Displacement Measurement using Digital Speckle Multi-frequency Harmonic Wave Correlation Method

Hongmao Zhu¹, Lili Hu^{2*1}

¹*Wenhua College, Wuhan 430074 China*

²*School of Civil Engineering and Mechanics, Huazhong University of Science and Technology, Wuhan 430074 China*

Abstract

In the displacement measurement using digital speckle correlation method, significant measuring error due to the multi-peak nature of the correlation coefficient function is a critical issue as sometimes the secondary peak can be wrongly determined as the maximum instead of the highest peak. In the multi-frequency harmonic wave correlation method (MHCM) as proposed in this work, for speckle patterns before and after the displacement, correlation calculations were performed on corresponding sets of data matrices assembled by amplitudes/intensities obtained from harmonic waves with different frequencies. The calculated maximum correlation coefficients were compared with each other to locate the maximum in order to measure the displacement. Both ultrasonic speckle MHCM and laser speckle MHCM were applied to the displacement measurement respectively. Measured results indicated that MHCM effectively helps to ensure the correct determination of the maximum correlation coefficient, to avoid random errors caused by other factors and improves the measurement accuracy in the meanwhile.

Key words: harmonic wave; digital speckle pattern; correlation method; displacement measurement

1 Introduction

In early 1980s, I Yamaguchi in Japan and W.H. Peters and W.F. Rason etc. in the US proposed the idea of Digital Speckle Correlation Method (DSCM) [1][2]. It utilizes digital computational technique to replace the traditional calculation method based on laser speckle interference fringes, and was successfully applied to rigid body displacement measurements [3]. After that, DSCM was intensively studied and improved to be a widely accepted engineering measuring method [4][5]. Similarly, speckle patterns can also be generated at the interface (or inside the media) by the interference of scattered waves when an ultrasonic wave is incident onto a rough interface or is propagating in an inhomogeneous media. Compared with laser light, ultrasound can propagate not only in gases and liquids, but also in solids, which enables underwater displacement measurements and inner surface deformation measurements in nontransparent objects. Based on this

* Corresponding author, email: llhu@hust.edu.cn

advantage, the ultrasonic speckle measurement method may open up a new field for underwater engineering measurement as well as the study of the interior deformation in solids. Earlier works on ultrasonic speckles mainly focus on medical imaging [6][7]. Engineering based applications attracted more attention in recent years. Experimental and analytical studies have been performed on the statistical characteristics of the first and the second order ultrasonic speckle field reflected at rough surfaces [8][9], and the ultrasonic DSCM was used to measure the underwater displacement, underwater strain and vibration coefficients [10][11][12].

The correlation coefficient in laser or ultrasonic DSCM is a multi-peak function. Because of the multi-peak nature when searching for the position of the maximum correlation coefficient, from time to time the secondary peak was wrongly determined as the highest peak and resulting in significant measurement errors. To avoid such misjudgments, a digital speckle multi-frequency harmonic correlation method (MHCM) is proposed in this work, that a spectrum analysis is performed on the collected ultrasonic speckle patterns. Data matrices consisting of harmonic wave amplitudes at different frequencies are assembled, and correlation calculations are performed respectively on set of data matrices before and after the displacement to obtain comparable results. By taking average of results from multiple groups of measurements, the measurement accuracy can be improved. For laser speckle MHCM on the other hand, different monochromatic speckle patterns can be obtained when laser light with different wavelengths are used, correlation calculations can be performed respectively before and after the displacement with each color. Comparing different groups of results helps to minimize the possibility of significant measurement error and improves the accuracy in the meantime.

In this way, significant measuring error can be avoided from both the misjudgment of the highest peak of correlation coefficient function and other random factors, including the electrical noise and the mechanical error (error from the stepping size and from the stepping motor in fixing the position of the sampling point). Besides, for underwater applications, the influence of the water fluctuation is a very important issue. Water is incompressible most of the time, so the fluctuation of water will not change the density of water. Therefore the index of refraction in water can be treated as a constant. Ultrasonic waves travel in water at a speed around 1500 m/s, so the refraction index of ultrasonic waves in water can be hardly influenced by a water speed less than 10 m/s. In this way, the water fluctuation is considered a minor influence for underwater applications for MHCM.

