A Compression Model for Ultimate Postbuckling Shear Strength

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Abstract: Tension field theory has traditionally been used to determine the ultimate postbuckling shear strength of steel plates. More than a dozen theories have been proposed in the last nine decades to describe and predict this phenomenon, and all are based on the tensile response of the web plate, referred to as tension field action. Alternatively, in this paper a compression approach for determining the ultimate postbuckling shear strength is studied. First, an experimentally-validated finite element model is used to examine the mechanics of plate shear buckling. The response is shown to be similar to axially compressed plates, but in this case the axial compression is acting on a diagonal. Then a physical model and formulation based on the compressive strength of the plate is developed for predicting the ultimate postbuckling shear strength of a plate. For common design parameters of most bridge and building structures, this compression approach produces strengths that are closer to experimental and finite element results than the best and commonly accepted formulation based on tension field action. Overall, the results of this study show that a compression approach to predicting the postbuckling shear capacity of plates is an honest representation of shear buckling mechanics and has good correlation to extensive experimental results, where in many cases improved correlation is seen compared to formulations based on tension field action.

Keywords: tension field, shear buckling, postbuckling, plate girder, plate buckling

1. Introduction

Plate shear buckling is an intriguing phenomenon because, unlike compressive structural elements (a column, for example) that lose their load-carrying capacity once elastic buckling has occurred, plates that elastically buckle due to shear still possess a significant amount of postbuckling shear strength. This postbuckling behavior has attracted the attention of researchers and engineers since the 1880s. Attempts to quantify this behavior led to the historical development of tension field theory, which argues that the source of this postbuckling shear strength is the development of tensile stresses in a defined diagonal field that is mobilized post-elastic shear buckling. Beginning with Wagner in 1931 [5], more than a dozen proposals have been developed to explain and predict the postbuckling shear strength based on tension field action. Previous authors have provided extensive discussions on the various proposed plate shear buckling models throughout the literature [1, 2, 3], and all are based on tension field action. As Yoo and Lee [3] point out, this fact "... is testimony to the complexity of tension field action. This may be the largest number of failure theories devoted to a single topic in structural mechanics." While a tension field does form, compression is always the catalyst to buckling, and further, many of the assumptions inherent to tension field theory have since been proven to be inaccurate as will be discussed later. These observations motivated the authors to examine postbuckling shear behavior based on a compression approach.

Tension field theory is based on the key assumptions that (1) compressive stresses cease to increase in the postbuckling range [3, 4], and (2) the stiffeners, flanges, or both anchor the tension field. The first assumption was proposed by H. Wagner in 1931 and stated that once elastic shear buckling occurred in the web, compressive stresses do not increase and any additional (or postbuckling) shear strength is mobilized due to the formation of diagonal tensile

stresses [1, 5, 6, 7]. This assumption is simultaneously the most important and most controversial because accepting it transforms a buckling problem, which is inherently a compression problem, into one based on tension. It is a fascinating historical irony to formulate a buckling problem in terms of tension, but one rooted in compelling experimental observations. However, the assumption that compressive stresses do not increase in the postbuckling range was shown not to be accurate based on findings from numerous authors using finite element (FE) analysis [2, 3, 8, 9, 10, 11]. In fact, near the edges, compressive stresses increase considerably [3].

The second assumption has its basis in the earliest recorded explanation for the postbuckling shear strength of stiffened plate girder webs offered by J. M. Wilson in 1886 [1, 12]. His explanation arguably established the foundation of tension field theory by suggesting that the transverse stiffeners act like the posts of a Pratt truss, with the web handling the duty of carrying the diagonal tensile stresses. However, previous researchers have found that the large axial loads expected to develop in the transverse stiffeners due to anchoring the diagonal tension field were smaller than expected for a truss action to develop as assumed [3, 8, 9, 13]. Similarly, Yoo and Lee [3] found that the vertical stresses reduce to zero near the flange, thus indicating that the flange does not act as an anchor to the tension field.

This paper draws parallels to axially compressed simply supported plates as shown in Figure 1. Comparing Figures 1(a) and (b) we note that, in an elastic state, a plate in pure shear is similar to an axially compressed plate except that it is loaded on a diagonal with equal tensile stresses acting perpendicularly. In Figures 1(c) and (d) we note that in the postbuckling range, both the axially compressed plate and the pure shear loaded plate develop a field of tension perpendicular to the field of compression. It is this tension field, in both cases, that allows postbuckling strength to develop. In both cases, the compressive stresses increase at the edges leading to

similar patterns of stress distributions. Physically this distribution can be explained by considering that the out-of-plane deformations are largest at the center of the plate. The larger the out-of-plane deformations the smaller the axial rigidity, therefore near the center, the stresses will not increase much (or at all) after elastic buckling. Near the edges, the out-of-plane deformations are much smaller and therefore the stresses continue to increase after elastic buckling.

This paper presents a novel approach to predicting the postbuckling shear capacity of plates based on the compressive behavior dominating the mechanical response. The proposed approach does not ignore the tension field, which is actually shown to provide additional stability such that postbuckling shear capacity may be mobilized. Closed-form equations are presented to predict the postbuckling (ultimate) shear buckling capacity of steel plates. Note that it is not the authors' intention to propose that these equations be directly incorporated into design codes. To meet such a goal, the equations may need to be simplified and perhaps also developed for a broader range of parameters. The fundamental goal of the work presented in this paper is to show the strong promise of a compression-based model for predicting the ultimate buckling load of a plate loaded in shear.

Since, as just illustrated, the behavior of a plate under pure shear is similar to that under pure axial load, the proposed approach is based on considering the plate acting as a column on a diagonal. The parallel of this approach to axially loaded plates is illustrated by comparing Figures 1(e) and (f). In an axially loaded plate, the “equivalence condition” for evaluating the postbuckling strength of the plate is done by transforming the postbuckling stress distribution shown in Figure 1(c) into a stress distribution acting over an equivalent width, b_e [14]. For plates loaded in pure shear, we propose instead to make the equivalence condition based on the length

of the diagonal column. The equivalent column length, L_e , is equal to the diagonal length of the plate minus the width of the tension field. The challenge is then to predict (1) the width of the tension field, (2) the axial load on this equivalent column, and (3) convert that axial load to a shear load. This paper illustrates this procedure to arrive at a formulation to predict the postbuckling shear capacity of plates. This compression approach is compared to the best and most well-known formulation for postbuckling shear strength of plates developed by Basler in 1961 [4], which is based on tension field theory. The next section therefore describes Basler's approach in detail.

Figure 1. Plate behavior (simply supported) under pure axial load compared to pure shear load.

2. Basler's Tension Field Approach

White and Barker [2] compare the shear resistance of the twelve “most promising” tension field based models to the experimental results of 115 to 129 tests of steel I-girders. They found the model by Basler [4], which is implemented in AASHTO [15] and AISC [14], to have the “best combination of accuracy and simplicity”. This section therefore focuses on this Basler model, which will be used as a comparison to a compression model that will be developed in this paper.

