The performance prediction and optimization of the fiber-reinforced composite structure with uncertain parameters

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Abstract

The fiber-reinforced composites display the random fiber orientations and uncertain material parameters because of the manufacturing error and scattering feature. For this problem, the uncertain prediction and optimization based on the transformed perturbation stochastic method, the edged-based smoothing technique and the optimization theory is presented. In this method, the non-Gaussian probability density functions and the cumulative distribution functions of multi-variables for stochastic static responses of fiber-reinforced composite structure are explicitly exhibited as the prediction result compared with the Monte Carlo solution. Unlike the direct MCs, it only needs an iteration as the FEM and has the potential in the complex construction. In order to improve the efficiency and capability to resist the mesh distortion compared with the traditional stochastic FEM, the edged-based smoothing technique is introduced into the present framework. Furthermore, the stable performance feature of the fiber-reinforced composite in the uncertain working condition is presented. Consequently, objectives of the structural stability and insensitivity criteria based on the second-order perturbation expansion are proposed, and overall uncertain conditions coupled by uncertain fiber orientation, external load, material parameters and geometry sizes are analyzed and optimized within this framework, respectively.

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To verify the effectiveness and accuracy of the uncertain analysis and optimization in this study, three examples including the composite plates, the composite shell and the composite top cover of automobile are provided.

Key words: The uncertain prediction and optimization; edged-based smoothing technique; fiber-reinforced composite structure; transformed perturbation stochastic method.

1. Introduction

Advanced composites have some superior characters, such as light weight, high strength, good anti-fatigue feature and environment resistance. As a consequence, it has been applied into aviation, aerospace, shipbuilding and automobile industry. In practical applications, material properties of the composite laminated plates and shells are usually subjected to a certain amount of scattering [1]. Furthermore, the current manufacture technology can only control the certainty of the laminated material parameters within 20% and the stochastic character of measurement procedure should also been considered. In addition, there exist the tolerance and the stochastic angle uncertainty of technology when laying fibers on each layer. Besides, geometry sizes, external loads and environment conditions are also uncertain variables which affect the normal use of laminated plates and shells. Therefore, it is reasonable to treat materials, fiber orientations, geometry sizes and external loads as stochastic parameters and formulate the uncertain analysis, especially in the precise reliable problems.

Composite structures are overdesigned due to a lack of reliable predictive methods for the complicatedly uncertain structure. As a consequence, many researches [2,3] based on the probabilistic predication are recently presented. M.F. Ngah [4] proposed the performance prediction of composite structures based on the spectral stochastic finite element method (SSFEM). Souvik Chakraborty et.al [5] proposed the polynomial correlated function expansion (PCFE) to predict stochastic

free vibration analysis using the 1200 samples compared with MCs and N. Carrere [6] proposed the efficient structural computations to analyze composite applications with parameter uncertainty. The stochastic analysis of composite structures based on material microstructure is also implemented by George Stefanou et.al [7], M.M.S. Dwaikat [8], and T.S. Mesogitis [9]. The study about predication in vibrational behavior of the long fiber-reinforced composites under uncertainty in fiber orientations is presented by K.Sepahvand [10]. However, it is still essential and vital to introduce the efficient method in prediction based on the feasible probability theory to visually and explicitly exhibit the probability density and cumulative functions of multi-variables instead of MCs which needs a large amount of computational cost. In this paper, a unified framework [11] integrating the classical perturbation stochastic finite element method (PSFEM) and the change-of-variable technique together is taken into the macroscopic structure predication of fiber-reinforced composite, and consequently the probability density and cumulative functions of multi-variables of Gaussian and non-Gaussian can be calculated. Unlike the direct MCs, it only needs an iteration as the FEM and has the potential in the complex construction, especially into the large-scale problems. These existing uncertainties can be defined as random variables whose probability density functions can be determined unambiguously based on the statistical analysis of experimental or sample data. The classic perturbation stochastic method (PSFEM) [12-14] and the spectral stochastic method (SSFEM) [15,16] are two main branches of the stochastic finite element (SFEM) [17-19]. In perturbation stochastic finite element method, the random system matrix

and loading vector are expanded by the Taylor series. Due to the disproportional increase of the computational cost, the first order approximation is the most feasible strategy [20]. Eventually, response formulas of the stochastic structure are translated into linear functions of random variables based on the first order Taylor expansion. Stochastic perturbation method has been significantly developed these years. Kaminski [21] et al. applied the generalized stochastic perturbation method to thermal stresses and deformation analysis of spatial steel structures. Recently, the dual iterative stochastic perturbation-based finite element was also proposed by Kamiński [22] to replace the linearization procedure with the iterative one. Li et al. [23] applied the mean-centered first order perturbation technique to compute the second-order statistics (mean and standard deviation) of the critical thermal buckling temperature and an efficient C0 stochastic finite element based on the first-order perturbation technique (FOPT) is proposed by Mohammad Talha et al. [24].

The high-precise determination of reliable problems for the perturbation method based on the first order Taylor expansion is still an open problem in computational mechanics. Reference [21] emphasize that the usage and precision strongly rely on the input random field types, their correlations as well as numerical technique used to solve the basic deterministic problem. It obviously indicates that there are two major approaches to obtain the high precise results: the higher orders expansion technology and the basic deterministically high-precision numerical method. To improve the accuracy of the results, Marcin Kamiński [26,27] presented the generalized n th order stochastic perturbation technique to test the procedure on simple problems with

analytical solutions sacrificing a plenty of computational cost. Though the higher order stochastic perturbation technique is employed to get the fine result, the potential defect of low accuracy in traditional FEM occurs to greatly influence and even eliminate the high accuracy which brought by the n th order technique. For the purpose of significantly improving accuracy without increasing computational cost, it is necessary to introduce the high efficient numerical method instead of the finite element method (FEM). In some practical occasions especially in complex structures, the result of FEM is seriously deteriorated by the easily-distorted mesh. In other words, it is the fatal limitation that the result heavily relies on the quality of mesh. Thus it is much better that the numerical method should have the capability to resist the mesh distortion.

Many modified numerical methods have appeared to amend the shortcomings of FEM. The conforming radial point interpolation method was introduced into the static and free vibration analysis of laminated composite plates by Zhao et al. [28]. Liu et al. [29] applied the cell-based smoothed finite element method (CS-FEM) and the generalized n th order stochastic perturbation technology into stochastic problems. This method can improve the accuracy significantly for stochastic problem. However, the determined method of CS-FEM is more suitable to the regular mesh and the accuracy also has potentials to improve further. Concerning the determined method, Liu [30] developed the edge-based smoothed finite element method (ES-FEM) formulation and made a major contribution to expand the use of the ES-FEM in various application fields, for instance, fluid-structure interaction [31], crack problems

[32,33], acoustic problems [34] etc. ES-FEM has its advantage that it is insensitive to the mesh distortion, moreover, it just uses the triangle elements to get highly accurate solutions without costing extra time, which is much more efficient than traditionally stochastic FEM. Recently, ES-FEM has been applied into laminated plates by Cui [35] and CAD/CAE integrated system by Wang et.al [36]. It has been gradually proven an accurate and reliable tool.