2. Theories

In MHCM, speckle patterns are recorded on the object surface before and after deformation. Information including displacement and strain at different positions can be extracted based on the probability statistical correlation on the amplitude/intensity distributions of speckles in small regions.

Ultrasonic and laser speckle MHCM shares similar calculation criteria but with discrepancies. In laser speckle MHCM, laser as an excellent monochromatic light source is adopted. A CCD camera records speckle patterns. Speckle intensity is represented by the grey scale of pixels, with which data matrices can be

determined. In ultrasonic speckle MHCM, a string of short pulses with poor monochromaticity is used instead. The speckle intensity is resulted from the superposition of interference waves with all different frequencies within the bandwidth. As a result, the intensity difference at different positions is not high enough to define a data matrix suitable for correlation calculation. On the other hand, receiving ultrasonic speckle signals simultaneously over the entire measuring surface is not currently possible due to instrument limitations. Therefore in ultrasonic speckle MHCM for the time being, 2D step by step scan of the testing surface is conducted using a focusing probe to collect the speckle signal at different positions. Then a discrete fast Fourier transform (DFFT) is performed on the collected speckle signals to obtain the amplitude spectrum. Amplitude at specific frequencies at different positions were extracted as the amplitude spectrum of that very frequency. As shown in the example in Figure 1 that the amplitude at frequency F_1 is determined as the characteristic value of the signal, and a data matrix can be assembled after interpolation.

Correlation calculations can be performed on matrices before and after the displacement using standard correlation coefficient $C_1(u, v)$ as the following:

$$C_1(u, v) = \frac{\sum_{i=1}^n \sum_{j=1}^n [f_1(x_i, y_j) - \bar{f}_1] [g_1(x'_i, y'_j) - \bar{g}_1]}{\sqrt{\sum_{i=1}^n \sum_{j=1}^n [f_1(x_i, y_j) - \bar{f}_1]^2} \sqrt{\sum_{i=1}^n \sum_{j=1}^n [g_1(x'_i, y'_j) - \bar{g}_1]^2}} \quad (1)$$

in which, $f_1(x_i, y_j)$ ($i, j = 1, 2, \dots, n$) is the amplitude distribution matrix m_1 at a subdivision with harmonic frequency F_1 before the displacement; $g_1(x'_i, y'_j)$ ($i, j = 1, 2, \dots, n$) is the amplitude distribution matrix M_1 at the corresponding subdivision with harmonic frequency F_1 after the displacement; $\bar{f}$ and $\bar{g}$ are the average amplitude of the subdivision before and after the displacement, with $\bar{f}_1 = \sum_{i=1}^n \sum_{j=1}^n f_1(x_i, y_j) / n^2$, $\bar{g}_1 = \sum_{i=1}^n \sum_{j=1}^n g_1(x'_i, y'_j) / n^2$; $x'_i = x_i + u$, $y'_j = y_j + v$, with u and v the displacement in x and y directions in the corresponding subdivision respectively; correlation coefficient C_1 is in the range $[-1, 1]$. When matrices m_1 and M_1 are so similar to each other, $C_1(u, v)$ is very close to or equals to 1, indicating the overlap of the center of m_1 with the center of M_1 after the displacement. In application of MHCM, it is very important that the subdivision M_1 associated with m_1 needs to be located after the displacement. Therefore the position of the highest peak needs to be located, which involves the proper searching protocol and the determination of the highest peak.

The correlation coefficient $C_1(u, v)$ is usually a multi-peak function as shown in the spatial distribution in Figure 2. The secondary peak can sometimes be misjudged as the highest peak leading to a significant measurement error. To avoid such mistakes, MHCM is proposed in this work. In MHCM, besides

current measuring results, another set of amplitude is chosen with frequency F_2 from the ultrasonic speckle amplitude spectrum at a sampling point before and after the displacement as shown in Figure 1, based on which new amplitude distribution matrices m_2 and M_2 can be set up as $f_2(x_i, y_j)$ ($i, j = 1, 2, \dots, n$) and $g_2(x'_i, y'_j)$ ($i, j = 1, 2, \dots, n$). Correlation calculation is also performed using Equation 1 to find the maximum $C_2(u, v)$ and the u and v of the corresponding position of the maximum $C_2(u, v)$. If the value of u and v are close to or equal to that at maximum $C_1(u, v)$, both calculations are considered correct. The accuracy of displacement measurement can be further improved by taking average of the two calculated displacements. However if the value of u and v dramatically deviate from the corresponding value at maximum $C_1(u, v)$, a third set of amplitude needs to be chosen with frequency F_3 as shown in the example in Figure 1. New amplitude distribution matrices m_3 and M_3 can be set up for calculations. The iteration proceeds until true values of u and v are obtained. By averaging multiple similar displacement calculation results from MHCM, measurement error from the misjudgment of the highest peak can be avoided, and also the measurement accuracy can be improved.

