarized in Figure 11. The results indicate that up to a point axial load may be independently optimized separate from bending, but beyond a certain front this becomes inefficient and the two become tied. To better understand the optimal lipped channel results two sets of sections, called out in Figure 11, are studied in detail. The two sets cover the set of optimal sections with M_r varied but $P_r=0$ (bending dominant) and the set with P_r varied but $M_r=0$ (axial dominant). The dimensions from these two classes of optimal lipped channels are provided in Figure 12.

The optimal dimensions that the PSO algorithm arrives at for bending dominant and axial dominant members is summarized in Figure 12. For bending members the web depth and thickness are systematically increased to carry greater bending demand. Interestingly, as Figure 12c shows the web width-to-thickness ratio (H/t) is held at a constant for the bending members.

For local buckling, at this H/t no reduction occurs, thus the algorithm is providing the “fully effective” bending capacity. Similarly, for bending the flange B/t , the overall shape, H/B , and the length of the lip compared with the flange, D/B , are all at nearly fixed values. For the axial members the optimal solution is generally square (H/B near 1) and the thickness increases for higher demands, but the flange, web, and lip dimensions vary more significantly as the optimal condition between local (largely driven by H/t and B/t) and distortional (more closely related to D/B) buckling is found by the PSO algorithm.

Level Two: Family Optimization for Lipped Channels in the P-M Space

The PSO-based optimal lipped channels from the level one optimization have significant material efficiency gains compared with the standard SFIA shapes. However, the solution is unconstrained by manufacturing realities and provides 880 unique solutions, one for each unique (P_r, M_r) investigated. The second level optimization problem is to determine, for a limited subset of lipped channels, i.e., a family of lipped channel sections, how many different unique sections are required to perform equal to or better than the current family of SFIA shapes? To answer this question the “fitness” or overall baseline efficiency of the current SFIA family needs to be determined. With this baseline metric established, the fitness of any new optimal family can be assessed and the number of unique sections required to meet or exceed the baseline fitness determined.

Baseline fitness of the optimal SFIA shapes

The ratio of areas between the optimal SFIA shapes and the level-one PSO-based optimal lipped channel shapes are utilized to gauge baseline fitness. Simple summation of the area ratios over the full P-M space is employed as the baseline fitness:

$$F_{\text{SFIA}} = \sum_i^{m \times p - 1} \frac{1}{m \times p} \frac{A_{\text{SFIAopt}[i]}}{A_{\text{LCopt}[i]}} \quad (8)$$

where, $A_{\text{SFIAopt}[i]}$ is the optimal SFIA shape at grid point i and $A_{\text{LCopt}[i]}$ is the level-one PSO-based optimal lipped channel shape at grid point i , m is the number of regularized grid points in bending capacity (i.e., 55 in this case) and p is the number of grid points in axial capacity (i.e., 16 in this case).

The baseline fitness in Eq. (8) assumes that each nominal demand over the P-M space has the same importance. In design, members are frequently categorized either as primarily a column or a beam; therefore, the demands for nominal pure axial or bending strength could have higher importance. In view of this, an alternative weighting scheme is also proposed:

$$\bar{F}_{\text{SFIA}} = \sum_i^{m \times p - 1} w_i \frac{A_{\text{SFIAopt}[i]}}{A_{\text{LCopt}[i]}} \quad (9)$$

where, w_i is the weighting factor at grid point i . Keeping in mind that typical end-use applications for cold-formed steel framing will be prone to pure axial or bending members, heavier weight should be assigned toward high axial and high bending capacity demand. Therefore, the proposed weighting coefficients are 0.25/16 for the 16 pure axial (P_r , 0) cases, 0.25/55 for the 55 pure bending strength (0, M_r) demands and 0.5/809 for the remaining beam-column demand (P_r , M_r and $P_r > 0$, $M_r > 0$) cases.

Optimization formulation for selecting reduced families of members

For the second-level optimization the objective is to minimize the fitness (F) given a family size (n) of optimal lipped channel sections and try to find the smallest family size that provides $F \leq F_{\text{SFIA}}$, specifically:

$$\min F(x_n) = \sum_i^{m \times p - 1} w_i \frac{A_{\text{LC-family-opt}[i]}}{A_{\text{LCopt}[i]}} \quad (10)$$

where, x_n is the design variables containing the dimensions for the optimal lipped channel shapes and indicating this optimal shapes is selected into the new family. $A_{LC-family-opt[i]}$ is the least-weight shape at grid i available in the new family of optimal lipped channel shapes. Family size n may be varied from 1 up to the maximal number ($m \times p - 1$) of available optimal lipped channel sections. The fitness converges to 1.0 if the family size is equal to the maximal number, since this is effectively the level-one optimization problem.

