1
2
3
4
5
6
7
8
9
10
11
12 THE MECHANICS OF INTERSECTING ECHELON VEINS AND
13 PRESSURE SOLUTION SEAMS IN LIMESTONE

14 Solomon Seyum^{1,2*} and David D. Pollard¹
15
16
17
18
19

20 **Keywords:** echelon veins, pressure solution seams, fracture mechanics,
21 limestone stiffness, finite element modeling, tectonics
22

23 **Journal:** *Journal of Structural Geology*
24
25
26
27
28
29
30
31

32 ¹Geological and Environmental Sciences, Stanford University, Stanford, California, 94305, USA

33 ²Current address: Danish Hydrocarbon Research and Technology Centre, Danmarks Tekniske Universitet,
34 2800 Kongens Lyngby, Denmark
35

36 *Corresponding author: solsey@dtu.dk

Abstract

Many studies that describe the formation of echelon vein arrays relate the causative stresses implicitly to the deformation, reliant on simple shear kinematics, such that the vein-to-array angle and the array width are the primary physical quantities. In contrast, we identify twelve physical quantities to describe echelon veins in two dimensions, including coeval, vein-intersecting, pressure solution seams. A finite element method is used to reproduce vein shapes in linear elastic and elastic-perfectly plastic model limestone. Model vein geometries are designed using values within the range of geometries measured from echelon veins at Raplee Anticline and Comb Monocline, Utah.

Four physical quantities are significant for describing echelon vein shapes: vein spacing, vein-array angle, limestone elastic stiffness, and closing of orthogonal pressure solution seams. Pressure solution seam closing influences the mechanical interaction between adjacent veins, and for a range of conditions, causes a nearly linear vein opening distribution (triangular shapes) and encourages straight vein propagation, both of which approximate field measurements. Model results show that small spacing of veins with seams and large vein-array angles promote straight vein traces in limestone with stiffness typical of laboratory measurements, given the physical geologic conditions inferred from the burial history of the limestone strata.

1. INTRODUCTION

Arrays of echelon veins with intersecting echelon pressure solution seams are common features in deformed limestone rocks (Figure 1). *Echelon*, commonly termed *en échelon* in structural geology literature, refers to the step-like geometry of closely spaced, similar length veins or seams. Here, the term *array* refers to the arrangement of vein and pressure solution seam midpoints along an approximately straight line over decimeter to meter distances. The systematic

geometries of these vein and seam arrays has encouraged structural geologists to relate their formation to a tectonic stress state (e.g. Jackson, 1991; Roering, 1968; Wiltschko et al., 2009). Inferences about the physical processes for opening and propagation of echelon veins in conjunction with pressure solution seams are mostly made from field maps and descriptions (Beach, 1975; Jackson, 1991; Kelly et al., 1998; Peacock and Sanderson, 1995; Roering, 1968; Smith, 1999; Willemse et al., 1997; Wiltschko et al., 2009).

Motivated by geologic field measurements and interpretations of echelon veins and seams in limestone at Raplee Ridge, Utah, our hypothesis is that echelon vein shapes are the result of deformation in limestone caused by remote tectonic stress, fluid pressure on the surfaces of echelon veins, closing of pressure solution seams that intersect contemporaneously opening veins, and plastic yielding of limestone near vein tips. Mechanical models can relate these applied stresses and material behavior to the shapes of veins (displacements of the vein surfaces) explicitly through material constitutive relations. In a mechanical model, physical quantities, like vein-array angle, can be varied to record their effect on vein aperture and stresses near vein tips. The ranges of acceptable values for the physical quantities are constrained by values that result in model vein shapes closely approximating measured vein shapes, given the values are defensible with geologic reasoning.

A large number of scientific studies invoke a simple shear kinematic method to describe the formation of echelon veins (e.g. Bons et al., 2012; Lisle, 2013; Seyum, 2015; and publications cited therein), as introduced by Ramsay (1967). Models using this method treat veins as passive markers: as imaginary lines that rotate and lengthen within the shear zone (Figures 2a, b). In a simple shear kinematic model, the angle of shear and the shear strain are estimated from the change in orientation of echelon veins, ignoring perturbations of the local

strain field (Figure 2d) (Seyum, 2015). Vein opening is inferred to be parallel to the axis of greatest extension at 45° to the shear direction within the simple shear zone (Ramsay, 1967 pp. 83-88). Subsequent shearing rotates the passive marker that is a proxy for the veins, and each additional increment of vein propagation is perpendicular to the direction of greatest extensional strain; thereby forming sigmoidal vein shapes. Echelon vein positions, modes of fracturing, the number of veins initiated, vein lengths, echelon vein spacing, and vein opening cannot be explained using a kinematic analysis (Figure 2b) because no choice is made of the material properties that control the deformation response to the applied stresses.

