Highly anisotropic corrugated laminates deflection under uniform pressure

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Abstract

Corrugated laminates with large corrugation amplitude have highly anisotropic stiffness properties. Here, the bending response of rectangular corrugated laminates to homogeneous transverse pressure is investigated. The center-point deflection of thin laminates with high corrugation amplitude is controlled by the higher bending-stiffness value alone. Simple formulas for calculating the center-point deflection are developed from beam formulas by replacing beam bending stiffness with plate bending stiffness and the line load with pressure. A finite-element model is used to verify the results and to illustrate the peculiar load-carrying action of the highly anisotropic plate. Further an error analysis is presented to indicate for which stiffness and aspect ratios the beam-equation modeling approach is valid and can be used for preliminary design.

Keywords: Corrugated laminates, highly anisotropic plates, flexible skins

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1. Introduction

Morphing wing solutions are a promising approach to improve the flight performance of an aircraft using lower weight than conventional airplanes. To achieve an adaptive behavior of the wing, elements are needed that have a high compliance in chord direction while they behave stiff in span direction [1, 2, 3]. The anisotropy is needed to guarantee that a good compliance in chord direction is achieved with applying only low actuation forces. On the other hand the structure still needs to sustain structural and aerodynamic loads which requires high bending stiffness about the chord direction. Corrugated structures are ideal candidates to fulfill these requirements [4, 5, 6]. They provide the required stiffness behavior and they can achieve high global axial strains since the local strains still remain relatively small thanks to the curved structure [7]. Flexible skins need to be designed in a way that they can withstand the external pressure during flight and too high out-of-plane deflections must be prevented [8, 9]. Beside the anisotropy between the compliance in and the bending stiffness about chord direction, there is also an anisotropy between the bending stiffness about span and chord direction especially if the corrugation pattern have high amplitudes. This anisotropy also must be analyzed and considered to achieve optimal designs. To the authors' knowledge this has not been addressed in other studies so far. This also rises the question where and how the corrugated panels must be supported in order to keep the deflection due to aerodynamic pressure in a feasible range. Finite element analyses of corrugated plates made from anisotropic material as suggested by other authors [10] can be very costly if the structure contains many periods and consists of many layers. Various models exist to analyze

the equivalent stiffness of a corrugated laminate [11, 12, 6, 13, 14]. In the present paper we combine the model suggested by Kress and Winkler [13] with a plate model for rectangular anisotropic plates under uniform pressure [15] to estimate the maximum deflection due to aerodynamic pressure which is crucial for the design of corrugated laminates for morphing wings. However, a closed-form solution of the plate model could only be found for the simply supported load case. For clamped edges on the other hand, a new approach is needed that allows a fast and easy use for preliminary design. The objective of the present work is to find methods that allow preliminary design of corrugated laminates under uniform pressure for morphing wings. We seek for procedures that are easy to apply and hence allow a fast first design estimation. In the following section we present the design problem, then an approach for simply supported plates is presented. Then we show how the solution for plates converges to beam theory for highly anisotropic plates. In the last section the suggested modeling approach is validated using commercial finite element method.

2. Theory and investigation

2.1. *Corrugated laminate as flexible skin*

The geometric effects of corrugation induce highly anisotropic stiffness properties to corrugated sheets. For instance, the membrane stiffness along the corrugations is much smaller than that along the direction transverse to them. The corrugated sheet is thus a candidate for a flexible skin as part of a morphing-wing design for aircraft if the corrugations are along the wing chord direction. Then, the airfoil shape can change thanks to the high com-

pliance along the chord direction whilst the same corrugated sheet makes a significant contribution to the required structural stiffness along the span direction. Fig. 1 indicates the principle of including a flexible skin into an

Figure 1: Wing with flexible skin

airplane-wing design, defines the span and the chord directions, and indicates the orientation of the corrugated sheet. Even though the corrugation geometry induces much higher degrees of anisotropy than fiber-reinforced composite materials possess, the use of the latter for a base sheet material is recommended as the limit strain of composites is significantly higher than that of metals. The higher limit strain allows for larger changes of length along the corrugations.