In addition, Porter et al. [16], and Höglund [17, 18] will be briefly discussed in this section since these currently serve as the basis for the European design codes [1, 15, 16, 17].

In 1961, Konrad Basler suggested a model to calculate the ultimate postbuckling shear strength of steel plate girders derived from the development of diagonal tension field stresses and based on the assumptions previously discussed [1, 4]. Figure 2 shows a schematic for the tension field of a simply supported plate at the ultimate postbuckling shear stress, τ_u .

Figure 2. Orientation of the diagonal tension field at τ_u for a plate with simple supports (SS) assumed for all four edges. D is the depth of the plate, and a is the span between transverse stiffeners.

The flanges were assumed to be too flexible to resist any vertical loading from the diagonal tension field, thus this field was oriented such that the vertical component of the tensile stresses was resisted by the transverse stiffeners [1, 4]. This hypothesis corresponded with the Pratt truss observation made by Wilson in 1886 [1, 12].

Based on the model shown in Figure 2, Eqn. (1) was developed to calculate the τ_u value of a simply supported steel plate. Eqn. (1) is a modification by Gaylord, Fujii, and Selberg [1, 19, 20, 21] of the equation originally-proposed by Basler.

$$\tau_u = \tau_{cr} + \sigma_y \left(1 - \frac{\tau_{cr}}{\tau_y} \right) \left(\frac{\sin \theta_d}{2 + \cos \theta_d} \right) \quad (1)$$

The ultimate postbuckling shear load, V_u , can be calculated by multiplying τ_u by Dt_w , where t_w is the thickness of the plate and D is the depth of the plate as shown in Figure 2. In Eqn. (1), σ_y and τ_y are the yield and shear yield strengths of the plate, respectively, θ_d is the angle of the panel diagonal, and τ_{cr} is the elastic shear buckling stress. τ_y may be calculated as $0.6\sigma_y$ [14], while τ_{cr} may be calculated from [1, 22]:

$$\tau_{cr} = \frac{k\pi^2 E}{12(1-\nu^2) \left(D/t_w \right)^2} \quad (2)$$

where E is Young's modulus, ν is Poisson's ratio, D/t_w is the slenderness ratio, and k is the shear buckling coefficient. The k value depends on the span-to-depth ratio, a/D (see Figure 2), as well as the assumed boundary conditions. For the four-sided simply supported plate shown in Figure 2, $k = k_{ss}$ may be calculated as [1, 22]:

$$k_{ss} = 4.00 + \frac{5.34}{\left(a/D \right)^2} \text{ for } a/D < 1 \quad (3a)$$

$$k_{ss} = 5.34 + \frac{4.00}{\left(a/D \right)^2} \text{ for } a/D \geq 1 \quad (3b)$$

A simplification of Eqn. (3) is used in AASHTO [15] and AISC [14] as proposed originally by Vincent [2] by using the integer 5.00 in lieu of both 5.34 and 4.00, thus resulting in one equation for all a/D ratios. A maximum difference of 5% is noted in this case [2].

For situations in which the flange is explicitly considered, equations to calculate k were developed that interpolate the value between two extreme cases - a flange modeled as providing

either a simple or a fixed support to the web (the stiffeners are typically treated as simple supports [1]). These equations for k can be found in [6, 7].

From Eqn. (1), τ_u is a summation of τ_{cr} plus the second term that represents the stress that develops in a defined tension field. This underscores the use of Wagner's assumption in Basler's model: once τ_{cr} has been reached, the only source of postbuckling strength up to τ_u is derived from the diagonal tension field stresses.

Basler's model is related to the Cardiff tension field theory model proposed by Porter et al. [16], except that the latter model explicitly accounts for the presence of the flanges in anchoring the tension field and does not impose a restriction on the orientation of the diagonal tension field. This means that the tension field could be anchored by both the transverse stiffeners and the flanges [1]. An interesting characteristic of the Cardiff model is that several other proposed tension field theories, including the Basler model, are actually specialized cases of this model [1, 16]. By assuming that the flanges cannot develop moment resistance, the Cardiff model reduces to Basler's model.

T. Höglund's theory does consider compression through the principal stresses, but it is still largely based on tension field action [17, 18] and it is not similar to that developed by the authors. Their model assumes that the ultimate postbuckling shear strength of a girder is the sum of (1) V_w , the web shear resistance and (2) V_f , the shear resistance offered by the flanges. Höglund assumes that V_w develops due to what he called a rotated stress field (i.e., the decreasing angle of the principal stresses as the shear load increases beyond elastic buckling) and it considers both the tension and compression principal stresses. V_f is assumed to develop due to a diagonal tension stress field, which is very similar to the diagonal tension field observed in the

Cardiff model except that in Höglund's model it is oriented between the top and bottom flanges only.

3. Finite Element Models

In the sections that follow, the postbuckling mechanics of steel plates loaded in pure shear will be examined through finite element analysis. Therefore, this section describes the development of the finite element models and validates these models by comparison to experimental results.

3.1. Boundary conditions and mesh

Figure 3(a) shows the assumed boundary conditions, while Figure 3(b) shows the stresses in the plate before the elastic shear buckling stress, τ_{cr} , is reached. A state of pure shear was observed in the model since the angle of the principal stress, φ , was 45° and the principal stresses in tension and compression (σ_{Max} and σ_{Min} , respectively) were equal to one another and also equal to $\tau = V/(D \cdot t_w)$, where V is the applied load.

The FE mesh and first mode buckled shapes are shown in Figure 4 for FE models with $D = 1.47$ m, $a = 1.47, 2.06, 2.21, 2.94, 3.68$, and 4.41 m, and $t_w = 0.011$ m. The FE models were all meshed using S4 (doubly curved, general-purpose, finite membrane strains) shell elements [23]. Mesh convergence studies were conducted using an eigenvalue extraction analysis. Various mesh densities were studied, and the selected meshes typically had percent errors less than 2.5%. Percent errors were calculated by taking the difference of τ_{cr} based on FE from that based on Eqn. (2) and dividing by τ_{cr} based on Eqn. (2).

Figure 3. (a) Boundary conditions imposed in the finite element model to simulate pure shear loading, where a “•” indicates a restrained translational (U) or rotational (UR) degree of freedom; and stress state in an infinitesimally small element (b) before elastic buckling and (c) after elastic buckling.

Figure 4. Mesh densities for FE models 1 through 6.

3.2. FE model validation

Previous work by the authors validated the nonlinear postbuckling analysis procedures used for these FE models at both ambient and elevated temperatures [10]. In this paper, additional FE model validation studies were done by comparing the FE results to 16 experiments as presented in Table 1. In this table, the FE/Exp value equals the FE V_u value (V_u^{FE}) divided by the experimental V_u value (V_u^{Exp}). The dimensions and V_u^{Exp} values were taken from [17]. V_u^{FE} was determined by examining the load-displacement curve and selecting the point where the load-displacement curve becomes horizontal or reverses slope as discussed in detail in [10].

Table 1 represents the published results of numerous experiments conducted over several decades. Each researcher adopted a different nomenclature for their test specimens. To maintain consistency with the historical record, the names of each test specimen as originally published are preserved in Table 1.

