Dynamic modeling and dynamical analysis of pump-turbines in S-shaped regions during runaway operation

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Abstract: There is a region of pump-turbine operation, often called the S-shaped
region, in which one unit rotational speed corresponds to three unit flows or torques.

In this paper, the dynamic model of the pump-turbine in S-shaped regions is
established by introducing the nonlinear piecewise function of relative parameters.

Then, the global bifurcation diagrams of the pump-turbine are presented to analyze its
dynamic characteristics in the S-shaped regions. Meanwhile, a stability criterion of

runaway point is given based on the established theoretical model. The numerical
experiments are conducted on the model and the results are in good agreement with

the theoretical analysis. Furthermore, the effects of the characteristic curve slopes on the stability of the pump-turbine are studied by an innovative use of the three-dimensional bifurcation diagrams. Finally, the factors influencing the runaway stability of pump-turbines are also discussed, based on the dynamic analysis.

Keywords: Pump storage power plant, Pump-turbine, S-shaped characteristics, Runaway operation

1. Introduction

Hydropower is a renewable energy source and it has high potential for economically viable utilization [1]. Since the pump storage power plant can store energy and generate power according to load fluctuations, it plays an important role in the grid system [2]. With the development of the modern technologies of the pump storage power plant, pump-turbines are moving toward the high head, large capacity and high speed to obtain better economic efficiency [3]. However, pump-turbines change frequently between different conditions and often operate in two directions. The servomotor often fails to close the guide vane when the pump-turbine rejects loads, which causes the runaway operations of the unit with rapid rotational speed. In addition, the S-shaped regions of pump-turbines prevent unit parameters from converging to constants as normal Francis turbines and results in the expansion of the accidents [4]. These conditions directly affect the steady operation of the Pump Storage Power Plant and the security of the grid system. Therefore, the unstable operating problems of the pump-turbines should be given more attention.

The safe and stable operation of the pumped-storage hydropower plants is tightly

1 connected to the S-characteristic of pump-turbines, and the pump-turbines are
2 seriously unstable in S-shaped regions [5]. In the S-characteristics curves, a
3 pump-turbine has three different corresponding unit flows at the same unit speed. The
4 rapid change of the flow will cause the opposite directions of torques, which easily
5 damages the pump-turbine components [6]. Hence, the stability study of the
6 pump-turbine in S-shaped regions is significant and profound.

7 At present, due to the existence of the complex S-characteristic, the stability of
8 the pump-turbine in the S-shaped regions attracts an enormous amount of research.
9 Sivakumar et al. [7] introduced the variable speed technology for improving the
10 power station performance and the existing conventional synchronous machines of the
11 plant could be operated as variable speed machines by adopting static frequency
12 converters. Results show that the variable speed technology has considerably
13 increased the plant performance. Ghoneim et al. [8] presented the results of
14 performance optimization of a photovoltaic powered water pumping system in the
15 Kuwait climate. The pumped water is desired to satisfy the domestic needs of 300
16 persons in a remote area. Lara et al. [9] employed an AC-DC-AC direct-drive power
17 converter for a wind electric pumping system and experimental trials at different
18 pump pressures were conducted. The results show that efficiency of the power
19 converter ranges between 70% and 95% with a power factor between 0.4 and 0.85,
20 depending on the operating pressure. To overcome the problem of nonlinear dynamics
21 of hydro-turbien governing system with sloping ceiling tailrace tunnel, Guo et al. [10]
22 used the Hopf bifurcation theory to study the stability of the system. A novel and

1 rational model of the system was proposed and the existence and direction of Hopf
2 bifurcation of the system were analyzed. Zeng et al. [11-12] theoretically explored the
3 root cause of the pump-turbine instability under no-load conditions and the dominant
4 factors influencing it. Furthermore, they provided a mathematical expression for the
5 no-load oscillation and analyzed the oscillation characteristics. Xu et al. [13] proposed
6 an adaptively fast fuzzy fractional order PID control method for the pumped storage
7 hydro unit and they used a novel bacterial-foraging chemotaxis gravitational search
8 algorithm to get the optimized parameters. Schleicher and Oztekin [14] investigated a
9 novel modular pumped-storage scheme that uses elevated water storage and cement
10 pools as the upper and lower reservoirs. Moreover, Li et al. [15] analyzed the regions
11 of the hump characteristics. Their results showed that there are three vortex groups
12 which distribute in the tandem cascade passages when entering the hump region. Jain
13 et al. [16] carried out experimental investigations on centrifugal pump running in
14 turbine mode to optimize its geometric and operational parameters. Micha Premkumar
15 T and Dhiman [17] numerically investigated the flow over a cascade of S-shaped
16 hydrofoils, finding that the cascade pressure loss was significantly more for the pump
17 cascade than the turbine cascade and this loss increased as the blade spacing reduced.

