Heterogeneous Fracture Mechanics for Multi-Defect Analysis

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Abstract

Discrete flaw computational damage mechanics is often used to certify the life of composite aircraft parts made from continuous fiber reinforced composite laminates. Other modes and combinations of damage that define changes in the defect state and the attendant loss of integrity and increased risk of operation of composite structures throughout the life of a component do not yet have a corresponding general representation. The present paper postulates a more general “heterogeneous fracture mechanics” concept based on an experimental observable and a multiphysics analysis used to interpret such observations, for multi-defect representations. Experimental and analytical foundations for the new concept are presented. The method can provide a foundation for the conception and design of durable and damage tolerant heterogeneous material systems for a myriad of applications, including vehicular and civil structures, medical prosthetics, and energy conversion and storage devices.

1. Introduction

Classical fracture mechanics (CFM) was created as a conceptual physical understanding and modeling approach to represent the local singularity at a crack tip in terms of physical observables (state variables), and thereby to enable the prediction of remaining strength and life of materials and structures. With various additions of elastic-plastic response, anisotropy, and non-local mechanics it has become a universal foundation for design and the basis for countless codes the world over. It has also become the industry standard for model-based predictive monitoring and supervisory control in which real time information (including uncertainty) is used to alter operational variables and greatly increase system efficiency, output, and reliability with greatly reduced cost.^{1,2}

Fracture mechanics was conceived for homogeneous materials, and is usually discussed for single cracks or countable assemblies of cracks. When one speaks of “fracture toughness” in this context, the global material observable is used. Modern applications of CFM to discrete defect analysis in heterogeneous (composite) materials have followed these precepts, typically using anisotropic physical properties to account for heterogeneity or micromechanics to discuss specific damage mechanisms.¹ But the physical foundation of CFM is one mechanism (a single self-similar crack) in a single homogeneous material (or an interface between two dissimilar materials). When applied to heterogeneous materials, the simple connection to global behavior is lost; crack initiation is not “failure;” single cracks almost never “act alone” to control strength and life; and we have no single measureable “crack length” or “toughness” to characterize damage tolerance – the most important single mechanical characteristic of heterogeneous materials (man-made and natural).

Historically, this barrier has been widely recognized. Perhaps the single most frequent proposed solution to the conundrum is what has become “classical damage mechanics” (CDM) which argues that a proper physical observable for the multi-scale evaluation and interpretation of heterogeneous damage is stiffness change.^{3,4} The appeal of this approach is strongly based; the original papers of Irwin⁵ and Kies⁶ and the subsequent work of Griffith⁷ (and many others) showed that multiple defect initiation and growth can be discussed in terms of total strain energy release rate which is proportional to global compliance change, now a standard method for evaluating crack growth rates and global toughness, and universally used in countless standard analysis codes (e.g., DARWIN, ABAQUS, ANSYS, etc.).⁸ Unfortunately, stiffness /compliance change is not unique to specific defects or mechanisms, so it is not possible to discriminate the specific contribution of different material constituents or defect types to a given stiffness change observation unless much more information is available. And it is very difficult to measure stiffness locally, in a specific material volume.

In the present work, we construct a new foundation for heterogeneous fracture mechanics (HFM) based on a different material property / observable, the dielectric material response to a time variable vector electric field. In earlier work, we have shown evidence that dielectric physical data directly relate to *individual* specimen strength (including uncertainty);⁹ that the multi-scale multiphysics analysis that relates the micro-response to global observations correctly predicts relationships between *constituent* responses (including defects) and their interactions and the global mechanical behavior such as strength and life;^{10,11} and that the resulting electrical/mechanical HFM (unlike damage mechanics) can be inverted (the inverse problem can be solved, at least in some cases) as a foundation for the design of materials and systems, and for supervisory control.¹² These results suggest that a new discipline can be constructed for heterogeneous materials that is an analogue to fracture mechanics for homogeneous materials. It is the purpose of the present paper to explore that possibility.

