Direct Strength Prediction of Web Crippling Failure of Beams under ETF Loading

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Abstract

This paper proposes a new approach to predict the web crippling failure load of cold-formed steel beams under External Two Flange (ETF) loading using the Direct Strength Method (DSM). After the description of the existing test data, Eurocode 3 [1] and the North-American Specification [2] are used to predict the corresponding web crippling failure loads and the accuracy of such predictions is briefly assessed. In order to obtain additional information on the web crippling behaviour of each test specimen, non-linear numerical simulations are performed. Since the calibration of the DSM-based design expressions requires the previous calculation of (i) elastic buckling loads and (ii) plastic strengths/loads, two procedures are presented to achieve these goals. While the buckling loads are determined using the GBTWEB software, specifically developed for this purpose, the plastic loads are calculated by means of analytical expressions based on yield-line models. By adopting a non-linear regression procedure, the coefficients of a DSM-based formula are determined on the basis of the set of 128 experimental results available and the corresponding buckling and plastic load values. In spite of the different cross-sections, fastening conditions and test set-ups considered in the calibration procedure, it is possible to establish a clear relationship between the web crippling slenderness and the nominal-to-plastic load ratio. Finally, it is shown that the developed/proposed DSM-based formula for ETF web crippling design yields reliable predictions, since they are associated with a LRFD (Load and Resistance Factor Design) resistance factor $\phi=0.81$, which is located within the range of values currently adopted in the 2012 NAS [2].

Keywords: Web Crippling Design; Direct Strength Method (DSM); Buckling load; Plastic load; Ultimate load; Failure mode

Notation

b_f	flange (flat) width
b_r	lip (flat) depth
c_1, c_2	auxiliary coefficients
f_y	yield stress
h	web (total) height
h_w	web (flat) height
k_1 - k_3	DSM-based formula coefficients
L	specimen length
L_s	load bearing length
N_m	length of yield line
N^*	auxiliary parameter (length in yield-line method)
N_{vert}	height of vertical yield line
P_{cr}	elastic buckling load
P_n	nominal value of web crippling strength
P_{SFE}	ultimate load obtained from SFE non-linear analysis
P_{test}	ultimate load obtained from experimental test
P_y	plastic strength/load
r_{ext}	external corner radius
r_m	midline corner radius
t	thickness
χ	reduction factor (ultimate-to-plastic load ratio)
λ_{wc}	web crippling slenderness
ϕ	resistance factor (LRFD)
β_0	target reliability index (LRFD)

1. Introduction

The strength of cold-formed steel members may be governed by the application of localized transverse loads, support reactions or a combination of both, which involve very high compressive stresses acting transversally to the member axis. Failure will be caused by either crushing, crippling or buckling of the web panel, and the associated behaviour is highly complex, since it is influenced by a large number of material and geometrical parameters. Indeed, a comprehensive theoretical model is yet to be developed and such task appears to be extremely complicated. The seminal investigation by Winter and Pian[3] preceded a large number of experimental studies [4-9], which included the calibration of design expressions. With the advent of very powerful computers, the recent trend is the performance of numerical investigations to advance the knowledge on the web crippling behaviour of cold-formed steel members [7,9-14].

Considering the two most relevant design steel codes, it is possible to conclude that the guidelines for web crippling design are based on substantially different approaches. The design approach presented in Eurocode 3 – Part 1-3 (EC3)[1] contains several formulae that depend on a fair number of coefficients. The appropriate formula must be carefully selected on the basis of

cross-section type and load configuration. The design approach presented in the AISI North American Specification (NAS)[2] is based on the Waterloo expression proposed by Prabakaran[15] and the coefficients obtained by Beshara and Schuster [6], which were calibrated against a large amount of web crippling test results. Unlike the EC3 approach, the NAS approach relies on a single formula and a huge number of coefficients—those adopted depend on the cross-section type, load configuration and degree of rotational restraint imposed to the flange. Overall, the application of the NAS approach appears to be more straightforward. Nevertheless, both approaches share an empirical background and are strongly restricted by the limited range of application (geometrical, material and loading parameters). In the case of EC3, it was not possible to identify the origin of the current design formulae and coefficients, while it was possible to verify that the coefficients proposed in the NAS were sometimes obtained from very small data samples [6]¹.

Nowadays, the Direct Strength Method (DSM) is a reliable, consistent and well established design approach for cold-formed steel structures, which has been adopted by the NAS [2]. Despite being increasingly used, the method is still limited to structural problems involving longitudinal normal stresses [16] (global, distortional and local buckling) and shear stresses [17] (shear buckling). In light of the previous considerations (on design guidelines and their respective limitations), the development of a DSM-based approach for web crippling would have an undeniable practical interest. Following the DSM philosophy, the calibration of a formula requires the use of three sets of data: (i) experimental (or numerical) collapse loads, (ii) elastic critical loads associated with the appropriate buckling mode and (iii) plastic loads based on idealised failure mechanisms. The main advantage of the DSM is that the critical and plastic loads contain the “ingredients” pertaining to the structural member capacity, since they concern the member elastic buckling response (geometrical non-linear behaviour) and cross-section strength (physical non-linear behaviour). In the context of web crippling, the DSM-based formula should be applicable regardless of the cross-section type, number of webs and additional specifications (restrained flange rotation and/or reduced beam length), mostly because these factors must be effectively considered in the numerical (or analytical) models adopted to calculate the elastic buckling and plastic loads. Unlike other empirical methods, the DSM adopts these rational procedures. Once these loads have been determined, the slenderness is calculated and the ultimate strength is obtained using a design curve previously calibrated through the comparison with experimental (or numerical) failure loads.

¹ The researchers who proposed these formulae invariably mentioned that additional testing should be carried out in order to improve its accuracy, through a better choice of the coefficients.

The authors [18,19] have previously attempted to develop a DSM-based approach to web crippling design of cold-formed steel beams, in which the elastic buckling load was obtained using Generalised Beam Theory (GBT) [20]. The calibration presented in the present work follows a similar procedure, with the GBT buckling loads verified against SFE results. Simultaneously, SFE non-linear results are validated against experimental data consisting of ultimate loads and load-displacement curves. These SFE non-linear results are pivotal in (i) the analysis of the failure mechanism and (ii) the definition of a yield-line model to use in the calculation of the plastic load. Therefore, this paper aims to propose a DSM-based formula to estimate web crippling failure loads for the case of External Two Flange (ETF) loading. First, the experimental data collected from the literature is described and reviewed. Then, the methodologies to calculate the buckling and plastic loads are presented. Next, the DSM-based formula is calibrated and proposed, and a resistance factor is calculated according to the NAS [2] guidelines. Finally, the set of pre-qualified sections, i.e. cross-sections that may be designed according to the proposed formula, is presented and their geometrical and material limits identified.