18 However, few researchers have focused on the dynamic modeling of the
19 pump-turbine in the S-shaped regions. The conventional autonomous model of the
20 pump-turbine can only describe the system characteristics at a specific condition. The
21 autonomous model of the pump-turbine is able to describe the system characteristics
22 in small fluctuation transient process. However, the dynamic characteristics of the

1 pump-turbine in S-shaped regions are so complex that the autonomous model cannot
2 reflect the dynamic behaviors of the system. To research and improve the global
3 dynamic characteristics of pump-turbines in S-shaped regions, the dynamic relative
4 parameters are introduced by using the S-characteristic curves, and the nonlinear
5 dynamic mathematical model of the pump-turbine in S-shaped regions is established.
6 Further, using the global bifurcation diagrams, the dynamic characteristics of the
7 pump-turbine are studied, when it rejects load moving from the turbine condition to
8 the S-shaped regions. The stability criterion of the runaway operation is given and
9 proved by using numerical experiments. Finally, the factors affecting the stability of
10 the runaway operation are analyzed in detail from the point of view of dynamics.

11 The rest of this paper is organized as follows. In Section 2, the dynamic model of
12 the pump-turbine in the S-shaped regions is established by introducing the dynamic
13 relative parameters, and the global dynamic characteristics of the pump-turbine in the
14 S-shaped regions are analyzed through the bifurcation diagrams. Section 3 gives the
15 stability condition of the pump-turbine at the runaway point and investigates the
16 stability of the above system using numerical experiments. Section 4 closes this paper.

17

18 **Nomenclature**

D_1	the diameter of runner inlet
H	water head
n	static rotational speed
Q_1	penstock flow
Q'_1	unit flow

Q_2	tailrace flow
M	static torque
M'_1	unit torque
L_1	the length of penstock
α_1	the friction coefficient of the penstock
α_2	the friction coefficient of the tailrace
A_1	area of the penstock
A_2	area of the tailrace
Z_{up}	the water level of upstream
Z_s	the water level in the surge tank
h_{f1}	head loss of penstock
F	area of the surge tank
g	gravitational acceleration
α_1	frictional coefficient value of the penstock
α_2	frictional coefficient value of the tailrace
T_a	time scale of machinery
a	slope of the n_{ed} - Q_{ed} characteristics
b	slope of the n_{ed} - M_{ed} characteristics

Subscripts and Superscripts

ed	nondimensional parameters
N	optimal condition
f	friction loss

2. Mathematical model of the pump-turbine in the S-shaped regions

To analyze the dynamic characteristics of the pump-turbine in the S-shaped regions, a dynamic model of the pump-turbine is established by introducing nonlinear piecewise functions of relative parameters.

2.1. The definitions and standardizations of the unit parameters

The sketch map of a pumped-storage plant with a surge tank installed at the downstream is shown in Fig. 1.

Fig. 1. The sketch map of a pumped-storage plant

To simplify the derivation of the unit parameters, the unit parameters were normalized with the non-dimensional parameters defined as

$$n_{ed} = \frac{n'_1}{n'_{1N}}, \quad Q_{ed} = \frac{Q'_1}{Q'_{1N}}, \quad M_{ed} = \frac{M'_1}{M'_{1N}} \quad \text{and} \quad H_{ed} = \frac{H}{H_N}. \quad (1)$$

The relative deviations of the unit parameters are defined as

$$x = \frac{n - n'}{n_N} = \frac{d(n)}{n_N}, \quad z_s = \frac{Z_s - Z'_s}{H_N} = \frac{d(Z_s)}{H_N}, \quad q_1 = \frac{Q_1 - Q'_1}{Q_N} = \frac{d(Q_1)}{Q_N},$$

$$q_2 = \frac{Q_2 - Q'_2}{Q_N} = \frac{d(Q_2)}{Q_N}, \quad h = \frac{H - H'}{H_N} = \frac{d(H)}{H_N} \quad \text{and} \quad m = \frac{M - M'}{M_N} = \frac{d(M)}{M_N}. \quad (2)$$

2.2. Linearization of pump-turbine characteristic curves

The conventional model of the pump-turbine is usually expressed as differential algebraic equations. They cannot denote the effect of the slopes of the curves. The transfer functions of the characteristic curves, which can reflect the influences of the slopes of the curves, are obtained by linearizing the characteristic curves of the pump-turbine. In this paper, suppose $a = \frac{dQ_{ed}}{dn_{ed}}$, which represents the slope of the characteristic curves on the operation point, and the linear treatment of an arbitrary operation point (n_{ed}, Q_{ed}) is shown as

$$Q_{ed} - Q'_{ed} = a(n_{ed} - n'_{ed}). \quad (3)$$

The complete differential equation of Eq. (3) is

$$\frac{d(Q)}{D_1^2 \sqrt{H}} - \frac{Qd(H)}{2D_1^2 H \sqrt{H}} = \frac{Q'_{1N}}{n'_{1N}} \left(\frac{aD_1 d(n)}{\sqrt{H}} - \frac{anD_1 d(H)}{2H \sqrt{H}} \right), \quad (4)$$

which can be simplified by Eq. (2). Hence, the transfer function of an arbitrary point (n_{ed}, Q_{ed}) on the $n_{ed} - Q_{ed}$ characteristic curves can be obtained as

$$q_1 = ax + \frac{Q_{ed} - an_{ed}}{2\sqrt{H_{ed}}} h. \quad (5)$$

Similarly, $b = \frac{dM_{ed}}{dn_{ed}}$ and the linear treatment of an arbitrary operation point (n_{ed}, M_{ed}) on the $n_{ed} - M_{ed}$ characteristic curves is shown as

$$M_{ed} - M'_{ed} = bn_{ed} - bn'_{ed}. \quad (6)$$

The complete differential equation of Eq. (6) is

$$\left(\frac{dM}{D_1^3 H} - M \frac{dH}{D^3 H^2} \right) = b \frac{M'_{1N}}{n'_{1N}} \left(\frac{dn}{D \sqrt{H}} - n \frac{dH}{2DH \sqrt{H}} \right). \quad (7)$$