2.2. Deflection due to aerodynamic pressure

The aspect which seems to not yet have been addressed in the literature is the deflection, or out-of-plane displacement, of the flexible skin due to aerodynamic pressure. For simplicity the flexible skin is considered with a rectangular shape with side lengths a and b and the aerodynamic pressure p_0 is assumed to be homogeneous. A measure of the skin deformation is the maximum deflection w_{max} . As indicated in Fig. 1, it occurs at its center $x = a/2$ and $y = b/2$, where x and y are the coordinates along the span and chord directions, respectively. The deflection depends on the pressure, the skin size, the boundary conditions, and the bending stiffness values. The bending stiffness values in turn depend on the sheet material and thickness and on the corrugation shape. The corrugation shifts material fibers away from the

neutral plane which is also the basic principle of sandwich constructions [16]. It is therefore anticipated that shear deformation may contribute significantly to deflection.

2.3. Corrugated laminate deflection analysis

The combination of simulation tools used for calculating the deflections of rectangular plates made of corrugated laminates is depicted in Fig. 2. The exact solution to the problem of calculating the bending deflection w_b of rectangular anisotropic plate, simply supported at all four edges, under homogeneous pressure is found in the textbook by Jones [15]:

$$w_b = \frac{16p_0}{\pi^6} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{mn} \frac{\sin(m\pi \frac{x}{a}) \sin(n\pi \frac{y}{b})}{D_{11} \left(\frac{m}{a}\right)^4 + 2(D_{12} + 2D_{66}) \left(\frac{mn}{ab}\right)^2 + D_{22} \left(\frac{n}{b}\right)^4}, \quad (1)$$

where D_{11} and D_{22} are bending stiffness values with respect to the x and y directions of the plate, D_{12} reflects the stiffening effect of the constrained Poisson-ratio effect, and D_{66} is torsional stiffness. It holds for laminates without coupling between bending and twist, or $D_{16} = D_{26} = 0$, so that variables can be separated into functions of plate coordinate x alone times functions of y alone [17]. The textbook by Zenkert [16] provides, in addition, an exact solution for the transverse-shear deflection w_s :

$$w_s = \frac{16p_0}{\pi^4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{mn} \frac{\sin(m\pi \frac{x}{a}) \sin(n\pi \frac{y}{b})}{K_{44} \left(\frac{m}{a}\right)^2 + K_{55} \left(\frac{n}{b}\right)^2}, \quad (2)$$

where K_{44} and K_{55} are transverse shear stiffness values with respect to the x and the y axes of the substitute homogeneous plate. For deflections being

much smaller than plate in-plane dimensions, the concept of partial deflections known from sandwich theory [16] may be used :

$$w = w_b + w_s. \quad (3)$$

The formulas (1) and (2) require the data input indicated at the right of Fig. 2. The bending and the transverse-shear stiffness values of the corrugated sheet are marked with a tilde. They characterize the structural load response

Figure 2: Corrugated Laminate deflection analysis tools

of a corrugated plate, which on a large scale behaves like a homogeneous plate having the same stiffness values. The substitute-plate stiffness values of a corrugated laminate are calculated with the formulas given by Kress and Winkler [13], for which a FORTRAN program CORLAM (Corrugated Laminates) exists at our Lab *CMAS*. This model considers corrugation shapes consisting of circular sections as Fig. 3 indicates. The shape shown in the

Figure 3: Maximum-amplitude corrugation shape

figure corresponds with the maximum amplitude c for a given unit-cell length P , see Fig. 4 for definition of c and P . The amplitude is constrained by the fact that the laminate may not penetrate itself; it gives the highest degree of anisotropy and, at the same time, smoother aerodynamic surfaces than lower amplitudes, for instance the amplitude corresponding to semi-circles. The data required by the model include unit-cell length and said corrugation

amplitude and the stiffness properties of the base laminate.

The stiffness properties of the base laminate are contained in the ABD matrix. The ABD matrix connects mid-plane strains ϵ^0 and bending curvature κ with line forces $\mathbf{N}$ and line moments $\mathbf{M}$. It consists of the membrane stiffness matrix $\mathbf{A}$, the bending stiffness matrix $\mathbf{D}$, and the coupling stiffness matrix $\mathbf{B}$. In the case of a symmetric laminate the coupling stiffness matrix $\mathbf{B}$ vanishes, and for a cross-ply laminate coupling exists neither between extension and in-plane shear nor between bending and twist, which is reflected by the sub-matrices $\mathbf{A}$ and $\mathbf{D}$ having the following population structure:

$$\mathbf{A} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \quad (4)$$

The ABD matrix of the base laminate is calculated with the classical theory of laminates plates which is also found in the textbook by Jones [15]. A proprietary FORTRAN program *CAP* (Composites Analysis Program) for evaluating the classical theory of laminates plates is available at our lab *CMAS*.