For continuous fiber reinforced composites, the general nature of the development of defects during quasi-static and fatigue loading is well established and widely accepted.^{13,14} A typical representation of that sequence is shown in Figure 1.

Fig. 1. Mechanisms and stages of damage development, accumulation, and fracture for continuous fiber reinforced polymer matrix composites.

An example of these damage modes is shown in the sequence of surface replicas in Figure 2. In that figure, scanning electron microscope images were taken of the same area on the edge of a coupon specimen during quasi-static 45 degree off-axis loading of a plain weave continuous glass fiber reinforced epoxy coupon to successively higher load levels until fracture occurred. The initial patterns (a,b) show no damage; matrix cracking is clearly seen in replicas (c,d), coupling of cracks is found near the end of loading, (e), and just before fracture a through-the-thickness coupling of cracks is resolved in (f).¹⁵ One focus of the present work is an effort to construct a non-invasive assessment method of discriminating the mechanisms, modes, and rates of the development of such damage.

Fig. 2. Example of progressive damage development in a plain weave, continuous fiber reinforced polymer composite loaded to failure in quasi-static tension at 45 degrees to the fiber axis, rendered by edge replication.

Similar damage patterns develop during cyclic (fatigue) loading of such materials. Fazzino recorded damage development patterns during the end-loaded bending fatigue of glass-epoxy materials, which showed that progressive damage grew from the two surfaces towards the interior, as expected. An example of such damage is shown in Figure 3.

Fig. 3. Example of surface crack growth in a plain-weave glass-epoxy specimen during end-loaded bending fatigue loading.¹⁶

Fazzino also recorded the changes in the dielectric properties (through the thickness measurements of the capacitance and material permittivity) for his specimens and discovered that there is a unique and logical relationship between the changes in dielectric response during fatigue damage as a function of the fraction of life, a discovery that earned the Silver Prize from the Royal Aeronautical Society in 2010.¹⁰ An example of those results is shown below.

Fig. 4. Changes in the impedance to alternating current through the thickness of composite specimens during end-loaded fatigue as a function of frequency of the AC signal for several fractions of life, for three different loading orientations relative to the fiber direction.

From Figure 4 it is seen that the impedance response begins with a straight line with monotonic negative slope as one would expect for a dielectric material between two conducting plates (the response of a parallel plate capacitor). As damage develops, in this case through the thickness, in the presence of ambient air which has some finite moisture content, the response becomes flat, i.e., independent of frequency, which is characteristic of a conductor. So the development of damage (even in the very early stages of life) is clearly detected, and the development of a fracture plane is clearly defined by the data.

More generally, our research (and a foundation of other work) has taught us that when micro-damage that creates new internal surfaces or volumes is formed in a material subjected to a vector electric field applied through the thickness of the damaged section, changes in dielectric polarization properties of the material are caused, precisely, uniquely, and directly by the new internal surfaces and volumes that are formed, and that the rate of change of those dielectric properties is controlled by the rate of new surface and crack volume formation. The ‘new work of polarization’ caused by the new surfaces and volumes created by progressive damage is directly related to the strain energy release rate from that same damage in a fundamental way. Since, for heterogeneous structural materials, it is arguably difficult or impossible to measure point-wise strain energy release rate (e.g. local compliance changes, or all local crack lengths) but demonstrably straight forward to measure local dielectric compliance changes and polarization in a material or structure, our approach is to use through-thickness changes in dielectric properties as a measure of the collective rate of degradation and to interpret those rates, directly, in terms of strength and life.

Griffith (and Eshelby, and others) have taught us that when cracks are formed in a material subjected to a tensor stress field, the (strain) energy released by the crack phase region is used to form the new surfaces of the crack, and that the amount of energy released is determined uniquely by the material properties, the crack (surface) area, the geometry of the specimen, and the tensor values of the applied stress field. This “strain energy release rate” (change per unit area crack surface increase) uniquely defines the crack formation event, and a material property called the “critical strain energy release rate” defines the onset of the fracture event (final failure, end of life, etc.). This discipline is generally referred to as Linear Elastic Fracture Mechanics (LEFM). In the present work, we formulate a dual concept based on an observable material state variable, the dielectric permittivity of the material, and postulate that it is possible to develop a heterogeneous fracture mechanics discipline for multiple defects that functions in the same general way as LEFM does for single defects.