The transfer function of (n_{ed}, M_{ed}) on the $n_{ed} - M_{ed}$ curves can be obtained by inserting the relative deviations of the parameters as

$$m = b\sqrt{H_{ed}}x + \left(M_{ed} - b\frac{n_{ed}}{2}\right)h. \quad (8)$$

2.3. Channel model

For the water transfer system in Fig. 1, the dynamical equation of the pressure conduits is

$$Z_{up} - Z_s - H - h_{f1} = \frac{L_1}{gA_1} \frac{dQ_1}{dt}, \quad (9)$$

in which $h_{f1} = \alpha_1 |Q_1| Q_1$. Substituting Eq. (1)-(2) into Eq. (9), Eq. (10) is obtained as

$$h = -T_{w1} \frac{dq_1}{dt} - K_1 |Q_{ed}| \sqrt{H_{ed}} q_1 - z_s, \quad (10)$$

where $T_{w1} = \frac{L_1 Q_N}{gA_1 H_N}$ and $K_1 = 2\alpha_1 D_1^4 (Q'_{1N})^2$.

The linearization continuity equation of the surge tank is

$$q_1 - q_2 = T_F \frac{dz_s}{dt}, \quad (11)$$

where $T_F = \frac{H_N F}{Q_N}$.

The linearization of the momentum equation for the tailrace tunnel is as follows

$$z_s - K_2 |Q_{ed}| \sqrt{H_{ed}} q_2 = T_{w2} \frac{dq_2}{dt}, \quad (12)$$

where $T_{w2} = \frac{L_2 Q_N}{gA_2 H_N}$ and $K_2 = 2\alpha_2 D_1^4 (Q'_{1N})^2$.

2.4. Generator model

The third-order and the first-order differential equations of the generator have little impact on the analysis of its stability [18-20]. Therefore, this paper uses the first-order differential equation of the generator which is described as

$$T_a \frac{dx}{dt} = m, \quad (13)$$

1 where $T_a = \frac{Jn_N}{375M_N}$.

2 2.5. The mathematical model in matrix form

3 From Eq. (5), Eq. (8) and Eq. (10)-(13), the simplified matrix form of the
4 mathematical model is obtained as

$$5 \begin{bmatrix} \frac{dq_1}{dt} \\ \frac{dq_2}{dt} \\ \frac{dz_s}{dt} \\ \frac{dx}{dt} \end{bmatrix} = \begin{bmatrix} j_{11} & 0 & j_{13} & j_{14} \\ 0 & j_{22} & j_{23} & 0 \\ j_{31} & j_{32} & 0 & 0 \\ j_{41} & 0 & 0 & j_{44} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ z_s \\ x \end{bmatrix}. \quad (14)$$

6 In the above matrix,

$$7 \quad j_{11} = -\frac{K_1 |Q_{ed}| \sqrt{H_{ed}}}{T_{w1}} - \frac{2\sqrt{H_{ed}}}{\left(Q_{ed} - \frac{dQ_{ed}}{dn_{ed}} n_{ed}\right) T_{w1}},$$

$$8 \quad j_{13} = -\frac{1}{T_{w1}}, \quad j_{14} = \frac{2\sqrt{H_{ed}} \frac{dQ_{ed}}{dn_{ed}}}{T_{w1} \left(Q_{ed} - n_{ed} \frac{dQ_{ed}}{dn_{ed}}\right)}, \quad j_{22} = -\frac{K_2 |Q_{ed}| \sqrt{H_{ed}}}{T_{w2}}, \quad j_{23} = \frac{1}{T_{w2}}, \quad j_{31} = \frac{1}{T_F},$$

$$9 \quad j_{32} = -\frac{1}{T_F}, \quad j_{41} = \sqrt{H_{ed}} \frac{2M_{ed} - n_{ed} \frac{dM_{ed}}{dn_{ed}}}{T_a \left(Q_{ed} - n_{ed} \frac{dQ_{ed}}{dn_{ed}}\right)} \quad \text{and} \quad j_{44} = \frac{\sqrt{H_{ed}} Q_{ed} \frac{dM_{ed}}{dn_{ed}} - 2M_{ed} \frac{dQ_{ed}}{dn_{ed}}}{T_a \left(Q_{ed} - n_{ed} \frac{dQ_{ed}}{dn_{ed}}\right)}.$$

10 (15)

11

12 2.6. Dynamic model of the pump-turbine in S-shaped regions

13 For the pump-turbine model, the constant relative parameters can only denote a
14 specific condition [21-22]. However, the characteristics of the pump-turbine in the
15 S-shaped regions are so complex that any one of the conditions in the S-shaped

1 regions cannot be ignored. So the dynamic model of the pump-turbine in S-shaped
 2 regions is established to study its global dynamic characteristics by introducing the
 3 dynamic relative parameters.