2. Experimental Data on ETF Loading

Considering the methodology adopted in this study, the calibration of a DSM-based formula for web crippling design required a literature survey of the existing experimental studies on beams under ETF loading conditions. Upon completion of the literature search, the database for this study includes 128 tests and data reported by Hetrakul and Yu [4], Young and Hancock [5,21], Beshara and Schuster [6] and Macdonald *et al.* [7,8]. These five groups of tests are described next:

- *Group (i)*: Hetrakul and Yu [4] carried out an extensive study on the web crippling behaviour of cold-formed steel beams, which included 30 ETF web crippling tests of built-up I-sections (two web-fastened back-to-back C-sections with stiffened flanges), unfastened to the bearing plates. The experimental set up is illustrated in Figure 1.
- *Group (ii)*: The same authors [4] performed also 30 tests of single unreinforced web C-sections, both with and without flange lips. Two C-sections were tested simultaneously. Albeit the flange-support connection is unfastened, lateral bracing elements may increase the flange rotational stiffness and web crippling capacity, due to the reduced specimen length. The test arrangement is shown in Figure 2.
- *Group (iii)*: Young and Hancock [5,21] studied the web crippling behaviour of stocky C-sections with plain flanges (no lips). The test set-up for ETF loading conditions consisted of a single specimen placed between two identical bearing plates, as depicted in Figure 3. Half

rounds were adopted to simulate a hinged support, which may directly impact the web crippling strength capacity.

- *Group (iv)*: Beshara and Schuster [6] reported web crippling tests on single-web sections subjected to two-flange loading conditions. Flange rotation was prevented by adopting a fastened connection with the bearing plates. This study included 36 ETF tests of lipped C- and Z-sections. Different combinations of parameters were considered (3 web depths, 3 corner radii, 2 thicknesses, 2 steel grades and 3 bearing plate widths). The test set-up is very similar to the one adopted by Hetrakul and Yu [4] (see *Group (ii)*) and the cross-section geometries are depicted in Figure 4.
- *Group (v)*: Heiyanthuduwa [7] and Macdonald *et al.* [8] tested lipped C-sections. The results of 18 ETF web crippling tests were reported – the C-section flanges were fastened to the bearing plates, thus preventing their rotation. Figure 5 shows the cross-section geometry and the experimental set-up. Tables 1 and 2 provide a brief characterization of the 5 groups of tests and indicate the corresponding ranges of geometrical and material data.

Figure 1 – Built-up I-sections with stiffened flanges tested by Hetrakul and Yu [4] under ETF loading conditions:
(a) cross-section geometry and (b) test set-up

Figure 2 – C-sections with stiffened flanges tested by Hetrakul and Yu [4] under ETF loading conditions: (a) cross-section geometry and (b) test set-up

Figure 3 – Unfastened C-sections with unstiffened flanges tested by Young and Hancock [5,21] under ETF loading conditions: (a) cross-section geometry and (b) test set-up

Figure 4– Fastened channel sections with stiffened flanges tested by Beshara and Schuster [6]: (a) C and (b) Z sections

Figure 5 – Flange-fixed (fastened) channel sections with stiffened flanges reported by Macdonald *et al.* [8]: (a) cross-section geometry and (b) schematic of test set-up

Table 1 – Summary of the experimental test data considered for calibration of DSM-based formula for ETF web crippling design

Group	Authors	Nr Tests	Description	t [mm]	h _w [mm]	b _f [mm]	r _m [mm]
(i)	Hetrakul& Yu [4]	30	Built-up I-sections	1.17 – 2.74	129.2 – 305.2	27.8 – 90.5	2.97 – 4.15
(ii)		30	Unfastened Lipped/Plain C-sections	1.17 – 1.31	117.4 – 304.7	9.8 – 73.4	1.81 – 3.80
(iii)	Young & Hancock [5,21]	16	Unfastened Un-lipped C-sections	3.83 – 6.01	58.8 – 269.7	31.9 – 76.8	5.82 – 11.40
(iv)	Beshara & Schuster [6]	18	Fastened Lipped C-sections	1.16 – 1.45	87.1 – 283.1	45.6 – 61.0	7.58 – 14.73
		18	Fastened Lipped Z-sections	1.16 – 1.45	89.1 – 283.1	44.8 – 60.7	7.58 – 14.73
(v)	Macdonald <i>et al.</i> [7,8]	18	Fastened Lipped C-sections	0.78	65.2 – 98.2	26.8 – 46.7	1.99 – 5.39

Table 2 – Summary of geometrical ratios and material yield stress of the ETF test data

Group	Authors	Nr Tests	h _w /t	L _s /t	r _m /t	f _y [MPa]
(i)	Hetrakul& Yu [4]	30	46.8 – 261.0	13.3 – 65.2	1.51 – 3.22	230.7 – 325.0
(ii)		30	89.9 – 255.2	19.4 – 63.8	1.46 – 3.22	250.0 – 324.9
(iii)	Young & Hancock [5,21]	16	15.3 – 45.1	5.2 – 19.4	1.39 – 1.90	275.0 – 450.0
(iv)	Beshara & Schuster [6]	18	60.1 – 195.2	20.7 – 70.0	5.33 – 12.57	328.0 – 448.0
		18	61.5 – 195.2	20.7 – 54.7	5.33 – 12.57	323.0 – 446.0
(v)	Macdonald <i>et al.</i> [7,8]	18	83.6 – 126.0	32.1 – 128.2	2.55 – 6.91	220

3. Evaluation of Web Crippling Design Approaches

Before proposing the new DSM-based approach for the web crippling design of cold-formed steel members, it is deemed relevant to assess the applicability and accuracy of the existing design approaches. For this purpose, both the EC3[1] and AISI-NAS[2] methodologies are considered.

The web crippling design guidelines prescribed by EC3 include different formulae for single web sections, multiple web sections and stiffened web sections. Although no specific mention is made to fastened flange cases, it seems acceptable to assume that the formulae for single-web sections incorporate some degree of web rotational restraint. The NAS approach is based on the unified Waterloo expression, first described by Prabakaran[15]. The coefficients found in Tables C.3.4.1-1 to C.3.4.1-5 of NAS were later calibrated by Beshara and Schuster [6]. It should be noted that *Groups (i)-(iii)* were previously considered by these authors to calibrate the coefficients, while *Group (iv)* was part of their own experimental programme. Therefore, the comparison made by Heiyanthuduwa[7] between his own experimental programme and the corresponding NAS estimates, which is also presented here, is of utmost importance.

As discussed in the introduction, the applicability of the existing design approaches is often limited by their range of validity. Indeed, it was observed that some specimens in each test group fell outside the range of application of the EC3 approach. In the case of NAS, a smaller number of cases in these circumstances were also detected. Even though the EC3 and NAS approaches cannot be applied to evaluate the failure loads of specimens in these conditions, the experimental and predicted ultimate loads were considered for all the specimens in each group. Table 3 presents the mean and standard deviation of the P_{test}/P_n ratio, where P_{test} is the experimental test failure load and P_n denotes the nominal value of the web crippling failure load obtained from either EC3 or NAS. Overall, the approach proposed in EC3 overestimates the web crippling strength of the majority of the tested specimens. In view of their publication dates, it is very likely that only *Group (i)* and *Group (ii)* have been included in the calibration of the EC3 formulas.