2. Analysis and Methods

In a series of papers over the last five years or so, the authors have reported several advances in the understanding of the dielectric response of heterogeneous materials, based on an innovative through-the-thickness experimental characterization method and first principles analysis. In classical dielectrics the relation between the applied vector electric field $\mathbf{E}$ and the dielectric displacement $\mathbf{D}$ is linear and can be expressed as,

$$\mathbf{D} = \epsilon_0 \epsilon_r \mathbf{E} \quad 1$$

where, ϵ_0 is the permittivity of the free space and ϵ_r is the relative permittivity of the dielectric material. If ρ is the charge density, from Maxwell’s equations we know that the dielectric displacement follows the following relationship

$$\nabla \cdot \mathbf{D} = \rho \quad 2$$

For current density $\mathbf{J}$ we can state the following from the continuity equation

$$\nabla \cdot \mathbf{J} = -\frac{d\rho}{dt} \quad 3$$

Also, from Ohm's law we know

$$\mathbf{J} = \sigma \mathbf{E} \quad 4$$

Here σ is the conductivity of the material. And the corresponding charge displacement associated with the application of an electric field in a dielectric material with a relative dielectric constant of ε is,

$$\mathbf{D} = \varepsilon \bullet \mathbf{E} \quad 4.1$$

where the dielectric constant of the material is normalized by the dielectric constant of a vacuum, ε_0 . So from equations 2 and 3 we obtain

$$\nabla \cdot \left(\mathbf{J} + \frac{d\mathbf{D}}{dt} \right) = 0 \quad 5$$

Now using 1, 4 and 5 we can write the following

$$\nabla \cdot \left(\sigma \mathbf{E} + \frac{d(\varepsilon_0 \varepsilon_r \mathbf{E})}{dt} \right) = 0 \quad 6$$

In case of a sinusoidal applied electric field $\mathbf{E}$ of angular frequency ω

$$\nabla \cdot (\sigma + i\omega \varepsilon_0 \varepsilon_r) \mathbf{E} = 0 \quad 7$$

By definition

$$\mathbf{E} = -\nabla \Phi \quad 8$$

so that from equations 7 and 8 we get

$$\nabla \cdot [(\sigma + i\omega \varepsilon_0 \varepsilon_r) \nabla \Phi] = 0 \quad 9$$

From equation 9, we can see that in a heterogeneous material the product of the physical properties (some form of the conductivity and permittivity) and the slope of the electric potential must be a constant as we cross material boundaries. For the quasi-static case with harmonic input fields, the gradient of that product vanishes. The interacting field is a result of the charge difference at the interface, and unless the conductivity and permittivity of adjacent material phases are identical, there is a disruption of charge transfer at the material boundary which results in internal polarization.

To solve equation 9, we can set the potential on one surface to be,

$$\Phi = U = U_0 e^{-i\omega t} \quad 10$$

and on the opposing boundary surface electrode to be,

$$\Phi = 0 \quad 11$$

The boundary conditions on all of the internal interfaces are,

$$\Phi_1 = \Phi_2 \quad 12$$

$$\varepsilon_1 \hat{\mathbf{n}} \cdot \nabla \Phi_1 = \varepsilon_2 \hat{\mathbf{n}} \cdot \nabla \Phi_2 \quad 13$$

where $\hat{\mathbf{n}}$ is the unit vector normal to the interface surface and the subscripts indicate phase 1 and 2.

Many phases and interfaces may be present. To eliminate fringe effects on the side planes, we set

$$\hat{\mathbf{n}} \cdot \nabla \Phi = 0 \quad 14$$

here $\hat{\mathbf{n}}$ is the unit vector normal to the free surfaces.