The transverse-shear stiffness values K_{44} and K_{55} connect plate shear angles γ_{xz} and γ_{yz} with transverse line forces T_x and T_y , respectively. As shear stress distributions must satisfy zero-traction natural boundary conditions at the laminate surfaces and are stack-up sensitive, the calculation of K_{44} and K_{55} according to first-order shear theory can be quite inaccurate and a more refined theory has been provided by Rolfes and Rohwer [18]. We do not yet have developed a method for exact calculation of the corrugated laminate homogenized substitute plate transverse-shear stiffness values $\tilde{K}_{44}$ and $\tilde{K}_{55}$.

Our estimate bases on the rough model illustrated with Fig. 4, which com-

Figure 4: Transverse shear stiffness model

bines the contributions of the base laminate in-plane and out-of-plane shear deformations: which suggests two straight profiles per corrugation period. The homogenized transverse stiffness $\tilde{K}_{44}$ is then estimated with:

$$\tilde{K}_{44} = 2 \frac{t_{equ}}{P} A_{66} + \frac{L_{equ}}{P} K_{44} , \quad (5)$$

where

$$t_{equ} = 4R \int_{\psi=0}^{\psi_0} \sin\psi d\psi \quad (6)$$

and

$$L_{equ} = 2R \left(\left| \int_{\psi=0}^{\pi/2} \cos\psi d\psi \right| + \left| \int_{\psi=\pi/2}^{\psi_0} \cos\psi d\psi \right| \right) . \quad (7)$$

It is taken from [13] that radius R and circular-sections angle ψ_0 depend on specified periodic length P and corrugation amplitude c ,

$$c > \frac{P}{4} \Rightarrow R = \frac{16c^2 + P^2}{32c} \quad \psi_0 = \arccos\left(\frac{P}{4R}\right) + \frac{\pi}{2}. \quad (8)$$

2.4. Deflection estimate reference situation

It is important to note that this study is not aiming at designing a structure for given concrete loads and requirements. However, the data are chosen to be similar in scale to what would be expected when considering the wing of commercial aircraft. Typical values for the aerodynamic pressure for large aircraft (Boeing 747 / Airbus A380) are about 700 kp/m^2 or 0.007 MPa . The parameter study considers a slightly higher value of 0.01 MPa to be typical.

The size of the flexible skin embedded in a typical commercial aircraft wing is assumed to be 1 m along the chord and 20 m along the span directions. As an arbitrary choice, it is aimed to limit the deflection due to aerodynamic pressure to 1 mm . The geometry of the corrugation is shown in Fig. 4. The base sheet is a symmetric cross-ply laminate $[0, 90]_s$ whose layers with thickness $h = 0.125\text{ mm}$ consist of carbon-fiber-reinforced plastics (CFRP) with high-strength fibers *T300* and an epoxy matrix. The engineering constants of the transverse isotropic composite material are given in Table 1, where the subscripts L and T refer to properties along the fiber and transverse directions. The numerical values of the base-laminate ABD matrix are listed in

Table 1: T300/Epoxy elastic constants at $v_f = 60\%$.

E_L	135000 [MPa]	E_T	10000 [MPa]
G_L	5000 [MPa]	G_T	3846 [MPa]
ν_L	0.27	ν_T	0.3

Table 2. The corrugation pattern consists of circular segments as illustrated in Figs. 3 and 4. The periodic length is $P = 100\text{ mm}$ and the corrugation amplitude $c = 93\text{ mm}$ is a limiting value above which the corrugated sheet would penetrate itself, and the circular-section angle is $\psi_0 = 164^\circ$. The numerical evaluation of (6) and (7) gives the equivalent shear thickness $t_{equ} = 120\text{ mm}$ and shear line length $L_{equ} = 15.2\text{ mm}$. The bending stiffness matrix $\tilde{\mathbf{D}}$ of the homogenized substitute plate, representing the structural properties of the corrugated laminate on a large scale, is given in Table 3. As a consequence