In macroscopic mechanical properties analysis, fiber composites are modeled as anisotropic materials. Therefore, the anisotropic material optimization [37-39] lays the foundation for composite optimization. In stiffness laminate design, the fiber orientation angles are usually chosen as the design variables. According to the macroscopic mechanical properties analysis of fiber composite, the variation of fiber orientation angle can be treated as the modification of material property matrix. Many research works have been carried out to optimize composite laminated plates to obtain a maximum stiffness or strength. Kim et al. [40] investigated the optimal stacking sequence of a symmetrical laminated plate under in-plane loading to maximize the load bearing ability. Park [41] also investigated a laminated plate under in-plane loading and gave an optimal design method. Kam and Chang [42] used the multi-start global optimization technique to optimize the ply orientation of a laminated plate for a maximum stiffness. Tauchert and Adibhalta [43] optimized the arrangement of a composite laminated plate for a maximum bending stiffness. Refs [44-46] studied the multilevel optimization of the laminated plates in which the ply orientation and thickness were designed at each level of optimization. In these methods, the material

property of the laminated plate was always regarded as deterministic and whereby the design process was formulated as a deterministic optimization problem. Thereafter, some studies on the uncertain optimization are gradually appeared. Jiang [47] proposed the uncertain optimization of composite plates based on the interval model. The probabilistic analysis and optimization of nanofiber-enhanced composite are presented by M. Rouhi et.al [48,49]. In this paper, the optimization of fiber-reinforced composite based on the transformed perturbation stochastic method and edged-based smoothing element method is proposed. In most engineering cases, the structure stability and the maximum stiffness character in complex conditions simultaneously coupled by uncertain angle orientations, external loads, material parameters and geometry sizes, are the hotspot to be studied. The objectives in the stability and sensibility criteria based on the second-order perturbation expansion are proposed and the overall uncertain condition coupled by various variables is analyzed using this strategy. The initial and optimization results are visually illustrated by the probability density function and cumulative distribution functions.

The organization of this paper is listed as follows: In Section 2, the basic equations of composite plates and shells are briefly formulated, and the strain smoothing technique and distinctly superior features of edged-based finite element method (ES-FEM) are also illustrated. The perturbation stochastic finite element method combining with mathematical theories based on the change-of-variable technique is discussed in Section 3. In section 4, the main object optimization method is considered and two optimization objectives are presented based on the perturbation

method. Three numerical examples including the composite plates, the composite shell and the composite top cover of automobile are provided to verify the effectiveness and accuracy of the uncertain analysis and optimization in Section 5. In Section 6, some vital conclusions are given.

2. Edge-based smoothing finite element method (ES-FEM) for the composite structure

2.1. Basic equation of the laminated structure

A typical shell model is shown in Fig. 1. Based on the first-order shear deformation laminated theory, the displacement field can be expressed as:

$$\begin{aligned} u &= u_0 + z\theta_y \\ v &= v_0 - z\theta_x \\ w &= w_0 \end{aligned} \quad (1)$$

Where u_0, v_0 and w_0 are the displacement of the mid-surface in the direction of x, y, z.

θ_x and θ_y are the rotation angles towards the x-axis and y-axis, respectively.

The strain vector $\boldsymbol{\varepsilon}$ can be defined in terms of the mid-plane deformations:

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{\varepsilon}_m \\ \mathbf{0}_{2 \times 1} \end{Bmatrix} + z \begin{Bmatrix} \boldsymbol{\varepsilon}_b \\ \mathbf{0}_{2 \times 1} \end{Bmatrix} + \begin{Bmatrix} \mathbf{0}_{3 \times 1} \\ \boldsymbol{\varepsilon}_s \end{Bmatrix} \quad (2)$$

Where $\boldsymbol{\varepsilon}_m$ stands for the membrane strain, $\boldsymbol{\varepsilon}_b$ is the bending strain and $\boldsymbol{\varepsilon}_s$ denotes the shear strain, which are given as the following way:

$$\boldsymbol{\varepsilon}_m = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix} \quad \boldsymbol{\varepsilon}_b = \begin{Bmatrix} \frac{\partial \theta_x}{\partial x} \\ -\frac{\partial \theta_y}{\partial y} \\ \frac{\partial \theta_y}{\partial y} - \frac{\partial \theta_x}{\partial x} \end{Bmatrix} \quad \boldsymbol{\varepsilon}_s = \begin{Bmatrix} \frac{\partial w}{\partial x} + \theta_y \\ \frac{\partial w}{\partial y} - \theta_x \end{Bmatrix} \quad (3)$$

The constitutive relationship of laminated plates and shells can be expressed as:

$$\hat{\boldsymbol{\sigma}} = \hat{\mathbf{D}} \hat{\boldsymbol{\varepsilon}} \quad (4)$$

Where

$$\hat{\boldsymbol{\sigma}} = \left\{ \hat{\boldsymbol{\sigma}}_m^T \quad \hat{\boldsymbol{\sigma}}_b^T \quad \hat{\boldsymbol{\sigma}}_s^T \right\}^T \quad \hat{\boldsymbol{\varepsilon}} = \left\{ \hat{\boldsymbol{\varepsilon}}_m^T \quad \hat{\boldsymbol{\varepsilon}}_b^T \quad \hat{\boldsymbol{\varepsilon}}_s^T \right\}^T \quad \hat{\mathbf{D}} = \begin{bmatrix} \mathbf{A} & \mathbf{B} & 0 \\ \mathbf{B} & \mathbf{D} & 0 \\ 0 & 0 & \mathbf{S} \end{bmatrix} \quad (5)$$

Where $\hat{\boldsymbol{\sigma}}_m^T = \{N_x, N_y, N_{xy}\}^T$ is the force vector in the mid-surface, $\hat{\boldsymbol{\sigma}}_s = \{Q_x, Q_y\}^T$ is the transverse shear force vector and $\hat{\boldsymbol{\sigma}}_b^T = \{M_x, M_y, M_{xy}\}^T$ is the bending moment vector. Besides, the membrane stiffness ($\mathbf{A}$), the bending stiffness ($\mathbf{D}$), the bending-extensional coupling stiffness ($\mathbf{B}$) and the transverse shear stiffness ($\mathbf{S}$) are respectively illustrated:

$$\mathbf{A} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \quad (6)$$

$$\mathbf{D} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \quad \mathbf{S} = \begin{bmatrix} \kappa_1 S_{44} & \sqrt{\kappa_1 \kappa_2} S_{45} \\ \sqrt{\kappa_1 \kappa_2} S_{45} & \kappa_2 S_{55} \end{bmatrix}$$

In which κ_1 and κ_2 denote shear correction factors.

