Statistical Adaptive Metric Learning in Visual Action Feature Set Recognition

Shuanglu Dai^{a,*}, Hong Man^a

^a*Department of Electrical and Computer Engineering, Stevens Institute of Technology, 1 Castle Point on Hudson, Hoboken, New Jersey, NJ07030, U.S.A.*

Abstract

Great variances in visual features often present significant challenges in human action recognitions. To address this common problem, this paper proposes a statistical adaptive metric learning (SAML) method by exploring various selections and combinations of multiple statistics in a unified metric learning framework. Most statistics have certain advantages in specific controlled environments, and systematic selections and combinations can adapt them to more realistic “in the wild” scenarios. In the proposed method, multiple statistics, include means, covariance matrices and Gaussian distributions, are explicitly mapped or generated in the Riemannian manifolds. Typically, d-dimensional mean vectors in R^d are mapped to a $R^{d \times d}$ space of symmetric positive definite (SPD) matrices Sym_d^+ . Subsequently, by embedding the heterogeneous manifolds in their tangent Hilbert space, subspaces combination with minimal deviation is selected from multiple statistics. Then Mahalanobis metrics are introduced to map them back into the Euclidean space. Unified optimizations are finally performed based on the Euclidean distances. In the proposed method, subspaces with smaller deviations are selected before metric learning. Therefore, by exploring different metric combinations, the final learning is more representative and effective than exhaustively learning from all the hybrid metrics. Experimental evaluations are conducted on human action recognitions in both static and dynamic scenarios. Promising results demonstrate that the proposed method performs effectively for human action recognitions in the wild.

Keywords: Feature Set Classification; Hybrid statistic modeling; Metric Learning; Manifolds selection

1. Introduction

Set classification has been studied within computer vision communities for a long period of time. In gait recognition, for example, frame by frame static features of a certain object are considered as a feature set. Similarly in human action recognition, spatial-temporal features uniformly extracted from frames of an action atom are considered as a feature set [1]. In addition, image sets have been commonly used in face recognitions

*Corresponding author: sdai1@stevens.edu

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[2, 3]. The task of feature set classification is to classify an input feature set to one of the sets in the training gallery [4]. Compared to image sets, feature sets are more diverse. They can not be easily assumed to follow certain distribution or lie in some scale and affine invariant linear subspace. One of the effective techniques handling such problem is by using statistical representations to substitute the original feature samples. For action recognition in the wild scenarios, combinations of statistics from lower-order to higher-order have shown promising representation capabilities, while how to combine these multiple statistics in a near optimal way remains a technical challenge [4, 5].

In general, three types of statistics have been commonly applied on set modeling, i.e. sample-based statistics (SAS) [6, 7, 8, 9], subspace-based statistics (SUS) [10, 11, 12, 13, 14, 15, 16] and distribution-based statistics (DIS) [2, 3]. Utilizing affine transformation and centroid of samples, sample-based statistics represent d-dimension feature sets with first-order statistics in the R^d space. A great advantage of SAS is that samples are considered as vectors so the nearest neighbor (NN) classification can be easily implemented with unified distance measures. But sample tests performed at every individual sample are often computationally expensive. Well-known sample-based methods include Minimum Mean Discrepancy (MMD)[17], Affine (Convex) Hull based Image Set Distance (AHISD, CHISD)[18], Set-to-Set Distance Metric Learning (SSDML)[9] and Information Theoretic Metric Learning (ITML)[19]. Differing from sample-based statistics, subspace-based statistics analyze sets lying on a specific Riemannian manifold. By learning the kernel functions or statistical metrics, the subspaces are projected back to the Euclidean spaces. The distance measures from the Riemannian manifold to the Euclidean space is often considered as the true geodesic distances, which lie in a Hilbert space [20]. Distance discriminant functions are then performed on the Hilbert space, and recognitions can finally be achieved by using Nearest Neighbors (NNs) method. Second-order statistics based methods have better representation of the data, but it is hard to design a discriminant function with a unified distance measure for the manifolds. Typical subspace-based methods include Mutual Subspace Method (MSM) [11], Discriminant Canonical Correlations (DCC)[10], Manifold Discriminant Analysis (MDA) [14], Grassmann Discriminant analysis (GDA)[13], Covariance Discriminative Learning (CDL)[15], Localized Multi-Kernel Metric Learning (LMKML)[16] etc. Distribution based statistics model each sample in the feature set with a distribution, which can be expressed as an expansion of the Riemannian manifold from the 2nd-order statistic space Sym_d^+ to Sym_{d+1}^+ . Such methods are often with 3rd-order statistics and may lead to complex parametric distribution comparison. Typical examples include Single Gaussian Models (SGM) [2], Gaussian Mixture Models (GMM) [3] and kernel version of ITML with DIS-based set model (DIS-ITML). Although 3rd-order statistics model sets with more consolidated representations, the hypothesis tests often require significant amount of computation in distribution comparisons.

More adaptive forms of set modeling methods have been proposed by combining multiple statistical metrics in certain heuristic ways. Some of the recent hybrid statistical models include Projection Metric Learning (PML) on Grassmann manifold and hybrid Euclidean-and-Riemannian Metric Learning (HERML) [21]. The main idea of multiple statistics combination is to project measurements from multiple heterogeneous spaces into high-dimensional Hilbert spaces. The key issue then becomes the learning of unified discriminant functions from the training sets. Due to the simplicity of the subspace mapping, discriminant functions can frequently be found from single statistics. For instance,

Set-to-Set Distance Metric Learning (SSDML) learns a proper metric between pairs of
55 single vectors in Euclidean space to obtain more accurate set-to-set affine hull based
distance for classification; Localized Multi-Kernel Metric Learning (LMKML) maps the
3rd-order statistics into Euclidean spaces by learning a unified metric discriminant func-
tion through reproducing kernel Hilbert spaces (RKHS). While hybrid multiple statistics
60 perform well in realistic outdoor scenarios, but their unified discriminant functions are
often more difficult to design. Addressing the subspace discrimination problems, Shao et
al. proposed a kernelized multiview projection (KMP) for action feature set recognition.
KMP discriminatively assigns weights to multiple kernelized sets with a single feature
to achieve a low-dimensional subspace. However, weighting kernels directly in the linear
subspaces is not an optimal way for learning kernelized sets with multiple features in
65 different scales. [22]

Aiming at classification for sets with multiple features, this paper proposes an adap-
tive subspaces analysis method for learning hybrid statistical metrics. The analyzed
single or multiple statistics can be used to classify sets through various feature combina-
tions in different scales. Inspired by the discriminant function design in the second-order
70 based methods, LogDet divergence is introduced as a unified discriminant function for
our metric learning. With this discriminant function, our method effectively unifies d-
ifferent statistics into a common measurement. Thus nearest neighbor method can be
easily performed for classification. The whole process of modeling and learning consist-
s of several steps. Firstly, heterogeneous statistics including mean, covariance matrix
75 and Gaussian distribution are introduced to project data into high-dimensional Hilbert
spaces. Typically, d-dimensional mean vectors represent samples from R^d to Sym_d^+ ex-
panded by a Point-to-Set projection; covariance matrices lie in Riemannian manifold
 Sym_d^+ and multivariate Gaussian distributions expand the second order statistics into
Riemannian manifold Sym_{d+1}^+ expressed by relative entropy. Secondly, by embedding
80 the heterogeneous spaces into high-dimensional Hilbert spaces, the Mahalanobis distance
is introduced as our discriminant metric. Then, the Hilbert spaces selection is conducted
based on the minimum Hilbert subspaces. The hybrid statistics are then reduced to sin-
gle or multiple statistics combination. Finally, with LogDet divergence that maps all the
Hilbert space points into R^d , a constrained kernel learning is performed. Recognitions
85 are mainly conducted on video sequences in both static and dynamic scenarios using
spatial image features such as edges, SIFT, HOG, and texture features.

