A residual strain model for progressive fatigue damage analysis of composite structures

Libin Zhao^{1,*}, Meijuan Shan¹, Haiming Hong², Dexuan Qi³, Jianyu Zhang^{4,**}, Ning Hu⁴

1 School of Astronautics, Beihang University, Beijing 100191, China

2 Shenyang Aircraft Design & Research Institute, AVIC, Shenyang 110035, China

3 China Aero-Polytechnology Establishment, AVIC, Beijing 100028, China

4 College of Aerospace Engineering, Chongqing University, Chongqing 400044, China

Abstract: A residual strain model was presented based on the fatigue-accumulated damage mechanisms of composite materials. By combining the proposed residual strain model with a residual stiffness model, the gradually degraded material properties due to the damage in fatigue cycles were characterized. Accompanying the residual strain based gradual material degradation model with a micromechanics-based sudden material degradation rule for describing the stiffness degradation in one fatigue cycle and an extended maximum strain criterion for evaluating failure of materials, a progressive fatigue damage model (PFDM) of a typical double-lap three-bolt joint made of T800 carbon/epoxy composites was developed. Compared with the residual strength model in a traditional PFDM, the residual strain model has consistent parameters with those in the residual stiffness model, which could be determined by nondestructive experiments. Fatigue tests of the double-lap three-bolt joint specimens under a stress ratio $R=-0.2$ and three-level load ratios $q=0.7, 0.8, 0.9$ were performed. Good consistency between the experimental results and numerical predictions of the PFDM validates the efficiency of the proposed residual strain model and PFDM in accurately predicting the residual strength and fatigue life of composite structures.

Keywords: Composite bolted joints; Residual strain model; Progressive fatigue damage model; Residual strength; Fatigue life.

***Corresponding author:** Libin Zhao, E-mail address: lbzhao@buaa.edu.cn

****Corresponding author:** Jianyu Zhang, E-mail address: jy Zhang@cqu.edu.cn

Nomenclature

$E(n)$	Residual stiffness or fatigue modulus
$\varepsilon^R(n)$	Residual strain
ε_{ult}	Ultimate strain
$\varepsilon(0)$	Initial strain
$\varepsilon(n)$	Resultant strain
$\Delta\varepsilon(n)$	Irreversible fatigue-accumulated strain
E_{11}, E_{22}	Longitudinal and transverse moduli
G_{12}	Shear modulus
ν_{12}	Poisson ratio
X_T, Y_T	Longitudinal and transverse tensile strengths
X_C, Y_C	Longitudinal and transverse compressive strengths
S_{12}	Shear strength
$\varepsilon_{11,t}^R(0), \varepsilon_{22,t}^R(0)$	Longitudinal and transverse ultimate tensile strains
$\varepsilon_{11,c}^R(0), \varepsilon_{22,c}^R(0)$	Longitudinal and transverse ultimate compressive strains
$\varepsilon_{12,s}^R(0)$	Shear ultimate strain
R	Stress ratio
q	Load ratio
P_{max}	The maximum load
P_{min}	The minimum load
P_r	Residual strength
N_f	Fatigue life

1. Introduction

Carbon fiber reinforced polymers (CFRPs) have been widely utilized in aircraft structures due to their high specific stiffness/strength and superior fatigue performance, etc. Initially, CFRPs were mainly used in inessential or secondary aircraft structures with lower load capability. Thus, the fatigue in composite structures was generally enveloped by the static strength [1]. With the increasing application of CFRPs in primary aircraft structures, a high demand for structure efficiency and loading capability makes the fatigue properties of composite structures a focus of attention. Since the mechanical joints are always weak parts of composite structures, their fatigue behaviors have attracted much attention in the last decades.

The investigations on fatigue of composite mechanical joints can be classified as experimental tests and numerical simulations. Up to now, great efforts have been made to study the effects of different factors on the fatigue properties of composite mechanical joints with experimental tests [2-9]. These studies mainly focused on the fatigue life of the joints and only several of them qualitatively evaluated fatigue damage evolution and failure mechanisms of the joints through the limited measurement techniques, such as the strain gauge and extensometer method [2], the DIC method [3] and the infrared thermography [5, 8]. In order to deeply explore the fatigue damage propagation and reveal the fatigue damage mechanism of composite mechanical joints, the numerical simulation techniques have been developed since the late 1980s [10-18]. Initially, a critical element model [10-11] was presented to predict fatigue behavior of composite hole laminates. In this model, residual strength of critical elements as well as residual stiffness of sub-critical elements needs to be calculated. In later studies, a progressive fatigue damage model (PFDM) [12-13] was established to analyze fatigue failure of composite pin-loaded and bolt-loaded laminates. The PFDM can determine the damage state at different fatigue loading levels and different fatigue cycles, and predict the residual stiffness, residual strength and fatigue life of composite laminates with arbitrary geometry and lay-ups. To reduce the huge computational cost of