The various elements of stiffness matrixes in Eq. (6) are defined as:

$$\begin{aligned}
A_{ij} &= \sum_{k=1}^{N_l} \bar{Q}_{ij}^{(k)} (t_k - t_{k+1}) \quad B_{ij} = \frac{1}{2} \sum_{k=1}^{N_l} \bar{Q}_{ij}^{(k)} (t_k^2 - t_{k+1}^2) \\
D_{ij} &= \frac{1}{3} \sum_{k=1}^{N_l} \bar{Q}_{ij}^{(k)} (t_k^3 - t_{k+1}^3) \quad S_{ij} = \sum_{k=1}^{N_l} \bar{Q}_{ij}^{(k)} (t_k - t_{k+1})
\end{aligned} \tag{7}$$

Where subscripts i and j of A_{ij} , B_{ij} and D_{ij} are defined as: $i, j = 1, 2, 6$, while subscripts concerning S_{ij} are defined as: $i, j = 4, 5$. t_k and t_{k+1} denote the distance from the reference mid-surface to the outer and inner surfaces of the k th layer, respectively, which are shown in Fig. (2).

2.2. Edge-based strain smoothing technique

In the present method, the problem domain is discretized into triangular elements as the means of FEM. The whole domain Ω is divided into N_e triangular elements with N_{edge} edges. To perform a smoothing operation, a smoothing domain of each edge is constructed by sequentially connecting two corner points of each edge with centroids of surrounding triangle elements.

The displacement in problem domain is interpolated using the element shape functions, which are constructed as same as the way in the linear standard FEM. The generalized mid-plane displacements vector $\mathbf{u} = \{u_{oi}, v_{oi}, w_i, \theta_x, \theta_y\}^T$ can be expressed as :

$$\mathbf{u}(\mathbf{x}) = \sum_{i=1}^3 \mathbf{N}_i(\mathbf{x}) \mathbf{d}_i \tag{8}$$

For interior edges which shown in Fig. (3), the integration domain of edge k is built by assembling two sub-domains which situated in two neighbor elements. On the other hand, the integration domain of the boundary edge is constructed by only one

sub-domain of the corresponding element, as shown in Fig. (4). Then introducing the smoothing operation, the smoothed strain of the smoothing area $\Omega_k (\Omega_{k1} + \Omega_{k2} = \Omega_k)$ is given as:

$$\bar{\varepsilon}_k = \frac{1}{A_k} \left(\int_{\Omega_{k1}} \varepsilon_{k1}(\mathbf{x}) d\Omega + \int_{\Omega_{k2}} \varepsilon_{k2}(\mathbf{x}) d\Omega \right) \quad (9)$$

Where A_k denotes the area of a smoothing domain, and ε_{k1} and ε_{k2} are compatible strains defined in elements Ω_{k1} and Ω_{k2} , respectively.

Because of the linear shape function is used, the compatible strain is a constant in each smoothing sub-domain. As a consequence, the formula can be rewritten as:

$$\begin{aligned} \bar{\varepsilon}_k &= \frac{1}{A_k} (\varepsilon_{k1} A_{k1} + \varepsilon_{k2} A_{k2}) \\ &= \frac{A_{k1}}{A_k} \mathbf{B}_m^{k1}(\mathbf{x}_{ck1}) \mathbf{d}_{k1} + \frac{A_{k2}}{A_k} \mathbf{B}_m^{k2}(\mathbf{x}_{ck2}) \mathbf{d}_{k2} = \bar{\mathbf{B}}_{mk} \mathbf{d}_k \end{aligned} \quad (10)$$

Where A_{k1} and A_{k2} are areas of two neighbor sub-domains Ω_{k1} and Ω_{k2} , $\mathbf{d}_{k1}$ and $\mathbf{d}_{k2}$ are the nodal displacement vectors of corresponding neighbor elements. $\mathbf{d}_k$ is the displacements of the nodes associated with the edge k , and $\mathbf{B}_m^{kl}(\mathbf{x}_{ckl})$ is the compatible strain matrices of the smoothing sub-domain Ω_{kl} ($I=1,2$), which are given as:

$$\mathbf{B}_m^{kl}(\mathbf{x}_{ckl}) = \begin{bmatrix} B_{m1}^{kl} & B_{m2}^{kl} & B_{m3}^{kl} \end{bmatrix}, \quad (I=1,2) \quad (11)$$

Where

$$\mathbf{B}_{mi}^{kl} = \begin{bmatrix} \frac{\partial N_i(\mathbf{x}_{ckl})}{\partial x} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial N_i(\mathbf{x}_{ckl})}{\partial y} & 0 & 0 & 0 \\ \frac{\partial N_i(\mathbf{x}_{ckl})}{\partial y} & \frac{\partial N_i(\mathbf{x}_{ckl})}{\partial x} & 0 & 0 & 0 \end{bmatrix}, \quad (i=1,2,3) \quad (12)$$

Then the Eq. (10), the smoothed membrane strain matrix $\mathbf{B}_{mi}^{kl}$ can be rewritten as:

$$\bar{\mathbf{B}}_{mk} = \frac{A_{k1}}{A_k} \mathbf{B}_m^{k1} + \frac{A_{k2}}{A_k} \mathbf{B}_m^{k2} \quad (13)$$

Similarly, the bending strain matrix $\bar{\mathbf{B}}_{bk}$ is described as follows:

$$\bar{\mathbf{B}}_{bk} = \frac{A_{k1}}{A_k} \mathbf{B}_b^{k1} + \frac{A_{k2}}{A_k} \mathbf{B}_b^{k2} \quad (14)$$

The smoothed bending strain can be expressed as:

$$\bar{\boldsymbol{\varepsilon}}_{bk} = \bar{\mathbf{B}}_{bk} \mathbf{d}_k \quad (15)$$

In order to eliminate the shear locking, the discrete shear gap (DSG) method [50] is used. In the same way, the smoothed shear strain $\bar{\boldsymbol{\varepsilon}}_{sk}$ of the integration domain Ω_k can be expressed as:

$$\bar{\boldsymbol{\varepsilon}}_{sk} = \bar{\mathbf{B}}_{sk} \mathbf{d}_k \quad (16)$$