In general, all of the physical properties in field equations 1 – 14 can be complex, i.e., they can have a real part that is a 'direct' response to the applied fields and an imaginary part that is out of phase with the applied fields indicating an internal degree of freedom, e.g., the motion of charge in the volume of the constituents or at the interfaces of constituents.⁹ These internal events are the key to the utility of this response for sensing and interpreting the changes in material state and the development, accumulation and interaction of defects in composite materials. Our previous work has identified several features of the dielectric response that are useful parameters (observables) for the interpretation and quantification of various specific changes. Some examples that we have examined in detail are:

1. “Dielectric strength:” the absolute difference between the maximum and minimum value of the real part of the dielectric permittivity
2. Minimum value of the imaginary part of the dielectric constant
3. Maximum value of the imaginary part of the dielectric constant
4. Low frequency limiting value of the real part of the permittivity
5. Global capacitance (total stored charge as a function of applied voltage)
6. Impedance (ratio of amplitude of the alternating voltage to the amplitude of the induced current)
7. Plot of the magnitude of the imaginary part of the impedance vs. the real part of the impedance (known as a Bode plot)

An example of the Bode plot is shown for fatigue data reported in Figure 5.⁷

Fig. 5. Plot of real vs. imaginary part of the dielectric impedance measured through the thickness of a composite coupon specimen during end-loaded-fatigue bending at different fractions of total life, showing remarkable sensitivity to the initiation, development, and coalescence of damage resulting in final failure.

These measurements were made on the global surface of a sample, i.e. they are measurements of the total impedance or capacitance across the thickness of a sample. An illustration of such an experimental arrangement is shown in Figure 6.

(a)

(b)

Fig. 6. Global experimental measurement method (a), and schematic of charge accumulation at interfaces by polarization, (b) in a composite material.

$$\Delta W = \int F(\text{on } Q \text{ by } E) * dr = \int_0^a Q * E * dx = \int_0^a E * dp \quad 15$$

$$\Delta W = \int_0^a E * dp = \int_0^{E(\text{final})} E_i * d(\alpha_{ij} E_j) = \int_0^{E(\text{final})} \alpha_{ij} E_i dE_j \quad 16$$

The field equations (1) – (14) can be solved for any configuration of heterogeneous phases (any morphology); we will look at some examples of those solutions in a section below. However, the data shown in Figs. 4 and 5 were recorded from the global surfaces of coupons, and represent a collective effect of many internal details. Figure 6(a) shows a schematic of that test arrangement in which the dielectric material to be examined is inserted between two contact plates (which can be conductive foils, etc.) by which an alternating voltage with variable frequency is applied across the thickness of the test material. (Numerous variations of this arrangement have been reported.^{6,7,9,13,14,15}) The complex impedance of the sample and the capacitance measured by the test equipment are remarkably sensitive to the presence of internal surfaces, as shown by Figs. 4 and 5, and reported in our earlier publications. But understanding how such data can be used to not only detect, but to interpret the global dielectric response in terms of the details of the internal microstructure (including defects) requires some first principles of the operating physics, as shown in Figure 6(b). The vector electric field, E , is shown acting across a volume with dimension, a , in the direction of that field. By definition, that will induce some charge displacement in a dielectric material, which results in a charge, $+Q_1$ on the left face and $-Q_1$ on the right face that creates a “back field” of E in the other direction. We describe the work done by those actions by “reading” equations (15) and (16), from left to right. Equation (15) states that the work done by the field, E , on that charge, Q , moving through distance, a , can be written as the integral of the field times the polarization, p , of the material (defined by the displacement of all charge). Then equation (16) reminds us that the polarization is a tensorial material property, α_{ij} , defined by the amount of charge displacement per unit field (typically normalized by the dielectric constant of a vacuum, ϵ_{0ij}).¹⁷ The last of equation (16) has the form of a strain energy density if the field were mechanical rather than electrical, with the dielectric constant playing the role of compliance, i.e., the amount of displacement resulting from a unit applied field.