For laser speckle MHCM, laser diode with multiple wavelengths is used as the light source. When monochromatic laser light with different wavelengths is used in the same optical path to shine onto the object surface, speckle patterns with different distributions can be obtained. Digital correlation calculations can then be performed on each set of speckle patterns at corresponding wavelength before and after the displacement. In this way, three measurements (e.g. three different wavelengths are used) determined by the maximum correlation coefficient can be obtained at the same testing point. The consistency of the three results is checked. If their difference exceeds the setting limit, it can be treated as the error result caused by the misjudgment of the maximum correlation coefficient or by other random factors. Averaging the consistent measuring results yields a more accurate displacement measurement.

During the measurement, object displacement is not usually the integer multiple of the pixel pitch, so it is necessary to obtain a sub pixel displacement. In laser speckle MHCM, interpolation and fitting of the correlation coefficient is often convenient to obtain the sub pixel displacement. Besides that, grey level interpolation is also an alternative, which interpolates between two adjacent sampling points on grey level speckle image, namely the integer pixels. To accomplish the required measuring accuracy at sub pixel level, the grey level at the position of the sub pixel can be obtained prior to the correlation calculation. In this way, higher accuracy can be obtained and with relatively stable computing results, only with a higher computational expense. However due to the limitation of instruments, stepping mode sampling of the ultrasonic speckle is adopted with a single focusing probe, yielding very few sampling data, which facilitates the usage of grey level interpolation. A two-variable cubic spline interpolation is applied to elements in amplitude matrices assembled

by the speckle signal amplitude spectrum with frequencies F_1 , F_2 and F_3 in the testing zone, a corresponding smooth and 2D continuously distribution of amplitude can be obtained to improve the measuring accuracy.

3 Application of ultrasonic speckle MHCM

Ultrasonic speckle MHCM is applied to an underwater object for 2D displacement measurement. The experimental setup is shown in Figure 3. The sample S is a thick aluminum plate (80mm×60mm×10mm) coated with 80 mesh aluminum powder. The powder-coated rough surface is favored by speckles. An alternative way is to polish the surface with course sand paper. S is placed horizontally on a manual translation stage D_1 . D_1 is movable in X-Y plane with an accuracy of 0.01 mm. The displacement reading from D_1 is considered as the true displacement. Ultrasonic transmitting probe T ($\Phi=40\text{mm}$, $f=5.0\text{MHz}$) is driven by an ultrasonic signal generator/receiver (CTS230A, Guandong Goworld Co.,Ltd.) to transmit ultrasound that is incident obliquely onto the surface of the aluminum plate. Ultrasonic focusing probe R ($\Phi=25\text{ mm}$, $f=5.0\text{MHz}$, $F=40\text{mm}$) focuses onto the aluminum surface to receive speckle signals. Its 6db focal spot has a diameter of 0.68 mm. Probes T and R are fixed on the stepping translation stage D_2 (TS202, Beijing Optical Instrument) driven by a stepping motor to allow movement in X and Y directions.

The average lateral dimension of the ultrasonic speckles on the sample surface is 0.92 mm according to reference [8] in the above experimental setup. D_2 is driven to scan the central region of the sample to obtain 40*40 speckle signals using R. The step size is 0.46 mm, half of the average lateral dimension of speckles. The scanning region is also named as the sampling zone (zone B) as shown in Figure 4. An ultrasonic signal receiver receives speckle signals. These signals are then A/D converted in a digital storage oscilloscope (TDS 2012B, Tektronix), and analyzed in a computer. The amplitude at harmonic frequencies $F_1=5.0\text{ MHz}$, $F_2=4.9\text{ MHz}$ and $F_3=5.1\text{ MHz}$ are chosen to assemble the three different 2D data matrices before the displacement. To improve the accuracy, a two-variable cubic spline interpolation is also applied on the 40*40 speckle data matrices. Two points are interpolated between adjacent two sampling points, yielding a 118*118 data matrix. A smaller rectangular region consisting of 21*21 measuring points is chosen as shown in the measurement zone (zone C) in Figure 4 from the 118*118 data matrix from zone B. This experiment measures displacements of these 21*21 measuring points in zone C. It is needed to note that the maximum measured displacement is limited by the size of zone B.