Table 3– Mean and standard deviation values of P_{test}/P_n ratio

Group	Authors	Nr Tests	EC3 - P_{test}/P_n		NAS - P_{test}/P_n	
			Mean	Sd. Dev	Mean	Sd. Dev
(i)	Hettrakul & Yu [4]	30	1.08	0.121	0.89	0.198
(ii)		30	1.11	0.115	1.03	0.125
(iii)	Young & Hancock [5,21]	16	0.72	0.083	1.02	0.105
(iv)	Beshara & Schuster [6]	18	0.68	0.053	1.03	0.119
		18	0.88	0.118	1.00	0.122
(v)	Macdonald <i>et al.</i> [7,8]	18	0.75	0.130	1.41	0.248

Because the coefficients currently prescribed by the NAS were calibrated using test data contained in *Groups (i)-(iv)*, it is easy to justify its good accuracy. However, if the NAS formula is applied to predict the web crippling strength of the most recent data set (*Group (v)*), it becomes clear that this approach appears to be severely inaccurate and unsafe for these fastened C-sections, even though they meet the pre-established range of application.

Figures 6 and 7 show comparisons between the nominal web crippling strength prediction P_n determined with the EC3 (Figure 6) and NAS (Figure 7) formulae, and the failure loads (P_{test}) obtained from the tests reported. These graphs provide clear information on the accuracy of each design method. While the NAS approach is consistently more accurate than the EC3 approach, it is also visible that, on the basis of experimental results included in *Group (v)*, the NAS approach may not be reliable at all. The EC3 approach is quite unsafe for the majority of the experimental data, particularly for *Groups (iii)*, *(iv)* and *(v)*. Furthermore, the EC3 approach lacks an appropriate distinction between C- and Z-sections, which have been proven to have different web crippling strengths. Despite the clause regarding the rotational restraint imposed to the web, the distinction between fastened and unfastened flanges is not explicitly addressed in EC3. In view of these results, the development of a DSM-based formula is necessary and will be very useful.

Figure 6 – Web crippling strength estimates: EC3 (P_n) vs experimental (P_{test}) failure loads (values divided by t)

Figure 7 – Web crippling strength estimates: NAS (P_n) vs experimental (P_{test}) failure loads (values divided by t)

4. Non-Linear SFE Analysis and Results

In the introduction, compelling arguments regarding the difficulties found in the development of fully analytical models to estimate the web crippling strength were presented. In 1989, Sivakumaran[10] completed a pioneer SFE investigation on the web crippling behaviour of cold-formed steel beams and discussed some of the issues pertaining to the high complexity of the physical phenomenon, namely the (i) non-uniform stress distributions due to the applied loading, (ii) steel strain hardening, (iii) simultaneous elastic and inelastic buckling of the web panel, (iv) load eccentricity between the load application and the web plane (due to the round corners), (v) web imperfections, (vi) restraint imposed by the flange on the web rotation (due to external connections), (vii) web inclination and (viii) flange curling.

Besides these factors, non-linear SFE models include both material and geometrical non-linearity, and are able to accurately simulate the web crippling failure of cold-formed steel beams. For these reasons, the performance of non-linear SFE analyses is extremely important. Static implicit analyses have traditionally been used to study web crippling and many other relevant structural problems. However, these analyses often encounter convergence difficulties for problems involving complex situations (e.g., contact and friction), as discussed by Kaitila[22]. Alternative models, using quasi-static (dynamic) explicit analyses, were previously implemented and successfully adopted by the authors [11,12]. These works present in-depth discussions on the advantages of quasi-static analyses and illustrate different options and parameters involved in the performance of non-linear SFE analyses, in the context of the

ABAQUS[23] finite element software. In the current work, an optimized SFE model was implemented in order to be conveniently adapted for different situations (accounting for several cross-section types and loading configurations). Next, a brief description of the SFE model implemented in ABAQUS (Version 6.10) [23] is presented. Three issues were deemed relevant: (i) the correct definition of the material and geometric properties of each specimen, (ii) the accurate simulation of external supports, bracing systems and load application, and (iii) the reliable contact formulation between the surfaces of the specimen and the bearing plates. The following general remarks summarise the characterisation of the SFE models:

- The SFE models adopted S4R elements to model the steel sheets, which are “all-purpose” quadrilateral elements, and rigid shell elements (R3D4) were used to model the load bearing and support plates. Mesh refinement was thoroughly investigated because it was important to balance the result accuracy with the computational effort required by each analysis. A finer mesh was adopted in the regions under the applied loading. It was further concluded that it was of the utmost importance to discretise the cross-section corners using a considerable number of elements, in spite of their high aspect ratios.
- The material was simulated considering either bi-linear or multi-linear stress-strain curves, depending on the available experimental data. The yield strength was assumed as the one corresponding to 0.2% strain. Following the ABAQUS User’s manual [23] recommendations, a true stress-logarithmic strain law was input to the model, considering the expected high plastic strains. The influence of residual stresses and plastic strains was disregarded, as well as the potential influence of web imperfections. Further discussion on both topics is presented in [11,12].
- Contact surfaces were defined for the beam and bearing plate elements, which account for the expected deformation of the beam under the transverse concentrated loading. The general contact algorithm [23] was adopted and a surface pair was assigned to each contact definition. Surfaces were separated by a distance equivalent to half the flange thickness, in order to prevent initial surface over-closure. A “hard” relation between the pressure and the over-closure was assumed, i.e. surfaces may freely separate, while surface penetration is limited, regardless of the contact pressure applied.
- The boundary conditions of the model were applied to the reference nodes of each bearing plate. In general, these nodes are fixed in the three spatial directions, except for the vertical displacement of the load bearing plate. Rotational restraints were optional and depended on the particular experimental set-up. The rigid plate elements were massless and an artificial

rotary inertia value was assigned to the reference node, in order to prevent the dynamic system becoming unstable.

- Additional support conditions, namely fastened flange-bearing plate connections and lateral bracing systems (experimental set-ups in which two beams are braced and tested simultaneously), were included in the model. The *TIE connection, between the rigid plate reference node and the flange node, simulated each fastener. Lateral displacement restraints were imposed to the flange nodes in order to represent lateral bracing systems.
- Specimen loading is simulated through the imposition of a vertical displacement to the reference node of the load bearing plate. A displacement function is defined by a total displacement ($\delta_{\max}$) and its time-dependent amplitude function. This function is smoothed in the initial stages of loading to ramp up the load rate and prevent “response noise” arising from initial kinetic energy peak.
- The non-linear quasi-static analyses were performed using an explicit integration scheme, based on the ABAQUS/Explicit package. This choice was due to the fact the implicit integrator adopted by default in ABAQUS [23] (ABAQUS/Standard) requires a very significant computational effort for non-linear problems dealing with large numbers of degrees of freedom and complex contact definitions. Indeed, the implicit integrator is based on the inversion of the stiffness matrix in all iterations. Although a larger number of time increments is required, each calculation step in the explicit integration scheme involves much less computational effort.