The tensor nature of these forms suggest that the global measurements carry information not only about the surfaces and volumes of the internal phases (including the defect phases), but also reports information about the orientation and shapes of those phases (or defects). If that is correct, then solutions to equations (1-14) for a given distribution of those internal shapes (or defects) should reflect their character in some way. To test this hypothesis, we have conducted a series of numerical experiments. First, surface and volume effects were investigated for non-dilute situations (representing

“real” composites) using variable volume fraction of second phase materials, as represented in Figure 7. The Physalis method of embedded boundaries was used to achieve non-dilute solutions which included interaction of the internal “back fields,” as defined above.¹⁸

Fig. 7. Volume fraction RVEs modeled for three volume fractions and two inclusion geometries.

Figure 7 shows examples of the variation of volume fraction modeled for two geometries. Figure 8 shows several example results of those calculations.

Fig. 8 Effective permittivity (real part) across a representative volume for various shapes, orientations, and volume fractions (including particle-particle field interactions, with an AC frequency of 10 Hz).

Polarization associated with both surface and volume effects enter that predictive analysis.^{19,20} Surface / volume effects, second phase shape and orientation (relative to the direction of the applied vector electric field), and volume fraction effects are clearly delineated by the interpretive analysis. Results in Figure 8 are shown for low frequency excitation, so that nearly static polarization is dominant in the results, in contrast to many other published methods which focus on high frequency excitation which does not resolve the polarization. One interesting feature of these results is the non-intuitive maximum in many of the predicted variations with volume fraction. To test this predicted result, quasi-static, off-axis loading of plain-weave glass-epoxy and carbon-epoxy coupons was conducted with continuous monitoring of the dielectric response (through the thickness of the coupons) during the tests. An example of the results is shown in Figure 9.

Fig. 9. Predicted and observed changes in global through-thickness permittivity during uniaxial loading of an off-axis plain-weave composite coupon.

These data, and the data from hundreds of similar tests in our laboratories, show the characteristic early increase followed by a decrease for these combinations of materials. An early increase followed by a more persistent increase or decrease is found for all material characteristics. A comprehensive analysis of this behavior will be presented in a subsequent paper. At this point we will only state that the early change is associated with internal defect surface formation, and the secondary changes are typically driven by volume effects, another clear distinction that is revealed by the method (experimental and

analytical). In Figure 9 a predicted result is also shown for that specific material and specimen type. The prediction was done by combining matrix crack density predictions based on ply-level Tsai-Wu failure criteria with predictive analysis as described above, using constituent properties taken from the literature and our measurements (c.f., ref. 20). Generally, the measured stress-strain response was matched point by point by reducing the stiffness of the off-axis plies, and those fractional reductions were applied to the permittivity values of the matrix in the models of global permittivity. The results are surprisingly close to the observed results.

In order to further explore the capability of the method to discriminate specific damage modes, a combined mechanical-dielectric analysis was constructed. It was decided to use ABAQUSTM for the large deformation, nonlinear analysis of mechanical effects, including defect development. The results of that analysis, at various increments of loading and damage development, were used to set dielectric analyses conformally on the deformed and defected microstructures in COMSOLTM to analyze the corresponding response to applied AC voltages using equations (1-14). Vadlamudi has conducted several such analyses, which we refer to for our present discussion. For this purpose we used the calculated deformation patterns directly in a conformal multiphysics analysis with COMSOLTM to calculate the expected changes in measured dielectric properties of the composite elements as a result of those mechanical deformations.²¹ Figure 10 shows an example of such results for a very large uniform tensile deformation of an epoxy coupon with included carbon fiber second phase particles. The vector electric field is applied transverse to the direction of the mechanical field, as shown in Figure 6. While the capacitance curve is monotonic increasing (as one would expect as the contact plates move closer together), the capacitance has distinct changes as revealed by successive derivatives of the slope of the capacitance curve. Those changes reflect changes in the internal distribution of the electric field caused by capacitive charge displacement, in the same sense as the changes in strain energy distribution, as suggested by equation (16).

