Analysis of nonlinear dynamic responses for functionally graded beams resting on tensionless elastic foundation under thermal shock

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Abstract This paper investigates the nonlinear transient thermal responses of functionally graded beams resting on tensionless foundation under unsteady heat conduction. The interaction between functionally graded beams and tensionless foundation is only compressive without tensile reaction. The whole model is established based on Euler beam theory and solved by the combination of differential quadrature method (DQM), Newmark method and Newton-Raphson method. Some detailed numerical results are carried out to reveal the transient heat conduction procedure and the mechanical behaviors of functionally graded beam resting on tensionless foundation.

Keywords Functionally graded beams, tensionless foundation, transient temperature field, nonlinear dynamic responses

1. Introduction

Functionally graded materials (FGMs) are novel non-uniform composites at certain directions with spatially continuous material properties. They are common made up of ceramic and metal in order to make full use of the heat-resistance of ceramic and the ductility of metal. As a result, FGMs are usually used as heat insulation coating and structural components in some extremely high temperature environments. The temperature in these areas usually varies quite fast within a limited time which must be taken as a transient heat conduction procedure. In this condition, the studies on thermal behaviors for FGM structures in transient thermal environment are significant and important to put them into application well. Fu et al.[1] studied the nonlinear dynamic responses of FGM shallow spherical shells subjected to mechanical load and transient temperature filed. Shariyat[2] used a nonlinear Hermitian transfinite element method to study the transient behaviors of FGM hollow cylinders with temperature-dependent material properties

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under thermo-mechanical loads. Jin and Paulino[3] investigated the transient thermal problem of FGM strip with an edge crack. A system of researches [4-6] are carried out for FGM structures like strip[4], plate[5] and hollow cylinder[6] by Ootao and Tanigawa. Asgari and Akhlaghi[7] presented a two-dimensional transient thermal analysis for finite FGM hollow cylinder. It should be mentioned that the settlement process for transient heat conduction must take time variable into account. Moreover, due to the non-uniform distribution of the reinforcement phase, the thermal properties of FGMs are different along the variational direction of the material attribute. The two points make it extremely different but full of significance to establish an efficient tool to solve the transient heat conduction of the FGMs.

On the other hand, beams, plates and shells resting on elastic foundations[8, 9] are common structures in engineering environment. Some engineering structures like railway, concrete road and piles embedded into soil can both be treated as structures resting on elastic foundation. Moreover, embedded carbon nanotubes[10] and microtubules in living cells[11] can also be handled in this way. In order to characterize the elastic foundation more briefly, attached elastic foundation which is the conventional one providing compressive and tensile forces to the structures is adopted in most studies[12-15]. It should be noted that engineering structures will never be separated from this foundation. In fact, tensionless foundation which can only provide compression or can provide small tension is more common and applicable. In this condition, engineering structures might be separated from the elastic foundation. Due to the engineering requirement, a growing number of studies are carried out to reveal the mechanical interactions between structures and tensionless foundation. Shen et al.[16-18] presented investigations on nonlinear bending, buckling and dynamic responses for shear deformable plates resting on tensionless elastic foundations. Zhang[19] introduced a more generalized model for a finite beam resting on tensionless Reissner foundation and studied the bending problem of the beam subjected to a concentrated load. Ma et al[20] proceeded an analysis for the static response of an infinite beam resting on a tensionless Pasternak foundations under complex loads including self weight. Coşkun[21] studied the forced vibration of a beam resting on nonlinear tensionless Winkler foundation subjected to a concentrated dynamic load. Even tensionless foundation is more suitable for some certain fields of engineering practices, the interaction between tensionless foundation and structures is much more complicated because the contact time and regions can not be determined at the outset and an iteration procedure is needed. Even a linear mechanical problem will lead to the nonlinear solving process.

This paper studies the nonlinear transient thermal responses of FGM beams resting on tensionless elastic foundation based on the Euler beam theory. The differential quadrature method (DQM), Newmark method and Newton-Raphson method are introduced to solve the whole problem. Numerical results show that the non-uniform material properties of FGMs make the heat conduction process and mechanical behaviors of the beam totally different from that of a homogenous beam. It also shows that the Characteristics of tensionless foundation may change the contact condition between foundation and beam.