the PFDM, a similar regional elements model [14] was further developed, where a 2D stress analysis model of representative region, residual strength of critical layers and residual stiffness of sub-critical layers were involved. Recently, Kaminski et al. [15] briefly introduced the PFDM developed by ONERA for predicting the fatigue of 3D woven composite structures and suggested the promising future of PFDM in fatigue analysis of composite structures. In order to overcome the deficiencies of the macro theories in aforementioned PFDM-type models, the micro-mechanics of failure (MMF) theory was extended to predict progressive fatigue failure of composite bolted joints [16], in which numerous fatigue tests of unidirectional laminates are required to obtain MMF critical parameters. Furthermore, to minimize the need for experimental material characterization, a model based on the kinetic theory of fracture for polymer matrices was developed to predict fatigue bearing failure of the hole laminates under the assumption that the initiation of the fatigue damage takes place in the matrix [17-18].

As discussed above, the PFDM is one of the most popular numerical techniques to investigate the fatigue of composite mechanical joints under arbitrary fatigue loading. In general, the PFDMs are combined by different stress analysis models, fatigue failure criteria and material degradation models to obtain accurate predictions. In particular, the material degradation models commonly involved the gradual material degradation stemmed from continuous fatigue cycles and the sudden material degradation resulted from the excessive strain/stress in one fatigue cycle, in which the latter has been maturely developed in static progressive failure analyses of composite joints [19-23]. With regard to the gradual material degradation rules, the researchers have proposed many residual stiffness models [24-30] and several residual strength models [31-33]. Generally, the basic parameters in the residual stiffness models are gained by nondestructive tests, while those in the residual strength models are difficult to be determined because the tests for strength are destructive and usually have a large dispersion.

To overcome this knottiness, a residual strain model was proposed based on the

fatigue-accumulated damage mechanism of composite materials. In the proposed model, the parameters are always consistent with those in the residual stiffness model, for which only nondestructive tests are required. Moreover, a PFDM was established to predict the residual strength and fatigue life of a typical double-lap three-bolt joint made of T800 carbon/epoxy composites. In the PFDM, a detailed 3D finite element (FE) model of the joint was built to obtain accurate stress distribution. The proposed residual strain model accompanied by a residual stiffness model was utilized to represent the gradual material degradation during the fatigue cycles and a micromechanics-based material degradation rule [23] was adopted to characterize the sudden material degradation in one fatigue cycle. The maximum strain criterion was extended to describe fatigue failure of composite materials. To validate the proposed residual strain model and the developed PFDM, fatigue tests of composite double-lap three-bolt joints were carried out.

2. Residual strain based material degradation model in fatigue cycles

A residual stiffness model proposed by Hwang and Han [24] was introduced and a residual strain model replacing the traditional residual strength model was presented, both of which are essential components to establish a gradual material degradation model in fatigue cycles.

2.1 Residual stiffness model

Most researchers agree on the gradual stiffness degradation measured nondestructively is an important aspect to describe the material degradation properties in the fatigue cycles of composite materials. A typical fatigue modulus proposed by Hwang and Han [24] to describe the residual stiffness of composite materials is:

$$E(n) = \frac{\sigma_{\max}}{\varepsilon(n)} = \frac{q\sigma_{\text{ult}}}{\varepsilon(n)} \quad (1)$$

where $E(n)$ and $\varepsilon(n)$ represent the fatigue modulus and resultant strain at the n^{th} cycle. $\sigma_{\max}$ is the maximum stress level in the fatigue loading spectrum and σ_{ult} is the ultimate strength. q is the load ratio, defined as the ratio of $\sigma_{\max}$ to σ_{ult} .