Table 1. Comparison of FE and experimental V_u values.

Specimen	a (mm)	D (mm)	t_w (mm)	a/D	D/ t_w	t_f/t_w	σ_y (MPa)	V_u^{FE} (kN)	V_u^{Exp} (kN)	FE/Exp
G6-T1 ^[24]	1905	1270	4.9	1.50	259	4.04	253	542	516	1.05
G7-T1 ^[24]	1270	1270	4.98	1.00	255	3.92	253	649	623	1.04
G7-T2 ^[24]	1270	1270	4.98	1.00	255	3.92	253	649	645	1.01
G8-T1 ^[24]	3810	1270	5.08	3.00	250	3.76	263	403	378	1.07
G8-T2 ^[24]	1905	1270	5.08	1.50	250	3.76	263	562	445	1.26
G8-T3 ^[24]	1905	1270	5.08	1.50	250	3.76	263	562	516	1.09
2.2 ^[25]	1440	600	2	2.40	300	3.00	255	75	75	1.00
US3/5 ^[26]	788	359	2.7	2.19	133	4.44	257	99	90	1.10
STG1 ^[27]	551	279	2	1.97	140	3.95	255	53	60	0.89
STG4 ^[27]	498	251	1.25	1.98	201	5.12	246	25	35	0.71
RTG1 ^[27]	305	305	1.27	1.00	240	3.54	244	40	40	1.01
RTG2 ^[27]	305	305	1.27	1.00	240	3.70	244	40	41	0.99
MSO ^[28]	947	608	2.01	1.56	302	5.02	261	100	94	1.06
CPI/1 ^[29]	747	500	2.04	1.49	245	3.92	246	88	88	1.00
S-2 ^[30]	581	319	3.2	1.82	100	3.28	352	158	161	0.98
S-3 ^[30]	577	477	3.2	1.21	149	3.28	317	208	198	1.05

Note: Superscripts [24], [25], [26], [27], [28], [29], [30] are references to publications.

All of the FE models in Table 1 used the boundary conditions from Figure 3. Results in Table 1 give confidence that the FE models can capture the ultimate postbuckling shear strength of plates. It is seen that the FE models predicted V_u values to within about 10% of the published experimental values with only two notable exceptions. For specimen G8-T2, the FE/Exp value was 1.26, however specimen G8-T3, which was a repeat of experiment G8-T2 had a FE/Exp value of 1.09. For specimen STG4, the flange-to-web thickness ratio (t_f/t_w) was quite large compared to other tests. It is hypothesized that the flanges, while neglected in the FE model, would have contributed to the ultimate postbuckling shear strength in the experiment and, thus, the correlation of FE to experimental results was not as accurate as the other data.

4. Plate Stresses at the Ultimate Postbuckling State

Yoo and Lee [3] present a study of the mechanics of plate buckling using finite element results. Their study, however, is only for one square plate ($a/D=1.0$). To expand the discussion of shear buckling mechanics, plates with a/D values of 1.0, 2.0, and 3.0 are discussed here. Both the normal and principal stresses in the plate are considered in this study. The normal stresses are referred to as σ_{11} and σ_{22} , which relate to S11 and S22 outputs in the Abaqus finite element results. σ_{11} and σ_{22} represent the horizontal and vertical directional stresses, respectively. The principal stresses relate to SMAX and SMIN outputs in Abaqus, referred to as σ_{Max} and σ_{Min} in this paper. Since tension is positive, σ_{Max} is related to the maximum tensile stress and σ_{Min} the maximum compression stress. All of the finite element stresses shown in this paper represent the average stress through the plate thickness (the average of SP:1 and SP:5 in Abaqus notation).

4.1. Normal stresses

At and before the shear load reaches the elastic shear buckling load, V_{cr} , σ_{11} and σ_{22} equal zero, which is consistent with a plate loaded in pure shear. After buckling these stresses increase and a new state of equilibrium exists in the plate. Figures 5(b) through (d) plot σ_{11} and σ_{22} , normalized by the yield stress σ_y , at the ultimate postbuckling load (V_u) for a horizontal strip at the center of the plate (Figure 5(a)). Results for a/D equal to 1.0, 2.0, and 3.0 and D/t_w values of 134 are plotted. Figures 5(f) through (h) plot a similar result, but for a vertical strip at the center of the plate (Figure 5(e)). Note that for all a/D values the stresses at the location of the flanges are essentially zero. While this result is influenced by the boundary conditions of the finite element model, these boundary conditions reflect a pure shear loading state and thin flanges, where ‘thin’ will be defined later and shown, by validation with experimental results, to be within a common range for design.

Figure 6 is similar to Figure 5 except that the normal stresses are plotted along the tension diagonal and compression diagonal. Again it is seen that at the postbuckling stage, the stresses increase significantly. Similar to Figure 5, the horizontal stresses, σ_{11} , are larger than the vertical stresses σ_{22} . In both normal stress plots, σ_{11} and σ_{22} increase with increasing a/D values.

Figure 5. Normal stresses in horizontal and vertical strips at V_u .

Figure 6. Normal stresses in diagonal strips at V_u .

4.2. Principal stresses

The normal stresses discussion was important for understanding the change in stress state that develops in a plate after buckling (i.e., it transitions from the stress state shown in Figure 3(b) to that shown in Figure 3(c)). In addition to normal stresses in both the tension and compression fields, it is important to examine the principal stresses since these represent the maximum stresses on a plate. Figures 7(a) and (b) plot σ_{Max} from corner to corner of a plate with a/D equal to 2.0 and D/t_w equal to 147. This strip represents the tension field area. Figures 7(a) and (c) plot σ_{Min} along the same diagonal. It is seen in Figure 7(b) that the maximum tensile stresses (i.e., σ_{Max}) at V_u equals on average about 40% of σ_y and the maximum compression stresses (i.e., σ_{Min}) at V_u equals on average about half of σ_{Max} (Figure 7(c)). If one considers the von Mises yield function separately on the upper and lower surfaces of the plate thickness, the plate reaches yield along this diagonal (recall that the stresses shown are averages through the plate thickness). σ_{Max} remains essentially constant, but at V_u σ_{Min} decreases below that at V_{cr} near the center of the plate, and more than doubles the V_{cr} value at the edges. Also, note that the shape of the stress distribution is similar to that of an axially loaded plate as illustrated in Figure 1(c).

Figure 8(a) is similar to Figure 7(a) except that it plots σ_{Min} along the compression diagonal. The intent is to begin to observe the plate acting as a column. Superimposed on Figure 8(b) is the

width of the tension field, w . This width is drawn to scale based on the compression model that will be described later. Note that within this width, σ_{min} reduces significantly as the plate is loaded from V_{cr} to V_u . Also, at the edges, σ_{min} increases as the plate is loaded from V_{cr} to V_u .

Figure 7. Principal stresses (maximum, σ_{max} , and minimum, σ_{min}) plotted along the tension field (corner to corner) for $a/D=2.0$, $D/t_w=147$, and $\sigma_y=250$ MPa.