2. Basic equations

Consider a FGM beam with length L in x direction, width b in y direction and thickness h in z direction, resting on tensionless foundation as shown in Fig.1. For the tensionless foundation, k_1 denotes the Winkler foundation stiffness and k_2 is the shearing layer stiffness of the foundation. The type of foundation can only provide counterforce only when the beam and foundation contact each other. The FGM beam is made up of ceramics and metals, and the material properties are assumed to vary only in thickness direction. In order to characterize this behavior, the Voigt model is adopted which can be expressed as[22, 23]

$$P_f(z) = P_c + (P_m - P_c) \left(\frac{h + 2z}{2h} \right)^N \quad (1)$$

Where P_f , P_m and P_c are the material properties of FGMs, metal and ceramics, respectively. N represents the volume fraction exponent.

At the initial time $t = 0$, a thermal shock, namely $T(-h/2, t) = T_m(1 - e^{-at})$, is uniformly applied on the top surface of the FGM beam, where T_m is the amplitude of the thermal load and a is a parameter which determines the speed of the temperature change. Then the thermal heat conduction will proceed along the thickness direction. In this condition, the transient heat conduction equation [1, 24] is given as

$$\frac{\partial}{\partial z} \left[\kappa(z) \frac{\partial T(z, t)}{\partial z} \right] - \rho(z)c(z) \frac{\partial T(z, t)}{\partial t} = 0 \quad (2)$$

where k , ρ and c are the thermal conductivity, mass density and specific heat, respectively. Here we assume heat exchange will exist between the bottom surface of FGM beam and outer environment whose temperature will not change denoted as T_e . Assume that the beam is in thermal equilibrium at the initial state $t = 0$ and the initial temperature is set as the reference state. Therefore, the initial condition and boundary condition can be written as

$$T|_{t=0}=0, \quad T|_{z=-h/2}=T_m(1-e^{-at}), \quad -\kappa(z)\frac{\partial T}{\partial z}|_{z=h/2}=h_c(T-T_e) \quad (3)$$

in which h_c is the convection coefficients of the bottom surface of FGM beam

Based on Euler beam model, the displacement (u_1, u_3) of the FGM beam can be expressed as

$$\begin{aligned} u_1 &= u - zw_{,x} \\ u_3 &= w \end{aligned} \quad (4)$$

where u and w are the displacement components of an arbitrary point on the midplane along x and z directions; the subscript “,” denotes the derivative with respect to coordinate x .

Base on the von Kármán-type nonlinear strain-displacement theory, the geometrical relation of the corresponding point can be obtained as

$$\varepsilon_x = \varepsilon_x^0 + z\kappa_x \quad (5)$$

where

$$\varepsilon_x^0 = u_{,x} + \frac{1}{2}w_{,x}^2, \quad \kappa_x = -w_{,xx} \quad (6)$$

Considering the effect of thermal load, the stress-strain relation of the FGM beam can be expressed as

$$\sigma_x = E\varepsilon_x - E\alpha_x T \quad (7)$$

Where E and α_x are Young's modulus and thermal expansion of FGM beam.

Based on Euler beam model, the nonlinear governing equations for the FGM beam can be written as

$$\begin{aligned} N_{x,x} &= 0 \\ M_{x,xx} + N_x w_{,xx} + q &= I_0 \ddot{w} \end{aligned} \quad (8)$$

The force N_x and moment M_x are defined as

$$(N_x, M_x) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x(1, z) dz \quad (9)$$

Substituting Eq.(7) into Eq.(9) and combining Eq.(4) and Eq.(5), N_x and M_x can be expressed as

$$\begin{Bmatrix} N_x \\ M_x \end{Bmatrix} = \begin{bmatrix} A_{11} & B_{11} \\ B_{11} & D_{11} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \kappa_x \end{Bmatrix} - \begin{Bmatrix} N_T \\ M_T \end{Bmatrix} \quad (10)$$

in which N_T and M_T are the force and moment produced by thermal load which are defined as

$$(N_T, M_T) = \int_A E\alpha_x T dA \quad (11)$$

Other coefficients are defined as

$$I_0 = \int_A \rho dA, \quad (A_{11}, B_{11}, D_{11}) = \int_A E(1, z, z^2) dA \quad (12)$$