The boundary conditions are

$$\begin{cases} E(0) \approx E_0 \\ E(N) = \frac{q\sigma_{\text{ult}}}{\varepsilon(N)} = \frac{q\sigma_{\text{ult}}}{\varepsilon_{\text{ult}}} = qE_0 \end{cases} \quad (2)$$

where E_0 denotes the initial stiffness and ε_{ult} is the ultimate strain. They assumed that the final fatigue failure occurs when the resultant strain $\varepsilon(n)$ stemmed from fatigue-accumulated damage arrives at the ultimate strain ε_{ult} .

The stiffness degradation rate is described by [24]

$$\frac{dE(n)}{dn} = -Acn^{c-1} \quad (3)$$

where A and c are material constants.

The integration of Eq. (3) is written as:

$$E(n) - E(0) = -An^c \quad (4)$$

When the fatigue cycle reaches the fatigue life N , A can be expressed as:

$$A = -\frac{E(N) - E(0)}{N^c} \quad (5)$$

Substitute Eq. (5) into Eq. (4), and the residual stiffness model is rewritten as:

$$\frac{E(n)}{E(0)} = 1 - (1 - q) \left(\frac{n}{N} \right)^c \quad (6)$$

2.2 Residual strain model

According to Eq. (1), the relationship between the resultant strain and fatigue cycles can be obtained and illustrated in Fig. 1. The resultant strain can be divided into two parts: the initial strain $\varepsilon(0)$ caused by the static load and the irreversible fatigue-accumulated strain $\Delta\varepsilon(n)$ resulted from the fatigue cycles under the maximum stress level σ_{max} . Thus, the resultant strain at the n^{th} cycle is

$$\varepsilon(n) = \varepsilon(0) + \Delta\varepsilon(n) \quad (7)$$

Fig. 1 also shows the fatigue modulus $E(n)$, which declines with the increase of fatigue cycles.

According to Eq. (1) and Eq. (6), the relationship between the resultant strain at the n^{th} cycle and the initial strain (when $n = 0$) was established, which can be expressed as

$$\frac{\varepsilon(n)}{\varepsilon(0)} = \frac{E(0)}{E(n)} = \frac{1}{1 - (1-q)\left(\frac{n}{N}\right)^c} \quad (8)$$

According to Eq. (7) and Eq. (8), the irreversible fatigue-accumulated strain can be expressed as:

$$\Delta\varepsilon(n) = q\varepsilon_{\text{ult}}\left(\frac{1}{1 - (1-q)\left(\frac{n}{N}\right)^c} - 1\right) \quad (9)$$

Define the fatigue residual strain $\varepsilon^R(n)$ at the n^{th} cycle as:

$$\varepsilon^R(n) = \varepsilon_{\text{ult}} - \Delta\varepsilon(n) \quad (10)$$

Thus, the fatigue residual strain can be obtained:

$$\varepsilon^R(n) = \varepsilon_{\text{ult}}\left(1 + q - \frac{q}{1 - (1-q)\left(\frac{n}{N}\right)^c}\right) \quad (11)$$

It satisfies the boundary conditions:

$$\begin{cases} \varepsilon^R(0) = \varepsilon_{\text{ult}} \\ \varepsilon^R(N) = \varepsilon(0) \end{cases} \quad (12)$$

Further, normalizing the fatigue residual strain according to $\varepsilon^R(0) = \varepsilon_{\text{ult}}$ in Eq. (12), a residual strain model can be obtained:

$$\frac{\varepsilon^R(n)}{\varepsilon^R(0)} = 1 + q - \frac{q}{1 - (1-q)\left(\frac{n}{N}\right)^c} \quad (13)$$

Obviously, the relationship between the normalized fatigue residual strain and fatigue cycles can be characterized by the load ratio q , the fatigue life N and the parameter c . It is clear that these parameters are always consistent with those in the residual stiffness model, for which only nondestructive tests are required. Therefore, the proposed residual strain model accompanied with

the residual stiffness model can be utilized to efficiently describe the gradual material degradation during the fatigue cycles.

3. Progressive fatigue damage model

In this section, a PFDM was developed to predict the residual strength and fatigue life of a typical composite double-lap three-bolt joint. In the PFDM, a 3D FE model was built to obtain the stress distribution in the joint. The residual strain based material degradation model and a micromechanics-based sudden material degradation rule [23] were adopted to characterize the material degradation. Parameters in the residual strain based material degradation model were determined by corresponding fatigue tests of composite laminates. The maximum strain criterion was extended to describe fatigue failure of composite materials. At last, a flowchart for predicting fatigue of composite joints was described in detail.