The following remarks address particular modelling issues of each test group:

- *Group (i)*: both C-sections were included in the model and the location of the web fasteners was considered as reported by the authors [24]. A *TIE constraint was imposed between the corresponding node in one of the C-section beams and the web contact surface of the opposite beam. Penetration between the webs was also prevented by an additional contact definition. Since the contact definitions account for the thickness, the web mid-line of each beam was located at a distance equivalent to the web thickness ($t/2 + t/2$), in order to prevent initial overlap.
- *Group (ii)*: The test programme consisted of 26 lipped C-section and 4 plain C-section beams. Two beams were tested simultaneously and braced at mid-span (in the edge opposite to the load application). A single C-section was included in the SFE model and the flange lateral displacement was restrained in the location of the lateral braces.
- *Group (iii)*: The SFE model is identical to that presented by Natário *et al.* [12] for the ITF case, except for the load configuration, which was ETF. The bearing plate rotation was

released and replaced by a rotary inertia value, in order to capture the effect of the half-round supports.

- *Group (iv)*: The model of lipped C and Z-sections is similar to that adopted for *Group (ii)*. The main difference is the inclusion of the flange-bearing plate fastened connection.
- *Group (v)*: Natário *et al.* [12] presented a thorough discussion of the SFE model adopted for this test group, which was based on a specimen taken from this group. Overall, the model is similar to that adopted for *Groups (ii)* and *(iv)*, with the exception of the lateral bracing system, which is absent in this case.

Overall, the quasi-static non-linear SFE analyses yielded reasonable results for the majority of the tests considered. Figure 8 compares the experimental (P_{test}) and numerical (P_{SFE}) failure loads. The following remarks on the SFE results are appropriate:

- *Group (i)*: The results for the built-up I-sections are the least accurate, most likely due to the complex contact behaviour of the back-to-back C-sections and the probable poor adjustment of the specimens in the beginning of loading. Nevertheless, the model successfully captures the web buckling collapse mechanism, as illustrated in Figure 9. The SFE results were particularly discrepant for the 2 specimens of the I-12' series – for this reason, they were not considered in the calibration of the DSM-based formula.
- *Group (ii)*: In this case, the influence of the lateral bracing systems on the overall web crippling behaviour was studied. Indeed, there was photographic evidence [4] to justify that the bracing angles located in the top flange limited the flange rotation, due to the small length of the specimens. Flange rotation was restrained in the sections corresponding to the location of the bracing systems, which was found to significantly influence the failure mechanism – this appeared to look more like what would be expected for a flange-fastened case. Note that Beshara and Schuster [6] included these results in the calibration of the NAS formula for unfastened C-sections.
- *Group (iii)*: In this case, the SFE results are very accurate in terms of both the failure mechanism and ultimate load. This test group is particularly interesting because it shows very clearly that the web thickness has more influence on the crippling strength than the web slenderness ratio. Figure 10 provides an overview of the SFE model and presents the expected web buckling failure mode, characterised by the formation of a single web yield line.
- *Group (iv)*: The results confirm the higher (25 to 40%) web crippling strength of the Z-sections, in comparison with twin C-sections. This difference is due to the different failure mechanism of the Z-sections, as the web plastic hinge moves from the mid-height towards

the top flange (in contact with the displaced load bearing plate). This evidence is due to the different bending moment distributions in the web for Z- and C-sections [24].

- *Group (v)*: The quasi-static analyses were able to accurately capture the influence of the flange fastened connection on the failure mode. The failure loads are also reasonably well estimated by the SFE analyses, even if some noticeable differences are observed in certain cases. It is not possible to be sure about the source of such discrepancies, since the numerical options were strictly identical and the test set-up was consistently the same for the different specimens.

Figure 8– Comparison between the ultimate loads obtained numerically (P_{SFE}) and the experimental collapse loads (P_{test}) – values divided by t

Figure 9– Failure mechanism of specimen I-6-ETF-1 (*Group (i)*):(a) experimental observation (from [4]) and (b) SFE model

Figure 10– SFE model of the S1ETF200N75 specimen – Group (iii):(a) modelled structural system and (b) failure mode and von Mises stress distribution at collapse.

5. Determination of Elastic Buckling and Plastic Loads

The calibration of the DSM-based formula for the web crippling design of cold-formed steel beams subjected ETF loading is based on a non-linear regression model applied to the calculated distribution of data points (χ, λ_{wc}) , where χ stands for the web crippling strength reduction factor and λ_{wc} is the slenderness parameter associated with the web failure. They are given by

$$\chi = \frac{P_{test}}{P_y} \quad \lambda_{wc} = \sqrt{\frac{P_y}{P_{cr}}} \quad (1)$$

where P_{test} stands for the experimental test load, and P_y and P_{cr} are the plastic load/strength² and elastic buckling load, respectively. In this context, it is indispensable to obtain numerical estimates of P_y and P_{cr} , associated with the expected failure mode. Ideally, this determination should be extremely accurate and account for the stiffness of the different cross-section elements and the influence of any other structural parameters, such as the flange-bearing plate connections, lateral bracing systems, flange lips, web stiffeners and specimen length. Both P_y and P_{cr} can be obtained from SFE models, using elastic buckling analyses (no plasticity) for P_{cr} and elastic-plastic 1st order analyses (no 2nd order effects) for P_y . However, these analyses should be avoided as much as possible, mostly because the SFE models are currently too complex for every-day computation in design practice. Moreover, the difference between performing these buckling and 1st order SFE analyses and fully non-linear ones is not so significant, with the latter having the advantage of leading directly to the required web crippling strength estimate. Therefore, it is necessary to search for easier-to-use numerical and/or analytical tools (not

² Do not confuse with the yield load (P_y), usually adopted for global, distortional and local buckling.

SFEmodels) to evaluate (even if approximately) P_{cr} and P_y . This issue is addressed in the following sections: Section 5.1 (buckling load P_{cr}) and Section 5.2 (plastic load P_y).

5.1 Buckling Load (P_{cr})

The application of the DSM to the design of beams, columns and beam-columns has been made much easier by the existence of the freely available software CUFSM [25], which is based on the constrained finite strip method. In the case of web crippling, the buckling loads can be generally obtained using SFE models/software. In this work, the buckling loads are determined by means of the freely available GBTWEB software [26], based on a GBT formulation previously developed by the authors [20]. Its main features are the accurate calculation of every first-order plane stress component (either uniform or non-uniform), which may then be included in the geometric stiffness matrix for the ensuing buckling analysis. Due to the nature of the web crippling phenomenon, which is potentially influenced by combinations of all stress components, GBT is viewed as a good alternative method to handle this problem, combining a reasonably low pre-processing effort with an accurate (and illuminating³) structural analysis.

The GBTWEB software [26] is based on three main windows that follow the corresponding main steps of the buckling analysis in the GBT framework: (i) cross-section analysis, (ii) first-order and buckling analysis and (iii) buckling solution. The current application is limited to cross-sections and flange-fastened arrangements identical (or very similar) to reviewed in Chapter 2. Figure 11 shows the graphical user interface for the cross-section and structural member data input and analysis.

Figure 11– GBTWEB [26] main windows: (a) cross-section analysis; (b) first-order and buckling analysis

³ In the sense that, unlike the SFE method, GBT has a modal nature.