Where $\bar{\mathbf{B}}_{sk}$ is assembled by $\mathbf{B}_s^{k1}$ and $\mathbf{B}_s^{k2}$ as follows:

$$\bar{\mathbf{B}}_{sk} = \frac{A_{k1}}{A_k} \mathbf{B}_s^{k1} + \frac{A_{k2}}{A_k} \mathbf{B}_s^{k2} \quad (17)$$

Then the smoothed generalized strain in the k th smoothing domain is illustrated according to the Eqs. (10), (15) and (16):

$$\bar{\boldsymbol{\varepsilon}} = \left\{ \bar{\boldsymbol{\varepsilon}}_{mk}^T \quad \bar{\boldsymbol{\varepsilon}}_{bk}^T \quad \bar{\boldsymbol{\varepsilon}}_{sk}^T \right\}^T = \bar{\mathbf{B}}_k \mathbf{d}_k \quad (18)$$

In which

$$\bar{\mathbf{B}}_k = \left[\bar{\mathbf{B}}_{mk}^T \quad \bar{\mathbf{B}}_{bk}^T \quad \bar{\mathbf{B}}_{sk}^T \right]^T \quad (19)$$

2.3. Distinctive features of the ES-FEM

The ES-FEM formulation is implemented based on the triangle elements and a linear FEM interpolation. Rather than the FEM which calculating the strain within each element, the smoothed strains of ES-FEM are calculated by the smoothing

integration domains formed on each edge. According to the initial researches, the favorable features can be concluded as following:

1. The stiffness of the discretized model is softer compared with FEM model, and it could get upper bound solutions with respect to FEM and exact solution.
2. The ES-FEM based on the triangular elements can produce more accurate solutions than the FEM results based on the quadrilateral elements.
3. The continuity of the trial and test functions can be relaxed, allowing us to use more methods in building shape functions.
4. The solution of ES-FEM is insensitive to the mesh distortion, and it lowers the limitation of mesh quality. It could continue to work efficiently, even when the mesh is overly distorted.
5. The discrete systems equations of ES-FEM are symmetrical and sparse, and have the same dimensions as the FEM does. Thus, it is identical compared with the FEM in terms of computational cost, whereas it produces more accurate solutions.

3. Transformed perturbation stochastic finite element method (TPSFEM) for the stochastic structure

3.1. Perturbation analysis of stochastic structure

The statically determined structure function is considered:

$$\mathbf{K}\mathbf{u} = \mathbf{F} \quad (20)$$

Where $\mathbf{K}$ stands for the stiffness matrix of the structure, $\mathbf{F}$ is the external loading vector and $\mathbf{u}$ is the static response of displacement.

In most cases, the parameters of geometry and material are imprecise and random caused by the effects of manufacturing/construction tolerances, uncertain environmental conditions, and the unpredictable loads. For purpose of getting reliable solutions, the stochastic character must be introduced into the specific formula.

Utilizing the perturbation stochastic method, the first order Taylor expansion of the random matrix $\mathbf{K}(\mathbf{x})$ and the random loading vector $\mathbf{F}(\mathbf{x})$ can be expanded as:

$$\begin{aligned}\mathbf{K} &= \mathbf{K}_0 + \sum_{i=1}^n \frac{\partial \mathbf{K}}{\partial \alpha_i} \bigg|_{\alpha = \alpha^e} (\alpha_i - \alpha_i^e) = \mathbf{K}^e + \Delta \mathbf{K} \\ \mathbf{F} &= \mathbf{F}_0 + \sum_{i=1}^n \frac{\partial \mathbf{F}}{\partial \alpha_i} \bigg|_{\alpha = \alpha^e} (\alpha_i - \alpha_i^e) = \mathbf{F}^e + \Delta \mathbf{F}\end{aligned}\quad (21)$$

Where $\alpha^e = \{\alpha_1^e, \alpha_2^e, \dots, \alpha_n^e\}$ is the expectation of respective random variables; $\mathbf{K}^e$ and $\mathbf{F}^e$ are the expectation of stiffness and load matrix. $\Delta \mathbf{K}$ and $\Delta \mathbf{F}$ are the deviation value of $\mathbf{K}^e$ and $\mathbf{F}^e$, respectively.

Substituting Eq. (21) into Eq. (20), then Eq. (20) can be translated into:

$$\mathbf{u} = (\mathbf{K}^e + \Delta \mathbf{K})^{-1} (\mathbf{F}^e + \Delta \mathbf{F}) \quad (22)$$

Based on the Neumann expansion, $(\mathbf{K}^e + \Delta \mathbf{K})^{-1}$ can be expressed as:

$$(\mathbf{K}^e + \Delta \mathbf{K})^{-1} = \mathbf{K}^e + \sum_{k=1}^{\infty} (\mathbf{K}^e)^{-1} (-\Delta \mathbf{K} (\mathbf{K}^e)^{-1})^k \quad (23)$$

Then Eq. (23) is introduced into Eq. (22). Neglecting the higher orders, the function Eq. (22) can be simplified as following:

$$\mathbf{u} \approx (\mathbf{K}^e)^{-1} \mathbf{F}^e + (\mathbf{K}^e)^{-1} \Delta \mathbf{F} - (\mathbf{K}^e)^{-1} \Delta \mathbf{K} (\mathbf{K}^e)^{-1} \mathbf{F}^e \quad (24)$$

Expanding the ΔF and ΔK , the formula of $\mathbf{u}$ can be expressed into:

$$\mathbf{u} \approx (\mathbf{K}^e)^{-1} \mathbf{F}^e + (\mathbf{K}^e)^{-1} \left(\sum_{i=1}^n \frac{\partial \mathbf{F}(x)}{\partial \alpha_i} \bigg|_{\alpha = \alpha^e} (\alpha_i - \alpha_i^e) \right) - (\mathbf{K}^e)^{-1} \left(\sum_{i=1}^n \frac{\partial \mathbf{K}(x)}{\partial \alpha_i} \bigg|_{\alpha = \alpha^e} (\alpha_i - \alpha_i^e) \right) (\mathbf{K}^e)^{-1} \mathbf{F}^e \quad (25)$$