Table 2: Base sheet membrane $[N/mm]$ and bending stiffness $[Nmm]$

A_{11}	$=$	$3.6447 \cdot 10^4$	D_{11}	$=$	$1.2502 \cdot 10^3$
A_{12}	$=$	$1.3573 \cdot 10^3$	D_{12}	$=$	$2.8278 \cdot 10^1$
A_{22}	$=$	$3.6447 \cdot 10^4$	D_{22}	$=$	$2.6838 \cdot 10^2$
A_{66}	$=$	$2.5000 \cdot 10^3$	D_{66}	$=$	$5.2083 \cdot 10^1$

Table 3: Corrugated laminate membrane $[N/mm]$ and bending stiffness $[Nmm]$ values

$\tilde{A}_{11}$	$=$	$1.9057 \cdot 10^{+5}$	$\tilde{D}_{11}$	$=$	$7.1374 \cdot 10^8$
$\tilde{A}_{12}$	$=$	$5.0967 \cdot 10^{-4}$	$\tilde{D}_{12}$	$=$	$5.4006 \cdot 10^0$
$\tilde{A}_{22}$	$=$	$1.3686 \cdot 10^{-2}$	$\tilde{D}_{22}$	$=$	$5.1256 \cdot 10^1$
$\tilde{A}_{66}$	$=$	$4.7746 \cdot 10^{+2}$	$\tilde{D}_{66}$	$=$	$9.9472 \cdot 10^0$

of the here considered corrugation geometry the bending stiffness values $\tilde{D}_{11}$ and $\tilde{D}_{22}$ differ from each other by seven orders of magnitude. Estimated transverse shear stiffness of the base K_{ij} and the corrugated $\tilde{K}_{ij}$ laminates are listed in Table 4, where the shear stiffness $\tilde{K}_{55}$ is estimated to be very

Table 4: Transverse shear-stiffness values [N/mm]

K_{44}	$=$	$2.0000 \cdot 10^3$	$\tilde{K}_{44}$	$=$	$1.2289 \cdot 10^{-4}$
K_{55}	$=$	$2.0000 \cdot 10^3$	$\tilde{K}_{55}$	$=$	$1.0000 \cdot 10^{-5}$

much smaller than $\tilde{K}_{44}$. The data give a center-point deflection of 29.320 meters where the shear contribution is only 41 millimeters. The bending deflection is so unrealistically high because the linear theory on which formula (1) is based only describes small deformations correctly. Instead of invoking nonlinear theory, which couples bending strains with membrane strains, deflections are reduced by introducing additional supports. The supports are intended to reduce the deformations to small values; then, they can be described with linear theory.

2.5. Corrugated laminates support rails

Fig. 5 suggests a design principle for fixing the corrugated laminate with

Figure 5: Flexible-skin support-design principle

respect to the vertical direction but allowing movement along the chord direction. The principle is similar to that of a curtain rail. The supporting

rails indicated in Fig. 5 subdivide the length of the rectangular plate into several smaller sections, as can be seen in Figs. 7 and 10. For a realistic design of flexible skins it has to be considered that the corrugated skin might curve which could be achieved with more sophisticated kinematics. However, such a design proposal is out of scope of this paper. Fig. 6 indicates that the

Figure 6: Length a influence on maximum deflection

deflection, calculated after linear theory, decreases quickly with decreasing plate length and that shear contribution to total deflection becomes significant at smaller segment lengths. At a segment length of $1.25m$ the maximum deflection is $w = 0.604mm$ with a shear contribution of $w_s = 0.159mm$. Even though the supporting rails design principle shown in Fig. 5 does not provide clamping in the sense that it can suppress a rotation about the chord direction, the corrugated laminate will not rotate because of symmetry, which is illustrated with Fig. 7. Provided that the pressure distribution is ho-

Figure 7: Symmetries of loading and support suggest clamped-edge plate modeling. At the left and right edge a double rail guarantees clamped conditions also for the most outer panels.

mogeneous, the rails as well as the segment centers lie on symmetry lines. Therefore, the plate segments deform as if they were clamped at the supports. This is in contrast to the previous modeling assumption that all four edges are simply supported, and deflections will be much smaller than solutions (1) and (2) would predict. However, calculation formulas for clamped

plates are much more complex than the one for simply supported plates (1). It will be shown in section 2.7 that the clamping at the segmenting supports can be taken into account if the bending stiffness anisotropy is high enough to justify prediction of centerline deflection by beam theory. At the two edges in span direction the symmetry condition, suppressing rotation about the chord direction, does not exist. In order to prevent rotation there, we suggest double rails, or a pair of rails with small distance away from each other, which are indicated in Figs. 7 and 10.