3.1 Joint descriptions

A typical composite double-lap three-bolt joint was designed, as shown in Fig. 2. The joint was made of T800 carbon/epoxy composites with the stacking sequence of $[45/0/-45/0/90/0/45/0/-45/0]_s$. The geometrical dimensions of the joint are also depicted in Fig. 2. The thickness of a unidirectional lamina is 0.188 mm. Fasteners with hexagon head bolts HB8015-8 and self-locking thick nuts HB8026-MJ8 $\times$ 1.25 were made of titanium alloy TC16. The nominal diameter of the bolt is 8mm. An assembly tightening torque of 25N·m was applied. The material properties of T800 carbon/epoxy composites provided by the manufacturer are listed in Table 1, among which the values marked with ‘*’ are determined by the transversely isotropic assumption and engineering experience. The elastic modulus and Poisson ratio of titanium alloy are 108 GPa and 0.29, respectively.

3.2 FE model for stress analysis

A detailed 3D FE model of the double-lap three-bolt joint was built with solid elements C3D8 in the software ABAQUS, as shown in Fig. 3. In order to acquire accurate stress and simultaneously

reduce the computational cost, a refined radial mesh scheme was designed around the fastener hole and a relatively coarse mesh scheme was adopted at the non-overlap section of the laminates. Four elements were partitioned in the inter-laminar direction of each composite laminate. Moreover, contact pairs were established at the contact surfaces between the central laminate and outer laminates as well as between the fastener and laminates. The corresponding friction coefficient was 0.3 and 0.1 for tangential behavior, and “Hard” contact was chosen for normal behavior.

Three load steps were set. Displacement constraints in three directions x , y and z were applied to the section A at all steps. A preload of 15.63kN was applied to the fasteners by bolt load method in ABAQUS at step-1. As for the fatigue loading spectrum, a simplified trigonal fatigue loading spectrum was applied in the direction x on the section B at step-2. The damage induced by the compressive load was ignored because the compressive load was much smaller than the tensile load when the stress ratio $R=-0.2$. Moreover, to calculate the residual strength of the joint, a displacement load was utilized to replace the fatigue load at step-3.

3.3. Material degradation models

The residual strain based material degradation model for gradually-degraded material properties during fatigue cycles and a micromechanics-based material degradation rule for suddenly-degraded material properties in one fatigue cycle were used in the PFDM. Fatigue tests of composite laminates were conducted to determine the parameters in the residual strain based material degradation model.

3.3.1 Parameter determination for residual strain based material degradation model

Fatigue tests of composite laminates were performed to obtain q - N curves and residual stiffness during fatigue cycles, from which the parameters in the residual stiffness and residual strain models can be determined. According to ASTM D3410 [34], the laminates made of T800 carbon/epoxy composites with the lamina thickness of 0.188mm were designed, as shown in Fig. 4. The dimensions of the specimens are listed in Table 2. Table 2 also presents the fatigue test schemes of

these specimens. The fatigue tests were performed by using a MTS 880 testing machine with the loading capacity of 50kN and dynamic error of 0.5%. Two groups of compressive fatigue and one group of tensile fatigue tests were designed to determine longitudinal, transverse and shear fatigue properties of T800 carbon/epoxy composites. The fatigue loading frequency was not higher than 15Hz to reduce the temperature effect.

From the fatigue tests, q - N curves for the longitudinal, transverse and shear fatigue properties of T800 unidirectional lamina were obtained, as shown in Fig. 5, which were $q = 0.933 - 0.024 \lg N$, $q = 1.149 - 0.08 \lg N$ and $q = 1.062 - 0.073 \lg N$, respectively. In addition, the gradual longitudinal, transverse and shear stiffness degradations during the fatigue cycles were automatically recorded in the tests, from which the parameters c in the residual stiffness and residual strain models can be determined as 0.15, 0.12 and 0.2, respectively. Typical stiffness degradation curves are depicted in Fig. 6.