4 This paper studies the case that the pump-turbine rejects load with servo motor
 5 failure, and moves from the turbine condition to the S-shaped regions. The transient
 6 time is 10s. According to the actual change law of rotational speed [11-12], suppose
 7 that the relative unit speed is changing in the three conditions as shown in Fig. 2.

8
 9 Fig. 2. The $n_{ed}-t$ curve of the pump-turbine in the transient process.

10 From Fig. 2, the relationship between n_{ed} and t is got as

11
$$n_{ed} = 2.84 \sin(0.275t - 0.017) + 1.982 \sin(0.414t + 2.384) + 0.354 \sin(0.67t + 4.274),$$

 12
$$(0 \leq t \leq 10). \quad (16)$$

13 According to the change laws of the relative unit speed, the transient process is
 14 divided into three parts, namely,

15 The turbine condition $(0-5s)$, the turbine braking condition $(5-8s)$ and the
 16 reverse pump condition $(8-10s)$.

17 The S-characteristic curves of the pump-turbine are shown in Fig. 3 [12].

Fig. 3. The S-characteristic curves of the pump-turbine.

From Fig. 3, using the method of numerical fitting, the relationships between n_{ed} , Q_{ed} and M_{ed} are obtained in the turbine condition, the turbine braking condition, and the reverse pump condition, respectively.

The turbine condition:
$$\begin{cases} Q_{ed} = -4.417 \times 10^{-13} n_{ed}^{134.1} + 0.956 \\ M_{ed} = -1.923 \times 10^{-6} n_{ed}^{62.8} + 0.883 \end{cases}, \quad (17)$$

The turbine braking condition:
$$\begin{cases} Q_{ed} = 2.871 \sin(1.875 n_{ed} + 41.82) \\ M_{ed} = 0.307 \sin(17.05 n_{ed} + 29.3) \end{cases}, \quad (18)$$

The reverse pump condition:
$$\begin{cases} Q_{ed} = 0.403 \sin(16.82 n_{ed} + 15.21) \\ M_{ed} = 1.064 \sin(14.45 n_{ed} + 18.22) \end{cases}. \quad (19)$$

In order to simplify the calculation, suppose $\frac{dQ_{ed}}{dn_{ed}} = -0.075$, $\frac{dM_{ed}}{dn_{ed}} = -1.5$ in

the turbine condition, $\frac{dQ_{ed}}{dn_{ed}} = 6$, $\frac{dM_{ed}}{dn_{ed}} = 4$ in the turbine braking condition

and $\frac{dQ_{ed}}{dn_{ed}} = -1.7$, $\frac{dM_{ed}}{dn_{ed}} = -2$ in the reverse pump condition, respectively.

Finally, the dynamic nonlinear model of the pump-turbine in the S-shaped regions is established by substituting Eq. (17)-(19) into Eq. (14)-(15).

3. Numerical simulation

In the following contents, the dynamic model of the pump-turbine is used to

study the dynamic characteristics of the system. Here, the numerical simulation results were carried out by using algorithm Adams-Bashforth-Moulton. The fixed step is $2\pi/500$, the initial values are (0.001, 0.001, 0.001, 0.001). The values of system parameters are as follows:

$$K_1 = 1, \quad K_2 = 0.02, \quad T_a = 3.93s, \quad T_F = 0.03s, \quad T_{w1} = 0.26s, \quad T_{w2} = 0.06s.$$

In dynamical systems, a bifurcation diagram shows the values visited or approached asymptotically (fixed points, periodic orbits, or chaotic attractors) of a system as a function of a bifurcation parameter in the system. For the sake of having an overview of the essential dynamics of the pump-turbine, the global bifurcation diagrams are used to analyze the dynamic characteristics of the relative deviation of the water level in the surge tank (z_s) and the relative deviation of the rotational speed (x) in the S-shaped regions, as follows.

(a) Global bifurcation diagram of the relative deviation of the water level in the surge tank (z_s)

with $(0 \leq t \leq 10)$.

(b) Global bifurcation diagram of the relative deviation of the rotational speed (x) with $(0 \leq t \leq 10)$.

Fig. 4. Global bifurcation diagrams of the relative deviation of the system parameters with $(0 \leq t \leq 10)$.

As shown in Fig. 4(a), the relative deviation of the water level in the surge tank (z_s) is relatively stable in the turbine condition and the reverse pump condition. However, it is worth noting that z_s experiences multi-amplitude violent oscillations in the turbine braking condition from $t = 5$ to $t = 8$, which is harmful to the surge tank. Fig. 4(b) shows the dynamic characteristics of the relative deviation of the rotational speed (x) in the transient process. Similarly, x is stable in the turbine condition and the reverse pump condition, but it becomes volatile at the beginning of the turbine braking condition and its oscillating amplitude gradually attenuates as time increases.

Fig. 4 shows the global dynamic characteristics of the pump-turbine in the S-shaped regions. From the above analysis, the system is stable in the turbine condition and the reverse pump turbine condition. The violent vibrations concentrate in the turbine braking condition. To verify the correctness of the simulation results,

the theoretical conclusions in Ref. [12] are presented in Fig. 5.