This second level family optimization problem is solved using a genetic algorithm (GA) schema. GA is a popular stochastic search algorithm that operates on populations and mimics evolutionary theory in its algorithm and vocabulary [32], [33]. GA belongs to a larger class of algorithms using techniques inspired by natural evolution, such as inheritance, mutation, selection, and crossover. GA operates on a population of designs and iterates generation by generation. The designs are then evaluated and ranked according to their objective function performance. New populations are created through crossover and mutation. Designs with higher rank have a higher probability of being selected for crossover, and thus the performance of the population as a whole should improve as the optimization progresses. Typical termination criteria include a maximum number of generations, or detecting a predefined convergence tolerance. For example, convergence is defined here as occurring when the elite design does not change over a large number of iterations (n_{max}).

A summary of the GA implemented here for this discrete unconstrained optimization problem is provided in the flow chart of Figure 13. Here, the population $np=40$, $k_{max} = 2000$, $n_{max} = 200$, the mutation rate is 0.08, and the crossover rate is 0.8.

Results for optimization as a function of family size

The fitness, F , for the optimal lipped channel shapes as a function of the family size, n , is provided in Figure 14. The results show that for uniform weighting of the fitness function a family of 14 unique lipped channels has an equal to or better fitness than the 108 (72 unique optimal) SFIA shapes across the P-M space. If the alternative (proposed) beam and column weighting is employed on the fitness function then only 12 unique sections are required to equal or better the fitness of the SFIA shapes.

The dimensions from the $n=12$ family of optimal lipped channel sections are listed in Table 2 and are compared against the SFIA shapes in the P-M space in Figure 15. For comparison, an $n=30$ family of optimal shapes is also highlighted so the convergence of Figure 14 can be judged in terms of how the optimal shapes cover the P-M space with limited family size. Further manufacturing constraints can be enforced to provide optimal sections that are more regularized than the current groups, but the potential for equaling or exceeding current performance with vastly reduced numbers of profiles is real. Note, as detailed in footnote (a) of Table 2 American standards [3] allow delivered material thickness to be 95% of the designated thickness, as a result member naming conventions as used by manufacturers such as SFIA in [2] use a designation thickness and also provide minimum and design thickness values.

Even though the new family of sections meets the fitness provided by the SFIA sections across the entire P-M space, at any particular point, i , (or region) the new family may provide an increased area. A contour plot of the area ratios, similar in spirit and scale to that provided in Figure 10 is provided in Figure 16 for the $n=12$ member optimal family of lipped channel

sections. In the most efficient case the SFIA optimal section is 1.75 times heavier than the $n=12$ member optimal family section, in the least efficient case the ratio is 0.55, the mean area ratio is 0.98, and the standard deviation across the space is 0.15. Figure 16 exhibits different regimes of high or low efficiency – in particular the optimal family of shapes is generally more efficient under high axial demands and less efficient for sections with high combinations of axial and bending demand.

Discussion

The optimization results presented herein demonstrate that there is still room to improve upon even the conventional shapes used in cold-formed steel construction. Further, it is shown that it is possible to develop small groups of members, i.e., families, that can provide equivalent or better performance as large classes of members in current use. Such an improvement could lead to decreased manufacturing costs and potentially broaden applications of use.

The member optimization provided covers only axial and bending strength. Shear strength, web crippling strength, (with appropriate combinations) and stiffness are also common and need to be incorporated. The optimization was performed assuming global bracing (i.e., considering only local and distortional buckling) is provided, consistent with typical end use applications for cold-formed steel framing. However, unbraced lengths of 2 to 4 ft (0.6 to 1.2 m) are common and should be incorporated. Distortional buckling was assumed to be unbraced in the optimization performed. This is conservative, but some restraint is typical and could be established and employed if end use details (sheathing, spacing, etc.) are incorporated into the optimization.

The member optimization in this paper has only minimal manufacturing constraints. Practical ranges were established, but discrete dimensions are usually employed in practice and could be incorporated. In addition, some manufactures aim to use light portable roll formers, these constraints can be introduced as greater limitations on slit width and thickness etc., but still arriving at an optimal family of efficient members.