To sum up, parameters A_i are introduced into the Eq. (25) and the simpler form is defined as:

$$\mathbf{u} = \mathbf{A}_0 + \sum_{i=1}^n \mathbf{A}_i \alpha_i \quad (26)$$

Where

$$\mathbf{A}_0 = (\mathbf{K}^e)^{-1} \mathbf{F}^e - \sum_{i=1}^n (\mathbf{K}^e)^{-1} \left(\left. \frac{\partial \mathbf{F}(\alpha)}{\partial \alpha_i} \right|_{\alpha=\alpha^e} - \left. \frac{\partial \mathbf{K}(\alpha)}{\partial \alpha_i} \right|_{\alpha=\alpha^e} (\mathbf{K}^e)^{-1} \mathbf{F}^e \right) \alpha_i^e \quad (27)$$

And

$$\mathbf{A}_i = (\mathbf{K}^e)^{-1} \left(\left. \frac{\partial \mathbf{F}(\alpha)}{\partial \alpha_i} \right|_{\alpha=\alpha^e} - \left. \frac{\partial \mathbf{K}(\alpha)}{\partial \alpha_i} \right|_{\alpha=\alpha^e} (\mathbf{K}^e)^{-1} \mathbf{F}^e \right), i=1, \dots, n \quad (28)$$

3.2. Transformed technique

Although the expectation and standard deviation are easy to obtain, unfortunately, the probability density and cumulative distribution of random static response vector are impossible to achieve unless all stochastic variables are obeyed to Gaussian random field. However, in most practical cases, the uncertain variables can't be modeled as the Gaussian distribution. On the contrary, various variables may obey to different probability distribution, especially for non-Gaussian models. In this section, a hybrid stochastic method is introduced to achieve the probability density function of non-Gaussian random response.

Eq. (26) can be rewritten in matrix way:

$$\begin{Bmatrix} u_1 \\ \vdots \\ u_j \\ \vdots \\ u_m \end{Bmatrix} = \begin{Bmatrix} A_{0,1} \\ \vdots \\ A_{0,j} \\ \vdots \\ A_{0,m} \end{Bmatrix} + \sum_{i=1}^n \begin{Bmatrix} A_{i,1} \\ \vdots \\ A_{i,j} \\ \vdots \\ A_{i,m} \end{Bmatrix} x_i \quad (29)$$

According to the algorithm of the linear equation, the j th degree freedom of $\mathbf{u}$ is

calculated:

$$u_j = A_{0,j} + \sum_{i=1}^n A_{i,j} \alpha_i, \quad j = 1, \dots, m \quad (30)$$

Step 1: Rewriting the Eq. (30) as:

$$u_j = A_{0,j} + \sum_{i=1}^n y_{i,j} \quad (31)$$

Where $y_{1,j} = A_{1,j} \alpha_1$ ($i = 1, \dots, n$). The probability density functions of $y_{1,j}$ can be expressed as:

$$f_{y_{1,j}}(y_{1,j}) = \frac{1}{|A_{1,j}|} f_{\alpha_1} \left(\frac{y_{1,j}}{A_{1,j}} \right), \quad i = 1, 2, \dots, n \quad (32)$$

Step 2: Imagine $Z_{1,j} = y_{1,j} + y_{2,j}$, Eq. (31) can be rewritten as:

$$u = A_{0,j} + Z_{1,j} + \sum_{i=3}^n y_{i,j} \quad (33)$$

Where the probability density function of the variable $Z_{1,j}$ is unknown, however, it can be calculated as:

$$\begin{aligned} f_{Z_{1,j}}(Z_{1,j}) &= \int_{y_{1,j}=\underline{b}_1}^{\bar{b}_1} f_{y_{1,j}}(Y_{1,j}) f_{y_{2,j}}(Y_{2,j}) dy_{1,j} \\ &= \frac{1}{|A_{2,j}|} \int_{\alpha_2=\underline{a}_1}^{\bar{a}_1} f_{\alpha_1}(\alpha_1) f_{\alpha_2} \left(\frac{Z_{1,j} - A_{1,j} \alpha_1}{A_{2,j}} \right) d\alpha_1 \end{aligned} \quad (34)$$

Where $\underline{b}_i$ and $\bar{b}_i$ are the lower and upper bounds of integrate domain of variable $y_{i,j}$. $\underline{a}_i$ and $\bar{a}_i$ are the lower and upper bounds of the integration domain of the random variable α_i .

Step 3: Assume $Z_{2,j} = Z_{1,j} + y_{3,j}$, Eq. (33) can be rewritten as:

$$u_j = A_{0,j} + Z_{2,j} + \sum_{i=4}^n y_{i,j} \quad (35)$$

Where the probability density function of the variable $Z_{2,j}$ is described as:

$$f_{Z_{2,j}}(Z_{2,j}) = \int_{\alpha_3=\underline{a}_3}^{\bar{a}_3} f_{\alpha_3}(\alpha_3) f_{Z_{1,j}}(Z_{2,j} - A_{3,j}\alpha_3) d\alpha_3 \quad (36)$$

Step 4: Similar to above steps, the probability density function of the specific random variable $Z_{r,j}$ ($r = 2, \dots, n-2$) can be expressed as:

$$f_{Z_{r,j}}(Z_{r,j}) = \int_{\alpha_{r+1}=\underline{a}_{r+1}}^{\bar{a}_{r+1}} f_{\alpha_{r+1}}(\alpha_{r+1}) f_{Z_{r-1,j}}(Z_{r,j} - A_{r+1,j}\alpha_{r+1}) d\alpha_{r+1} \quad (37)$$

Where $Z_{r,j} = Z_{r-1,j} + y_{r+1,j}$

Step 5: After obtaining the probability density function of the variable $Z_{n-2,j}$, the last step is to analyze the probability density function of variable u_j . The Eq. (35) can be translated as:

$$u_j = A_{0,j} + Z_{n-2,j} + y_{n,j} \quad (38)$$

And the probability density function of variable u_j is expressed as:

$$f_{u_j}(u_j) = \int_{\underline{a}}^{\bar{a}} f_{x_n}(x_n) f_{Y_{n-2,j}}(u_j - A_{0,j} - A_{n,j}x_n) dx_n \quad (39)$$

Based on the probability density function of u_j , the corresponding cumulative distribution function $F_{u_j}(u_j)$ can be expressed as:

$$F_{u_j}(u_j) = \int_{-\infty}^{u_0} f_{u_j}(t) dt \quad (40)$$

4. Optimization formulation

Considering the manufacturing uncertainty of fiber path, the linear fiber of laminated plates and shells may not obey the designable linear angles. As a result, it unavoidably leads to the unstable and random solution, and even unreliable results. Besides, many other inevitably uncertain factors severely affect common use of laminated structures, for instance geometry parameters, material parameters, external loads and even working environments. It is necessary to utilize optimization theory to

redesign angles of linear fiber paths. The multi-objective problems based on the perturbation theory are considered into this work. With regard to the various destinations, the optimization objectives are certainly different. Due to concentrate on the model stiffness feature, the displacement fields, especially some specific key-points, are studied. In order to obtain the most stable solution and eliminate the influences brought by the uncertain variables as far as possible, the stable displacement function is constructed:

$$\min \text{ object}_1 = \sum_{i=1}^n \sum_{j=1}^n \frac{\partial \mathbf{u}}{\partial \alpha_i} \frac{\partial \mathbf{u}}{\partial \alpha_j} \text{Cov}(\alpha_i \alpha_j) \quad (41)$$