Next, the GBT model for the buckling analysis is briefly explained. For further details on the formulation, numerical implementation and illustration of GBT, the interested reader is referred to [20]. In the earlier stages of this investigation [19,20], the GBT analyses employed a careful discretisation of cross-section round corners (sheet folds), which included several wall segments. However, the additional independent displacements (degrees-of-freedom) associated with the discretised corner nodes were responsible for a substantial increase in the computational effort required by each calculation. Since the pre-processing simplicity and fast calculation are very desirable traits of the DSM, indispensable to promote it as a viable alternative amongst design engineers, simplified cross-sections with sharp corners were implemented in GBTWEB – the simplified cross-section dimensions are obtained by extending the flats until they intersect. Although this simplification removes the influence of the corner bend radius from the buckling analysis, its impact on the buckling load P_{cr} has been found to be quite limited [24] – negligible P_{cr} differences between cross-sections with round and sharp corners were found for most configurations. Any web crippling strength calculation based on the proposed DSM-based formula should adopt a P_{cr} value obtained from the buckling analysis of a cross-section with sharp corners.

Bearing in mind that the elastic steel properties were based on experimental measurements, the particular definitions of each GBT model are presented in the following paragraphs:

- *Group (i)*: the GBT analyses have limitations regarding the simulation of back-to-back C-section beams because of the web contact. Instead, an equivalent lipped I-section section was considered (see Figure 9(b)) by assuming a web with double thickness. In GBT, the null flange rotation is achieved by using relative displacement restraints, which are also described in [24].
- *Group (ii)*: the GBT model is straightforward in this case. The load is applied to the web-flange corner and the lateral bracing system is replicated by imposing null lateral displacements to both flanges. The flange restraint is handled by imposing a common vertical displacement to all the flange nodes.
- *Group (iii)*: The GBT model is very similar to the one adopted for *Group (ii)*, without the lateral bracing systems and the flange rotational restraint.
- *Group (iv)*: In this case, it was important to include the flange rotational restraint imposed by the fastened connection. The procedure adopted is similar to that used for *Group (i)* and described in [24]. In brief, the flange region corresponding to the location of the bearing plates is discretized into a nodal mesh. Then, the vertical displacement of each node is assumed identical to that of the node corresponding to the location of the flange-bearing

fastener, using a relative displacement restraint. Lateral braces are simulated as described for *Group (ii)*.

- *Group (v)*: These specimens were analysed in GBT using a similar approach to that considered for *Group (iv)*. Naturally, no lateral restraints were included.

In order to validate the GBT results, the SFE models developed to carry out the non-linear analyses (previously presented in Section 4) were adapted to perform the corresponding elastic buckling analyses. In particular, it was important to revise the contact specifications, since the ABAQUS SFE model ties the surfaces together in the case of static analysis, i.e., the surfaces cannot separate in the buckling analysis. This limitation was deemed acceptable in the flange-bearing plate interface for fastened specimens. Conversely, the contact specifications were removed for the unfastened cases. Direct loads and nodal boundary conditions were applied to the flange nodes shared with the corner element, i.e. the load eccentricity due to the corner radius (with respect to the web plane) was included.

In general, the GBT and SFE analyses yield similar results, not only in terms of the web buckling mode configuration but also concerning the buckling load (P_{cr}) values, as shown in Figure 12. It is possible to conclude that both models are quite performing in terms of capturing the influence of the geometrical parameters (e.g., thickness, web height and bearing plate width) on P_{cr} .

Figure 12– Comparison between buckling loads obtained with SFE ($P_{cr,SFE}$) and GBT ($P_{cr,GBT}$) analyses – values divided by t^2

5.2 Plastic Load (P_y)

Besides P_{cr} , P_y (plastic load) is the other key ingredient of the DSM-based design approach. For columns, beams and beam-columns failing in global, distortional and local buckling modes, P_y merely corresponds to the first yield load/moment, which can always be obtained in a very straightforward manner. In the case of web crippling, the non-uniform stress state and highly localised deformation pattern preclude an immediate definition for P_y (unlike for the first yield load/moment) and, therefore, also the possibility of its simple calculation. A rational basis to calculate P_y is to view it as the load associated with the idealized plastic mechanism, akin to the true failure mode. For this purpose, rigid plastic analysis, namely the yield-line method, must be employed. In this section, formulae to calculate P_y are derived, using the principle of virtual work, on the basis of the observed yield-line mechanisms. Unfortunately, there is little information in the literature regarding the application of the yield-line method to web buckling failure modes. Zhao and Hancock [27,28] and Zhou and Young [29] proposed analytical models to estimate the bearing yield capacity of RHS and SHS sections subjected to interior and end bearing loads. The selected yield-line mechanism for the derivation of a P_y formula depends on the observation of experimental (if available) and/or numerical (non-linear SFE) results. Both are instrumental to the definition of the failure mechanism. The non-linear analyses were particularly important in describing the progressive development of the mechanism, from the formation of the first yield line until the post-failure regime. Naturally, the yield-line method leads to a P_y value that is an upper bound of the real plastic load – this fact is crucial for the validation of the proposed analytical models. Next, the rigid plastic analysis models adopted for each test group and the corresponding formula to calculate P_y are briefly described⁴:

- *Group (i)*: According to the non-linear results presented in Section 4, the web buckling failure of the built-up I-sections involves a rather dissimilar behaviour of the two C-sections. For the typical web buckling failure, the failure mechanism was based on the development of a yield line in the web of both beams. Two additional yield lines are also formed near the web ends of one beam. Figure 13 shows the von Mises stress distribution after collapse of a built-up I-section, on the basis of which it is possible to validate the proposed mechanism (the orange regions indicate the yield line location). Figure 14 shows a schematic representation of the idealised mechanism. The stress percolation path in the web was inspired by the models proposed by Zhao and Hancock [27,28]. The formulae to calculate P_y read

⁴ The derivation of these formulae has been reported in detail in [20].

$$P_y = \frac{4}{3} f_y N_m \left[\sqrt{r_m^2 + \left(\frac{3}{2}t\right)^2} - r_m \right] \quad , \quad (2)$$

$$N_m = \min \left\{ L; L_s + 2.5 r_{ext} + \frac{h_w}{2} \right\} \quad .(3)$$

Figure 13 – Von Mises stress distribution after failure for specimen I-3-ETF-5[4]

Figure 14 – Yield-line mechanism for the built-up I-section beams subjected to ETF loading conditions.