Thus, the residual stiffness and residual strain models of T800 carbon/epoxy composites can be expressed as:

$$\begin{cases} \frac{E_{11}(n)}{E_{11}(0)} = 1 - (1 - q) \left(\frac{n}{10^{(0.933-q)/0.024}} \right)^{0.15} \\ \frac{E_{22}(n)}{E_{22}(0)} = \frac{E_{33}(n)}{E_{33}(0)} = 1 - (1 - q) \left(\frac{n}{10^{(1.149-q)/0.08}} \right)^{0.12} \\ \frac{G_{12}(n)}{G_{12}(0)} = \frac{G_{13}(n)}{G_{13}(0)} = \frac{G_{23}(n)}{G_{23}(0)} = 1 - (1 - q) \left(\frac{n}{10^{(1.062-q)/0.073}} \right)^{0.2} \end{cases} \quad (14)$$

$$\left\{ \begin{array}{l} \frac{\varepsilon_{11,t}^R(n)}{\varepsilon_{11,t}^R(0)} = \frac{\varepsilon_{11,c}^R(n)}{\varepsilon_{11,c}^R(0)} = 1 + q - \frac{q}{1 - (1-q) \left(\frac{n}{10^{(0.933-q)/0.024}} \right)^{0.15}} \\ \frac{\varepsilon_{22,t}^R(n)}{\varepsilon_{22,t}^R(0)} = \frac{\varepsilon_{22,c}^R(n)}{\varepsilon_{22,c}^R(0)} = 1 + q - \frac{q}{1 - (1-q) \left(\frac{n}{10^{(1.149-q)/0.08}} \right)^{0.12}} \\ \frac{\varepsilon_{33,t}^R(n)}{\varepsilon_{33,t}^R(0)} = \frac{\varepsilon_{33,c}^R(n)}{\varepsilon_{33,c}^R(0)} = 1 + q - \frac{q}{1 - (1-q) \left(\frac{n}{10^{(1.149-q)/0.08}} \right)^{0.12}} \\ \frac{\varepsilon_{12,s}^R(n)}{\varepsilon_{12,s}^R(0)} = \frac{\varepsilon_{13,s}^R(n)}{\varepsilon_{13,s}^R(0)} = \frac{\varepsilon_{23,s}^R(n)}{\varepsilon_{23,s}^R(0)} = 1 + q - \frac{q}{1 - (1-q) \left(\frac{n}{10^{(1.062-q)/0.073}} \right)^{0.2}} \end{array} \right. \quad (15)$$

where t , c and s denote the tensile, compressive and shear properties, respectively. $E_{33}(n)/E_{33}(0)$ 、 $G_{13}(n)/G_{13}(0)$ 、 $G_{23}(n)/G_{23}(0)$ 、 $\varepsilon_{33,t}^R(n)/\varepsilon_{33,t}^R(0)$ 、 $\varepsilon_{33,c}^R(n)/\varepsilon_{33,c}^R(0)$ 、 $\varepsilon_{13,s}^R(n)/\varepsilon_{13,s}^R(0)$ 、 $\varepsilon_{23,s}^R(n)/\varepsilon_{23,s}^R(0)$ are determined by the transversely isotropic assumption and engineering experience. $\varepsilon_{ii,t}^R(n)/\varepsilon_{ii,t}^R(0) = \varepsilon_{ii,c}^R(n)/\varepsilon_{ii,c}^R(0)$ ($i=1, 2, 3$) were obtained by assuming that the tensile and compressive residual strain degradations are accordant. The initial residual strains $\varepsilon_{ij,k}^R(0)$ ($1 \leq i \leq j \leq 3; k=t, c, \text{ or } s$), namely the corresponding ultimate strains, were obtained from the static tests of the laminate specimens.

The effect of the load ratio on the residual stiffness and residual strain was further discussed here. Fig. 7 illustrates the longitudinal residual stiffness and strain degradations under different load ratios. Obviously, the longitudinal residual stiffness and residual strain decrease with the increase of the load ratio. Fig. 8 shows the longitudinal, transverse and shear residual stiffness and strain degradations under $q=0.7$. Clearly, the longitudinal residual stiffness and strain are almost unchanged with the increase of the fatigue cycles; the transverse residual stiffness and strain decrease remarkably; and the shear residual stiffness and strain fall more steeply. Besides, the residual strain degradation is more severe than the residual stiffness degradation.

3.3.2 Sudden material degradation model

A sudden material degradation rule based on micromechanical analyses [23] was adopted in the PFDM to characterize the material degradation in one fatigue cycle. The corresponding degradation factors for each failure mode are listed in Table 3.