Figure 8. Minimum (compression) principal stresses plotted along the compression diagonal (corner to corner) for $a/D=2.0$, $D/t_w=147$, and $\sigma_y=250$ MPa.

Figure 7 shows that the stress distribution on the compression field at V_u has a similar pattern to that of an axially compressed plate (Figure 1(c)). Figure 9 examines this stress pattern (and magnitude) of σ_{Min} across various cuts of the plate. In all cases it is seen that the stresses increase towards the edges and have a similar in pattern to that expected in an axially compressed simply supported plate.

Figure 9. Maximum compression stresses (σ_{Min}) across various cuts of the compression field for $a/D=2.0$ and $D/t_w=147$.

Figure 10 shows the value of the angle of principal stresses, θ_p , at various locations for plates with a/D of 1.0, 2.0, and 3.0 at V_u . These are the same plates previously discussed in Figures 5 and 6. Four points (A, B, C, and D) were selected at various locations with respect to the tension field and diagonal compression line as drawn in Figure 10(d). The θ_p values are tabulated in Figure 10(e) and show that at V_u these θ_p values are less than 45° , which is the θ_p value at V_{cr} . θ_p values fall within a range of 35° to 40° and do not appear to be significantly affected by a/D values or location.

Figure 10. Angle of principal stresses at various locations for a/D values of (a) 1.0, (b) 2.0, and (c) 3.0 at V_u ; (d) diagram of tension field, centerline of equivalent column, and angle α ; (e) α and θ_p values.

5. A Compression-Based Approach for Shear Buckling

Using the mechanics study of the previous section, a compression model is developed for defining and predicting the postbuckling capacity of a plate under shear. Since buckling is inherently a phenomenon based on compression, it is assumed that the compression field dominates the physical response instead of tension, but the tension field is not ignored. Figure 11 illustrates the concept of plate shear buckling using a compression approach. In the elastic state, the plate loaded in pure shear is assumed to have a concentrated band of compression as shaded and circled in Figure 11(a). It is assumed that the compression stresses are all concentrated in this region, which is converted to an equivalent column with an equivalent moment of inertia, I_{eq} , as shown in Figure 11(b). When the plate is loaded to the elastic shear buckling load, V_{cr} , the load on the equivalent column equals P_{cr} and the equivalent column buckles as shown in Figure 11(c).

At the ultimate shear load state, V_u , the plate has developed an additional postbuckling capacity as defined earlier. A clearly visible tension field has formed, with width w , as shown in

Figure 11(d). Just like in an axially compressed plate, this tension field is responsible for the post-buckling capacity as it pulls down the plate as it tries to deform out-of-plane after elastic buckling (see Figure 1(c) and (d)). It is assumed that in the postbuckling state, the equivalent column has the same equivalent moment of inertia as in the elastic state (Figure 11(e)). The tension field width, w , is assumed to be in sufficient tension so that it is flexurally rigid; therefore, when the plate reaches V_u , the equivalent column length, L , is reduced to a length L_e as illustrated in Figure 11(f).

Figure 11. The equivalent column model for the development of a compression approach to plate shear buckling.

P_{cr} in Figure 11(c) equals the elastic flexural buckling strength of a column:

$$P_{cr} = \frac{\pi^2 EI_{eq}}{(k_{el}L)^2} \quad (4)$$

In Eqn. (4), the moment of inertia, I , has been replaced with an equivalent moment of inertia, I_{eq} , to indicate that this value is for an equivalent structural system. The coefficient k_{el} is the effective length factor of the equivalent column (Figure 11(c)) and assumed to equal 1.0.

For the equivalent column representing the postbuckling capacity of Figure 11(f), it will be assumed that the elastic flexural buckling equation still applies, but for a reduced length:

$$P_u = \frac{\pi^2 EI_{eq}}{\left(k_{e2} \frac{L_e}{2}\right)^2} \quad (5)$$

Solving Eqn (4) for I_{eq} , and substituting $k_{e2}=2.0$ for a compression element that is pinned on one end and free to translate, but not rotate, at the other end, results in the following expression:

$$P_u = P_{cr} \left(\frac{L}{L_e}\right)^2 \quad (6)$$

The challenge now becomes to relate P_u and P_{cr} , back to V_u and V_{cr} , respectively. The stress, σ , acting over area A (Figure 12(a)) varies in that space as shown in Figures 12(b) and 9. The axial force, P , at any distance, r , from the tension field (see Figure 12(c)) equals $\int_0^s \sigma t_w ds$. With t_w constant, the average axial force, P_{avg} , acting over the area, A , in the realm where the equivalent column resides thus equals:

$$P_{avg} = \frac{t_w}{L_e/2} \int_0^r \int_0^s \sigma \cdot ds dr \quad (7)$$

Figure 12. (a) Area, A , over which P_{avg} is calculated, (b) schematic of varying stresses acting over area A , and (c) notations used in Eqn. (7).

At $V=V_{cr}$, it is known that $\sigma = \sigma_{cr} = \tau_{cr} = V_{cr} / (D \cdot t_w)$. Therefore, for $P = P_{cr}$, Eqn. (7)

becomes,

$$P_{cr} = V_{cr} \cdot \frac{1}{D} \int_0^s ds \quad (8)$$

At $V=V_u$, σ is equal to the minimum principal stress (i.e. the maximum axial compression stress):

$$P_u = \frac{t_w}{L_e/2} \int_0^r \int_0^s \sigma_{Min} \cdot ds dr \quad (9)$$

It will be assumed that Eqn. (9) collapses to something similar to Eqn. (8), where P_u becomes V_u times a function, call it $f(\bullet)$. It is expected that $f(\bullet)$ will be influenced by geometric parameters such as a/D , D/t_w , flange thickness, and perhaps other parameters such as material properties.

Substituting Eqn. (8) into Eqn. (6), and setting $P_u = V_u f(\bullet)$:

$$V_u = V_{cr} \left(\frac{L}{L_e} \right)^2 \cdot Q_v \quad (10)$$

where

$$Q_v = \frac{\frac{1}{D} \int_0^s ds}{f(\bullet)} \quad (11)$$

Due to the complex nature of the stress distributions and magnitudes in the postbuckling stage, it is beyond the scope of this paper to determine precisely what $f(\bullet)$ equals. A simplified assumption will be made by assuming $Q_v=1.0$, or in other words $f(\bullet) \approx \frac{1}{D} \int_0^s ds$. It will be shown by comparison to experimental results that this is a reasonable assumption. Future work will examine Q_v in more detail.

Eqn. (10) requires calculating L_e , which in turn requires solving for w since $L_e = L - w$ from Figure 11(f). In sections to follow, a discussion will be given on how to calculate w , which will be based on experimentally validated finite element models.

6. Formula for Width of Tension Field, w

6.1. FE models

Sixty finite element (FE) models (shown in Table 2) were used to develop an equation to calculate w . The first 6 models were each repeated 9 times with different D/t_w ratios. All FE models use Young's modulus = $E = 2e11$ N/m² and Poisson's ratio = $\nu = 0.3$. The yield stress was studied at both $\sigma_y = 250$ MPa and 345 MPa as will be discussed later.