- *Group (ii)*: Upon observing the results of the non-linear SFE analyses, it was concluded that the lateral bracing system can influence the web buckling failure mechanism, depending on the restraint level imposed to the flange rotation. Figure 15 compares the picture of a tested specimens, taken from [6], with the corresponding von Mises stress distribution and deformation pattern near the peak load, obtained with SFE analysis. Overall, although the failure mechanism involves the development of a main yield line in the web, the flange rotation also appears to be restrained to some extent. The bracing system would be irrelevant if it was located further away from the edge where the load is applied – this does not happen in this case due to the small length of the tested specimen. The yield-line mechanism shown in Figure 16 was first suggested by Natário [24] for the calculation of P_y in single-web sections with flanges fastened to the bearing plates. This mechanism agrees reasonably well with the stress distribution presented in Figure 15(b), even if the bearing

plates were not effectively fastened to the flanges. Thus, it will be assumed that the bracing system causes the behaviour of these specimens to shift from unfastened to fastened, which contradicts the classification previously adopted by Beshara and Schuster [6]. The plastic load P_y is given by

$$P_y = f_y N_m \left(-2 r_m + \sqrt{4 r_m^2 + t^2 \frac{N^*}{N_m}} \right) \quad , \quad (4)$$

$$N^* = 2N_m + \frac{4\sqrt{3}}{3} (h_w + 2 r_m) \quad , \quad (5)$$

$$N_m = L_s + 2.5 r_{ext} + \frac{h_w}{2} \quad . \quad (6)$$

Figure 15 – Failure mechanism of the SU-4-ETF-3 specimen: (a) photograph taken from [6] and (b) von Mises stress distribution obtained from SFE analysis.

Figure 16– Yield-line mechanism for C-section beams with fastened flanges

- *Group (iii)*: The yield-line mechanism proposed for this group is depicted in Figure 17. On the basis of the derivations included in [24], it is possible to obtain the expression proposed for the calculation of the plastic load P_y in C-sections with unrestrained flange rotation,

$$P_y = f_y N_m \left(\sqrt{4 r_m^2 + t^2} - 2r_m \right) \quad .(7)$$

Figure 17 – Yield-line mechanism for C-section beams with unfastened flanges

- *Group (iv)*: In this group, the C- and Z-section specimens must be considered separately. The influence of the flange-bearing plate fastened connection on the web crippling strength and resulting plastic mechanism was previously discussed in the context of *Group (ii)*. The von Mises stress distribution and failure mode of a C-section specimen are presented in Figure 18. The triangular-shaped mechanism in the web of C-section specimens with restrained flange rotation is notorious. Thus, the yield-line mechanism presented in Figure 16 should be considered for C-section specimens in this group and the plastic load P_y may be computed using Eq. (4).

Figure 18 – SFE stress distribution and failure mode of the C-300-7-60 specimen

There is substantial experimental evidence [24] to confirm the higher web crippling capacity of the Z-sections, when compared with similar C-sections. Figure 19 compares the configurations of the failure mode obtained experimentally and numerically—they are in excellent agreement. The main difference is the position of the web yield line, which appears to be shifted towards the top flange, where the vertical displacement is imposed. This

behaviour could be justified by the different bending moment diagram imposed to the web of the Z-section, due to the eccentricity and location of the bearing plates. In order to provide a better grasp of the argument presented, consider the scheme presented in Figure 20, where it can be observed that the support load causes a moment in the web with opposite sign, due to the different cross-section shape. It is likely that this difference causes the web yield line to shift towards one of the flanges. It was further observed that the web yield line tends to shift towards the top flange as the cross-section depth decreases. This aspect is disregarded in the plastic analysis model presented next, as the web yield line is always located at a distance equivalent to 1/3 of the web height from the top edge of the web panel. The proposed yield line mechanism is presented in Figure 21. The formulae to estimate P_y for fastened Z-sections are

$$P_y = \frac{f_y N_m}{2(1 + c_1)} \left(-4 r_m (1 - c_1) + \sqrt{[4 r_m (1 - c_1)]^2 + 4 (1 + c_1) t^2 \frac{N^*}{N_m}} \right) , \quad (8)$$

$$N_{\text{vert}} = h_w + 2r_m \quad N_m = L_s + 2.5 r_{\text{ext}} + \frac{h_w}{3} , \quad (9)$$

$$c_1 = \frac{\frac{h_w}{3} + r_m}{\frac{2h_w}{3} + r_m} \quad c_2 = \frac{\frac{h_w}{3} + r_m}{\frac{\sqrt{3}}{2} N_{\text{vert}}} , \quad (10)$$

$$N^* = \left(2N_m + \frac{\sqrt{3}}{2} N_{\text{vert}} \right) (1 + c_1) + \frac{\sqrt{3}}{2} N_{\text{vert}} \left[c_1 + \frac{2\sqrt{3}}{3} c_2 + c_2^2 \left(1 + \frac{1}{c_1} \right) + 1 \right] . \quad (11)$$

Figure 19 – Failure modes of Z-section beam: (a) photograph of the experimental test [6]; (b) von Mises stress distribution and failure mode of specimen Z-200-10-30.

Figure 20 – Different bending moment diagrams in the web of C- and Z-sections

Figure 21– Yield-line mechanism for Z-sections with fastened flanges

- *Group (v)*: The specimens in this group are classified as C-sections with stiffened flanges, fastened to the supports. In this context, the yield-line mechanism (Figure 16) and Eqs. (4)-(6) presented for *Group (ii)* shall be considered also for *Group (v)*. Notice the excellent agreement with the failure mode experimentally observed, displayed in Figure 22.

Figure 22– Photograph of an unspecified ETF test reported by MacDonald et al. [9]

In summary, this section presented yield-line models for different web buckling failure mechanisms observed. Overall, the main objective was to capture the experimental collapse mechanisms as closely as possible, with the help of the non-linear SFE investigation presented in Section 4. Although these models are based on several simplifying assumptions, which obviously constitutes a limitation, they are suitable to be considered for the application of the DSM to web crippling design.

6. DSM-Based formula for ETF Web Crippling Design

The current DSM design formula for the design of columns, beams and beam columns [2] exhibit a general format, which is also adopted herein for web crippling design,

$$\frac{P_n}{P_y} = k_1 \left[1 - k_2 \left(\frac{P_{cr}}{P_y} \right)^{k_3} \right] \left(\frac{P_{cr}}{P_y} \right)^{k_3} \quad , \quad (12)$$

where (i) P_{cr} is the elastic buckling load, calculated with the GBTWEB software [26] (Section 5.1), (ii) P_y is the plastic load, determined using the yield-line mechanism formulae presented in Section 5.2, and (iii) P_n is the nominal value of the web crippling strength.

The k_1 , k_2 and k_3 coefficients were calibrated through a non-linear regression procedure, fitting the set of ratios P_{test}/P_y to the right hand side of Eq. (12), and using the computed value of P_{cr} and P_y for all the tested specimens belonging to Groups (i)-(v) [24]. These coefficients were obtained via the minimization of the sum of squared differences and led to a coefficient of determination equal to $R^2=0.928$. The DSM-based formula to calculate the web crippling strength of sections under External Tow Flange loading is then given by

$$P_n = 0.474 P_y \left[1 - 0.115 \left(\frac{P_{cr}}{P_y} \right)^{0.728} \right] \left(\frac{P_{cr}}{P_y} \right)^{0.728} \quad .(13)$$

Figure 23 shows the corresponding DSM-based design curve and all test data points used for its calibration. It is possible to confirm graphically that the different Groups included in this calibration exhibit a clear trend that is captured by the DSM formula. There is some dispersion in the low web crippling slenderness range (up to 2). Moreover, there are specimens with very high slenderness, namely those corresponding to the fastened cases, due to the large yield-to-buckling load ratio.