In addition, q is the counterforce produced by the tensionless elastic foundation which is applied on the bottom surface of the FGM beam. It should be pointed out that the tensionless foundation can generate force to FGM beam only when the beam contacts the foundation. In this case, the counterforce is assumed to be

$$q = -(k_1 w - k_2 w_{,xx}) H(w) \quad (13)$$

where $H(w)$ is a Heaviside step function defined as

$$H(w) = \begin{cases} 1, & w \geq 0 \\ 0, & w < 0 \end{cases} \quad (14)$$

The form of q in Eq.(13) is not an ideal one to solve the whole problem so that it should be transformed. Ref[25] presents a useful transformational relation, namely.

$$H(w) = [1 + \tanh(Aw)] / 2 \quad (15)$$

In order to satisfy the accuracy of the solution, A should be chosen as a large number, like $A = 10^{15}$. Therefore, the force can be rewritten as

$$q = -(k_1 w - k_2 w_{,xx}) [1 + \tanh(Aw)] / 2 \quad (16)$$

Substituting Eq.(10) into Eq.(8), the governing equations expressed by the displacements can be obtained as

$$\begin{aligned} & A_{11}(u_{,xx} + w_{,x} w_{,xx}) - B_{11} w_{,xx} = 0 \\ & B_{11}(u_{,xxx} + w_{,xx} w_{,xx} + w_{,x} w_{,xxx}) - D_{11} w_{,xxxx} \\ & + [A_{11}(u_{,x} + \frac{1}{2} w_{,x}^2) - B_{11} w_{,xx} - N_T] w_{,xx} - (k_1 w - k_2 w_{,xx}) [1 + \tanh(Aw)] / 2 = I_0 \ddot{w} \end{aligned} \quad (17)$$

Simply supported FGM beam with one in-plane immovable end and one in-plane movable end is considered here whose boundary conditions can be described as

$$\begin{aligned} x = 0: & \quad u = w = M_x = 0 \\ x = L: & \quad N_x = w = M_x = 0 \end{aligned} \quad (18)$$

3. Solution methodology

It hard to get analytical solutions to heat conduction equations and nonlinear governing equations for FGM beams resting on tensionless elastic foundation, so the differential quadrature method (DQM) [26, 27], the Newmark method [25] and Newton-Raphson method are used to solve these equations numerically.

As for the derivative with respect to the spatial variable, it is discretized by the DQM. The

essence of DQM is that the function and its derivative with respect to space variables at a given point are approximated by a weighed linear summation of the function values at all discrete points in domain. Thus, for a function $F(x)$ with the domain $0 < x < L$, it is discretized by N points dividing the beam into $N-1$ parts. The r th order derivative of $F(x)$ at any point x_i can be expressed approximately as

$$\frac{\partial^r F(x_i)}{\partial x^r} = \sum_{k=1}^N A_{ik}^{(r)} F_{kj} \quad (j=1,2,\dots,N) \quad (19)$$

in which, $A_{ik}^{(r)}$ stands for the r th weighed coefficient, and the weighed coefficient be determined by the following forms:

$$\begin{aligned} A_{ij}^{(1)} &= \frac{\Pi(x_i)}{(x_i - x_j)\Pi(x_j)} \quad , (i, j=1,2,\dots,N \text{ and } j \neq i) \\ A_{ij}^{(r)} &= r[A_{ii}^{(r-1)} A_{ij}^{(1)} - \frac{A_{ij}^{(r-1)}}{(x_i - x_j)}] \quad , 2 \leq r \leq N-1 \\ A_{ii}^{(m)} &= - \sum_{j=1, j \neq i}^N A_{ij}^{(m)} \quad , 1 \leq m \leq N-1 \end{aligned} \quad (20)$$

where

$$\Pi(x_i) = \prod_{j=1, j \neq i}^N (x_i - x_j) \quad (21)$$

The sampling points of DQM along thickness and length can be taken as [26, 27]

$$\begin{aligned} x &= \frac{1 - \cos[(i-1)\pi / (N-1)]}{2} L, \quad (i=1,2,\dots,N) \\ z &= -\cos[(i-1)\pi / (N-1)]h / 2, \quad (i=1,2,\dots,N) \end{aligned} \quad (22)$$