Where i, j are the numbers of stochastic variable, $\mathbf{u}$ is the displacement field, and else components are listed as follows:

$$\frac{\partial \mathbf{u}}{\partial \alpha_i} = \mathbf{K}_0^{-1} \left(\frac{\partial \mathbf{F}}{\partial \alpha_i} - \mathbf{u}_0 \frac{\partial \mathbf{K}}{\partial \alpha_i} \right) \quad (42)$$

And

$$\text{Cov}(\alpha_i \alpha_j) = \int_{j=1}^n \int_{i=1}^n (\alpha_i - \alpha_i^0)(\alpha_j - \alpha_j^0) f(\alpha_i, \alpha_j) d\alpha_i d\alpha_j \quad (43)$$

Where α_i^0 and α_j^0 are the expectations of α_i and α_j , respectively.

The displacement of key-points based on the 2nd-Taylor expansion of perturbation technique is also expressed as:

$$\min \text{ object}_2 = \mathbf{u}_0 + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \frac{\partial^2 \mathbf{u}}{\partial \alpha_i \partial \alpha_j} \text{Cov}(\alpha_i \alpha_j) \quad (44)$$

Where

$$\frac{\partial^2 \mathbf{u}}{\partial \alpha_i \partial \alpha_j} = \mathbf{K}_0^{-1} \left(\frac{\partial^2 \mathbf{F}}{\partial \alpha_i \partial \alpha_j} - \mathbf{u}_0 \frac{\partial^2 \mathbf{K}}{\partial \alpha_i \partial \alpha_j} - \frac{\partial \mathbf{u}}{\partial \alpha_i} \frac{\partial \mathbf{K}}{\partial \alpha_j} - \frac{\partial \mathbf{K}}{\partial \alpha_i} \frac{\partial \mathbf{u}}{\partial \alpha_j} \right) \quad (45)$$

And $\mathbf{u}_0$ is the determined solution of displacement field.

In this paper, the main object method is introduced into the multi-objective problems. The $object_2$ denoting the level of stiffness feature is assumed as the additional objective, and $object_1$ denoting the stable extent of solution and capability to resist uncertain distortion is considered as the major objective. According to the main object method, the additional subject is considered as the constrained condition while $object_1$ is the only objective.

$$\begin{aligned} \min \quad object_1 &= \sum_{i=1}^n \sum_{j=1}^n \frac{\partial \mathbf{u}}{\partial \alpha_i} \frac{\partial \mathbf{u}}{\partial \alpha_j} Cov(\alpha_i \alpha_j) \\ object_2 &= \mathbf{u}_0 + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \frac{\partial^2 \mathbf{u}}{\partial \alpha_i \partial \alpha_j} Cov(\alpha_i \alpha_j) \leq \mathbf{U}_{Initial} \end{aligned} \quad (46)$$

Where $\mathbf{U}_{Initial}$ is the initial displacement field.

$$\begin{aligned} Boundary1: -\pi &\leq \theta_1 \leq \pi \\ Boundary2: -\pi &\leq \theta_2 \leq \pi \end{aligned} \quad (47)$$

Where θ_1 and θ_2 are optimization variables.

5. Problems analysis

5.1. Composite plate with uncertain external loads and fiber orientations

The fiber-reinforced composite plate is a typical engineering component and frequently used in the practical engineering area. Due to the complexity of working condition, in most cases, the external loads are difficult and impractical to deal as the exact values, and instead, it is more practical to define it as the interval and probability model. Therefore, the external pressure is dealt as the stochastic variable, and considered as the Gumbel distribution. Besides, the Gaussian distribution is introduced to describe the manufacturing uncertain of fiber orientation, shown in

Table 1.

A composite plate with the size $L \times L$ is showed in Fig.5 which subjected to a uniform surface pressure P on the top. Other geometry and material parameters as [35] for the problem are: $E_{11} = 3.0 \times 10^7$ psi, $E_{22} = 0.75 \times 10^6$ psi, $G_{12} = 4.5 \times 10^5$ psi, $G_{23} = 3.75 \times 10^5$ psi, $G_{13} = 4.5 \times 10^5$ psi, $\nu_{12} = \nu_{13} = \nu_{23} = 0.25$, $L = 20$ in, $t = 0.02$ in and $P = 100$ N. The number of fiber plies is 9, the thickness of odd pile is $t/10$ and others are $t/8$. The nine-layer plate is symmetrically laminated with [60/-60/60/-60/60/-60/60/-60/60]. In order to compare with the determined solution, the reference result is obtained by ABAQUS with S8R5 element using the 4961 nodes. The results are dealt as $w_c E_{22} t^3 \times 10^3 / (P \times L^4)$ and listed in the Table 2.

Then the transformed perturbation stochastic method is employed to illustrate results of multi-variables of non-Gaussian distributions. To verify the accuracy of present method, it is compared with the reference solution from Monte Carlo method. The Monte Carlo algorithm is the most stable and reliable method to get the accurate results as the reference solution by sacrificing a large amount of time (at least 100000 iterations), especially for complex problems. In distinct contrast, the transformed perturbation stochastic method just needs an iteration of algorithm and saves plenty of time. As a consequence, it has the large potential and superiority in large-scale engineering problem.

In probability theory, both the probability density function and the cumulative distribution function can be used to estimate the failure probabilities and reliabilities which act as the role to optimize stochastic structure. The probability density

functions and the cumulative distribution functions of the deflection of the key-point which calculated by MCM with 100,000 samples and the present method are both plotted in Fig.6. It can be seen from Fig. 6 that the probability density function of the deflection of key-point is non-Gaussian remarkably. This indicates the static response of the stochastic structure is non-Gaussian when some random variables are non-Gaussian variables. Therefore, it is unreliable or even wrong to approximate the responses of stochastic structures as Gaussian fields if some random variables are non-Gaussian variables. Moreover, it can be found from Fig. 6 that the probability density function and the cumulative distribution function of the deflection of key-point yielded by this strategy match the reference results obtained by MCM well. Thus, the accuracy of this strategy is effective, when used for predicting the probability density function and the cumulative distribution function of the static response of the stochastic structure whose analytical solution is hard to obtain.