3.4 An extended maximum strain criterion

With regard to the fatigue failure criteria, several static stress-based failure criteria such as the maximum stress criterion, Tsai-Hill criterion, Hashin-type criterion, etc, have been extended to play important roles in estimating the fatigue failure. Among the failure criteria for composite materials, the maximum strain criterion is the most popular one due to its simpleness and easiness [35]. Moreover, as discussed above, the fatigue failure results from the accumulation of irreversible strain during the continuous fatigue cycles. Thus, the maximum strain criterion was extended to evaluate the fatigue failure of composite materials:

Tensile or compressive failure:

$$\left\langle \frac{\varepsilon_{ij}}{\varepsilon_{ij,t}^R(n)} \right\rangle + \left\langle -\frac{\varepsilon_{ij}}{\varepsilon_{ij,c}^R(n)} \right\rangle = 1 \quad \text{if } i=j=1, 2, 3 \quad (16)$$

Fiber-matrix shear-out failure:

$$\left| \frac{\varepsilon_{ij}}{\varepsilon_{ij,s}^R(n)} \right| = 1 \quad \text{if } 1 \leq i < j \leq 3 \quad (17)$$

where $\langle x \rangle$ is the McCauley operator, defined as $\langle x \rangle := (x + |x|) / 2$. ε_{ij} and $\varepsilon_{ij,k}^R(n)$ ($k=t, c$, or s) represent the strains in the material coordinate system and corresponding gradually-degraded ultimate strains, which replaced the constant ultimate strains in the traditional maximum strain criterion and decreased with the increase of the fatigue cycles according to Eq. (15). Clearly, the extended maximum strain criterion degenerated to the traditional maximum strain criterion when $n=0$.

3.5 Model implementation

Based on the developed PFDM, a user defined subroutine UMAT, which includes the aforementioned material degradation models and extended maximum strain criterion, was embedded into ABAQUS[®] for predicting the fatigue of the double-lap three-bolt joint. Fig. 9 illustrates the flowchart and the main steps are described as follows:

Step 1: Set n and Δn , which depend on the expected fatigue life of the joint.

Step 2: Establish a detailed 3D FE model of the composite joint.

Step 3: Carry out stress analysis under the maximum load and obtain the stress distribution of the joint.

Step 4: Calculate the gradually-degraded material properties according to the residual stiffness and residual strain models.

Step 5: Apply an initial displacement δ and perform stress analysis of the damaged joint.

Step 6: Inspect the failure of the element with the extended maximum strain criterion.

Step 7: If element failure occurs, degrade the stiffness of the damaged element according to the sudden material degradation model; otherwise, increase the displacement with a small increment $\Delta\delta$ and jump to *Step 5*.

Step 8: If the displacement δ reaches the expected maximum value $\delta_{\max}$, acquire the P - δ curve, from which the residual strength P_r of the joint is obtained; otherwise, increase the displacement with a small increment $\Delta\delta$ and jump to *Step 5*.

Step 9: If the maximum load $P_{\max}$ in the fatigue loading spectrum is less than the residual strength P_r , increase the fatigue cycles and jump to *Step 3*; otherwise, output the corresponding fatigue cycle n .

Step 10: According to the residual strength under different fatigue cycles, acquire the fatigue life of the joint by using linearly interpolation.

4. Fatigue tests of the double-lap three-bolt joint

In order to validate the developed PFDM, fatigue tests of the double-lap three-bolt joints were conducted, from which both the residual strength and fatigue life of the joint were obtained.

Table 4 lists the fatigue test scheme of the joint. Limited to the number of the specimens, three-level load ratios, $q=0.7, 0.8$ and 0.9 , and a stress ratio $R=-0.2$ were set. Accordingly, the maximum and minimum loads were determined, as shown in Table 4. In addition, to reduce the high temperature effect in the fatigue tests, a loading frequency of 2Hz was applied.

The fatigue tests were performed by using a MTS 880 testing machine. In the tests for fatigue life, the experiments were stopped when the specimen broke or the number of the fatigue cycle reached 10^6 . As concerning the tests for residual strength, when the number of the fatigue cycle arrived at 25%, 50% and 75% of the fatigue life under the load ratio 0.8 respectively, the experiments were stopped and then static tensile tests were conducted to obtain residual strength of the damaged joint.

Table 5 lists the fatigue life of the joint in the tests. Obviously, the fatigue life decreases with the increase of the load ratio.