A complete physical model for this geologic phenomenon includes a geologically appropriate constitutive relationship between applied stresses and the resulting strain that, with other geologic and mechanical constraints, would reproduce the observed geometries. The underlying relationships are Cauchy's laws of motion, which are based on conservation of mass and momentum (Pollard and Fletcher, 2005 pp. 269-273).

Studies of echelon fractures that use two-dimensional deformation models based on the laws of motion include Dey and Wang (1981), Pollard and others (1982), Rogers and Bird (1987), Du and Aydin (1991), Fleck (1991), Olson and Pollard (1991), Mandal (1995), Ramsay and Lisle (2000 pp. 711, 715, 766-769, 1044-1046), and Chau and Wang (2001). Ramsay and Lisle (2000) use three distinct finite element models to: (1) show how the stress field is deflected across a narrow zone of linear elastic material that is relatively softer than the surrounding elastic material, to infer the orientation of echelon veins in a shear zone; (2) show how stress in a linear elastic material is distributed near echelon vein tips; and (3) describe sigmoidal vein shapes in a viscous material that is deformed in simple shear. Rogers and Bird (1987) illustrate the complexity of stress distributions surrounding a geometrically irregular set of echelon dikes

using a boundary element model with isotropic, linear elastic material. Dey and Wang (1981) record changes in magnitude of the maximum tensile stress at the tip of a crack in an array with changes in the initial crack positions. Pollard and others (1982) model propagation paths of echelon cracks and show that interaction between overlapping cracks causes curved propagation paths, increasing aperture near crack centers, and decreasing aperture near the crack tips. Chau and Wang (2001) describe interactions between echelon cracks and the limits on straight crack growth patterns using an analytical solution to solve for the critical ratios of crack length and crack spacing for a variety of boundary configurations. Du and Aydin (1991) use an analytical solution to show how the position of the maximum stress intensity near the tips of echelon cracks varies with distance and angle between neighboring crack tips. The results suggest that an array of echelon veins would promote the growth of new veins at the array ends with the same echelon step arrangement. Mandal (1995) uses an analytical solution for the stress field in an elastic material containing cracks to show how echelon crack spacing, or mechanical interaction of neighboring crack tips, affects the direction of crack propagation. Fleck (1991), using dislocation theory, and Olson and Pollard (1991), using a boundary element model, show that the crack surface displacements can be calculated given the stiffness of the material, and similar to Pollard and others (1982), Chau and Wang (2001), and Mandal (1995), show that crack propagation is a function of the near-tip stress field and the near-tip stress field is perturbed by neighboring cracks.

We build on these published numerical methods and results for model echelon vein formation by introducing closing of orthogonal, intersecting pressure solution seams, assigning an elastic-plastic material constitutive relationship to the limestone host rock, and comparing model results to vein and pressure solution seam array structures measured at Raplee Ridge,

Utah.

In the remaining text, *fracture* is used as a generic term without any specification of relative motion of those surfaces. The term *joint* refers to dominantly opening mode fractures identified in the field. *Vein* is a term used for a fracture that has been filled with mineral precipitates. The term *crack* is used when referring to a displacement discontinuity in mechanical models and consists of two surfaces. Model cracks are compared to veins observed in the field. *Pressure solution seam* refers to the field identification of a two-dimensional trace in limestone along which we infer soluble grains within the rock were dissolved and transported in solution, and insoluble grains remain. A *model seam* refers to the mechanical model representation of a pressure solution seam.

In this paper, we introduce a mechanical description for the formation of echelon veins and orthogonal pressure solution seams. The numerical model input values are derived from field-measured geometries of veins and seams, published values for the mechanical properties of limestone, remote stress and fracture (vein) surface pressure conditions inferred from the published burial history of the strata, and estimates for closing of pressure solution seams from images of sampled rock thin-sections. The shapes of veins are presented as a function of the measured and inferred physical variables, and are described by the relative displacements of the surfaces of model echelon cracks. The stresses near crack tips are also evaluated to constrain the physical conditions that would cause observed echelon veins to have relatively straight traces.

2. FIELD ATTRIBUTES OF VEIN AND SEAM ARRAYS

Two distinct sets of echelon vein and pressure solution seam arrays are exposed in marine limestone strata near Mexican Hat, Utah, within the bounds of Raplee Anticline and Comb Monocline folds. They were not observed in the same strata at locations off of the folds and they

are therefore inferred to have formed during folding (Seyum, 2015). The two sets are distinguished from each other based on the echelon step geometries of veins: right-stepping veins versus left-stepping veins (Figure 1).

Pressure solution seams coexist with veins in all of the arrays observed in the field, and the veins and seams in a given array are roughly orthogonal to one another in bedding plane view. Veins are interpreted to be fractures that opened approximately perpendicular to the local least compressive stress (Pollard and Aydin, 1988; Secor, 1965), and the fractures were subsequently filled via the precipitation of minerals (calcite, in this case). Pressure solution seams, on the other hand, are closing structures. They are interpreted to form approximately perpendicular to the local greatest compressive stress by dissolution of minerals in the presence of fluid that transports the dissolved material to areas away from the seam, perhaps to the adjacent, intersecting veins (Bathurst, 1975 p. 462; Nenna and Aydin, 2011; Weyl, 1959; Zhou and Aydin, 2010).