The plastic plateau ($P_n/P_y = 1$) is defined as the slenderness range for which the estimated web crippling load equals the plastic load. It is also interesting to assess the quality of the proposed DSM formula by observing the test data points in the vicinity $P_n/P_y = 1$ (unit value of the

strength reduction factor χ – see Eq. (1)). In theory, and apart from possible strain hardening effects, the ratio between the ultimate and plastic loads (P_n/P_y) should never exceed 1. With the exception of one case, all the remaining 127 specimens satisfy this condition. As expected, the results concerning stocky sections (*Group (iii)*) are closer to the unit value, due to the very high elastic buckling loads – their failure is mostly governed by yielding, rather than buckling. Therefore, the DSM-based equations for ETF web crippling design are completely defined by

$$P_n = \begin{cases} P_y & \text{for } \lambda_{wc} \leq 0.415 \\ 0.474P_y \left[1 - 0.115 \left(\frac{P_{cr}}{P_y} \right)^{0.728} \right] \left(\frac{P_{cr}}{P_y} \right)^{0.728} & \text{for } \lambda_{wc} > 0.415 \end{cases} \quad (14)$$

Figure 23 – Comparison between the proposed DSM-based formula and test data

In Figure 24, the test failure loads (P_{test}) are directly compared with their DSM-based predictions (P_n), obtained from Eq. (14). The mean and standard deviation of the P_{test}/P_n data are equal to 1.06 and 0.21, respectively. The higher differences were obtained for the C- and Z-sections in *Group (iv)*, which is probably due to an overestimation of the plastic load. Nevertheless, the main advantage (and also the main challenge) of adopting this methodology for web crippling design was the possibility of achieving a unified design formula for different cross-section types and fastening conditions. Unlike for beam or column design, the plastic load estimation for web crippling is based on idealised yield-line models that may not always accurately capture the plastic behaviour of every specimen in each *Group*.

Figure 24– Plot of the experimental P_{test}/P_n values against the web crippling slenderness

It was also deemed important to evaluate the resistance factor ϕ associated with the proposed DSM-based design approach. The load and resistance factor design (LRFD) methodology adopted by the NAS [2] is based on the condition

$$\phi P_n \geq P_u \quad , \quad (15)$$

where P_n stands for the nominal strength capacity and P_u is the factored load. The provisions of Chapter F of the NAS [2] were considered in the following calculations, namely clause (b) of section F.1.1 and table F1. The resistance factor ϕ is given by

$$\phi = C_\phi (M_m F_m P_m) e^{-\beta_0 \sqrt{V_M^2 + V_F^2 + C_P V_P^2 + V_Q^2}} \quad , \quad (16)$$

and a full explanation of the different parameters can be found in [2]. The coefficient β_0 defines the target reliability index, which is equivalent to 2.5 for structural members designed with the LRFD method. The correction factor C_P is defined as a function of the number of test specimens $n=128$ (*Group (i)* – 28 specimens; *Group (ii)* – 30 specimens; *Group (iii)* – 16 specimens; *Group (iv)* – 36 specimens; *Group (v)* – 18 specimens). The resulting resistance factor is $\phi=0.81$, a value that was found to be strongly influenced by the coefficient of variation V_P . In order to obtain the design web crippling strength, the nominal web crippling resistance P_n calculated according to Eq. (14) must be multiplied by this factor. The resistance factor $\phi=0.81$ is located within the

range of the coefficients that are proposed in the NAS for web crippling design, which vary between 0.75 and 0.90. For comparative purposes, note also that the current resistance factor for compression members has been established as 0.85. Finally, it should be mentioned that any beam is pre-qualified to be designed using the DSM approach if it satisfies a given set of geometrical and material conditions/limits. These limits, presented in Table 4, might be extended whenever additional test data becomes available.

7. Conclusion

This paper proposed a new approach to predict the web crippling failure load of cold-formed steel beams under External Two Flange (ETF) loading using the Direct Strength Method (DSM). After reviewing and selecting existing experimental data, the available design guidelines proposed in the EC3 [1] and NAS [2] were applied to all test data and their accuracy was assessed. It was concluded that there was undeniable potential in the development of a new alternative approach for web crippling design, based on the DSM philosophy. In order to obtain additional information on the web crippling behaviour of each test specimen, non-linear numerical analyses were completed using Shell Finite Element (SFE) models. The calibration of the DSM-based design curve involved the calculation of (i) elastic buckling loads, using the GBTWEB software, specifically developed for this purpose, and (ii) plastic loads, using analytical expressions based on yield-line models.

Overall, the DSM calibration for ETF web crippling design was successful. Despite the different cross-section types, several fastening conditions, and distinct experimental set-ups considered in the calibration, it was possible to establish a clear relationship between the web crippling slenderness and the strength reduction factor. In spite of some dispersion exhibited by the results, it was shown that the LRFD resistance factor obtained was within the expected range.

Despite its undeniable potential, it is reasonable to consider that this proposal may still be enhanced before it can be used as an alternative design method. In particular, it will be important to further investigate the plastic mechanisms associated with web crippling and their analytical expressions. In order to achieve its full potential, this method should also be extended to other cross-section types, such as single hats and multi-web deck sections. In light of the promising results presented in this study, the methodology was also applied to beams under Internal Two Flange (ITF) loading [30] and, in addition, may also be easily extended to External One Flange (EOF) and Internal One Flange (IOF) loading conditions in the future, by following similar calibration procedures.

Table 4 – Proposed limits for pre-qualified beams (ETF web crippling design)

<p>built-up I-sections:</p>	<p>built-up I-sections regarded as two lipped C-sections fastened back-to-back through the web, near the flanges. No rotational restraint imposed to the flanges. 90° angle between webs and flanges (vertical web) 90° angle between flanges and stiffeners (vertical lip)</p> $h/t < 270(b_1/t, b_2/t) < 66(d_1/t, d_2/t) < 15$ $2.3 < (h/b_1, h/b_2) < 8.4$ $0.20 < (d_1/b_1, d_2/b_2) < 0.46$ $13.3 < L_s/t < 65.2 \quad 1.2 < r_i/t < 2.7$ $f_Y < 325 \text{ MPa}$
<p>unfastened C-sections:</p>	<p>unfastened plain channels (without flange stiffener) and C-sections (with flange stiffener), without any flange rotational restraint. 90° angle between webs and flanges (vertical web) 90° angle between flanges and stiffeners (vertical lip)</p> $h/t < 263 \quad b/t < 65 \quad d/t < 15$ $1.8 < h/b < 16.2 \quad 0.19 < d/b < 0.46$ $5.2 < L_s/t < 63.8 \quad 0.9 < r_i/t < 2.7$ $f_Y < 450 \text{ MPa}$
<p>fastened C-sections:</p>	<p>C-sections (with flange stiffener), with an effective provision that prevents the flange rotation (e.g. fastener). 90° angle between webs and flanges (vertical web) 90° angle between flanges and stiffeners (vertical lip)</p> $h/t < 207 \quad b/t < 66 \quad d/t < 28$ $2.0 < h/b < 6.6 \quad 0.35 < d/b < 0.64$ $20.7 < L_s/t < 128.2 \quad 2.0 < r_i/t < 12.1$ $f_Y < 450 \text{ MPa}$
<p>fastened Z-sections:</p>	<p>Z-sections (with flange stiffener), with an effective provision that prevents the flange rotation (e.g. fastener). 90° angle between webs and flanges (vertical web) 90° angle between flanges and stiffeners (vertical lip)</p> $h/t < 207 \quad b/t < 52 \quad d/t < 28$ $2.3 < h/b < 6.7 \quad 0.37 < d/b < 0.66$ $20.7 < L_s/t < 54.7 \quad 4.8 < r_i/t < 12.1$ $f_Y < 450 \text{ MPa}$