Then the uncertain optimization of symmetric fiber angles is complemented using the main object method. The optimization angles of θ_1 and θ_2 are $0.8159(46^\circ 44')$ and $-0.8159(-46^\circ 44')$, showed in Table 3. Both the probability density function and the cumulative distribution function are calculated and compared with the initial results in Fig. 7. The probability density function of the optimization is very tall, thin and intensive compared with the initial probability density function. Besides, its displacement field is overall much less than the initial result, and it indicates that the optimization result performs much better than the initial result in the stiffness character and the capability of resisting the fluctuation in displacement. The result of

optimized and initial reliability indices based on displacement criteria are provided in Fig. 8. The displacement criterion acts as the boundary to calculate reliability index, and the structure would be in failure when key-point's displacement exceeds the displacement criterion. Once the displacement criteria change, the reliability indices would vary comparatively. The optimized reliability index β is much larger than the initial one, which means that the optimization structure is more stable and reliable once the displacement criteria are fixed. The results of *Object₁* and *Object₂* are listed in Table 4, respectively. Obviously, the optimization procedure largely improves the stiffness (*Object₂*) and the displacement stability (*Object₁*) of the key-point. The displacement of key-point becomes much less sensitive to the uncertain parameters of the composite structure. It is the key-feature for composite structure to keep the stable performance in uncertain working conditions (external load). It proves that the optimization procedure is effective, and can obtain a better result.

5.2. Composite shell with uncertain material parameters and fiber orientations

The composite Scordelis-Lo roof is a typical shell model and has the potential to apply into the advanced architecture. However, the material uncertainties of composite Scordelis-Lo roof can affect the performance characteristics and even lead to the failure condition. The composite material parameters are usually measured by corresponding experiments and, unavoidably, it causes the geodesic and experimental errors which may present the regularity somehow. It could be described as the probability distribution according to the experiment data. Currently, the material parameters E_1 and E_2 are treated as the stochastic variables which are both

non-Gaussian probabilistic model which can be actually determined by the experiment data. Meanwhile, the manufacturing uncertainty of fiber angle of each ply also affects the result, especially in high precise determination of composite reliable problems. And all the uncertain variables are listed in Table 5, defined as the different probability distributions.

The composite Scordelis-Lo roof is showed in Fig. 9, whose two curved edges are supported by rigid diaphragms and other two edges are free. The geometry and fiber material properties as [35] are listed: radius $R = 25$, the length of the roof $L = 50$, the thickness $t = 0.25$, Young's modules $E_{11} = 3.0 \times 10^7$, $E_{22} = 0.75 \times 10^6$, $G_{12} = 4.5 \times 10^5$, $G_{23} = 3.75 \times 10^5$, $G_{13} = 4.5 \times 10^5$, $\nu_{12} = \nu_{13} = \nu_{23} = 0.25$. The model is subjected to the self-weight $q = -90$ in the z-direction and the angle ϕ is 40° . The two-cross-ply shell is symmetrically laminated with $[0, 90]$. The reference solution is calculated by SR8 elements of ABAQUS using 14725 nodes in Table 6.

To investigate the accuracy of the present framework, the probability density functions and the cumulative distribution functions respectively obtained from the MCs with 100,000 samples and the present method are plotted in Fig. 10. It indicates that the probability density function of the deflection of node A is non-Gaussian remarkably. Furthermore, it can be seen that the probability density function and the cumulative distribution function of the node A acquiring by the present method match the reference results well. Then it also illustrates that the accuracy of the present method is sufficient when used as predicting the probability density function and the cumulative distribution function of composite shells with random variables.

The fiber orientations of optimization are listed in the Table 7. To verify the efficiency of optimization, values of optimization objectives are compared with the initial result in Table 8, and it is obvious that the results have been largely improved in the displacement field ($Object_2$) and in capability to resist the displacement fluctuation ($Object_1$). For more details about the result, the probability density functions and the cumulative distribution functions of initial and optimization are plotted in Fig. 11. The probability density function of the optimization is thinner and taller, and its displacement field is less than the initial result. It shows that the optimization result performs much better than the initial result in the stiffness character and in the capability to resist the fluctuation of displacement. Furthermore, the optimized and initial reliability indices are plotted in Fig. 12, and overall it proves that the optimization procedure is efficient and effective. The framework can perform the prediction and optimization role well for the fiber-reinforced composite structure with the uncertain multi-variables.

5.3. Composite top cover of automobile with uncertain geometry parameters and fiber orientations

In automobile industry, the fiber-reinforced composite is the important structure applied into cover panel of the automobile. In this practical example, the fiber-reinforced composite shell is designed as the top cover of the automobile showed in Fig. 13. Geometry parameters are not always exact values, especially considering the uncertainty of manufacture and tolerances. It is reasonable to build the probability model to describe this uncertainty according to the experimental data.

Besides, the lognormal distribution is selected to describe the uncertainty of the important geometry parameter t . All the probabilistic models of uncertain parameters are listed in Table. 10.

The geometry and fiber material properties as [35] are listed: thickness $t=0.0$, Young's modules $E_{11}=3.0\times 10^7$, $E_{22}=0.75\times 10^6$, $G_{12}=4.5\times 10^5$, $G_{23}=3.75\times 10^5$, $G_{13}=4.5\times 10^5$, $\nu_{12}=\nu_{13}=\nu_{23}=0.25$. The model is subjected to the concentrated force $q=-50$ in the middle area in the z -direction. The four-ply shell is symmetrically laminated with $[60/-60/60/-60]$. To correctly verify the determined solution, the result is compared with the reference solution calculated by S3 elements of ABAQUS using the same number of nodes (494), showed in Table 9.

To testify the accuracy of uncertain analysis using the present method, the probability density function and the cumulative distribution function of the key-point calculated by MCM with 100000 samples and the present method are plotted in Fig. 14. It shows that the probability density function and the cumulative distribution functions of the deflection in key-point calculated by the present method match the reference results well. As a consequence, it is a brilliant method to predict the probability density function and the cumulative distribution function instead of the low-efficient MCs.