Table 2. Matrix of Finite Element (FE) Models.

FE Model #	Girder length (m / ft)	D (m)	a (m)	a/D	t_w (mm) ^a	D/t_w ^b
1	27.4 / 90	1.47	1.47	1.0	5 to 13	113 to 294
2	27.4 / 90	1.47	2.06	1.4	5 to 13	113 to 294
3	27.4 / 90	1.47	2.21	1.5	5 to 13	113 to 294
4	27.4 / 90	1.47	2.94	2.0	5 to 13	113 to 294
5	27.4 / 90	1.47	3.68	2.5	5 to 13	113 to 294
6	27.4 / 90	1.47	4.41	3.0	5 to 13	113 to 294
7	33.5 / 110	1.85	2.78	1.5	14	132
8	33.5 / 110	1.85	5.56	3.0	14	132
9	39.6 / 130	2.24	3.35	1.5	17	131
10	39.6 / 130	2.24	6.71	3.0	17	131
11	45.7 / 150	2.62	3.92	1.5	20	131
12	45.7 / 150	2.62	7.85	3.0	20	131

(a) t_w values (mm) = 5, 6, 7, 8, 9, 10, 11, 12, 13

(b) D/t_w values = 113, 123, 134, 147, 163, 184, 210, 245, 294

The FE models listed in Table 2 were based on standard plans for typical steel girder highway bridges specified by the Federal Highway Administration (FHWA) for girder lengths of 27.4, 33.5, 39.6, and 45.7 m (90, 110, 130, and 150 feet, respectively) [31]. Different depths (D)

were selected to represent different girder lengths and to study the effects of scale. In FE models 1 through 6, web thickness, t_w , values were selected (ranging from 5 mm to 13 mm) to study a range of slenderness ratios, D/t_w , equal to 113 through 294. Six plate span-to-depth ratios (a/D) were selected to cover a wide range of typical design values: 1.0, 1.4, 1.5, 2.0, 2.5, and 3.0.

6.2. Formulation

To solve for V_u in Eqn. (10) one must solve for L_e , and in turn to solve for L_e , one must solve for w . Figure 11(f) shows that $L_e = L - w$. To develop an empirical formulation for w , V_u in Eqn. (10) is set equal to V_u^{FE} , Q_v is assumed to equal 1.0, and a value of L_e is solved for, labeled here as L_e^* :

$$L_e^* = \frac{L}{\sqrt{V_u^{FE} / V_{cr}}} \quad (12)$$

The width that the tension field in Figure 11(f) must be for V_u^{FE} to match exactly V_u of Eqn. (10), therefore, is $w^* = L - L_e^*$, which is then used to derive an empirical formulation for w as will be discussed next.

FE models 1 through 6 were used to study the effect of a/D , D/t_w , and σ_y , on w^* . Figure 13 plots w^* versus a/D for D/t_w values ranging from 113 to 294 and for σ_y equal to (a) 250 MPa and (b) 345 MPa. These two σ_y values were selected since they represent common values for steel plate girders. The linear regression of the data points is also presented in Figure 13 and shows that the relationship between w^* and a/D is nearly linear for each D/t_w value.

Figure 13. Plot of w^* versus a/D for different D/t_w values and σ_y of (a) 250 MPa and (b) 345 MPa. The linear regression results are plotted for each set of data points corresponding with (from top to bottom) D/t_w values of 294, 245, 210, 184, 163, 147, 134, 123, and 113.

A comparison of Figures 13(a) and (b) shows that as σ_y increases, w^* (and, therefore, the width of the tension field at V_u) also increases. This implies that a higher σ_y permits a wider tension field to form. As a/D increases from 1.0 to 3.0, w^* also increases in part because the angle of inclination of the compression diagonal decreases, therefore it will cross a wider portion of the diagonal tension field. In addition, Figure 13 shows that an increase in D/t_w results in a larger w^* value.

Since Eqn. (10) does not explicitly account for σ_y , the effect of σ_y is accounted for through formulations for w . Using only the linear regression results of models 1 through 6, which all have $D = 1.47$ m, results in the following value of w , in units of meters:

$$w^{(D=1.47)} = \left(0.4121 \ln \left(\frac{D}{t_w} \right) - 1.668 \right) \left(\frac{a}{D} \right) + 0.7768 \ln \left(\frac{D}{t_w} \right) - 3.8624 \quad (13a)$$

$$w^{(D=1.47)} = \left(0.4107 \ln \left(\frac{D}{t_w} \right) - 1.5914 \right) \left(\frac{a}{D} \right) + 0.6428 \ln \left(\frac{D}{t_w} \right) - 3.0566 \quad (13b)$$

Eqns. (13a) and (13b) are for $\sigma_y = 250$ MPa and 345 MPa, respectively. Eqn. (13) works well for plates with $D=1.47$ m (FE models 1 through 6) as shown in Table 3, which compares $w^{(D=1.47)}$

to w^* ; however, results for FE models 7-12 show that for deeper sections $w^{(D=1.47)}$ becomes much smaller than w^* . Eqn. (13) must, therefore, be modified to consider the effects of D .

Eqn. (1) calculates τ_u based on two geometric parameters: a/D and D/t_w . Therefore, two plates with equal a/D and D/t_w values, yet different D values, will have the same τ_u value.

However, V_u , and therefore P_u , are functions of D since $V_u = \tau_u D t_w$, thus D must be explicitly accounted for in the calculation of w . FE models 7 through 12 were used for this purpose.

Table 3. Comparing w^* to $w^{(D=1.47)}$ [from Eqn. (13)] and w [from Eqns. (14) and (15)].

FE Model	D/ t_w	$\sigma_y = 250$ MPa				$\sigma_y = 345$ MPa			
		w^* (m)	$w^{(D=1.47)}/w^*$	1.47/D	w/w^*	w^* (m)	$w^{(D=1.47)}/w^*$	1.47/D	w/w^*
1	134	0.27	1.08	1.00	1.07	0.50	1.03	1.00	1.02
2	134	0.45	0.96	1.00	0.95	0.70	0.97	1.00	0.97
3	134	0.48	0.98	1.00	0.97	0.74	0.98	1.00	0.98
4	134	0.62	1.04	1.00	1.03	0.90	1.04	1.00	1.04
5	134	0.86	0.96	1.00	0.95	1.17	0.97	1.00	0.98
6	134	0.98	1.02	1.00	1.01	1.35	1.00	1.00	1.01
7	132	0.576	0.78	0.79	0.96	0.895	0.79	0.79	0.98
8	132	1.193	0.81	0.79	1.00	1.653	0.80	0.79	1.01
9	131	0.677	0.64	0.66	0.96	1.062	0.65	0.66	0.98
10	131	1.412	0.67	0.66	1.00	1.968	0.67	0.66	1.01
11	131	0.778	0.56	0.56	0.97	1.227	0.56	0.56	1.00
12	131	1.632	0.58	0.56	1.02	2.284	0.57	0.56	1.02