2. Statistical Feature Set Modeling

2.1. Statistics and Subspaces Embedding

2.1.1. Data Statistics

90 Let $X = [X_1, \dots, X_N]$ denotes the training set formed by N feature sets, where $X_i =$
 $[x_1, x_2, \dots, x_M] \in R^{n_i \times d}$ indicates the i-th feature set, $1 \leq i \leq N$, and n_i is the number
of samples in this set. It is known that the kernel function is always defined by firstly
mapping the original features to a high dimensional Hilbert space, that is $\phi : R^d \rightarrow F$ or
 $Sym^+ \rightarrow F$, and then calculating the dot product of high dimensional statistics Φ_i and
95 Φ_j in the new space. Considering ϕ as an explicit mapping with the statistical kernels,
 Φ_i^r denotes the high dimensional feature of r-th statistics extracted from the feature set
 X_i . Here, $1 < r < R$ and R is the number of statistics being used.

We uniformly map feature set $X_i, 1 \leq i \leq N$ with following three statistics: sample-based, subspace-based and distribution-based statistics.

100 Sample-based statistics (SAS): Supported by Bregman divergence, mean vector is considered as one of the important properties describing the probability distributions. It is often used to measure the central tendency of set of samples. Given sample $x_k \in X_i, 1 \leq k \leq M$, the mean vector μ_i of X_i is computed as: $\mu_i = \frac{1}{M} \sum_{k=1}^M x_k$.

Subspace-based statistics (SUS): Within subspaces derived from eigen-decomposition, 105 set variances are influenced by covariant matrix. Given sample $x_k \in X_i, 1 \leq k \leq M$, the covariant matrix C_i of X_i is computed as: $C_i = \frac{1}{M-1} \sum_{k=1}^M (x_k - \mu_i)(x_k - \mu_i)^T$.

Distribution-based statistics (DIS): Gaussian distribution is a very commonly occurring probability distribution, which is a continuous distribution with the maximum entropy for a given mean and variance. Therefore, the d-dimensional distribution of set 110 X_i is modeled as a Single Gaussian Model (SGM) with an estimated d-dimensional mean vector $\hat{m}_{k,i}, 1 \leq k \leq d$ and a covariance matrix $\hat{C}_i$: $x \sim N(\hat{m}_i, \hat{C}_i)$

2.1.2. Subspaces Embedding and Canonical Correlation-based Selection

A point μ in the Euclidean space R^d can be mapped into a symmetrical positive definite matrix $\{\mu\mu^T | R^{d \times d}, |\mu\mu^T| > 0\} \in Sym_d^+$. For DIS, the space of Gaussian distribution 115 is able to be embedded into a Riemannian manifold Sym_{d+1}^+ . [20]

Theorem 1. Let $G = \{\gamma|dx|, x \in R^d\}$ be a space of normal distribution, where $|dx|$ is Lebesgue measurement. Then its positive definite affine space Aff_d^+ has an explicit embedding $Aff_d^+ \rightarrow Sym_{d+1}^+$ lying on the Riemannian symmetric space Sl_{d+1}/SO_{d+1} .

Proof. Denote an affine group of G in R^d : $Aff_d = \{(m, Q) | x \rightarrow Qx + m, Q \in G_d, m \in R^d\}$ acts transitively on G by $\gamma|dx| \rightarrow (m, Q) \otimes \gamma|dx|$, where $\otimes$ denotes the transitive operator. 120

Assume $\gamma_0|dx| = (2\pi)^{-d/2} e^{-\frac{1}{2}|x|^2} |dx|$ as an standard Gaussian distribution on R^d , where $\pi^{-1}(\gamma_0|dx|) = O_d$ is a positive measure. The transitive operation $(m, Q) \otimes \gamma|dx|$ can be explicitly written as

$$(m, Q) \otimes \gamma|dx| = (2\pi)^{-d/2} (detQ)^{-1} e^{-\frac{1}{2}|Q^{-1}(x-m)|^2} |dx| \quad (1)$$

only when $detQ > 0$. Therefore, we add a restriction θ to keep affine group positive definite $\theta : Aff_d^+ = \{(m, Q) | detQ > 0\}$. Since the restriction is also transitive on G , where $G_d = Aff_d^+/SO_d$. Considering Aff_d as a subgroup of a larger simple Lie group $SL_{d+1} = \{V | V \in R^{d+1}\}$, where $Aff_d \subset SL_{d+1}$, (m, Q) can be embedded as

$$Aff_d \rightarrow Sl_{d+1}, (m, Q) \rightarrow V, (m, Q) \rightarrow (detQ)^{-\frac{1}{d+1}} \begin{pmatrix} Q & m \\ 0 & 1 \end{pmatrix} \quad (2)$$

Since the simple Lie group Sl_{d+1} acts canonically on all symmetric definite positive matrices Sym_{d+1}^+ with determinant 1. In order to meet the restriction θ of the subgroup Aff_d , the simple Lie group Sl_{d+1} is mapped to a symmetric definite positive space $\tilde{\theta} : Sym_{d+1}^+ = \{VV^T | VV^T \in R^{(d+1) \times (d+1)}, |VV^T| > 0\}$, where

$$Sl_{d+1} \rightarrow Sym_{d+1}^+, V \rightarrow VV^T, (m, Q) \rightarrow (detQ)^{-\frac{2}{d+1}} \begin{pmatrix} QQ^T + mm^T & m \\ m^T & 1 \end{pmatrix} \quad (3)$$

Then the transitive action of the restriction for the simple Lie group Sl_{d+1} : $G_{d+1} = Sl_{d+1}/SO_{d+1}$ lies on the Riemannian symmetric space with SO_{d+1} -principle bundle $\tilde{\theta}: Sl_{d+1} \rightarrow Sym_{d+1}^+, V \rightarrow VV^T$. Hence, Theorem 1 is proven. $\square$