The fiber orientation of the initial and optimization are listed in Table 11. In order to prove the efficiency of optimization, the objective values of initial and optimization results are listed in Table 12. It is obvious that Object2 (the displacement criterion) is much less than the initial displacement of key-point and Object1 (the

displacement fluctuation criterion) is also much smaller than the initial result. Furthermore, the probability density functions and the cumulative distribution functions of initial and optimization are respectively plotted in Fig. (15). The probability density function of the optimization is very taller and thinner compared with the initial probability density function, and its displacement field is much less than the initial result. It indicates that the optimization result performs much better than the initial result in the stiffness character and the capability to resist the fluctuation of displacement. Also, the initial and optimized reliability index of top cover model of the automobile based on the displacement criteria is plotted in Fig. 16, and the optimized reliability index β is much larger than the initial one. It means that the optimized structure is more reliable and less likely to fail. Overall, the optimization procedure is efficient and effective, and other objectives, such as stress, can be explored in the same way.

6. Conclusion

The integration of performance prediction and optimization for the uncertain fiber-reinforced composite structure based on the transformed perturbation method, the edged-based smoothing technique and the optimization theory is presented. It proves that it is a brilliant method to explicitly predict and optimize the probability density function and the cumulative distribution function instead of the low-efficient MCs. The characteristics of this strategy are summarized as follows:

- (1) For the purpose of significantly improving accuracy without increasing the computational cost, the edged-based smoothing technique is introduced into the

framework. Consequently, it inherits the superior characters, such as the high-efficiency and the capability to resist the mesh distortion. Moreover, it just uses the triangle elements to get highly accurate solutions without costing extra time, which is much more efficient than traditionally stochastic FEM.

(2)Based on the perturbation theory, the non-Gaussian probability density functions and the cumulative distribution functions of stochastic multi-variables (external loads, material parameters, fiber orientations and geometry size) for stochastic static responses of fiber-reinforced composite structure are explicitly exhibited. Unlike the MCs, this strategy only needs one-iteration as the FEM and has the potential in the complex construction. It is more practical and easier to apply into the reliability problem of engineering as the prediction when analytic solution is difficult to obtain.

(3)In optimization procedure, the objectives in the structural stability and maximum stiffness criteria based on the second-order perturbation expansion are proposed, and the overall uncertain condition coupled by uncertain fiber orientation, external load, material parameters and geometry sizes is analyzed and optimized using this framework.

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Fig. 1. The typical shell model and its global coordinates.

Fig. 2. Cross-sectional view of the laminated plate.

Fig. 3. The integration domain of Ω_k of internal edge k .

Fig. 4. The integration domain of boundary edge k .

(a)

● Key-point ● Corner-point

(b)

Fig. 5. The composite plate subjected to the uniform surface moment.

(a)

(b)

Fig. 6. The probability density functions and cumulative distribution functions of the deflection of the key-point calculated by MCs and TPSEM.

(a)

(b)

Fig. 7. The probability density functions and cumulative distribution functions of the deflection of the key-point are calculated based on the initial and optimization fiber orientation.

Fig. 8. The initial and optimized reliability index of composite plate based on the displacement criteria.

Fig. 9. The composite Scordelis-Lo roof model subjected to the self-weight.

(a)

(b)

Fig. 10. The probability density functions and cumulative distribution functions of the deflection of the key-point calculated by MCs and TPSEM.

(a)

(b)

Fig. 11. The probability density functions and cumulative distribution functions of the deflection of the key-point are calculated based on the initial and optimization fiber orientation.

Fig. 12. The initial and optimized reliability index of composite Scordelis-Lo roof model based on the displacement criteria.

Fig. 13. The top cover model of the automobile.

(a)

(b)

Fig. 14. The probability density functions and cumulative distribution functions of the deflection of the key-point calculated by MCs and TPSEM.

(a)

(b)

Fig. 15. The probability density functions and cumulative distribution functions of the deflection of the key-point are calculated based on the initial and optimization fiber orientation.

Fig. 16. The initial and optimized reliability index of top cover model of the automobile based on the displacement criteria.

Table 1. The stochastic variables of external load and fiber orientation.

Variables	Distribution	Stochastic parameter of the variables
P	Gumbel (evpdf)	$\mu=100, \sigma=40$
θ_1	Gaussian	$\mu=\pi/3, \sigma=0.05$
θ_2	Gaussian	$\mu=-\pi/3, \sigma=0.05$

Table 2. The determination results compared with the Ref.

Point	81 nodes (%)	121 nodes (%)	169 nodes (%)	Ref.(mm)
Key-Point	2.639	2.6571	2.6674	2.694
Error	-2.04	-1.36	-0.987	/

Table 3. The fiber orientation results of the initial and optimization.

Condition	Initial	Optimization
layer	9	9
θ_1	$\pi/3$	0.8159
θ_2	$-\pi/3$	-0.8159

Table 4. The objective values of the initial and optimization.

Condition	Initial	Optimization
<i>Object₁</i>	0.00504502	6.4789e-5
<i>Object₂</i>	2.6387	2.25047

Table 5. The stochastic variables of material parameters and fiber orientation.

Variables	Distribution	Stochastic parameter of the variables
E ₁	Gumbel (evpdf)	$\mu=3.0e7, \sigma=1e6$
E ₂	Gumbel (evpdf)	$\mu=0.75e6, \sigma=0.5e5$
θ_1	Gaussian	$\mu=\pi/2, \sigma=0.02$
θ_2	Gaussian	$\mu=0, \sigma=0.02$

Table 6. The determination results compared with the Ref.

Point	961 nodes	Ref.(14725nodes)
Key-Point	25.42	28.20
Percent (%)	90.14%	/

Table 7. The fiber orientation results of the initial and optimization.

Condition	Initial	Optimization
layer	2	2
θ_1	$\pi/2$	-0.062831
θ_2	0	-0.125664

Table 8. The objective values of the initial and optimization.

Condition	Initial	Optimization
<i>Object₁</i>	0.37667	0.0816696
<i>Object₂</i>	22.9367	16.761779

Table 9. The determination results compared with the Ref.

Point	961 nodes	Ref.
Key-Point	-1.0067e-2	-1.0193e-2
Error (%)	-1.23%	/

Table 10. The stochastic variables of shell thickness and fiber orientation.

Variables	Distribution	Stochastic parameter of the variables
t	lognormal	$\mu=1.884$, $\sigma=0.06579$
θ_1	Gaussian	$\mu=\pi/3$, $\sigma=0.05$
θ_2	Gaussian	$\mu=-\pi/3$, $\sigma=0.05$

Table 11. The objective values of the initial and optimization.

Condition	Initial	Optimization
<i>Object₁</i>	1.45592e-004	6.708489e-5
<i>Object₂</i>	0.01027682	0.00883552

Table 12. The fiber orientation results of the initial and optimization.

Condition	Initial	Optimization
Layer	4	4
θ_1	$\pi/3$	$\pi/2$
θ_2	$-\pi/3$	$-\pi/2$