The derivative with respect to time is discretized by the Newmark method. Using this method, the variable t should be equally divided into time increment Δt . Therefore, the specific form of Newmark method in J th iteration procedure can be expressed as follows:

$$\begin{aligned} (F_{,tt})_J &= \frac{4(F_J - F_{J-1})}{(\Delta t)^2} - \frac{4(F_{,t})_{J-1}}{\Delta t} - (F_{,tt})_{J-1} \\ (F_{,t})_J &= (F_{,t})_{J-1} + \frac{\Delta t}{2} [(F_{,tt})_{J-1} + (F_{,tt})_J] \end{aligned} \quad (23)$$

On the basis of above work, heat conduction Eq.(2) can be solved directly. The nonlinear governing Eq.(17) can be solved by Newton-Raphson method. It should be pointed out the

contact condition between FGM beam and tensionless elastic foundation can not be determined in advance so that it must be approached by iteration.

4. Numerical results and discussion

To ensure the validity of analysis in this paper, linear transient thermal dynamic responses of FGM beam without taking the geometrical nonlinearity and the effect of tensionless elastic foundation into consideration are compared with Ref[24] Fig.2 and Fig.3 show the comparisons of the temperature and deflection responses for FGM beam under thermal shock, respectively. In these comparisons, the material properties and boundary conditions are chosen as Ref[24] and the dimensionless maximum deflection $W_m = w_m / h$ is introduced where w_m is dimensional maximum deflection. It can be concluded from the comparisons that present results is in good agreement with that in [24] which demonstrates the rightness of present research.

In the next part, FGM beams resting on tensionless elastic foundation are chosen to study the nonlinear dynamic behaviors under thermal shock. For all numerical results, the material components of FGM beams are selected as SiC and Ni whose material properties are listed in Table 1. Moreover, unless special mention, the geometrical size of the beams are taken as $h = 1\text{cm}$, $L = 20\text{cm}$. The thermal parameters are $T_m = 200\text{K}$, $a = 10$, $h_c = 50$. It should be noted that the upward and downward directions are set as the positive and negative deflections for FGM beams rather than using the coordinate system as a reference in order to leave readers an intuitive impression.

Fig.4 gives the effect of volume fraction exponent on temperature responses of FGM beams. It can be seen that when the heat conduction occurs the temperature at different point increases gradually and the temperature at top surfaces, mid-plane and bottom surfaces increases. Moreover, the effect of volume fraction exponent on heat conduction is not monotonous, because the variational rate of functionally graded material properties including thermal conductivity, mass density and specific heat are not the same which changes the speed of heat conduction.

Fig.5 reveals the temperature distribution of FGM beam at different time where $N = 1$. It can be known that the temperature through the thickness increases with heat conduction and gradually approaches the same line. The temperature distribution at $t = 20\text{s}$ and $t = 25\text{s}$ are exactly the same which shows the heat conduction comes to an end and the temperature field becomes steady.

Fig.6 presents the influences of volume fraction exponent on temperature distribution of

FGM beam through thickness at $t = 20s$. In this case, the temperature field is stable and it will not change with time. As can be found from the figure, the temperature field of isotropic beam presents linear variation while that of FGM presents nonlinear variation. The main reason is that the thermal conductivity of FGM which is the only factor determining the steady temperature distribution is gradient through thickness. In addition, the temperature reduces with the increment of volume fraction exponent, because the thermal conductivity will be smaller when the volume fraction exponent becomes bigger.

Fig.7 demonstrates the effect of foundation types on the nonlinear deflection of FGM beams, where $(K_1, K_2) = (100, 2)$. From the figure, one can find that the deflection of homogenous beam ($N = 0$) just occurs at one side which firstly increases to a maximum value and then decreases to zero. For FGM beam, the deflection increases reaching to the peak and gradually bends to the other side approaching to an extrema which is due to the non-homogenous material properties of FGMs. In addition, the deflection response of FGM beam resting on conventional foundation is smooth during heat conduction, because beam keeps touch with this foundation and the force provided by the foundation is smooth. However, the deflection of FGM beam resting on tensionless foundation is not always smooth. If the beam contacts elastic foundation throughout the heat conduction, like homogenous beam, the deflection is smooth. Once the beam begins to separate from the elastic foundation, the deflection becomes non-smooth because of the sudden vanished force from elastic foundation.