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References

- [1] Comité Européen de Normalisation (CEN). Eurocode 3, Design of steel structures – Part 1-3: General rules, Supplementary rules for cold-formed members and sheeting, Brussels, Belgium, 2006.
- [2] American Iron and Steel Institute (AISI). North American Specification for the design of cold-formed steel structural members, S100-12, Washington, USA, 2012.
- [3] Winter G., Pian R. Crushing strength of thin steel webs, Engineering Experiment Station, Cornell University, New York, USA, 1946.
- [4] Hetrakul N., Yu W. Webs for cold-formed steel flexural members; Structural behaviour of beam webs subjected to web crippling and a combination of web crippling and bending, University of Missouri-Rolla, Missouri, USA, 1978.
- [5] Young B., Hancock G.J. Design of cold-formed unlipped channels subjected to web crippling, Research Report No R794, The University of Sydney, Australia, 1999.
- [6] Beshara B., Schuster R.M. Web crippling data and calibrations of cold formed steel members, Research Report RP00-2, Canadian Cold Formed Steel Research Group, University of Waterloo (AISI Report, revised 2006), Canada, 2000.
- [7] Heiyanthuduwa M. Web crippling behaviour of cold-formed thin-walled steel lipped channel beams, Ph. D. Thesis, School of Engineering & Computing, Glasgow Caledonian University, Scotland, 2008.
- [8] Macdonald M., Heiyantuduwa M.A., Kotelko M., Rhodes J. Web crippling behaviour of thin-walled lipped channel beams, *Thin-Walled Structures*, 49, pp. 682-690, 2011.

- [9] Macdonald M., Heiyantuduwa M.A. A design rule for web crippling of cold-formed steel lipped channel beams based on nonlinear FEA, *Thin-Walled Structures*, 53, pp. 123-130, 2012.
- [10] Sivakumaran K. Analysis for web crippling behaviour of cold-formed steel members, *Computers & Structures*, 32 (3-4), pp. 707-719, 1989.
- [11] Natário P., Silvestre N., Camotim D. Web crippling failure using quasi-static FE models, *Thin-Walled Structures*, 84, pp. 34-49, 2014.
- [12] Natário P., Silvestre N., Camotim D. Computational modelling of flange crushing in cold-formed steel sections, *Thin-Walled Structures*, 84, pp. 393-405, 2014.
- [13] Ren W.X., Fang S.E., Young B. Finite element simulation and design of cold-formed steel channels subjected to web crippling, *Journal of Structural Engineering (ASCE)*, 132 (12), pp. 1967-1975, 2006.
- [14] Kaitila O. Web crippling of thin-walled cold formed steel cassettes, *Journal of Constructional Steel Research*, 63, pp. 766-778, 2007.
- [15] Prabakaran K., Schuster R.M. Web crippling of cold formed steel members, 14th International Specialty Conference on Cold-Formed Steel Structures, St. Louis, Missouri, USA, pp. 151-164, 1998.
- [16] Schafer B.W. Review: The direct strength method of cold-formed steel member design, *Journal of Constructional Steel Research*, 64 (7-8), pp. 766-778, 2008.
- [17] Pham C.H., Hancock G.J. Direct strength design of cold-formed C-sections for shear and combined actions, *Journal of Structural Engineering*, 138(6), pp. 769-768, 2012.
- [18] Natário P., Silvestre N., Camotim D. Rumor à aplicação do Método da Resistência Directa no dimensionamento de vigas ao esmagamento da alma (Towards the application of the Direct Strength Method in the design of beamssubjected to web crippling, in Portuguese), VIII Congresso de Construção Metálica e Mista, L. Simões da Silva et al. (Eds.), vol. II, pp. 437-446, Guimarães, Portugal, 2011.
- [19] Natário P., Silvestre N., Camotim D. Direct strength method for web crippling design of beams under ETF loading, Conference in Coupled Instabilities in Metal Structures (CIMS'2012), Joseph Loughlan, David H Nash, James Rhodes (Eds.), pp. 159-166, Strathclyde University, Glasgow, Scotland, 2012.

- [20] Natário P., Silvestre N., Camotim D. Localized web buckling analysis of beams subjected to concentrated loads using GBT, *Thin-Walled Structures*, 61, pp. 27-41, 2012.
- [21] Young B., Hancock G.J. Design of cold-formed channels subjected to web crippling, *Journal of Structural Engineering ASCE*, 127 (10), pp. 1137-1144, 2001.
- [22] Kaitila O. Web crippling of cold-formed thin-walled cassettes, Ph. D. Thesis, Laboratory of Steel Structures, Helsinki University, Espoo, Finland, 2004.
- [23] DassaultSystèmesSimulia Corp. Abaqus User's Manual and Abaqus Theory Manual (version 6.10), 2010.
- [24] NatárioP. Localized failure of thin-walled steel members subjected to concentrated loads: analysis, behaviour and design, PhD Thesis in Civil Engineering, IST, University of Lisbon, 2015.
- [25] Li Z., Abreu J.C.B., Leng J., Ádány S., Schafer B.W. Review: Constrained finite strip method developments and applications in cold-formed steel design, *Thin-Walled Structures*, 81, pp. 2-18, 2014.
- [26] Natário P., Silvestre N., Camotim D., GBTWEB, GBT-based code for web buckling analysis of members under localised loads, soon freely available at <http://www.civil.ist.utl.pt/gbtweb/>, 2015.
- [27] Zhao X.L., Hancock G.J. Square and rectangular hollow sections subjected to combined actions, *Journal of Structural Engineering (ASCE)*, 121 (9), pp. 1323-1329, 1992.
- [28] Zhao X.L., Hancock G.J. Square and rectangular hollow sections under transverse end-bearing force, *Journal of Structural Engineering*, 121 (9), pp. 1323-1329, 1995.
- [29] Zhou F., Young B. Yield line mechanism analysis on web crippling of cold-formed stainless steel tubular sections under two-flange loading, *Engineering Structures*, 28, pp. 880-892, 2006.
- [30] Natário P., Silvestre N., Camotim D., Direct strength prediction of web crippling failure of beams under ITF loading, submitted for publication, 2015.

GRAPHICAL ABSTRACT