Fig. 5. Stable regions for the S-shaped regions [12].

For the stable regions of the S-shaped regions, shown in Fig. 5, the pump-turbine is stable in turbine condition and reverse pump condition and the system loses stability in the turbine braking condition. The numerical simulation results (in Fig. 4) are consistent with the conclusion, which verifies the dynamic model can describe the dynamic characteristic of the pump-turbine in the S-shaped regions.. Moreover, from the global bifurcation diagrams of the relative deviation of system parameters, there are two amplitude jumps at $t=5$ and $t=8$, respectively. Based on the dynamic analyses and the numerical experiments, both the positive and negative variations of the slopes (i.e. $\frac{dM_{ed}}{dn_{ed}}$ and $\frac{dQ_{ed}}{dn_{ed}}$) can cause the amplitude jumps. Thus, the slopes of the characteristic curves which have important influences on the stability of the pump-turbine in the S-shaped regions, deserve further investigation and discussion. Since the runaway point is the demarcation point of the system stability between the turbine condition and the turbine braking condition, the runaway point is at $t=5$ by

1 observing Fig. 4. In addition, as displayed in Fig. 4, the violent vibrations of the
2 pump-turbine, which is detrimental to the system, appear primarily in the turbine
3 braking condition.

4 For the pump-turbine, the runner has forward and reverse rotation in turbine
5 condition and pump condition, respectively. The flow characteristic inside the
6 pump-turbine has significant difference in the forward and reverse rotation. For the
7 same parameter, it may have opposite effects on the characteristics of the runner in
8 turbine condition and pump condition [23]. In the S-shaped regions, the performance
9 behavior of the pump-turbine in turbine braking condition has a significant influence
10 on the dynamic characteristics of the system. The S-characteristic curves of the
11 pump-turbine show that the flow rate and head vary dramatically when the
12 pump-turbine moves from the turbine condition to the S-shaped regions, especially in
13 turbine braking condition [24]. This means that the hydrodynamic behavior of the
14 pump-turbine experiences dramatic change. The simulation results (in Fig. 4)
15 illustrate that the flow rate and head have significant influence on the dynamic
16 characteristics of the pump-turbine. Moreover, the pump-turbine loses stability in
17 turbine braking condition. This is accordance with the violent change of the
18 hydrodynamic behavior of the pump-turbine in turbine braking condition.

19 **4. Theoretical analysis and simulations on the runaway stability**

20 When a pump-turbine rejects its load with servo motor failure, the operating
21 point will pass along the characteristic curves at a constant opening, until reaching the
22 runaway point. The operating point may continue dropping into the turbine braking

condition if the pump-turbine cannot stabilize at the runaway point. When the pump-turbine meets the stable condition of the runaway point, the transient parameters such as the pressures, the rotational speed, and the flow will show a damped oscillation and stabilize at this point. Otherwise, the parameters would oscillate without convergence. Hence, it is important and necessary to analyze the stability of the runaway point.

The characteristic equation of the matrix equation (14) is

$$a_0\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0, \quad (20)$$

where $a_0 = 1$,

$$a_1 = -(j_{11} + j_{22} + j_{44}), a_2 = j_{11}j_{22} + j_{22}j_{44} + j_{11}j_{44} - j_{13}j_{31} - j_{14}j_{41} - j_{23}j_{32},$$

$$a_3 = j_{22}j_{13}j_{31} + j_{44}j_{13}j_{31} + j_{22}j_{14}j_{41} - j_{11}j_{22}j_{44} + j_{44}j_{23}j_{32} + j_{11}j_{23}j_{32} \text{ and}$$

$$a_4 = j_{14}j_{23}j_{32}j_{41} - j_{13}j_{22}j_{31}j_{44} - j_{11}j_{23}j_{32}j_{44}.$$

The system is stable when all the solutions of the equation are a negative or imaginary number containing a negative real number.

15

4.1. The stability condition of the runaway point

At the runaway point, suppose $M_{ed} = 0$ and $H_{ed} = 1$. According to the stability criteria, the necessary and sufficient condition of the stable for the system (20) is

$$a_i > 0, (i = 0, 1, 2, 3, 4) \text{ and } \Delta = \begin{vmatrix} a_1 & a_3 & 0 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix} > 0.$$

From $a_1 > 0$, we get

$$\frac{Q_{ed} \frac{dM_{ed}}{dn_{ed}} - 2 \frac{T_a}{T_{w1}}}{Q_{ed} - n_{ed} \frac{dQ_{ed}}{dn_{ed}}} < T_a Q_{ed} \left(\frac{K_1}{T_{w1}} + \frac{K_2}{T_{w2}} \right). \quad (21)$$

In general, $a_0 > 0$ and $a_4 > 0$, so they are not discussed below.