Table 3 shows that the $1.47/D$ ratios for FE models 7 through 12 are close to the $w^{(D=1.47)}/w^*$ ratios. Therefore, to account for scale effects, Eqn. (13) was modified by multiplying it by $D/1.47$. In addition, Eqn. (13) was modified for two ranges of D/t_w values to improve correlation between w and w^* . Splitting Eqn. (13) at $D/t_w = 164$ with different slope and y-intercept terms derived for each group provided the best fit. The best fit equations for the width of the tension field that crosses the compression diagonal, in units of meters, is therefore:

$$w = \left(\left(0.315 \ln \left(\frac{D}{t_w} \right) - 1.306 \right) \left(\frac{a}{D} \right) + 0.647 \ln \left(\frac{D}{t_w} \right) - 3.209 \right) \times D \quad D/t_w \leq 164 \quad (14a)$$

$$= \left(\left(0.278 \ln \left(\frac{D}{t_w} \right) - 1.124 \right) \left(\frac{a}{D} \right) + 0.376 \ln \left(\frac{D}{t_w} \right) - 1.792 \right) \times D \quad 164 < D/t_w \quad (14b)$$

$$w = \left(\left(0.278 \ln \left(\frac{D}{t_w} \right) - 1.072 \right) \left(\frac{a}{D} \right) + 0.622 \ln \left(\frac{D}{t_w} \right) - 2.99 \right) \times D \quad D/t_w \leq 164 \quad (15a)$$

$$= \left(\left(0.31 \ln \left(\frac{D}{t_w} \right) - 1.249 \right) \left(\frac{a}{D} \right) + 0.238 \ln \left(\frac{D}{t_w} \right) - 0.992 \right) \times D \quad 164 < D/t_w \quad (15b)$$

Eqns. (14) and (15) are for $\sigma_y = 250$ MPa and 345 MPa, respectively.

This formulation, therefore, considers scale, slenderness, aspect ratio, and material. Table 3 shows that Eqns. (14) and (15) for w correlate well to the predictions given by the FE solution, typically coming within 5% of w^* . This good correlation is not surprising since w is derived from FE results. The next section examines the “physical” correlation of w to FE results (i.e., the observed width of the tension field in the FE models). But since w is useful only for deriving V_u , the true validation of w will be based on how well V_u correlates to both FE and experimental results. These validations will be done in the sections that follow.

7.0. Validation of the Compression-Based Model

7.1. w versus FE observation of tension field

Figures 14(a) and (b) show 2 representative FE models ($a/D=1.0$ and 2.0) with $D/t_w=134$ and $\sigma_y=250$ MPa. A dashed black diagonal line is drawn on each plate representing the equivalent column location. The contour plots of the maximum in-plane principal stresses are drawn for $V=V_u$, where the red zone marks the region of highest tensile stresses. Superimposed to scale on

this contour is the width, w , based on Eqns. (14) and (15). It is seen that these equations predict reasonably well the width of the tension field that forms.

Figures 14(c) and (d) plot the out-of-plane displacements at V_u measured along the diagonal dashed line. It is seen that the peak out-of-plane displacements occur approximately at the center of the diagonal lines. Also superimposed in Figures 14(c) and (d) is the predicted width, w , drawn to scale. The plots show that this width develops in the regions of maximum displacement.

Figure 14. Contour plot of the maximum in-plane principal stresses at V_u for a/D of (a) 1.0 and (b) 2.0, with a black dashed diagonal line indicating the assumed location of the equivalent column. Plots (c) and (d) show the out-of-plane displacements measured along this diagonal line from the bottom left corner to the top right corner for $a/D=1.0$ and 2.0, respectively. Superimposed in these figures is the predicted width, w , drawn to scale based on Eqns. (14) and (15).

For $a/D = 3.0$, the physical response of the plate changes compared to $a/D=1.0$ and 2.0 . Two D/t_w ratios are examined in Figure 15 for $a/D=3.0$: $D/t_w=134$ and 294 . These two ratios represent the spectrum of stress contour plots for $a/D=3.0$. Figures 15(a) and (b) show contour plots of the maximum in-plane principal stresses at V_u for plates with $a/D=3.0$, $\sigma_y=250$ MPa, and D/t_w of (a) 134 and (b) 294. The out-of-plane displacements at V_u measured along the black dashed diagonal lines are shown in Figure 15(c) and (d) for D/t_w of 134 and 294, respectively. Superimposed in Figures 15(a) through (d) is the width, w , drawn to scale based on Eqns. (14) and (15).

Figure 15. Contour plot of the maximum in-plane principal stresses at V_u for a/D of 3.0 and D/t_w of (a) 134 and (b) 294, with a black dashed diagonal line indicating the assumed location of the equivalent column. Plots (c) and (d) show the out-of-plane displacement measured along this diagonal line from the bottom left corner to the top right corner for $D/t_w=134$ and 294, respectively. Superimposed in these figures is the predicted width, w , drawn to scale based on Eqns. (14) and (15).

From Figures 15(a) and (c), which show results for $D/t_w=134$, it is seen that w is centered approximately about the inflection point between the peak and trough of the out-of-plane displacements. From Figures 15(b) and (d), however, which show results for $D/t_w=294$, it is seen that w captures the width of the two tension fields that form. Eqns. (14) and (15) calculate a

larger w value as D/t_w increases, which is consistent with what is observed in Figure 15. Despite the tension field being slightly off-center from the diagonal for smaller D/t_w values (see Figure 15(a)), it will be shown in the next section that assuming w is centered at the mid-length of the compression diagonal will still result in an accurate prediction of V_u using the compression approach for $a/D=3.0$.

7.2. Comparison of V_u to finite elements

Using Eqns. (14) and (15), w is solved for from which $L_e=L-w$. This value of L_e is then used in Eqn. (10) to solve for V_u assuming $Q_v = 1.0$. Figures 16 and 17 and Table 4 compare V_u^{FE} (the finite element solution) to both V_u (derived based on Eqns. (10), (14), and (15) – the compression approach) and V_u^{BT} (the Basler-Thürlimann solution from Eqn. (1)). Models 1 through 6 (with $D = 1.47$ m) are presented in Figures 16 and 17, and Models 7 through 12 (with larger D values) are presented in Table 4. In both figures, the dashed red lines mark the 5% correlation values.

It is seen in Figures 16(a) and 17(a) that V_u based on the compression approach is generally within 5% of V_u^{FE} , and always within 10% of V_u^{FE} , except for 2 data points. Comparison to Basler in Figures 16(b) and 17(b) shows, however, that the V_u^{BT} values do not correlate as well to V_u^{FE} . As slenderness increases, V_u^{BT}/V_u^{FE} generally becomes smaller, reaching values as low as 0.63 for $a/D=3.0$, $D/t_w=294$, and $\sigma_y=345$ MPa (Figure 17(b)). These trends illustrated in Figures 16 and 17 for Models 1 through 6 are also seen in Table 4 for Models 7 through 12.