In DIS, $QQ^T = \hat{C}$. Therefore, after embedding, we have 3 statistical representations:

$$\Phi_{SAS} = \{\mu\mu^T | \mu\mu^T \in R^{d \times d}, |\mu\mu^T| > 0\} \quad (4)$$

$$\Phi_{SUS} = \{C | C \in R^{d \times d}, |C| > 0\} \quad (5)$$

$$\Phi_{DIS} = \{VV^T | VV^T = \begin{pmatrix} \hat{C} + \hat{m}\hat{m}^T & \hat{m} \\ \hat{m}^T & 1 \end{pmatrix}, VV^T \in R^{(d+1) \times (d+1)}, |VV^T| > 0\} \quad (6)$$

where $\Phi_{SAS} \in Sym_d^+, \Phi_{SUS} \in Sym_d^+, \Phi_{DIS} \in Sym_{d+1}^+$. The SPD matrices lie in a specific Riemannian manifold, where tangent vector of Sym^+ is often utilized as a projection to R^d . For the three different embedded statistics and all their possible combinations, we examine logarithm Hilbert subspace through an eigen-decomposition: $\log(\Phi) = U(\log(\Lambda))U^T$ where $\Phi = \{\Phi_{SAS}, \Phi_{SUS}, \Phi_{DIS}, \Phi_{SAS}\Phi_{SUS}, \Phi_{SAS}\Phi_{DIS}, \Phi_{SUS}\Phi_{DIS}, \Phi_{SAS}\Phi_{SUS}\Phi_{DIS}\}$ indicates the set of all possible statistic combinations; $\Lambda = \{\Lambda_{SAS}, \Lambda_{SUS}, \Lambda_{DIS}, \Lambda_{SAS}\Lambda_{SUS}, \Lambda_{SAS}\Lambda_{DIS}, \Lambda_{SUS}\Lambda_{DIS}, \Lambda_{SAS}\Lambda_{SUS}\Lambda_{DIS}\}$ indicates the set of all the possible eigen-value matrices of Φ , respectively; U is the decomposed projection matrices. Set Φ describes eigen-spaces Λ of all the possible statistics combinations. The last element utilizes all the statistics, while the first three have only single statistic. The logarithmic eigen-values measures the tangent subspaces of the symmetric positive definite Riemannian manifold. In the mutual subspaces method (MSM) and discriminant canonical correlations (DCC) method the diagonal eigen-matrix is also known as canonical correlations. [10][23]

In our work, subspace with minimum deviation of the canonical correlations is selected from set Φ . The canonical correlation is computed as: $\cos\Theta = \text{diag}(\log\Lambda)$. Subspaces are selected by $\min\{\text{var}(\cos\Theta)\}$, where $\text{var}(\cdot)$ calculates the deviation and $\text{diag}(\cdot)$ denotes the diagonal matrix.

2.2. Riemannian Distance Discriminant Function

After the subspaces embedding and selection, N labelled training set $X = \{X_1, \dots, X_N\}$ becomes $R \times N$ labelled statistical points $\Phi = \{\Phi_1, \dots, \Phi_R\} = \{\{\Phi_1^1, \dots, \Phi_N^1\}, \dots, \{\Phi_1^R, \dots, \Phi_N^R\}\}$ lying on Sym^+ Riemannian manifolds, where R is the number of selected statistics and N is the number of sets for training.

In the Lie group structure, the Log-Euclidean distance (LED) for $X_i, X_j \in Sym^+$ is defined as $LED(X_i, X_j, t) = \exp((1-t)\log(X_i) + t\log(X_j))$ where $\exp(\cdot)$ and $\log(\cdot)$ are matrix exponential and logarithm operator. Assume two Riemannian points for r -th statistic Φ_i^r, Φ_j^r has two linear discriminative mapping Y_i^r, Y_j^r on the domain of matrix logarithms where $\log(\Phi_i^r)Y_i^r \perp \log(\Phi_j^r)Y_j^r$, the geodesic distance between $\log(\Phi_i^r)$ and

Figure 1: Mapped orthogonal subspace for a cannot-link (i, j) pairs. Φ_i, Φ_j are two Riemannian points and $\Phi_i Y_i, \Phi_j Y_j$ are Riemannian points after mapping. After mapping, $\log(\Phi_i) Y_i \perp \log(\Phi_j) Y_j$ and $\theta = \angle u' O v' = 90^\circ$

$\log(\Phi_j^r)$ after mapping is then computed by Euclidean distance in the domain of matrix logarithms:

$$\begin{aligned} d(\Phi_i^r, \Phi_j^r) &= \|\log(\Phi_i^r) Y_i^r - \log(\Phi_j^r) Y_j^r\|_F \\ &= \text{tr}((\log(\Phi_i^r) Y_i^r - \log(\Phi_j^r) Y_j^r)(\log(\Phi_i^r) Y_i^r - \log(\Phi_j^r) Y_j^r)^T) \end{aligned} \quad (7)$$

where $\|\cdot\|_F$ is the matrix Frobenius norm and $\text{tr}(\cdot)$ is the matrix trace. Fig.1 shows the mapped subspaces graphically for a cannot-link (i, j) statistic pair. As Fig.1 shows, $\theta = \angle u' O v'$ is the principle angle. The Riemannian distance actually measures the principle angle $\|\theta\|_2$. If the two points are similar, the distance $d(\Phi_i^r, \Phi_j^r) = 1$. If they are dissimilar, distance $d(\Phi_i^r, \Phi_j^r) = -1$. The relationship holds only when the assumed linear mapping Y_i, Y_j are equally orthonormal where $Y = Y_i^r = Y_j^r$ and $Y_i^r Y_j^r = I_d$, I_d is a $d \times d$ identity matrix.

Given a scale invariant positive definite matrix $A_r = Y_i^r Y_j^r$. It can be approximated with an identity matrix $A_0 = I_d$ with the distance measurement $d_{A_r}(\Phi_i^r, \Phi_j^r)$ constrained by the must-link and cannot-link training pairs (Φ_i^r, Φ_j^r) for the r -th statistics.

$$d_{A_r}(\Phi_i^r, \Phi_j^r) = \delta_{ij} \text{tr}(A_r (\log(\Phi_i^r) - \log(\Phi_j^r)) (\log(\Phi_i^r) - \log(\Phi_j^r))^T) \quad (8)$$

As a distance discriminant function, $d_{A_r}(\Phi_i^r, \Phi_j^r)$ shows outstanding scale invariant properties ranging from $[-1, 1]$. δ_{ij} is an indicator function for (i, j) pairs. If (i, j) is must-link, $\delta_{ij} = 1$; if (i, j) cannot-link $\delta_{ij} = -1$. A_r is a Mahalanobis-like distance metric approximated with an identity matrix. After the approximation of A_r , the classification can be simply completed by nearest neighbor (NN) method using d_{A_r} as its distance discriminant function.