Fig. 10. Large deformations (up to 140%) in tension (A), electric potential distributions at internal phases (B), changes in through-thickness capacitance (C) and changes in global capacitance (D) for a carbon-epoxy system.

For the example in Figure 10, a unit composite cube was stretched to 125% strain in the horizontal plane (A) while COMSOLTM was used to calculate the through-thickness change in voltage and charge distribution at the local level (B) resulting in global capacitance changes (C) and distinctive features in the rate of change of capacitance (D) which show initiation (1), accumulation (2), and (clearly) failure onset (3). Further calculations were conducted to determine if discrete defects introduced into the microstructure also show distinct features of response. To determine this capability, we introduced a sequence of a few fiber-matrix separations, and then a few fiber failures (with moist air in the crack volumes) into the first principles calculations of dielectric response. The preliminary results are shown in Figs. 11 and 12. Figure 11 shows two modes of damage; (a) shows cracks simulated in the continuous phase, between the distributed phase regions (e.g., matrix cracks between fibers) and (b) shows additional damage in the form of rupture of the distributed phase (e.g., fiber fracture) simulated by introducing regions of moist air into the spaces (defect volumes) created by the fracture (in this view).

Fig. 11. Composite cross sections modeled with ABAQUSTM and COMSOLTM altered to include matrix cracks between the second phase regions (left) and “fiber fracture” simulated by replacing some second phase regions with moist air (right).

Fig. 12. Example of conformal computation of the distribution of electric field around internal defects (left) and changes in the global compliance with strain (right) for internal microcracking and fiber fracture simulations as shown in Figure 11.

Figure 12 shows the resulting distribution of the internal potential field as calculated with COMSOLTM (left) and a plot of the second derivative of the change in capacitance calculated for the introduction of those two damage modes. The incidence of separation of the second phase at the boundary to create internal voids between the two phases (a highly nonlinear failure mode calculated directly using ABAQUSTM) and subsequently the simulation of fiber fracture is clearly and distinctly indicated by the second slope (second derivative) of the capacitance, as shown in the right of Figure 12. In that figure, changes in the rate of capacitance change with strain clearly identify the two internal defect failure modes and the strain levels when they were introduced. This demonstrates that this science-based method has the capability to discriminate different kinds of micro-cracks and fiber fracture pattern formation.

3. Discussion

A foundation for the concept of heterogeneous fracture mechanics based on dielectric measurements, and the modeling and physical evidence cited above, follows. At the level of first principles, the association of response to a dielectric field with the response to a mechanical field for a material element is direct. For the case of linear response, from equations (14) and (15), the change in stored electrical energy, U_E , due to a change in defect area, dA , can be written (in scalar form) as

$$\frac{\partial U_E}{\partial A} = \frac{E^2}{2} \frac{\partial \epsilon}{\partial A} \quad (17)$$

which has the same form as the classical relationship conceived by Irwin and Kies for the change in stored strain energy for a mechanical applied force P ,

$$\frac{\partial U_M}{\partial A} = \frac{P^2}{2} \frac{\partial c}{\partial A} = G \quad (18)$$

where G is the strain energy release rate, P is the mechanical load, and c is the material mechanical compliance for linear response. In our experiments we typically measure total capacitance, C , across a material distance (specimen thickness) d , which measures the charge, Q , stored over the area of the contact points, i.e.,

$$C = \frac{A*\epsilon}{d} \quad (19)$$

so that the measurement reflects the effective dielectric constant of the material over the thickness, d . So in principle, our measurements of global capacitance are related by extensive variables of shape and size to the dielectric constant of the material volume.