2.6. *Corrugated laminates load-carrying action*

It is suspected that the contribution of the plate stiffness values D_{12} , D_{22} , and D_{66} is negligibly small if compared to the very large value of D_{11} , see Table 3. The chord-wise length of segment with length $a = 1.0\text{ m}$ is varied to investigate the load-carrying action of the smaller stiffness values. Fig. 8 shows that a load-carrying action is achieved by the lower stiffness values $\tilde{D}_{22}$

Figure 8: Maximum deflection versus chord length b at span length $a = 1.25\text{ m}$

and $\tilde{K}_{55}$ only at very small chord-wise plate extensions $b < 50\text{ mm}$. At larger chord-length values the large stiffness values $\tilde{D}_{11}$ and $\tilde{K}_{44}$ alone determine the deflection: The plate behaves like a beam. The result indicates that the plate center point deflection at larger values of b may be well approximated by thin-beam theory equations.

2.7. Plate center-line deflection and beam theory

Thin beam theory has developed equations for calculating center deflections of simply-supported as well as clamped beams under homogeneous line load q [19]:

$$w_{ss} = \frac{5}{384} q \frac{l^4}{EI}; \quad w_{cl} = \frac{1}{384} q \frac{l^4}{EI} \quad (9)$$

The formulas indicate that the maximum deflection is by a factor of 5 smaller for the clamped beam than for the simply supported beam. If the beam absolute bending stiffness EI is replaced with the bending stiffness per unit length D , and the line load q with the pressure p_0 , the same formulas can be used to describe the limiting cases of plate geometries:

$$a \rightarrow \infty : \quad w_{ss} = \frac{5}{384} p_0 \frac{b^4}{D_{22}}; \quad w_{cl} = \frac{1}{384} p_0 \frac{b^4}{D_{22}} \quad (10)$$

$$b \rightarrow \infty : \quad w_{ss} = \frac{5}{384} p_0 \frac{a^4}{D_{11}}; \quad w_{cl} = \frac{1}{384} p_0 \frac{a^4}{D_{11}} \quad (11)$$

According to the concept of partial deflections expressed with (3) shear-strain contributions w_s

$$a \rightarrow \infty : \quad w_s = \frac{1}{8} p_0 \frac{b^2}{K_{55}} \quad (12)$$

and

$$b \rightarrow \infty : \quad w_s = \frac{1}{8} p_0 \frac{a^2}{K_{44}} \quad (13)$$

can be added to the maximum deflections (10) or (11), respectively, as the shear angle distribution does not depend on end support conditions.

We can show that equation (1) at $x = a/2$ and $y = b/2$ describing the plate deformation in the middle of the plate converges to the beam equation (11) if either the D_{22} , D_{12} and D_{66} go to zero or if b goes to infinity. In that case

the equation simplifies to

$$w_b = \frac{16p_0}{\pi^6} \frac{a^4}{D_{11}} \sum_{m=1,3,5,\dots}^{\infty} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{m^5 n} \sin\left(\frac{m\pi}{2}\right) \sin\left(\frac{n\pi}{2}\right). \quad (14)$$

Using numerical tools we can show that

$$\frac{16}{\pi^6} \sum_{m=1,3,5,\dots}^{\infty} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{m^5 n} \sin\left(\frac{m\pi}{2}\right) \sin\left(\frac{n\pi}{2}\right) = \frac{5}{384} \quad (15)$$

meaning that we can expect accurate results using beam theory if either D_{11}/D_{22} is very big or $b \rightarrow \infty$.