2.3. Log-Determinant Divergence Based Learning Problem

As discussed in Section 2.2, the learning problem is to approximate R Mahalanobis-like metrics $A = \{A_1, \dots, A_R\}$ with identity matrix A_0 for R selected statistics. Two

Figure 2: Data space mapping

properties must be kept during the matrix approximation: First, $A_r, r \in [1, R]$ must be scale invariant; Second, A_r can be factorized as two linear mapping $A_r = Y_i Y_j^T$. Accordingly, Log-Determinant divergence is introduced as our measurement for the positive definite matrices approximation. Log-Determinant divergence is a typical measurement for positive definite matrices nearness problem proved in [24], which has also been proved to be convex and scale-invariant in [25]. The optimization problem is considered as seeking R discriminant metrics to describe the statistical characteristics of the $N \times R$ selected statistical feature sets. Finally, our metric learning problem is expressed as

$$\begin{aligned} \min_{A_1 \leq 0, \dots, A_R \leq 0, \epsilon} \frac{1}{R} \sum_{r=1}^R D_{ld}(A_r, A_0) + \gamma D_{ld}(\text{diag}(\epsilon), \text{diag}(\epsilon_0)) \\ \text{s.t.} \quad \frac{\delta_{ij}}{R} \sum_{r=1}^R d_{A_r}(\Phi_i^r, \Phi_j^r) \leq \epsilon_{ij} \forall (i, j) \end{aligned} \quad (9)$$

δ_{ij} denotes the similarity between i -th and j -th kernel. If they are similar, $\delta_{ij} = 1$, otherwise $\delta_{ij} = -1$. $D_{ld}(A_r, A_0)$ is the LogDet divergence measuring the correlation between matrix A_r and A_0 where $D_{ld}(A_r, A_0) = \text{tr}(A_r, A_0^{-1}) - \log \det(A_r A_0^{-1}) - d$. d is the dimension of A . ϵ_{ij} denotes the boundaries of similarity and dissimilarity, which is introduced as an upper/lower bound for the i -th and j -th sample covariance [26]. While the covariance between different classes may not always be the same, a trade-off is the self-deviation between ϵ and optimal slack ϵ_0 , which is supposed to be of unit at principle space. Thus, in order to control such a self-deviation, a second term is introduced as the trade-off correction term, where γ is a parameter arbitrarily set before the computation. In Eq.(13), the first term of the minimization corresponds to the maximum of inter-class and the second term corresponds to the minimum of intra-class.

Figure 2 gives a graphic illustration explaining the streamline of our methods. Firstly, various features set are mapped into their statistic subspaces; Secondly, by constructing the multiple statistic combinations, some combination is selected based-on its canonical correlation. Then, Riemannian distance is used as a discriminant function measuring the different statistics in the selected combination separately in the heterogeneous Hilbert space. Finally, all the statistics are measured in the Euclidean space through Mahalanobis-like identity matrices approximation with Log-Determinant divergence.

3. Optimizations

The form of Eq.(9) can be considered as Bregman optimizations for R selected statistics [25, 27]. With Bregman projection, Eq.(9) has the following Lagrange form:

$$\begin{cases} \nabla D_{ld}(A_r^{t+1}) = \nabla D_{ld}(A_r^t) + \delta_{ij}\alpha(\Phi_i - \Phi_j)^T A_r \\ \nabla D_{ld}(\epsilon_{ij}^{t+1}) = \nabla D_{ld}(\epsilon_{ij}^t) - \delta_{ij}\alpha/\gamma \end{cases} \quad (10)$$

where α is the Lagrange multiplier. The first and the second equation are the Bregman projection with respect to the minimizers $A_1, \dots, A_R$ and ϵ , respectively. Minimizer $A_r (r = 1, \dots, R)$ and ϵ are then iteratively updated by solving Eq.(15)

$$\begin{cases} A_r^{t+1} = A_r^t + \frac{\delta_{ij}\alpha}{1 - \delta_{ij}\alpha d_{A_r^t}(\Phi_i^r, \Phi_j^r)} A_r^t (\Phi_i, \Phi_j) (\Phi_i, \Phi_j)^T A_r^t \\ \epsilon_{ij}^{t+1} = \gamma \epsilon_{ij}^t / (\gamma + \alpha \epsilon_{ij}^t) \end{cases} \quad (11)$$

Solution to Eq.(15) is provided in Appendix A. For solving the Lagrange multiplier, multiple inequality constrains of Eq.(13) have to be satisfied $\frac{\delta_{ij}}{R} \sum_{r=1}^R tr(A_r(\Phi_i^r, \Phi_j^r)(\Phi_i^r, \Phi_j^r)^T) \leq \epsilon_{ij}$ for all (i, j) . Thus we introduce dual variables $\gamma \geq 0$. By meeting the KKT condition, α can then be derived into a Euler-Lagrange equation with non-negative maintaining

$$\begin{cases} \frac{\delta_{ij}}{R} \sum_{r=1}^R d_{A_r^{t+1}}(\Phi_i^r, \Phi_j^r) = \epsilon_{ij}^{t+1} \\ \alpha = \min(\lambda_{ij}, \alpha), \lambda_{ij} = \lambda_{ij} - \alpha \end{cases} \quad (12)$$

The Lagrange multiplier α can be solved with Eq.(16). The proof of uniqueness and solution of α are provided in Appendix A. According to the optimization, statistical kernel $A_r (1 \leq r \leq R)$ can be learned in the following algorithm:

Algorithm 1 Statistical Adaptive Metric Learning

Input: Training pairs $\{(\Phi_i^r, \Phi_j^r), \delta_{ij}\}$, input Initial Mahalabonis matrix A_0 , self-covariance co-efficient γ , distance thresholds ρ , margin parameter τ and tuning scale ς

Output:

- 1: $t \leftarrow 1, A_r^1 \leftarrow A_0$, for $r=1, \dots, R, \lambda_{ij} \leftarrow 0, \epsilon_{ij} \leftarrow \delta_{ij}\rho - \varsigma\tau, \forall (i, j)$
 - 2: **Repeat**
 - 3: Pick a constraint (i, j) , and compute the distances $d_{A_r}(\Phi_i^r, \Phi_j^r)$
 - 4: Solve α with Eq.(16)
 - 5: $\epsilon_{ij} \leftarrow \gamma \epsilon_{ij} / (\gamma + \alpha \epsilon_{ij})$
 - 6: $\lambda \leftarrow \lambda_{ij} - \alpha$
 - 7: $A_r \leftarrow A_r + \frac{\lambda_{ij}\alpha}{1 - \lambda_{ij}\alpha d_{A_r}(\Phi_i^r, \Phi_j^r)} A_r (\log(\Phi_i^r) - \log(\Phi_j^r)) (\log(\Phi_i^r) - \log(\Phi_j^r))^T A_r, r=1, \dots, R$
 - 8: Until convergence
 - 9: **return** Mahalanobis Matrices $A_1, \dots, A_R$
-

4. Relations with Hybrid Euclidean-and-Riemannian Metric Learning (HERML) and Information Theoretic Metric Learning (ITML)

It is essential to point out that our method, statistical adaptive metric learning (SAML) is an adaptive version of hybrid statistical metric learning, e.g. Hybrid Euclidean-and-Riemannian Metric Learning (HERML). HERML individually fixes the number of learning Mahalanobis matrices as 3 for all kinds of feature sets. In HERML,

$$\min\{\text{var}(\text{diag}(\log\Lambda))\} = \log\Phi_{SAS}\Phi_{SUS}\Phi_{DIS} = U(\log\Lambda_{SAS}\Lambda_{SUS}\Lambda_{DIS})U^T \quad (13)$$

Eq.(13) is arbitrarily set as an implicit condition. Only full statistics combination is used for their set modelling. Different from HERML, better statistic combinations are selected by analyzing the canonical correlations in SAML. Although the optimization looks similar, HERML is dealing with image sets while SAML is handling feature sets. Minor variations will usually not lead to large fluctuations in scale of images, but they may likely lead to large fluctuations in that of feature sets, e.g. HOG and SIFT. HERML individually combines all three metrics, while SAML explores the correlation among these metrics. Therefore, the subspaces scaling and effective metrics selection in SAML can achieve better performances for various feature sets.