Figure 16. Comparison of (a) V_u (compression approach - Eqn. (10)) and V_u^{FE} and (b) V_u^{BT} and V_u^{FE} for FE models 1 through 6 and for $\sigma_y = 250$ MPa.

Figure 17. Comparison of (a) V_u (compression approach - Eqn. (10)) and V_u^{FE} and (b) V_u^{BT} and V_u^{FE} for FE models 1 through 6 and for $\sigma_y = 345$ MPa.

Table 4. Comparison of V_u and V_u^{BT} for FE models 7 through 12.

FE Model	$\sigma_y = 250$ MPa		$\sigma_y = 345$ MPa	
	V_u/V_u^{FE}	V_u^{BT}/V_u^{FE}	V_u/V_u^{FE}	V_u^{BT}/V_u^{FE}
7	0.99	0.92	0.99	0.85
8	1.01	0.81	1.01	0.74
9	0.99	0.92	0.99	0.86
10	1.01	0.81	1.01	0.75
11	0.99	0.92	0.99	0.86
12	1.01	0.81	1.01	0.75

7.3. Comparison of V_u to experimental results

This section evaluates how the compression approach for predicting V_u compares with published experimental data [17, 32]. Experimental V_u results (V_u^{Exp}) were compared with values using the compression approach (V_u) calculated from Eqns. (10), (14), and (15) assuming $Q_v = 1.0$. Eleven specimens were removed from the data in [17, 32] since these had D/t_w values equal to 800,

which would not be used in the design of plate girders used in bridges and buildings (the focus of

this study). With these removed, in total 84 specimens were compared with V_u . To enable detailed evaluation, these 84 specimens were separated into two sets:

- Set A specimens (27 total) fall within the most common geometric and material parameter range for plate girders used in bridges and buildings. In addition, this set falls within the range of finite element model parameters used to calibrate the compression model. The range of parameters for Set A are thus: $180 \text{ MPa} \leq \sigma_y \leq 420 \text{ MPa}$; $100 \leq D/t_w \leq 300$; $1.0 \leq a/D \leq 3.0$; and $t_f/t_w \leq 5$. This data set is shown in Table 5.
- Set B specimens (57 total) have geometric parameters (a/D , D/t_w , t_f/t_w) and the material parameter (σ_y) that fall beyond that of Set A, such as $D/t_w > 300$ or $\sigma_y > 420 \text{ MPa}$.

Note that Equations (14) and (15), from which V_u is based, are derived for two discrete values of σ_y : 250 MPa and 345 MPa. However, most of the specimens studied (e.g., see Table 5) have different yield stresses. To enable direct comparison with experimental results, a linear interpolation and extrapolation of Eqns. (14) and (15) was done. Figure 18 presents the results of this interpolation and extrapolation for Set A. It is seen that the V_u/V_u^{Exp} values typically fall within 0.90 and 1.10 (as marked in the shaded region), regardless if an interpolation or extrapolation was done. One outlier does exist (corresponding to specimen G8-T2 [32]), however this specimen appears to have been re-tested (specimen G8-T3 [32]), which resulted in a V_u/V_u^{Exp} value close to 1.00. These results indicate that interpolation and extrapolation may be used to calculate w within the range $180 \text{ MPa} \leq \sigma_y \leq 420 \text{ MPa}$. Extrapolation, however, does not work well for yield stresses outside of this range as will be shown later when Set B is discussed.

Table 5 presents the results of Set A in more detail. Again, it is seen that the V_u and V_u^{Exp} values agree well, and generally within 10% (based on the V_u/V_u^{Exp} column) for all 27 specimens of this Set. Table 5 also compares the experimental results with the Basler-Thürlimann solution

from Eqn. (1). Two values are presented: V_u^{BT-FL} calculates the k value in Eqn. (2) considering the flange thickness, while V_u^{BT-SS} calculates this k value assuming simple supports for the flanges per Eqn. (3). Both the V_u^{BT-FL}/V_u^{FE} and V_u^{BT-SS}/V_u^{FE} values are generally less than 1.00 and in some cases offer predictions that are less than 70% of V_u^{Exp} .

Table 5. Set A specimens: comparison of compression approach, Basler (tension) approach, and experimental data from [17, 32].

Specimen	<u>Experimental</u>					<u>Compression approach</u>		<u>Basler approach</u>		<u>Performance ratios</u>		
	D (m)	a/D	D/t _w	σ_y (MPa)	V_u^{Exp} (kN)	w (m)	V_u (kN)	V_u^{BT-FL} (kN)	V_u^{BT-SS} (kN)	V_u/V_u^{Exp}	V_u^{BT-FL}/V_u^{Exp}	V_u^{BT-SS}/V_u^{Exp}
<i>G6-T1</i> ^[24]	1.270	1.50	259	253	516	1.180	533	428	393	1.03	0.83	0.76
<i>G7-T1</i> ^[24]	1.270	1.00	255	253	623	0.899	691	542	515	1.11	0.87	0.83
<i>G7-T2</i> ^[24]	1.270	1.00	255	253	645	0.899	691	542	515	1.07	0.84	0.80
2.2 ^[25]	0.600	2.40	300	255	75	0.876	80	58	52	1.06	0.78	0.70
<i>US3/5</i> ^[26]	0.359	2.19	133	257	90	0.168	99	108	84	1.10	1.21	0.94
<i>STG1</i> ^[27]	0.279	1.97	140	255	60	0.134	56	60	48	0.94	1.00	0.80
<i>S-3</i> ^[30]	0.477	1.21	149	317	198	0.191	189.9	199	176	0.96	1.01	0.89
<i>STG2</i> ^[27]	0.253	2.0	158	272	40	0.174	41.0	39	32	1.03	0.96	0.79
<i>RTG4</i> ^[27]	0.254	1.00	267	259	24	0.190	26.9	21	20	1.12	0.86	0.82
<i>TG5</i> ^[33]	1.000	1.00	400	200	308	0.981	295.7	151	146	0.96	0.49	0.47
<i>TG5-1</i> ^[33]	1.000	1.00	400	200	300	0.981	295.7	151	146	0.99	0.50	0.49
<i>U2/5</i> ^[26]	0.359	2.19	113	230	135	0.074	124.2	144	117	0.92	1.06	0.86
<i>TG22</i> ^[27]	0.305	1.00	150	229	79	0.084	75.0	72	64	0.95	0.91	0.82
<i>TG23</i> ^[27]	0.305	1.00	150	229	81	0.084	75.0	72	64	0.93	0.89	0.80
<i>TGV1-1</i> ^[34]	0.600	2.00	290	211	83	0.714	81.3	60	54	0.98	0.72	0.65
<i>TGV1-2</i> ^[34]	0.600	1.00	290	211	111	0.457	123.0	87	83	1.11	0.79	0.75
<i>TGV2-2</i> ^[34]	0.600	1.00	288	211	115	0.455	123.5	88	84	1.07	0.77	0.73
<i>TGV3-2</i> ^[34]	0.600	1.00	299	211	113	0.469	120.0	84	80	1.06	0.74	0.71
<i>LS1-PA</i> ^[35]	0.608	1.55	290	183	76	0.587	81.1	63	57	1.07	0.82	0.75
<i>LS3-PA</i> ^[35]	0.608	1.56	247	201	103	0.519	104.9	87	78	1.02	0.84	0.76
<i>RTG1</i> ^[27]	0.305	1.00	240	244	40	0.202	42.2	33	31	1.05	0.83	0.79
<i>RTG2</i> ^[27]	0.305	1.00	240	244	41	0.202	42.2	33	31	1.03	0.81	0.77
<i>CPI/I</i> ^[29]	0.500	1.49	245	246	88	0.439	87.7	71	65	1.00	0.81	0.74
<i>S-2</i> ^[30]	0.319	1.82	100	352	161	0.085	167.9	190	152	1.04	1.18	0.95
<i>G8-T1</i> ^[24]	1.270	3.00	250	263	375	1.960	432.4	323	275	1.15	0.86	0.73
<i>G8-T2</i> ^[24]	1.270	1.50	250	263	445	1.164	576.8	466	426	1.30	1.05	0.96
<i>G8-T3</i> ^[24]	1.270	1.50	250	263	516	1.164	576.8	466	426	1.12	0.90	0.83