After analysis, $\Delta > 0$ is the sufficient condition of $a_2 > 0$ and $a_3 > 0$. When $K_2 = 0$, the corresponding borders of Eq. (21) are basically coincident with the borders of $\Delta > 0$. Thus the stable condition of runaway point is

$$S = \frac{Q_{ed} \frac{dM_{ed}}{dn_{ed}} - 2 \frac{T_a}{T_{w1}}}{Q_{ed} - n_{ed} \frac{dQ_{ed}}{dn_{ed}}} - \frac{K_1 T_a Q_{ed}}{T_{w1}} < 0. \quad (22)$$

According to Eq. (22), it can be concluded that the most important factors affecting the runaway stability are the slopes of the characteristic curves, namely, $\frac{dM_{ed}}{dn_{ed}}$ and $\frac{dQ_{ed}}{dn_{ed}}$. Fig. 6 shows two conditions of the pump-turbine S-shaped curves, according to the slopes on the runaway point.

Fig. 6. Slopes on the runaway points

In Fig. 6, $1(a)$ and $1(b)$ denote the pump-turbines with low head and high specific speed, respectively. By contrast, $2(a)$ and $2(b)$ denote the pump-turbines with high head and low specific speed, respectively.

4.2. Numerical simulation of the runaway stability

1 The slopes of the characteristic curves for the pump-turbine have different forms

2 on the runaway points. In general, $\frac{dM_{ed}}{dn_{ed}}$ and $\frac{dQ_{ed}}{dn_{ed}}$ have the same identity,

3 meaning that $\begin{cases} \frac{dM_{ed}}{dn_{ed}} > 0 \\ \frac{dQ_{ed}}{dn_{ed}} > 0 \end{cases}$ or $\begin{cases} \frac{dM_{ed}}{dn_{ed}} < 0 \\ \frac{dQ_{ed}}{dn_{ed}} < 0 \end{cases}$. As $M_{ed} = 0$ at the runaway point,

4 according to the S-characteristic curves in Fig. 3, suppose the system parameters at
5 the runaway point are

$$6 \quad (M_{ed}, Q_{ed}, n_{ed}, H_{ed}) = (0, 0.45, 1.23, 1).$$

7 Meanwhile, three-dimensional bifurcation diagrams are presented to analyze the
8 effects of the slopes of the characteristic curves simultaneously.

9

10 (a) Three-dimensional bifurcation diagram of the relative deviation of the rotational speed x with

$$11 \quad \left(-5 \leq \frac{dM_{ed}}{dn_{ed}} \leq -1 \right) \text{ and } \left(-5 \leq \frac{dQ_{ed}}{dn_{ed}} \leq -1 \right).$$

(b) Three-dimensional bifurcation diagram of the relative deviation of the rotational speed x with $\left(1 \leq \frac{dM_{ed}}{dn_{ed}} \leq 5\right)$ and $\left(1 \leq \frac{dQ_{ed}}{dn_{ed}} \leq 5\right)$.

Fig. 7. Three-dimensional bifurcation diagram of the relative deviation of the rotational speed x with different coefficients $\frac{dM_{ed}}{dn_{ed}}$ and $\frac{dQ_{ed}}{dn_{ed}}$.

Fig. 7(a) shows the dynamic characteristic of the relative deviation of the rotational speed x with $\left(-5 \leq \frac{dM_{ed}}{dn_{ed}} \leq -1\right)$ and $\left(-5 \leq \frac{dQ_{ed}}{dn_{ed}} \leq -1\right)$. when

$\begin{cases} \frac{dM_{ed}}{dn_{ed}} < 0 \\ \frac{dQ_{ed}}{dn_{ed}} < 0 \end{cases}$ the pump-turbine is relative stable. However, the system will tend to the

oscillation state, if the absolute values of $\frac{dM_{ed}}{dn_{ed}}$ and $\frac{dQ_{ed}}{dn_{ed}}$ are too large. From Fig.

7(b), the system tends to be unstable with the increasing values of $\frac{dM_{ed}}{dn_{ed}}$ and $\frac{dQ_{ed}}{dn_{ed}}$.

In addition, these results agree with the stable condition of runaway point (see Eq.(22)).

From Eq. (22), the friction losses (k_l) , the water inertia in penstock (T_{w1}) and the rotational inertia (T_a) all affect the stability of the runaway point. Therefore, the

1 influences of the three factors with other parameters constant are studied, respectively.

2 The corresponding time waveforms are obtained by calculating Eq. (14).

3

5 (a) Time waveforms of the relative deviation of the rotational speed x with $\frac{dM_{ed}}{dn_{ed}} = -3$ and

6 $\frac{dQ_{ed}}{dn_{ed}} = -2$.

8 (b) Time waveforms of the relative deviation of the rotational speed x with $\frac{dM_{ed}}{dn_{ed}} = 3$ and

9 $\frac{dQ_{ed}}{dn_{ed}} = 2$.

10 Fig. 8. Time waveforms of the relative deviation of the rotational speed x with different slopes

of characteristic curves $\left(\frac{dM_{ed}}{dn_{ed}}, \frac{dQ_{ed}}{dn_{ed}}\right)$ and friction loss (k_1) .

From Fig. 8(a), when $\frac{dM_{ed}}{dn_{ed}} = -3$ and $\frac{dQ_{ed}}{dn_{ed}} = -2$, the relative deviation of the rotational speed x tends to be stable with different values of k_1 . The results show that the stability of the system will increase as the value of k_1 increases. When $\frac{dM_{ed}}{dn_{ed}} = 3$ and $\frac{dQ_{ed}}{dn_{ed}} = 2$, Fig. 8(b) shows that the system is stable with $k_1 = 5$ and $k_1 = 10$, while the system tends to be unstable with $k_1 = 1$. The simulation results show that the stability of the system is improved gradually with the increasing

value of k_1 , and the system is hard to be stabilized when $\begin{cases} \frac{dM_{ed}}{dn_{ed}} > 0 \\ \frac{dQ_{ed}}{dn_{ed}} > 0 \end{cases}$.