The same can be done for the shear deflection. If $b \rightarrow \infty$ or $K_{55} \rightarrow 0$ for $x = a/2$ and $b = y/2$ equation (2) simplifies to:

$$w_s = \frac{16p_0}{\pi^4} \frac{a^2}{K_{44}} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{m^3 n} \sin\left(\frac{m\pi}{2}\right) \sin\left(\frac{n\pi}{2}\right). \quad (16)$$

Using numerical tools we can show that

$$\frac{16p_0}{\pi^4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{m^3 n} \sin\left(\frac{m\pi}{2}\right) \sin\left(\frac{n\pi}{2}\right) = \frac{1}{8} \quad (17)$$

holds.

A convergence study, based on a simply-supported plate with size $a = 2.0 \text{ m}$ and $b = 1.0 \text{ m}$, shows the relation between the number of series terms in (1) and accuracy and confirms the result. The accuracy measure shown in Fig. 9 is defined as the relative deviation between the plate-center deflection and

Figure 9: Plate equation convergence

w_{ss} in (11),

$$e = \frac{|w_{plate} - w_{beam}|}{w_{beam}}, \quad (18)$$

so that the plots also show that the plate and the beam formulas give, in the limit of a very large number of series terms, the similar result of $w_{max} = 3.3260 \text{ mm}$ and $w_{max} = 3.3258 \text{ mm}$, respectively. The error depends on how exactly the assumptions $D_{11}/D_{22} \rightarrow \infty$ or $b \rightarrow \infty$ are fulfilled. A detailed analysis is presented in section 2.9. As we could show that it is eligible to use the beam model since we have a very high D_{11}/D_{22} -ratio in the presented problem, we now use equation (11) to analyze the clamped load case described in Fig. 7 where, to the authors' knowledge, a closed-form solution for the plate does not exist. The maximum deflection of a clamped plate of the same dimensions $a = 2.00 \text{ m}$ and $b = 1.0 \text{ m}$ is only $w_{max} = 0.99065 \text{ mm}$. Fig. 10 indicates the supporting rail arrangement

Figure 10: Supporting rails lattice limiting deflections to $w = 0.95 \text{ mm}$

where the flexible skin of a length of 20 m is subdivided into ten segments.

2.8. FEM structural analysis verification

We verify our preliminary design result, illustrated in Fig. 10, with a finite-element-method analysis. The shell element has four nodes with six degrees-of-freedom on each. It is capable of large-deflection analysis, where it considers coupling of membrane and transverse deformations and loads, and is fully integrated with additional incompatible deformation modes. The stiffness matrices $\mathbf{A}$ and $\mathbf{D}$ are imparted directly and independently. The element size is specified with 2 mm so that the mesh consists of 1000×500 finite elements. All degrees-of-freedom are specified to zero on all boundaries to simulate a plate clamped all around. The fine mesh allows mapping of the

deformation details revealed by the plots in Fig. 11. All plots illustrate that

Figure 11: Plate deflections after FEM

the stiffness term D_{11} resists the pressure, see the deflection line from the bottom plot, and hence that applying the beam theory is correct. The plot to the left shows how steeply the deflection increases close to the edges $\pm b/2$ and reveals that the maximum deflection ($w_{max} = 1.035 \text{ mm}$) occurs there and not at the plate center ($w_{cen} = 0.990645 \text{ mm}$)! The plate-center deflection agrees very well with the beam-theory prediction of $w_d = 0.99065 \text{ mm}$: the relative deviation is only $e \approx 1 \cdot 10^{-5}$.

2.9. Error estimation for design procedure

For the preliminary design of clamped highly anisotropic plates under pressure, we need to know in which cases the error between the plate and the beam theory is in an acceptable range. Further, we need to ask the question in which cases the assumption of linear theory are valid. The total relative error is given by the sum of the error from the bending and the shear:

$$e_{tot} = e_b + e_s \quad (19)$$

where e_b and e_s can be calculated using equation (18).