The translation stage D_1 is manually adjusted to provide the aluminum plate with a rigid movement of 3.0 mm both in the X and in the Y directions. The true displacements are then determined as $u=3.00\text{ mm}$, $v=3.0\text{ mm}$ for all 21*21 measuring points. Displacement measurements by MHCM are then performed for all points. Before the rigid displacement, 9*9 sampling points around the measuring point were chosen as a sample subdivision. After the rigid displacement, it is necessary to search for the most relevant object subdivision matrix assembled by also 9*9 sampling data in the whole zone B. Displacement of the measuring

point is the relative displacement of the center of the most relevant object subdivision with respect to the center of the sample subdivision.

Correlation measurements were performed on all 21*21 measuring points with their data matrices assembled with amplitude of harmonic waves with frequencies $F_1=5.0$ MHz, $F_2=4.9$ MHz and $F_3=5.1$ MHz respectively. During the measurement by MHCM, if the error of two successive measurements at a same point exceeds the setting limit, it is treated as a significant measurement error. In that case more measurements are needed to obtain a valid result. The measured displacement u in X direction is shown in Figure 5 in which the measured displacement difference is indicated by different colors. As indicated by white circles, two points in Figure 5(a), a single point in Figure 5(b) and a single point in Figure 5(c) can be observed to be significantly different with others and also deviate a lot from the preset true displacement. These differences are believed to be induced by random measurement errors including the misjudgment of the highest peak of correlation coefficient function. Remove all the four points and average the rest measuring results in the three figures of Figure 5 respectively, yields average displacement as shown in Figure 6(a). Figure 6(b) shows the true displacement in the X direction for comparison. The improved measurement accuracy can be noticed immediately as compared to Figure 5. The measuring error of only 19 testing points out of 441 exceeds 5%.

With the same procedure, displacement v in the Y direction was also measured. Significant measuring error was observed only in three points. Remove these points and average 3 sets of measurements yields measurement results with improved accuracy, of which error exceeding 5% were found only on 15 points.

4. Application of laser speckle MHCM

Laser has excellent monochromaticity compared with ultrasonic waves. Unlike ultrasonic speckle MHCM, it is difficult to perform frequency analysis and to assemble data matrices using amplitudes from different frequency harmonic waves in laser speckle MHCM. However, if a dual laser or a multi-wavelength laser is used to shed the object surface with different wavelengths, two or more speckle patterns can be observed with different distribution of speckle intensities. Digital correlation calculation can also be performed on the speckle patterns before and after the displacement by different frequency monochromatic laser light. The displacement results can be computed and compared with each other to avoid significant measuring error due to the misjudgment of the highest peak of correlation coefficient function and other random factors. The optical setup of the laser speckle MHCM is shown in Figure 7. A multi-wavelength laser (TLB-6700, Newport Company) is used as the light source with the wavelength ranging from 633 nm to 1630 nm. Speckle signals are collected by a CCD camera (MR4201-BH, Ximea Company) with a 1 inch target plane, 2048*2048 pixels and a pixel size as 7.4 μm . Measured results with improved accuracy are also obtained by laser speckle MHCM. The obtained displacement distribution figure is quite similar with Figure 5, so there is no need to provide a repeated figure here.

5. Conclusions

In ultrasonic and laser speckle correlation methods, it is important but also difficult to avoid the significant measurement error due to the misjudgment of the highest peak of correlation coefficient function. MHCM proposed in this work is an effective way to avoid such misjudgment during the correlation search and can also avoid other significant random errors. In addition, benefit from the averaging of the measured data sets, the accuracy can be greatly improved. 2D in-plane displacement of an underwater object was measured in this work using a lab setup device based on the proposed method. A good agreement of the measured results with true displacement was obtained. Only 19 testing points out of 21*21 showed measurement error greater than 5%.