The present work postulates a dual concept to classical linear elastic fracture mechanics (LEFM) for multiple defect analysis in heterogeneous materials based on changes in dielectric energy storage that are specific to the defects created. In the proposed concept, the dual of the strain energy released during defect development in LEFM is the increase in capacitive electrical energy density stored in the presence of an applied electrical field. To demonstrate this, we consider the nonlinear response measured and shown in Figure 9. First, we consider the change in strain energy with strain which is linear until damage develops; so we consider any nonlinear variation of strain energy to be specific to damage development (as shown above and confirmed in other publications). So we plot the second variation of strain energy vs. strain to see when damage is developing, and to what extent. Also shown in Figure 9 is the measured dielectric capacitance change, which we claim is linear with strain (from our discussion of a parallel plate capacitance wherein the plates change position and the dielectric material does not change) unless the creation of new interior surfaces and defect volumes causes a nonlinear variation that is specific to that damage. Again, using measured data from Figure 9 we consider the second variation of the measured capacitance to display any variation from that linear change. If we normalize both of those plots and put them together the results are shown in Figure 13. It is seen that the two variations are remarkably similar, as postulated from our analysis and discussion. We believe that the physics of individual and specific microcrack formation events drives corresponding changes in strain energy and dielectric response measured in the laboratory, i.e., that those observables are dual responses to the process of damage development.

Fig. 13. Comparison of the second variation (variation from linear slope) of the stored strain energy and dielectric capacitance (data from Figure 9).

We can interpret those plots directly in terms of the development of damage. For this highly nonlinear response the largest deviation from linear response (largest damage development) is in the early part of the loading, between 0 and 6% strain when matrix cracking causes major changes in the stiffness of the material for this tensile test of an off-axis plain weave composite material. But there is another damage development peak in the 10 to 12% strain region marking the “beginning of the end” of strength when damage coalescence (including the initiation of fiber fracture) lead to specimen fracture. Although this plot does not define the damage modes (without the addition of the interpretive analysis we have discussed), it does confirm, from experiments alone, that there is a duality in the mechanical and dielectric response for this cases discussed here. More comparisons confirming this claim will be discussed in subsequent papers.

4. Conclusions

The present work shows evidence that it is possible to formulate a dual concept to classical linear elastic fracture mechanics (LEFM) for multiple defect analysis in heterogeneous materials based on changes in dielectric energy. We also postulate (but have not yet shown) that the dual of the concept of material toughness defined by a critical strain energy release rate in LEFM is a critical rate of change of capacitive electrical energy release rate during defect state development. It is further postulated that the dual to different modes of mechanical defect development (crack opening, sliding, and out of plane shearing) is, in general, the sensitivity of an applied vector electric field to the shape, size, and orientation of defects that develop in the interior of a heterogeneous material, as we have clearly shown in the present paper and several other publications.

The utility of this new concept is self evident. Measuring the length and geometry of a single dominant flaw in a structural component can be difficult, especially during service. And measuring the structural integrity of such a structure at various points in the life of a component with a complex history of use is more difficult. And finally, finding all of the defects that occur during service in a composite material and assessing the changes in structural integrity changes caused by them is not presently possible. The current method offers a possible avenue to approaching all of those objectives. It is possible to (at least) imagine measuring the point-wise changes in the dielectric response of structural composites during service, continuously, and using the direct interpretation tools mentioned above to estimate the consequent changes in the structural reliability, integrity, strength, and life of composite structures.

The need for further work on this proposed concept is also evident. A comprehensive and systematic analysis of specific defect types is needed to create a robust interpretive model that does not depend on material parameters, per se. The postulate that there is, in fact, a material property that defines a critical change in dielectric permittivity / capacitance at fracture for a given material requires careful experimental and analytical scrutiny. (Other dielectric material properties, like the dielectric strength, are well established.) And convenient methods of implementation of this method must be developed. If these challenges are overcome, heterogeneous fracture mechanics for multiple defect analysis in composite materials will provide a new tool for the durability design and maintainability of composite structures for complex histories of applied conditions, and for the reduction of the risk of using heterogeneous materials in safety critical structures.

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