The model of the pump-turbine is manufactured based on the real pump-turbines in Baoquan station [12]. The load rejection conditions with servomotor failure were examined for this station. Figure 9 shows the changing process of the rotational speed during the runaway.

Fig. 9. Comparison of model data between partial valve closing and full opening [12].

From Fig. 9, the experiment with a partially closing valve was conducted at the same opening. The experiment results showed that the convergence rate increased,

which validates that the increase of the friction loss can improve the stability. The simulation results (in section 4.2) are in accordance with the model data. Therefore, the model data can validate the dynamic model of the pump-turbine is able to reflect the effects of the friction. Meanwhile, the theoretical analysis from Ref. [11] shows that when the friction loss in the conduits is considered, the stable regions are increasing, which also illustrates that the friction loss has a positive effect on the stability. The experimental and theoretical results are in accordance with the simulation results of the dynamic model of the pump turbine.

Moreover, in order to study the impacts of the water inertia in penstock (T_{wl}) and the rotational inertia (T_a) on the stability of the pump-turbine at the runaway point, suppose $\left(k_1, \frac{dM_{ed}}{dn_{ed}}, \frac{dQ_{ed}}{dn_{ed}}\right) = (1, 3, 2)$ and study the dynamic behaviors of the system with different T_{wl} and T_a . Fig. 8 shows the dynamic characteristics of the pump-turbine at the runaway point with different affecting factors.

(a) Time waveforms of the relative deviation of the rotational speed x with different water inertia (T_{wl}) in the penstock.

(b) Time waveforms of the relative deviation of the rotational speed x with different rotational inertia (T_a)

Fig. 10. Time waveforms of the relative deviation of the rotational speed x with different affecting factors.

For $T_{w1}=0.2, 0.25$ and 0.3 , as shown in Fig. 10(a), the system tends to be unstable in the three cases. With the increase of the T_{w1} , however, the stability of the system can be improved to some extent. In contrast to the influence of T_{w1} , T_a has a negative effect on the stability of the system. In addition, these results in Fig. 10 can also verify the correctness of the stable condition of the runaway point.

5. Conclusions

In this paper, the model of the pump-turbine is established by linearizing the characteristic curves. Then, to obtain an overview of the essential dynamic of the pump-turbine in the S-shaped regions, the dynamic relative parameters are introduced and the dynamic nonlinear model of the pump-turbine is established. This method provides a new perspective on the modeling of the pump-turbine in the transient

processes. Also, combining the numerical experiments with theoretical analysis, following conclusions can be obtained.

When the pump-turbine rejects load with servo motor failure, it moves from the turbine condition to the turbine braking condition, and further to the reverse pump turbine condition. According to the stable condition of the runaway point, the slopes of the characteristic curves have an important influence on the stability of the system. When $\frac{dM_{ed}}{dn_{ed}} < 0$ and $\frac{dQ_{ed}}{dn_{ed}} < 0$, the system is relative stable but it will tend to the oscillation state, if the absolute values of $\frac{dM_{ed}}{dn_{ed}}$ and $\frac{dQ_{ed}}{dn_{ed}}$ are too large. In addition, the system tends to be unstable with the increasing values of $\frac{dM_{ed}}{dn_{ed}}$ and $\frac{dQ_{ed}}{dn_{ed}}$, when $\frac{dM_{ed}}{dn_{ed}} > 0$ and $\frac{dQ_{ed}}{dn_{ed}} > 0$. In terms of the effects of k_1 , T_{w1} and T_a , the stability of the system is improved with the increase of k_1 and T_{w1} , whereas T_a has an opposite influence.

The conventional autonomous model of the pump-turbine can only reflect the characteristics of the system on a specific condition. However, the parameters vary dramatically in the S-shaped regions and the autonomous model cannot describe the dynamic characteristics of the pump-turbine. To study the dynamic characteristics of the pump-turbine in the S-shaped regions, the dynamic model of the pump-turbine is established in this paper. In the future, the effects of the hysteresis on the dynamic characteristics of the pump-turbine will be considered and the dynamic model will be developed to obtain better applicability.