Fig.8 investigates the influences of elastic foundation stiffness on the nonlinear dynamic responses of FGM beams resting on tensionless foundation where $N = 1$. It can be known that at the beginning of the heat conduction the deflections of FGM beams are the same no matter what the stiffness of elastic foundation is, because the beams do not contact the elastic foundation. However, with the development of the heat conduction the FGM beams begin to touch the elastic foundation and the bigger the stiffness of elastic foundation is, the smaller the deflection of FGM beam will be. The main reason is that elastic foundation with larger stiffness will provide bigger counterforces to the beams.

Fig.9 shows the effect of volume fraction exponent on the nonlinear deflection responses of FGM beams resting on tensionless elastic foundation whose stiffness is taken as $(K_1, K_2) = (100, 2)$. It is clear that the peak of the FGM beam will decrease to a small range if the volume fraction exponent increases while the maximum deflection on the other side will be bigger firstly and then be smaller if the volume fraction exponent keeps increasing. Moreover,

the time when the beams separate from the elastic foundation will change when the volume fraction exponent is different.

The effects of the amplitude of thermal shock and length-to-thickness ratio on nonlinear transient thermal responses of FGM beam resting on tensionless elastic foundation are depicted in Fig.10 and Fig.11, respectively, where $N=1$ and $(K_1, K_2)=(100, 2)$. From Fig.10, bigger amplitude of thermal shock makes larger deflection for FGM beam, because it will produce larger thermal forces and bending-moment. From Fig.11, the deflection of beams will become larger with the increment of length-to-thickness ratio since beam with bigger length-to-thickness ratio has lower whole stiffness.

It should be mentioned that the contact time in Fig.8, Fig. 10 and Fig.11 is same while that in Fig.9 is different. The main reason is that the deflection of FGM beams is produced by thermal forces and thermal bending-moment, namely heat conduction. The heat conduction process in Fig.8, Fig.10 and Fig.11 is the same while that in Fig.9 is not the same due to the gradient material properties of FGM.

5. Conclusions

This paper deals with the nonlinear transient responses of FGM beams resting on tensionless elastic foundations are dealt with based on Euler beam theory. The whole problem is solved by introducing DQM, Newmark method as well as Newton-Raphson method. According to the numerical results, the main conclusions can be obtained as follows.

The heat conduction will occur through thickness once the thermal shock is applied and the temperature field gradually becomes steady. The gradient material properties of FGM make the speed of heat conduction not be monotonous. Moreover, the temperature field of isotropic beams is linear while that of FGM beams is nonlinear.

When subjected to thermal shock, the deflection of FGM beams resting on conventional elastic foundation is smooth while that resting on tensionless elastic foundation is not smooth once the contact condition between beams and elastic foundation changes. When the FGM beams contact the tensionless elastic foundation, the deflection will be smaller if the foundation stiffness increases. The effect of volume fraction exponent on thermal deflection and contact time is not monotonous. When the amplitude of thermal shock and length-to-thickness ratio increase, the thermal deflection of FGM beams will increase.

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Table 1 Material properties of functionally graded materials [20]

Materials	$E(\text{GPa})$	$\rho(\text{Kg} / \text{m}^3)$	$\alpha(/K)$	$\kappa(\text{W/m} \cdot \text{K})$	$c(\text{J} / \text{Kg} \cdot \text{K})$
SiC	427	3100	4.3×10^{-6}	65.0	670
Ni	206	8890	13.3×10^{-6}	90.5	439

Figure captions:

Fig.1 Schematic configuration of a FGM beam resting on tensionless elastic foundation

Fig.2 Comparison of temperature responses for FGM beam under thermal shock

Fig.3 Comparison of deflection responses of FGMs under thermal shock

Fig.4 Effect of volume friction exponent on temperature responses of FGM beams

Fig.5 Temperature distribution of FGM beams through the thickness at different time

Fig.6 Effect of volume friction exponent on temperature distribution of FGMs beam through thickness

Fig.7 Effect of elastic foundation types on nonlinear deflection responses of FGM beams

Fig.8 Effect of elastic foundation stiffness on nonlinear deflection responses of FGM beams

Fig.9 Effect of volume friction exponent on nonlinear deflection responses of FGM beams

Fig.10 Effect of thermal shock amplitude on nonlinear deflection responses of FGM beams

Fig.11 Effect of length-to-thickness ratio on nonlinear deflection responses of FGM beams

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Fig.11 Effect of length-to-thickness ratio on nonlinear deflection responses of FGM beams