Since our methods is described as a Mahalanobis-like metric learning, it is also essential to note that our method clearly has three advantages over ITML. Firstly, the data scale is better kept by the subspaces mapping in our method. Secondly, multiple statistic combination makes our method more adaptive to various data than that of ITML. Thirdly, the measurement of Log-determinant divergence provides a more uniformed framework for identity matrices approximation than the distribution analysis in ITML.

In ITML, a multivariate Gaussian distribution is defined directly on data as $p(x; A) = \frac{1}{Z} \exp(-\frac{1}{2}x^T A x)$. Z is the normalizing constant. The probability density $p(x; A_0)$ is approximated by the initial density distribution, where A_0 is the identity matrix. In ITML, the distance between the two distributions is formulated as the KL-divergence measuring the relative entropy:

$$KL(p(x; A_0)||p(x; A)) = \int p(x; A_0) \log \frac{p(x; A_0)}{p(x; A)} dx \quad (14)$$

Then the problem identity approximation is derived as a KL-divergence minimization for $\forall(i, j)$ pairs.

$$\min_{A \leq 0} KL(p(x; A_0)||p(x; A)), \text{ s.t. } \delta_{ij}(x_i^T A x_j) \leq \epsilon_{ij}, \forall(i, j) \quad (15)$$

Finally, Eq.(15) is equivalent to the problem of Log-Determinant distance minimization:

$$\min_{A \leq 0} D_{ld}(A, A_0) + D_{ld}(\text{diag}(\epsilon), \text{diag}(\epsilon_0)), \text{ s.t. } \delta_{ij}(x_i^T A x_j) \leq \epsilon_{ij} \forall(i, j) \quad (16)$$

where ϵ_{ij} is the slack variable for the (i, j) pairs and ϵ_0 is identity. δ_{ij} is the characteristic function for (i, j) . If (i, j) are similar, $\delta_{ij} = 1$, if (i, j) are not similar, $\delta_{ij} = -1$. From Eq.(16), the minimization problem of Eq.(16) in ITML can only be used for learning a single Mahalanobis metric directly on input data. The expansion of Eq.(16) is basically

constrained by its distribution assumption in Eq.(14). Relative entropy can be computed only when all the data samples are in the same scale. Contrary to ITML, the learning problem proposed in ASML is handling multiple statistics in a uniformed minimization framework. Taking advantage of statistical subspaces embedding and log-determinant divergence, a scale invariant framework of Eq.(9) proposed in our method is capable of minimizing R Mahalanobis-like matrices and leading to better classification than that of ITML.

5. Experiments

5.1. Experimental Settings

5.1.1. Methods in the Comparative Study

As a statistic-based metric learning method, the proposed Statistical Adaptive Metric Learning (SAML) is mainly compared with two types of state-of-art methods in action visual feature set classification: 1) Statistic-based methods; 2) Metric learning methods. To obtain a comprehensive performance evaluation in action recognition, our method is also compared with some well-known Spatio-Temporal feature based methods.

The statistic-based methods include: 1) Sample-based methods: Maximum Mean Discrepancy (MMD)[17]; 2) Subspace-based methods: Covariance Discriminant Learning (CDL)[15] 3) Distribution-based methods: Gaussian Mixture Model(GMM)[3] and Single Gaussian Model (SGM)[2].

The metric learning methods include: Information theoretic metric learning (ITML)[19], Manifold Discriminant Analysis (MDA)[14], Hybrid Euclidean-and-Riemannian Metric Learning (HERML) [4] and Distribution-based ITML (DIS-ITML).

Some well-known methods using spatio-temporal feature for action recognition were proposed by Campos et al.[28], Wang et al.[29], Fathi et al., Jhuang et al.[30], Wu et al.[31], Kovashka et al.[32], Liu et al.[33], Shao et al.[34] and Zhen et al.[35].

Static computer vision features are extracted frame by frame. In case of set modeling, common scale invariant image features are employed, including Histogram of Gradients (HOG), Shift-invariant Feature Transformation (SIFT), Harris corners and Textures. All the features are clustered into 128 dimensions, while SIFT features are clustered into 128 points. Thus one frame generates a $128 \times k$ feature set. For videos with different resolutions, we uniformly resize each frame into 320×240 at QVGA resolution. When compared with other methods, feature combination of SIFT+HOG+Corners are uniformly used. Average recognition rate is used to measure the accuracy. For statistic-based methods and metric learning methods, all the source codes of the compared methods are obtained from their original authors, except for SGM and GMM. We implemented SGM and GMM meticulously. For methods using Bregman optimization like ours, distance thresholds ρ is set as the mean distances, margin parameter τ is set as the standard deviations and tuning scale $\varsigma \in [0.01 \ 10]$. Furthermore, since our experiments are tested on feature sets, we carefully tuned each models of all the methods to achieve the best possible recognition performance. For MMD, the number of iteration is set to 10. For ITML, we set the parameter of Bregman optimization as stated above. The parameter settings of MDA is according to [14]. For CDL we adopt the KLDA for the discriminant analysis and k-NN for classification. For Spatial-Temporal (ST) feature based methods, we obtain their best recognition rate from the referred publications.

Figure 3: Illustration of sampling procedure for large action video data set

5.1.2. Databases

Five famous human action data sets are used in our experiments: Weizmann human action, UCF101, KTH human action, YouTube and HMDB51. Among those five video data sets, Weizmann and KTH have very static backgrounds with different human actions, while YouTube and HMDB51 both have complex and varying backgrounds and dynamic scenarios with different humans. Experiments are conducted both on human verification and action recognition.

The KTH dataset is a simple human action dataset, which includes 600 black and white videos captured by stationary cameras in 6 classes of actions with resolution of 160*120. The Weizmann dataset include 10 classes of human actions with 90 videos at low resolution of 180*144. The UCF101 human action dataset is a large dataset with 13320 videos from 101 action categories. UCF101 provides a large diversity in term of action types and with large variations in camera motion, object appearance and pose, object scale, viewpoint, cluttered background, illumination conditions, etc. It is the most challenging data set to date. The action categories can be divided into five types: 1) Human-Object Interaction 2) Body-Motion Only 3) Human-Human Interaction 4) Playing Musical Instruments 5) Sports. In our experiments we randomly select classes from all of the UCF101 categories. The YouTube dataset is a challenging and widely used video dataset. It has 1,910 video clips of 47 subjects collected from YouTube. The HMDB51 dataset was collected by Kuehne et al., containing 6,766 video clips in total. There are 51 action classes, each with at least 101 positive samples.

For smaller datasets with less than 20 classes we test on all classes. For larger datasets with more than 20 classes, we take a statistical sampling approach to evaluate the average performance. We conduct 20 trials on each of these large datasets. In each trial we randomly select 14 classes; and inside each class, we randomly select 10 video clips, 5 for training and 5 for testing. Thus 280 random action classes and 2800 random video clips are evaluated for each of these large action datasets. Average recognition rate over 20 trials is then calculated as the measure of performance. Figure 3 illustrates the sampling procedure of our experiments on large datasets. A fixed number of frames and parameter values are selected in the experiments. The training gallery includes the same number of frames for each video sequence, and the test sets are extracted in the same way. The task is to recognize the input test set based on the training gallery.