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Reference

- [1] Chen CL, Chen HC, Lee JY. Application of a generic superstructure-based formulation to the design of wind-pumped-storage hybrid systems on remote islands. *Energy Conv. Manag.* 2016; 111: 339-351.
- [2] Kusakana K. Optimal scheduling for distributed hybrid system with pumped hydro storage. *Energy Conv. Manag.* 2016; 111: 253-260.
- [3] Schleicher WC, Oztekin A. Hydraulic design and optimization of a modular pump-turbine runner. *Energy Conv. Manag.* 2015; 93: 388-398.
- [4] Wang F, Jiang JG. A novel static frequency converter based on multilevel cascaded H-bridge used for the startup of synchronous motor in pumped-storage power station. *Energy Conv. Manag.* 2011; 52: 2085-2091.
- [5] Xu BB, Wang FF, Chen DY, Zhang H. Hamiltonian modeling of multi-hydro-turbine governing systems with sharing common penstock and dynamic analyses under shock load. *Energy Conv. Manag.* 2016; 108: 478-487.
- [6] Zhang XX, Cheng YG, Xia LS, Yang JD, Qian ZD. Looping Dynamic Characteristics of a Pump-Turbine in the S-shaped Region During Runaway. *J. Fluids Eng.-Trans. ASME.* 2016; 138: 091102.
- [7] Sivakumar N, Das D, Padhy NP. Variable speed operation of reversible pump-turbines at Kadamparai pumped storage plant – A case study. *Energy Conv. Manag.* 2014; 78: 96-104.
- [8] Ghoneim AA. Design optimization of photovoltaic powered water pumping systems. *Energy Conv. Manag.* 2006; 47: 1449-1463.

- [9] Lara D, Merino G, Salazar L. Power converter with maximum power point tracking MPPT for small wind-electric pumping systems. *Energy Conv. Manag.* 2015; 97: 53-62.
- [10] Guo WC, Yang JD, Wang MJ, Lai X. Nonlinear modeling and stability analysis of hydro-turbine governing system with sloping ceiling tailrace tunnel under load disturbance. *Energy Conv. Manag.* 2015; 106: 127-138.
- [11] Zeng W, Yang JD, Yang WJ. Instability analysis of pumped-storage stations under no-load conditions using a parameter-varying model. *Renew. Energy.* 2016; 90: 420-429.
- [12] Zeng W, Yang JD, Guo WC. Runaway instability of pump-turbines in S-shaped regions considering water compressibility. *J. Fluids Eng.-Trans. ASME.* 2015; 137: 051401.
- [13] Xu YH, Zhou JZ, Xue XM, Fu WL, Zhu WL, Li CS. An adaptively fast fuzzy fractional order PID control for pumped storage hydro unit using improved gravitational search algorithm. *Energy Conv. Manag.* 2016; 111: 67-78.
- [14] Schleicher WC, Oztekin A. Hydraulic design and optimization of a modular pump-turbine runner. *Energy Conv. Manag.* 2015; 93: 388-398.
- [15] Li DY, Wang HJ, Xiang GM, Gong RZ, Wei XZ, Liu ZS. Unsteady simulation and analysis for hump characteristics of a pump turbine model. *Renew. Energy.* 2015; 77: 32-42.
- [16] Jain SV, Swarnkar A, Motwani KH, Patel RN. Effects of impeller diameter and rotational speed on performance of pump running in turbine mode. *Energy Conv. Manag.* 2015; 89: 808-824.
- [17] Premkumar TM, Chatterjee D. Computational analysis of flow over a cascade of S-shaped hydrofoil of fully reversible pump-turbine used in extracting tidal energy. *Renew. Energy.* 2015; 77: 240-249.
- [18] Yuan XH, Chen ZH, Yuan YB, Huang YH. Design of fuzzy sliding mode controller for hydraulic turbine regulating system via input state feedback linearization method. *Energy.* 2015; 93: 173-187.
- [19] Sarasua JI, Perez-Diaz JI, Wihelmi JR, Sanchez-Fernandez JA. Dynamic

1 response and governor tuning of a long penstock pumped-storage hydropower
2 plant equipped with a pump-turbine and a doubly fed induction generator.
3 Energy Conv. Manag. 2015; 106: 151-164.

4 [20] Ademi S, Jovanovic M. High-efficiency control of brushless doubly-fed
5 machines for wind turbines and pump drives. Energy Conv. Manag. 2014; 81:
6 120-132.

7 [21] Benini E, Biollo R, Ponza R. Efficiency enhancement in transonic compressor
8 rotor blades using synthetic jets: A numerical investigation. Appl. Energy. 2011;
9 88: 953-962.

10 [22] Jansson I, Akerstedt HO, Aidanpaa JO, Lundstrom TS. The effect of inertia and
11 angular momentum of a fluid annulus on lateral transversal rotor vibrations. J.
12 Fluids Struct. 2012; 28: 328-342.

13 [23] Li ZJ, Bi HL, Wang ZW, Yao Z. Three-dimensional simulation of unsteady flows
14 in a pump-turbine during start-up transient up to speed no-load condition in
15 generating mode. Proc. Inst. Mech. Eng. Part A-J. 2016; 230: 570-585.

16 [24] Yang W, Xiao RF. Multiobjective optimization design of a pump-turbine impeller
17 based on an inverse design using a combination optimization strategy. J. Fluids
18 Eng.-Trans. ASME. 2014; 136: 014501.

Nonlinear piecewise relative parameters are introduced. In the S-shaped regions, the nonlinear dynamical behaviors of the system are studied in detail.

Fig. 1. Global bifurcation diagram of the relative deviation of the water level in the surge tank

Fig.2. Three-dimensional bifurcation diagrams with changing slopes of the characteristic curves.

Fig. 3. Time waveforms with $x-T_{w1}$

Fig. 4. Time waveforms with $x-T_a$

(zs: relative deviation of the water level ; x: relative deviation of the rotational speed)