Table 6 lists the residual strength of the joint obtained from the tests. The target cycle was calculated according to the fatigue life under $q=0.8$. It can be seen that the residual strength decreases as the number of fatigue cycle increases. In order to quantitatively describe the damage degree of the joint after different fatigue cycles, the retention η , defined as the residual strength divided by the static strength of the joint, was calculated. Clearly, η is still above 92% even though the number of fatigue cycle is up to 75% of the fatigue life, which indicates that the joint has an excellent capability of resisting the fatigue damage.

5. Results and discussions

Table 7 lists the residual strength predicted by the developed PFDM, which was compared with the experimental outcomes. Small numerical error less than 5.0% shows good agreement between the numerical and experimental residual strength, which give evidence of the accuracy of the proposed

PFDM in predicting the residual strength of the double-lap three-bolt joint.

Further, the residual strength obtained from the numerical simulation under the load ratio 0.8 was linearly interpolated, as depicted in Fig. 10, from which the logarithmic fatigue life of the joint 5.285 and the fatigue life 192,752 were gained. Compared with the experimental results, the corresponding errors are 2.6% and 36.8%, respectively.

Similarly, the logarithmic fatigue life of the joint under the load ratios 0.9 and 0.7 can also be obtained, which were 4.290 and 6.000 with errors of 2.3% and 2.8%, respectively. Fig. 11 presents the *S-N* curve predicted by the PFDM and the fatigue life obtained from experimental tests. The twice standard deviation bands for the fatigue life are also plotted. It can be seen that the *S-N* curve predicted by the PFDM has little difference from the experimental results, which shows that the developed PFDM can be utilized to predict fatigue properties of the double-lap three-bolt joint.

Moreover, the fatigue failure pattern of the joint was elaborated through the numerical simulation. Fig. 12 depicts the failure patterns in different layers of the middle laminate, where the gray elements represent the failed area. It can be observed that the damage concentrates around the third hole, which indicates the joint failed in tensile failure mode near the third hole. Fig. 13 illustrates the fatigue failure pattern of typical specimens in the fatigue tests. Tensile failure modes near the third hole are observed. Good consistency between the numerical and experimental failure pattern also indicates the efficiency of the proposed PFDM.

6. Conclusions

In this paper, a residual strain model was presented based on the fatigue-accumulated damage mechanism of composite materials. By combining the proposed residual strain model with a residual stiffness model, the gradually-degraded material properties due to the damage in fatigue cycles can be characterized. Besides, the parameters in the proposed residual strain model are always consistent with those in the residual stiffness model, for which only nondestructive experiments are required. Moreover, the residual strain based material degradation model for the

gradual material degradation during the fatigue cycles and a micromechanics-based material degradation rule for the sudden stiffness degradation in one fatigue cycle were utilized to develop a PFDM for a typical double-lap three-bolt joint made of T800 carbon/epoxy composites. An extended maximum strain criterion, which replaced the constant ultimate strains in the traditional maximum strain criterion with a set of gradually-degraded ultimate strain functions, was constructed for evaluating the failure of composites in the PFDM. Fatigue tests of the double-lap three-bolt joint specimens under three-level load ratios $q=0.7, 0.8, 0.9$ and a stress ratio $R=-0.2$ were performed and validated the developed PFDM. Good agreement in the numerical and experimental results verifies the efficiency of the proposed residual strain model and PFDM in accurately predicting the fatigue properties of composite bolted joints. It follows that the double-lap three-bolt joint made of T800 carbon/fiber composites has an excellent capability of resisting the fatigue damage. Fatigue failure pattern of the joint shows the damage concentrated around the third hole leads to the tensile failure of the joint.

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Figure Captions

Fig. 1 Schematic diagram of the resultant strain during the fatigue cycles.

Fig. 2 Configuration and geometrical dimensions of the joint (Unit: mm).

Fig. 3 Finite element model of the double-lap three-bolt joint.

Fig. 4 Configurations of the laminate specimens: (a) Compressive fatigue specimen; (b) Tensile fatigue specimen.

Fig. 5 q - N curves of T800 carbon/epoxy composites: (a) Longitudinal property; (b) Transverse property; (c) Shear property.

Fig. 6 Typical stiffness degradation curves during the fatigue cycles: (a) Longitudinal stiffness; (b) Transverse stiffness; (c) Shear stiffness.