To calculate the relative error due to bending between the plate and the beam theory for simply supported boundary conditions, we can introduce a stiffness ratio $\alpha = D_{11}/D_{22}$ and an aspect ratio $\beta = a/b$ and evaluate

equation (1) at $x = a/2$ and $y = b/2$:

$$w_b = \frac{16p_0}{\pi^6} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{mn} \cdot \frac{a^4}{D_{11}} \cdot \frac{\sin(\frac{m\pi}{2})\sin(\frac{n\pi}{2})}{(m)^4 + \frac{2}{\alpha}(\gamma + 2\delta)(mn\beta)^2 + \frac{(n\beta)^4}{\alpha}} \quad (20)$$

where $\gamma = D_{12}/D_{22}$ and $\delta = D_{66}/D_{22}$. We assume $\gamma = 1/10$ and $\delta = 1/5$ which is an average values of the corrugated laminate with different amplitudes. Hence $(\gamma + 2\delta)$ equals 0.5.

By substituting equation (20) and (11) in (18) we can estimate the error by calculating the deflection with the beam equation as a function of the stiffness and the aspect ratio:

$$e_b = \frac{6144}{5\pi^6} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\frac{1}{mn}\sin(\frac{m\pi}{2})\sin(\frac{n\pi}{2})}{m^4 + \frac{(mn\beta)^2}{\alpha} + \frac{1}{\alpha}(n\beta)^4} - 1 \quad (21)$$

Fig. 12 shows the relative error for different stiffness and aspect ratios. As expected, the relative error between the plate and the beam theory is very small for very high stiffness ratios. We assume that the error between the two models is in the same range for clamped boundary conditions where we do not have a closed solution for the deflection of the plate.

Figure 12: Relative error due to bending between beam and plate model as a function of stiffness and aspect ratio

The same can be done to calculate the error due to shear. This error depends on the shear stiffness and aspect ratio. We introduce a ratio between

the shear stiffnesses $\eta = K_{44}/K_{55}$ and we can write equation (2) at $x = a/2$ and $y = b/2$ as:

$$w_s = p_0 \frac{a^2}{K_{44}} \frac{16}{\pi^4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{mn} \frac{\sin(\frac{m\pi}{2}) \sin(\frac{n\pi}{2})}{m^2 + \frac{1}{\eta} (n\beta)^2} \quad (22)$$

And for the error we find:

$$e_s = \frac{128}{\pi^4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{mn} \frac{\sin(\frac{m\pi}{2}) \sin(\frac{n\pi}{2})}{m^2 + \frac{1}{\eta} (n\beta)^2} - 1 \quad (23)$$

Fig. 13 shows the relative error due to shear as a function of the stiffness ratio η and the aspect ratio. The error is high for high aspect ratios and low stiffness ratios. For stiffness ratios $\eta > 10^3$ the relative error between the plate and the beam model is smaller than 0.5%.

Figure 13: Relative error due to shear between beam and plate model as a function of stiffness and aspect ratio

Further, we need to consider that the models are only valid for small deflections since they assume linear behavior. Fig. 14 shows the deflection to length ratio w/a for different stiffness values D_{11} and sample length a . The authors suggest that the linear theories are valid below a w/a ratio of 5 %, everything above 5 % is red colored in the figure.

Figure 14: Ratio between deflection w and sample length a indicating where the linear theory is applicable

Figures 12, 13 and 14 can directly be used to estimate whether the beam equations are valid for a preliminary design of specific highly anisotropic

laminates as they indicate the range where the model can be used with blue and the range where the model might predict a misleading result with red.

3. Conclusion and outlook

Our preliminary sizing of a plate under homogeneous pressure shows that a flexible skin consisting of a high-amplitude corrugated thin laminate can be a feasible design solution for morphing wings. The analysis method relies on replacing the corrugated structure with a substitute homogeneous plate whose stiffness parameters can be used in simple formulas available for plates. The stiffness parameters are anisotropic to such a high degree that the deformation response to homogeneous pressure is cylindrical bending for most part of the plate except for the edge effects at chord edges $\pm b/2$. This allows for calculating the center-line and the maximum deflections with beam-theory formulas to a very high degree of accuracy.

The remaining question, whether local buckling may appear as a consequence of the thinness of the laminate and how the buckling, if it should occur, would influence deformation at a large scale and strength, we will address with future work.

The here considered corrugation geometry consisting of circular sections has recently be shown by us to cause lower interlaminar stressing than other corrugation shapes [20]. It also helps achieving very high degree of anisotropy if the corrugation amplitude is as high as indicated in Fig. 3. A method for fabricating such a design will be investigated in future work.

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