5.2. Results and Analysis

5.2.1. Performance Evaluation and Methods Comparisons

Performance comparisons are conducted with various statistic-based algorithms. Table 1 shows the performances comparison among different statistic-based methods. Three

benchmark video analysis datasets are in use: UCF101, YouTube and HMDB51. Note that the videos sequences in those datasets are not just with human action scenes, but also with more common and complicated shapes, streets and dynamic scenarios.

From Table 1, we can observe that the sample-based statistics have clear weakness on complex scenarios. Though supported by Bregman divergence, mean vectors focusing on the centroids are far from adequate to represent the feature set. With the addition of the 2nd-order statistic, subspace-based method achieves better recognition rates. Riemannian manifold and Grassmann affine spaces scatter the data with a scalable representation. 70.3% recognition rate is achieved by covariant discriminative learning (CDL). While the distribution-based methods perform better than sample-based methods on UCF101 data set with over 80% average recognition rate, and perform similar with sample-based methods on Youtube and HMDB51 data set with a relatively low recognition rate between 25% and 35%. The proposed SAML performs better than other statistic-based methods, with the highest average recognition rate on UCF101, Youtube and HMDB51 data set. Performances evaluated in Table 1 demonstrates that the proposed SAML performs better than statistic-based methods for action feature set recognition with complex background scenarios.

Table 2 shows the performances comparison among different metric learning methods. Average recognition rates are analyzed for action feature set recognition. UCF101, YouTube and HMDB51 are used. From Table 2, HERML and DIS-ITML have over 85% recognition rates on UCF101 dataset and over 70% recognition rates on YouTube dataset. The proposed SAML has better recognition performance than other metric learning methods on UCF101 and HMDB51. On YouTube data, SAML performs better than other metric learning methods, except HERML. The results in Table 1 and 2 show that our method is able to adaptively take advantage of both statistic-based methods and metric learning methods, and consequently boost their performance to some extent. SAML achieves 95.5% average accuracy on UCF101 dataset, the second best performance on YouTube dataset, and the best performance on HMDB51 dataset in the state-of-art statistic-based and metric learning-based methods.

Statistic-based methods		UCF101	YouTube	HMDB51
Sample-based	MMD[17]	73.5	51.2	32.4
Subspace-based	CDL[15]	93.5	67.7	70.3
Distribution-based	GMM[3]	87.5	61.2	32.7
	SGM[2]	82.3	50.3	26.4
Proposed	SAML	95.5	70.83	71.3

Table 1: Average recognition rate (%) compared with statistic-based methods on UCF101, YouTube and HMDB51 datasets.

Table 3 compares our method with some well-known Spatial-Temporal (ST) feature based action recognition methods on KTH, Weizmann and UCF101 data set. Average recognition rate is used as the measure of accuracy. From Table 3, we observe that the recognition performance of SAML is better than other well-known methods. For human action recognition, the spatial-temporal features have clear advantages over large scale feature sets on modeling and optimization computation. The subspace mapping in the proposed SAML method takes advantage of scale invariant feature space and statistic

Metric learning methods	UCF101	YouTube	HMDB51
ITML[19]	76.2	67.6	51.2
MDA[14]	88.9	65.3	63.3
HERML[4]	92.5	74.6	68.1
DIS-ITML[19]	86.1	70.3	47.8
SAML	95.5	70.83	71.3

Table 2: Average recognition rate (%) compared with metric learning methods on UCF101, YouTube and HMDB51 datasets.

ST-based Methods	KTH	Weizmann	UCF101
Campos et al.[28]	91.5	96.7	80
Wang et al.[29]	89	97.8	83.3
Fathi et al.[36]	90.5	100	-
Jhuang et al.[30]	91.7	98.8	-
Wu et al.[31]	94.5	-	91.3
Kovashka et al.[32]	94.5	-	87.3
Liu et al.[33]	95	-	-
Shao et al.[34]	95	-	93.9
Zhen et al.[35]	94.5	-	80.7
SAML	100	100	95.5

Table 3: Average recognition rate (%) compared with well-known Spatial-Temporal(ST) feature based methods on KTH, Weizmann and UCF101 human action benchmark datasets.

ST-based Methods	HMDB51	YouTube
Yu et al.[37]	46.9	85.2
Liu et al.[33]	48.4	82.3
Shao et al.[34]	37.3	-
Zhen et al.[35]	31.7	80.7
SAML	71.3	70.83

Table 4: Average recognition rate (%) compared with Spatial-Temporal(ST) feature based methods on HMDB51 and YouTube datasets.

combination, which leads to improved recognition accuracy. Our method is further compared with several ST feature based methods on HMDB51 and YouTube data set in Table 4. The proposed SAML performed 20% better than the best results from other ST feature based methods on HMDB51. But SAML does not perform well on YouTube dataset. This is because the proposed SAML is developed for subspace analysis on visual feature sets in different scales, and static visual features such as HOG, SIFT and corners are adaptively combined to construct scale variant feature sets. For YouTube video data, the ST features such as HOG3D and CNN generally have very good representation already, and thus there is no clear advantage from SAML. Overall the proposed SAML has 100% accuracies on KTH and Weizmann, 95.5% accuracy on UCF101, 71.3% accuracy on HMDB51 and 70.83% accuracy on YouTube data set. Such results are the best on KTH, Weizmann, UCF101, HMDB51, and the second best on YouTube. In summary, the proposed SAML outperforms most of the state-of-art methods, including statistic-based methods, metric learning methods and ST feature-based methods, on five benchmark databases.

Furthermore, as a hybrid statistical metric learning, the proposed SAML is compared with other hybrid methods, i.e. HERML and DIS-ITML, in term of class-based classification accuracy. 14 action classes were randomly chosen from UCF101. For each class we randomly sampled 10 actions, 5 for training and 5 for testing. The average recognition rate is computed by averaging 10 trials of sampling. Figure 4 shows the average classification accuracy over 14 randomly sampled action classes. For 14 classes with different statistic characters, the recognition rates of SAML is higher than that of others. More importantly, no class is classified with extremely low recognition rate. Such a result demonstrates that our method is robust to various kinds of action sets. In another words, SAML is more stable than other hybrid statistical metric learning methods, and would provide similar level of recognition rate uniformly over different human action types.

Figure 4: Classification comparison on UCF101 dataset among DIS-ITML, HERML and SAML in term of average class recognition rate(%). The last column show an average recognition rate over all the classes.

5.2.2. Performance Analysis

According to the experimental setup, two parameters of the algorithm and one variable of the video clips are to be evaluated. They are tuning scale ς , slack weights γ and frames in use for each video, respectively. In desirable circumstance, the classification performance should be robust to algorithm parameters. If more frames are used, which are considered as additional information, the recognition rate should also increase.

Figure 5 compares the robustness of tuning scale ς , slack weights γ in SAML and that of HERML, which has similar optimization and parameter settings in the algorithm design. 5 human action classes in UCF dataset were tested for recognition accuracy in this experiment. Average recognition rate is calculated on each value of the parameter. From Figure 5, we observe that the recognition rate of SAML becomes stable when zeta is no less than 0.3 and slack weights are no less than 0.1, so as the HERML. To sum up, both HERML and SAML have the similar level of parameter robustness.