The weighting factors for fitness determination deserve further consideration. Specifically, weighting factors based on the probability of end use make more economic sense than equally valuing all locations in the P-M space. Case studies of existing buildings or sales history of an existing manufacturer could be used to develop improved weighting factors.

The sensitivity of the solution was explored. The optimal families of members that results from the level two optimization are not unique, but do consistently follow the trends established in Figure 11. Sensitivity of the optimal family size to the search grid density was also investigated. For a discretization $\frac{1}{2}$ as fine as the original an optimal family size of 11 members (as opposed to 12) is required to beat the fitness of the SFIA shapes. Thus, the family size solution is regarded as reasonably robust in this respect.

The framework developed here to optimize families of cold-formed steel shapes for manufacturing is believed to have great potential. In the future, utilizing more detailed implementations of the member shape optimization can provide a means to assess and help encourage the introduction of new optimal cold-formed steel shapes that fully take advantage of the flexibility afforded in the manufacturing process.

Conclusions

Families of optimal (least weight) cold-formed steel lipped channel sections against local and distortional buckling with as little as 12 members can meet the overall axial and bending performance of current industry available lipped channel catalogs including as many as 108 different sections. The work herein provides a means to evolve cold-formed steel shape optimization from single members under single applied actions to families of members under multiple applied actions. Family optimization across multiple actions is more consistent with the optimization requirement for producers of cold-formed steel shapes. The optimization is performed in two levels. In the first level member optimization across an established axial and bending space is completed using a Particle Swarm Optimization algorithm. Objective function evaluation employs the Direct Strength Method and a finite strip based elastic buckling solution and is general enough to incorporate unusual cross-section geometries, but is implemented here for lipped channels. Using a targeted set of shapes, for example those in current industry use, a baseline fitness across the selected axial and bending space is established. In the second level optimization a genetic algorithm is used to determine the optimal fitness for a family of sections of a given population size. The smallest population size that meets or improves upon the baseline established fitness is the minimal family size needed for manufacturing. The established framework provides a means to employ optimization to develop minimal sets of sections required for manufacturing and is applicable for future work considering additional limit states, more general cross-sections, and improved fitness functions.

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Table 1: Dimensional and Property Limits of SFIA CFS Lipped Channel Sections

	H		B		D		$t^{(a)}$	
	(mm)	(in.)	(mm)	(in.)	(mm)	(in.)	(mm)	(in.)x1000
min	63.5	2.5	34.93	1.375	9.53	0.375	0.84	33
max	406.4	16.0	88.90	3.500	25.40	1.000	3.00	118

- (a) Designation thickness reported, designation thickness is 95% of design thickness
- (b) Corner radius $r = 2t$
- (c) Nominal yield stress, F_y , is either 33 ksi (227 MPa) or 50 ksi (345 MPa) depending on the thickness

Table 2: Dimension of elite design for n=12 family of sections

ID	H		B		D		$t^{(a)}$	
	(mm)	(in.)	(mm)	(in.)	(mm)	(in.)	(mm)	(in.)x1000
1	69.5	2.74	10.5	0.41	18.9	0.74	1.5	59
2	76.3	3.00	48.5	1.91	23.6	0.93	1.7	68
3	99.8	3.93	48.9	1.92	6.4	0.25	2.5	97
4	136.5	5.37	31.7	1.25	49.5	1.95	3.0	118
5	141.1	5.56	66.0	2.60	13.7	0.54	2.3	91
6	230.1	9.06	83.9	3.30	24.7	0.97	3.0	118
7	237.1	9.33	67.2	2.65	17.4	0.69	2.2	88
8	250.3	9.85	73.6	2.90	17.3	0.68	2.4	94
9	290.2	11.42	110.0	4.33	20.0	0.79	2.9	115
10	292.5	11.52	91.8	3.61	19.3	0.76	2.8	111
11	320.2	12.61	70.8	2.79	16.9	0.67	3.0	118
12	332.4	13.09	103.6	4.08	26.8	1.06	3.0	118

(a) Designation thickness reported, designation thickness is 95% of design thickness

(b) Nominal yield stress, $F_y=50$ ksi (345 MPa)

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(b) 1400S350-97, high bending
optimal ($M_r > 350$ kips-in. or 40 kN-m)

(c) 1400S200-118, optimal for
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