During the speckle correlation measurement, strain is calculated by taking derivative of the measured displacement. A great strain measurement error can be expected due to small displacement measurement error. Therefore, because of the improved accuracy in displacement measurement, the proposed method is able to set up a solid foundation for the accurate measurement of surface and inner surface strain for underwater objects using ultrasonic speckles. On the other hand, by replacing the single wavelength laser with the multiple wavelengths laser, the proposed method is also applicable for laser speckle correlation method to avoid significant error from random factors and improve the measurement accuracy.

6. Acknowledgements

This work is financially supported by the National Science Foundation of China (No. 10972085).

References

- [1] I Yamaguchi. Speckle displacement and decorrelation in the diffraction and image fields for small object deformation. *Optica Acta*, 1981, 28(10) :1359 - 1376
- [2] W H Peters, W F Ranson. Digital imaging techniques in experimental stress analysis[J]. *Optical Engineering*, 1982, 21(3) :427 – 431
- [3] W H Peters, W F Ranson, M A Sutton, et al. Application of digital correlation Methods to rigid body Mechanics. *Optical Engineering*, 1983, 22(6):738-742
- [4] M Shimizu, H Sawano, H Yoshioka, H Shinno. Surface texture assessment of ultra-precision machined parts based on laser speckle pattern analysis . *Precision Engineering*. 2014, 38(1):1-8
- [5] S Bjaerum, D Martens, K Kristoffersen, H Torp. Blood motion imaging — A new technique to visualize 2D blood flow. *Proc. of IEEE Ultra. Symp.*, 2002, 2:1507-1510
- [6] W Kosmala, R Plaksej and J M Strotmann *et al.*. Progress of left ventricular functional abnormalities in hypertensive patients with heart failure: An ultrasonic two-dimensional speckle tracking study. *J. of Ame. Soc. of Echocardiography*, 2008,21(12):1390-1417
- [7] D Abookasis, T Moshe. Feasibility study of hidden flow imaging based on laser speckle technique using multiperspectives contrast images. *Optics and Lasers in Engineering*. 2014, 62:38-45
- [8] H M Zhu, Q H Qin. Statistics of ultrasonic speckle reflected from a rough surface. *Archive of Applied Mechanics*, 2002, 72:189-198
- [9] H M Zhu, Y Y Wu, Y G Wang. Average longitudinal size of ultrasonic speckles back-scattered from an interface. *Archive of Applied Mechanics*, 2004, 73: 841-845
- [10] H M Zhu, Y Y Wu, W H Zheng . In-plane and out-of-plane displacement measurement by ultrasonic speckle correlation method (USCM), *Archive of Applied Mechanics*, 2006, 75: 521-526
- [11] Z H Luo, H M Zhu, J Chu, L Shen, L L Hu. Strain measurement by ultrasonic speckle technique based on adaptive genetic algorithm. *Journal of strain analysis for engineering design*, 2013,48(7):446-456

- [12] Z H Luo, J Chu, L Shen, P Hu, H M Zhu, L L Hu. Measurement of underwater vibration by ultrasonic speckle stroboscope technique. *Measurement*, 2014,47:938-945

List of figures:

Figure 1 Spectrum diagram of ultrasonic speckle signal.

Figure 2 Spatial distribution of correlation coefficient.

Figure 3 Displacement measurement system of ultrasonic speckle technique.

Figure 4 Relation between the specimen (zone A), the sampling zone (zone B) and the measurement zone (zone C).

Figure 5 Measured displacement u in X direction with wave frequencies (a)5.0MHz; (b) 4.9MHz; (c)5.1MHz.

Figure 6 Measured displacement u : (a) average displacement by MHCM; (b) true displacement.

Figure 7 Optical setup of the laser speckle MHCM for displacement measurement.

Figure 1 Spectrum diagram of ultrasonic speckle signal

Figure 2 Spatial distribution of correlation coefficient

Figure 3 Displacement measurement system of ultrasonic speckle technique

Figure 4 Relation between the specimen (zone A), the sampling zone (zone B) and the measurement zone (zone C)

Figure 5 Measured displacement u in X direction with wave frequencies (a)5.0MHz; (b) 4.9MHz; (c)5.1MHz

Figure 6 Measured displacement u : (a) average displacement by MHCM; (b) true displacement

Figure 7 Optical setup of the laser speckle MHCM for displacement measurement