Figure 5: Evaluation of the parameter robustness with SAML and HERML. The red solid line marked by circles is the average recognition rate (%) / parameter of SAML and blue dashed line marked by asterisks is the average recognition rate (%) / parameter of HERML. The recognition is conducted on 5 different action classes in UCF dataset. (a) Tuning scale zeta; (b) Slack weights.

Other than parameter robust evaluation, an evaluation of frames robustness is given by Figure 6 in term of average recognition rate w.r.t. number of frames. Three hybrid statistic metric learning methods are evaluated on three datasets: KTH, Weizmann and HMDB51. From Figure 6, we observe that all the recognition rates are increasing among all three of the data set as the number of frames increases with SAML and HERML, while the classification rates of DIS-ITML on HMDB51 and Weizmann remain the same. Therefore in the case of feature set classification, our method and HERML are robust to the number of frames while DIS-ITML is not. In addition, Figure 7 compares the average recognition rate w.r.t. number of frames among all the three hybrid statistic methods. Not only robust to frames, SAML also performs the best in this case.

Figure 6: Evaluation of the frame robustness with hybrid statistical methods. The red solid line marked by circles is the recognition rate (%) on KTH data; the blue dotted line marked by asterisks is the recognition rate (%) on Weizmann data and the green dashed line marked with plus signs is the recognition rate (%) on HMDB51 data. (a) SAML; (b) HERML; (c) DIS-ITML

Figure 7: Comparison of recognition rate with different number of frames among three hybrid statistic methods. The red solid line marked by circles is the average recognition rate (Arr) (%) of SAML; the blue dotted line marked by asterisks is the recognition rate (%) of HERML and the green dashed line marked with plus sign is the recognition rate (%) of DIS-ITML. The recognition is conducted on the human recognition for 5 different classes on UCF dataset.

Furthermore, we test our method on different feature combinations to evaluate the robustness of various features. Figure 8 shows an example of recognition rate with 14 randomly actions classes from Youtube database. With 5 available static features, the figure shows that the recognition rates of SAML are nearly the same among different combinations for each of the actions.

Figure 8: Robustness test of SAML with different feature combinations on YouTube database in terms of recognition rate(%). Feature combinations: SIFT, HOG+Corners, HOG+SIFT, Corner+SIFT, HOG+SIFT+Corner

5.2.3. Computational Costs

Finally, computational costs of our method on different data sets are also presented in Table 4 and Table 5. In comparison, the computational cost of HERML is provided as well. All the classifications were set up as stated in section 5.1. From Table 4 and 5, we observe that both the training time and testing time of the proposed SAML are faster than those of HERML. Such acceleration is mainly due to the statistic kernel reduction introduced by our minimum Hilbert subspace selection. Classifications with only necessary statistics combination efficiently improve the computation and recognition.

Table 5: Gallery training time (in seconds) of HERML and SAML in the experiments on five video databases: UCF101, KTH, Weizmann, YouTube and HMDB51.

Training time	UCF101	KTH	Weizmann	YouTube	HMDB51
HERML	227.2	765.737	280.57	366.99	567.36
SAML	77.43	123.676	71.35	100.77	83.49

In summary, our method performs well in human action recognition. It generally outperforms other hybrid statistic metric learning methods in terms of parameter robustness and computational efficiency.

Table 6: Recognition test time (in seconds) of HERML and SAML in the experiments on five video databases: UCF101, KTH, Weizmann, YouTube and HMDB51.

Testing time	UCF101	KTH	Weizmann	YouTube	HMDB51
HERML	0.03	0.037	0.0351	0.037	0.0342
SAML	0.0277	0.031	0.029	0.032	0.0335

6. Conclusions

In this paper, we proposed a statistical adaptive metric learning method for feature set modeling and classification. The extensive experiments have shown that our proposed method outperforms several state-of-the-art methods in human action recognition over a variety of public data sets. To our best knowledge, the optimal selection of metric learning combination in the Hilbert spaces has not been investigated before this work. In the future, it would be interesting to expand our method for other possible hybrid metric selections with more robust statistics modeling feature sets in different structures and real-world classifications.

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Appendix A Solution to Bregman Optimization

Consider the following constrained minimization

$$\begin{aligned}
& \min_{A_1 \leq 0, \dots, A_R \leq 0, \epsilon} \frac{1}{R} \sum_{r=1}^R D_{ld}(A_r, A_0) + \gamma D_{ld}(\text{diag}(\epsilon), \text{diag}(\epsilon_0)) \\
& s.t. \quad \frac{\delta_{ij}}{R} \sum_{r=1}^R d_{A_r}(\Phi_i^r, \Phi_j^r) \leq \epsilon_{ij} \forall (i, j)
\end{aligned} \tag{17}$$

where $A_r (r = 1, \dots, R)$ is additive separable for all R matrices. Simplify the minimization as

$$\begin{aligned}
& \min_{A \leq 0, \epsilon} D_{ld}(A, A_0) + \gamma D_{ld}(\text{diag}(\epsilon), \text{diag}(\epsilon_0)) \\
& s.t. \quad \frac{\delta_{ij}}{R} d_A(\Phi_i, \Phi_j) \leq \epsilon_{ij} \forall (i, j)
\end{aligned} \tag{18}$$

where $d_A(\Phi_i, \Phi_j) = \text{tr}(A(\log(\Phi_i) - \log(\Phi_j))(\log(\Phi_i) - \log(\Phi_j))^T)$. The Mahalabonis distance matrix A is the primal minimizer.

Firstly, with Bregman projection with respect to A ,

$$\nabla D_{ld}(A^{t+1}) = \nabla D_{ld}(A^t) + \delta_{ij} \alpha (\log(\Phi_i) - \log(\Phi_j)) (\log(\Phi_i) - \log(\Phi_j))^T A^t \tag{19}$$

where α is the Lagrange multiplier, since

$$\nabla D_{ld}(A) = \nabla_A(\text{tr}(AA_0^{-1})) - \nabla_A(\log \det(AA_0^{-1})) = A_0^{-1} - \frac{(AA_0^{-1})^*}{\det AA_0^{-1}} A_0^{-1} = A_0^{-1} - A^{-1} \quad (20)$$

Eq.(19) is derived to

$$(A^{t+1})^{-1} = (A^t)^{-1} - \delta_{ij}\alpha(\log(\Phi_i) - \log(\Phi_j))(\log(\Phi_i) - \log(\Phi_j))^T \quad (21)$$

$$A^{t+1} = ((A^t)^{-1} - \delta_{ij}\alpha(\log(\Phi_i) - \log(\Phi_j))(\log(\Phi_i) - \log(\Phi_j))^T)^{-1} \quad (22)$$