Fig. 7 Longitudinal residual stiffness and strain under different load ratios: (a) Residual stiffness; (b) Residual strain.

Fig. 8 Longitudinal, transverse and shear residual stiffness and strain ($q=0.7$): (a) Residual stiffness; (b) Residual strain.

Fig. 9 Flowchart for predicting fatigue of composite bolted joints.

Fig. 10 Residual strength after different fatigue cycles ($q=0.8$, $R=-0.2$).

Fig. 11 S - N curve of the double-lap three-bolt joint.

Fig. 12 Fatigue failure pattern in different layers obtained from the PFDM ($q=0.8$, $R=-0.2$).

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Table 1 Material properties of T800 carbon/epoxy composites

Items	E_{11} (GPa)	$E_{22}=E_{33}^*$ (GPa)	$G_{12}=G_{13}^*$ (GPa)	G_{23}^* (GPa)	$\nu_{12}=\nu_{13}^*$	ν_{23}^*
Values	195	8.58	4.57	2.90	0.33	0.48
Items	X_T (MPa)	X_C (MPa)	$Y_T=Z_T^*$ (MPa)	$Y_C=Z_C^*$ (MPa)	$S_{12}=S_{13}^*$ (MPa)	S_{23}^* (MPa)
Values	3071	1747	88	271	143	143

Table 2 Specimens and test schemes

No.	Configurations and geometrical dimensions					Test schemes			
	Layups	L (mm)	W (mm)	c (mm)	t (mm)	Test type	R	f (Hz)	Wave shape
P1-YP	[0] ₁₂	140	12	63.5	2.0	Compressive fatigue	10	15	Sine wave
P2-YP	[90] ₁₂	140	25	63.5	2.0	Compressive fatigue	10	15	Sine wave
P3-LP	[45/-45] _{4s}	250	25	/	/	Tensile fatigue	0.1	5	Sine wave

Table 3 Sudden material degradation for T800 carbon/epoxy composites

Failure mode	Degradation factors								
	E_{11}	E_{22}	E_{33}	G_{12}	G_{13}	G_{23}	ν_{12}	ν_{13}	ν_{23}
Fiber tension	0.008	1	1	1	1	1	1	1	1
Fiber compression	0.044	1	1	1	1	1	1	1	1
Matrix tension	1	0	1	0	1	0	0	1	0
Matrix compression	1	0	1	0	1	0	0	1	0
Fiber-matrix shear-out	1	1	1	0.08	0.08	0.12	0.08	0.08	0.12
Interlaminar tension	1	1	0	1	0	0	1	0	0
Interlaminar compression	1	1	0	1	0	0	1	0	0

Table 4 Fatigue test scheme of the double-lap three-bolt joint

q	R	$P_{\max}/\text{kN}$	$P_{\min}/\text{kN}$	f/Hz	Wave shape
0.7	-0.2	62.1	-12.4	2	Sine wave
0.8		71.0	-14.2		
0.9		79.9	-16.0		

Table 5 Fatigue life of the double-lap three-bolt joint ($R=-0.2$)

q	$P_{\max}/\text{kN}$	N_f	$\lg(N_f)$	Means	S_{n-1}
0.7	62.1	742,399	5.871	5.835	0.034
		633,899	5.802		
		682,025	5.834		
0.8	71.0	139,014	5.143	5.149	0.110
		145,169	5.162		
		190,666	5.280		
		102,851	5.012		
0.9	79.9	7,854	3.895	4.195	0.392
		18,543	4.268		
		40,712	4.610		
		4,943	3.694		
		32,187	4.508		

Table 6 Residual strength of the double-lap three-bolt joint ($q=0.8$, $R=-0.2$)

Percentage of life/%	Target cycle	P_r/kN	Means/kN	S_{n-1}	$\eta/\%$
25	35,000	91.02	88.09	2.53	99.3
		86.63			
		86.63			
50	70,000	78.32	81.90	3.81	92.3
		83.00			
		84.39			
75	105,000	79.76	82.43	3.77	92.9
		85.09			

Table 7 Comparisons of numerical and experimental residual strength ($q=0.8$, $R=-0.2$)

Cycle numbers	Experimental results/kN	Numerical results/kN	Error/%
35,000	88.09	91.36	3.7
70,000	81.90	83.65	2.1
105,000	82.43	78.64	-4.6
250,000	/	67.73	/