Notes: 1. Italicized specimens were also used for the FE model validation results presented in Table 1.
2. Superscripts [24], [25], [26], [27], [28], [29], [30], [33], [34], [35] are references to publications.

Figure 18. Plot of V_u/V_u^{Exp} values based on Set A specimens (Table 5). Solid black circles correspond with V_u values where w was calculated directly using Eqns. (14) and (15), while open black and red circles indicate V_u values calculated with an interpolated or extrapolated w value, respectively.

Figure 19. Box plots comparing V_u/V_u^{Exp} values for Set A, B, and A+B.

A statistical comparison of the data is made through box plots shown in Figure 19. On each box, the central mark is the median (the second quartile), the edges of the box are the 25th and 75th percentiles (i.e., the first and third quartiles, Q1 and Q3). The boxes enclose the interquartile

range (IQR). The lower whisker is a line from Q1 to the smallest V_u/V_u^{Exp} within $1.5 \cdot \text{IQR}$ from Q1. Similarly, the upper whisker is a line from Q3 to the largest V_u/V_u^{Exp} within $1.5 \cdot \text{IQR}$ from Q3. Data outside of the whiskers are considered outliers and plotted individually with a cross. For each data set (A, B, and A and B combined), Figure 19 shows box plots for the compression approach (“Compression V_u ”), Basler’s model calculated assuming simply supported boundary conditions for the flanges, V_u^{BT-SS} (“Basler V_u (SS)”), and Basler’s model calculated considering the flange thickness, V_u^{BT-FL} (“Basler V_u (FL)”).

It is seen in Figure 19 that the compression V_u in Set A, which contains 27 specimens within a common range of geometric and material properties, has Q1 and Q3 near the median, which is close to 1.0, and short whiskers. Thus, Set A has excellent fit with the experimental data. One outlier does exist, which is the same outlier discussed with Figure 18. The “Compression V_u ” model using the geometric and material constraints defined by Set A has a better fit to the experimental data than the Basler model, regardless of the value used for k .

Set B data of Figure 19, which has geometric and material parameters outside that of Set A, shows that the Basler V_u models have a better fit than the compression V_u model, which has a large spread in the data. This figure indicates that the compression model needs to be further developed for better fit to geometric and material properties outside the geometric and material constraints defined by Set A. The two outliers for the compression approach for V_u of Set B have $\sigma_y = 745$ MPa, which is well outside the σ_y range of Set A. These results indicate that the extrapolation of yield stress is not valid for σ_y outside of the range $180 \text{ MPa} \leq \sigma_y \leq 420 \text{ MPa}$.

When Sets A and B are combined in Figure 19, the results for the compression V_u model are reasonable. The median is closer to the experimental results than the Basler models, however, the Basler models have a smaller IQR and shorter whiskers, thus indicating a smaller spread in the

data – i.e., more consistent results. The four outliers for the compression approach for V_u in this combined Set correspond to specimens with a/D , D/t_w , and σ_y values that are well outside the common design parameters of Set A.

Overall, Figure 19 and Table 4 show that the compression V_u model has excellent correlation to experimental data for the geometric and material parameters most common to plate girders used in bridges and buildings. It also has a better fit to the experimental data than the Basler solution for this data set. However, the Basler solution is more consistent in predicting V_u for the entire data set and the predictions are conservative throughout, albeit sometimes perhaps too conservative.

8.0 Conclusions

Traditional approaches for calculating the ultimate postbuckling shear strength (V_u) have relied on tension field theory, which is based on the fundamental assumption that once elastic shear buckling has occurred, the postbuckling shear strength is due solely to the development of tensile stresses within a defined diagonal tension field. Since buckling is inherently a phenomenon based on compression, this paper presented a new approach where the compressive response dominates the physical model, and the tension field plays a secondary, yet critical, role.

The mechanics of shear buckling was examined via finite element analyses with models that were experimentally validated. It was shown that the response of a plate that buckles in shear is similar to a plate that buckles under axial compression: in the postbuckling phase, tensile stresses exist perpendicular to the compressive stresses, and the compressive stress distribution is larger on the edges than in the center for both types of loading. The results of this finite element study also disagreed with the key assumptions of tension field theory since it was found that (1)

compressive stresses continue to increase in the postbuckling range – in particular near the edges, and (2) the stiffeners and flanges do not necessarily anchor the tension field.

An equation for predicting V_u based on a compression model was developed, which considered scale (plate depth), plate slenderness, aspect ratio (distance between transverse stiffeners versus depth), and yield stress. The results of this compression model for V_u were compared to finite element models and 84 experimental results. The compression model for V_u was also compared to the solution for V_u , developed in 1961 by Basler, whose basis is the tension field approach.

Results show that the compression model for V_u has excellent correlation to experimental data for the most common geometric and material parameters with structural engineering applications: $180 \text{ MPa} \leq \sigma_y \leq 420 \text{ MPa}$; $100 \leq D/t_w \leq 300$; $1.0 \leq a/D \leq 3.0$; and $t_f/t_w \leq 5$. Within this range of parameters, the compression model for V_u also has a better fit to the experimental data than the Basler solution. However, the Basler solution is more consistent in predicting V_u for data outside of this set of parameters. Further, Basler predictions are conservative throughout, albeit sometimes too conservative.

Future work will further develop the compression model for better fit to geometric and material properties outside the geometric and material constraints defined above. This could be done via further study of the variable Q_v in the compression model and through additional finite element studies outside of the current range of study. Further, the authors are currently studying how to incorporate the effects of elevated temperatures into the compression model.

Overall, this work shows that a compression approach to predicting V_u is viable and leads to excellent correlation with design parameters common to plate girders used in bridges and

buildings. Further study is encouraged for applicability to a wider range of parameters and for the development of simpler equations that can be used in design.

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Figure 1. Plate behavior (simply supported) under pure axial load compared to pure shear load.

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