According to the Sherman-Morris inverse formula:

$$(B + uv^T)^{-1} = B^{-1} - \frac{B^{-1}uv^TB^{-1}}{1 + v^TB^{-1}u} \quad (23)$$

where u, v is semi-positive definite matrix and B is symmetric positive definite (SPD) matrix. Eq.(22) is then derived in form of

$$A^{t+1} = A^t + \frac{\delta_{ij}\alpha A^t(\log(\Phi_i) - \log(\Phi_j))(\log(\Phi_i) - \log(\Phi_j))^T A^t}{1 - (\log(\Phi_i) - \log(\Phi_j))^T A^t(\log(\Phi_i) - \log(\Phi_j))} \quad (24)$$

Simplify Eq.(24) as

$$A^{t+1} = A^t + \frac{\delta_{ij}\alpha}{1 - \delta_{ij}\alpha \text{tr}(A^t(\log(\Phi_i) - \log(\Phi_j))(\log(\Phi_i) - \log(\Phi_j))^T) \cdot A^t(\log(\Phi_i) - \log(\Phi_j))(\log(\Phi_i) - \log(\Phi_j))^T A^t} \quad (25)$$

$$A^{t+1} = A^t + \frac{\delta_{ij}\alpha}{1 - \delta_{ij}\alpha \text{tr}(A^t(\log(\Phi_i) - \log(\Phi_j))(\log(\Phi_i) - \log(\Phi_j))^T) A^t(\log(\Phi_i) - \log(\Phi_j))(\log(\Phi_i) - \log(\Phi_j))^T A^t} \quad (26)$$

The primal minimizer A is solved by Eq.(26). Secondly, after solving primal minimizer, look back at Eq.(18) for the solution of the other minimizer. With Bregman projection of Eq.(18) with respect to ϵ ,

$$\nabla D_{ld}(\epsilon_{ij}^{t+1}) = \nabla D_{ld}(\epsilon_{ij}^t) - \frac{\delta_{ij}\alpha}{\gamma} \quad (27)$$

$$\begin{aligned} \nabla D_{ld}(\epsilon) &= \nabla_\epsilon[\text{tr}(\text{diag}(\epsilon) \cdot \text{diag}(\epsilon_0)^{-1})] - \nabla_\epsilon[\log \det(\text{diag}(\epsilon)) \cdot \text{diag}(\epsilon_0)^{-1}] \\ &= \text{diag}(\epsilon_0)^{-1} - \text{diag}(\epsilon)^{-1} \end{aligned} \quad (28)$$

Then Eq.(27) becomes

$$\begin{aligned}
diag(\epsilon_{ij}^{t+1}) &= (diag(\epsilon_{ij}^t)^{-1} - \frac{\delta_{ij}\alpha}{\gamma})^{-1} \\
&= diag(\epsilon_{ij}^t) + \frac{\delta_{ij}\alpha \cdot diag(\epsilon_{ij}^t) \cdot diag(\epsilon_{ij}^t)}{\gamma - \delta_{ij}\alpha diag(\epsilon_{ij}^t)} = \frac{\gamma diag(\epsilon_{ij}^t)}{\gamma - \delta_{ij}\alpha diag(\epsilon_{ij}^t)} \\
\Rightarrow \epsilon_{ij}^{t+1} &= \frac{\gamma \epsilon_{ij}^t}{\gamma - \delta_{ij}\alpha \epsilon_{ij}^t}
\end{aligned} \tag{29}$$

The updating of two minimizers ϵ and A can be re-written together as

$$\begin{cases} A^{t+1} &= A^t + \frac{\delta_{ij}\alpha}{1 - \delta_{ij}\alpha d_{A^t}(\Phi_i, \Phi_j)} A^t (log(\Phi_i) - log(\Phi_j))(log(\Phi_i) - log(\Phi_j))^T A^t \\ \epsilon_{ij}^{t+1} &= \frac{\gamma \epsilon_{ij}^t}{\gamma - \delta_{ij}\alpha \epsilon_{ij}^t} \end{cases} \tag{30}$$

Finally, given the updated minimizers, Lagrange multiplier α can be solved by the following Euler-Lagrange equation with a non-negative maintaining equation for α handling the inequality constrains.

$$\begin{cases} \delta_{ij} tr(A^{t+1}(\alpha)(log(\Phi_i) - log(\Phi_j))(log(\Phi_i) - log(\Phi_j))^T) - \epsilon_{ij}^t(\alpha) = 0 \\ \alpha = \min(\lambda_{ij}, \alpha) = \lambda_{ij} - \alpha \end{cases} \tag{31}$$

405 where α is the Lagrange multiplier, $r = 1, \dots, R$, dual variable $\lambda_{ij} \leq 0$. Note that for semi-positive definite matrix z and SPD matrix B , $tr(Bzz^T) = z^T B z$. Thus, according to Eq.(31)

$$\begin{aligned}
\delta_{ij} [d_{A^t}(\Phi_i, \Phi_j) + \frac{\alpha \delta_{ij} d_{A^t}(\Phi_i, \Phi_j) d_{A^t}(\Phi_i, \Phi_j)}{1 - \alpha \delta_{ij} d_{A^t}(\Phi_i, \Phi_j)}] &= \frac{\gamma \epsilon_{ij}^t}{\gamma - \alpha \delta_{ij} \epsilon_{ij}^t} \\
\Rightarrow \delta_{ij} \frac{d_{A^t}(\Phi_i, \Phi_j)}{1 - \alpha \delta_{ij} d_{A^t}(\Phi_i, \Phi_j)} &= \frac{\gamma \epsilon_{ij}^t}{\gamma - \alpha \delta_{ij} \epsilon_{ij}^t}
\end{aligned} \tag{32}$$

410 Since all the elements in $d_{A^t}(\Phi_i, \Phi_j) = tr(A(\Phi_i - \Phi_j)(\Phi_i - \Phi_j)^T)$ are either semi-positive definite or positive definite, $d_{A^t} > 0$. By adding the dual variables $\lambda_{ij}, \epsilon > 0$. Therefore, with all these condition above, we can conclude that a unique solution of α exists. The Lagrange multiplier can be solved by Eq.(32).

In summary, the simplified single statistic minimization of Eq.(18) is solved by Eq.(30) and Eq.(32). For r -th ($r=1, \dots, R$) statistic, the solution of the minimizers can then be easily written as

$$\begin{cases} A_r^{t+1} &= A_r^t + \frac{\delta_{ij}\alpha}{1 - \delta_{ij}\alpha d_{A_r^t}(\Phi_i^r, \Phi_j^r)} A_r^t (log(\Phi_i^r) - log(\Phi_j^r))(log(\Phi_i^r) - log(\Phi_j^r))^T A_r^t \\ \epsilon_{ij}^{t+1} &= \frac{\gamma \epsilon_{ij}^t}{\gamma - \delta_{ij}\alpha \epsilon_{ij}^t} \end{cases} \tag{33}$$

With the Lagrange multiplier solved by

$$d_{A_r^t}(\Phi_i^r, \Phi_j^r) + \frac{\delta_{ij}}{R} \sum_{r=1}^R \frac{\alpha d_{A_r^t}(\Phi_i^r, \Phi_j^r)}{1 - \alpha d_{A_r^t}(\Phi_i^r, \Phi_j^r)} = \frac{\gamma \epsilon_{ij}^t}{\gamma - \alpha \delta_{ij} \epsilon_{ij}^t} \tag{34}$$

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