From vein-fossil intersections in thin-section, we infer that veins formed predominantly as opening fractures (Figures 3a, d), with few veins exhibiting a relatively small, measurable component of shear. We do not observe relative displacement markers across veins at the outcrop scale. The veins have approximately straight traces at both the outcrop and thin-section scales (Figures 1, 3b, c). Pressure solution seams have curved and jagged traces (Figure 1).

Veins and pressure solution seams mutually crosscut and abut one another (Figure 1). Mutually intersecting veins and pressure solution seams can be reconciled as structures responding to an identical local stress state with the greatest compressive stress approximately parallel to the vein and perpendicular to the pressure solution seam (Fletcher and Pollard, 1981;

Katsman, 2010). Thus, we infer that veins and pressure solution seams developed simultaneously.

Although individual veins do not extend through the thickness of limestone beds, additional veins and pressure solution seams exist beneath those exposed at the limestone bedding surface to continue the array perpendicular to bedding; often truncated by bedding planes or bed-parallel pressure solution seams. We have made no observations of echelon veins or pressure solution seams in adjacent sandstone and siltstone strata.

The age relationship between conjugate arrays has not been determined because intersection geometries are either complex or ambiguous. The current hypothesis is that the two sets formed in response to the same remote stresses, since the vein and seam geometries across the two sets are similar. The angle, γ , of an array to the remote greatest compressive stress direction is assumed to be half of the acute angle between intersecting arrays of each set. The range of γ for the array sets across the folds is 8° to 26° , with a mean value of 19° (Seyum, 2015).

Vein and seam geometries from the image in Figure 1 are measured in Adobe® Illustrator, and the values are presented in Supplementary Table 1 (Seyum, 2015). The measured geometries include vein and seam lengths ($2c$ and l , respectively), vein and seam spacing (s and s_s), vein apertures, angles between vein orientations and the array orientation (α), angles between seams and the array, and angles between veins and seams. Veins and seams are approximated as straight lines. The length and angle measurements of those lines have a ± 0.05 mm error and $\pm 0.5^\circ$ error, respectively. Vein and seam spacing is measured along scanlines oriented parallel to the array. The mean and range values of the measured geometric attributes are presented in Table 1.

3. MODEL METHODS

We assume plane strain deformation with the 2-D plane orthogonal to vein height and parallel to bedding. This assumption is used because echelon veins are relatively straight in the direction perpendicular to bedding. We include an initial arrangement of echelon cracks based on the idea that a pre-existing, self-organization of fractures from discrete discontinuities (flaws) form the echelon geometry (Olson and Pollard, 1989; Olson and Pollard, 1991; Seyum, 2015). Narrow zones of homogeneous, weaker material with dimensions similar to arrays, as modeled by Ramsay and Lisle (2000 pp. 766-768), were not observed in the field. An array of cracks localizes deformation as shown in Figure 2d. Because pressure solution seams facilitate the initiation and growth of complementary veins (Fletcher and Pollard, 1981; Katsman, 2010), similar reasoning is used to introduce echelon seams as part of the initial step in the following models.

The following subsections define the physical quantities needed to describe echelon veins and seams.

3.1 Crack Surface Displacements and Crack Tip Stress Field

For a 2-D, homogeneous, isotropic elastic material subject to uniform remote principal stresses, containing a single, pressurized, straight crack parallel to the y-axis (Pollard and Segall, 1987), the relative normal and tangential displacement components across the crack are

$$\Delta u_n^c = \Delta \sigma_n \frac{4(1 - \nu^2)}{E} \sqrt{c^2 - y^2}, \quad (1)$$

$$\Delta u_t^c = \Delta \sigma_t \frac{4(1 - \nu^2)}{E} \sqrt{c^2 - y^2}. \quad (2)$$

E is Young's modulus; ν is Poisson's ratio; c is the crack half-length; y is position along the crack surfaces. $\Delta\sigma_n$ and $\Delta\sigma_t$ are the normal and tangential components of the crack driving stress.

The driving stresses are related to the two remote principal stresses, σ_1^∞ and σ_3^∞ , the uniform pressure, P , applied to the crack surfaces, and the crack angle to the remote stresses (β) as

$$\Delta\sigma_n = P + \sigma_1^\infty \cos^2(\beta) + \sigma_3^\infty \sin^2(\beta), \quad (3)$$

$$\Delta\sigma_t = (\sigma_1^\infty - \sigma_3^\infty) \sin(\beta) \cos(\beta). \quad (4)$$

The driving stresses are derived from Cauchy's Formula for relating stress components to traction components resolved on the crack plane (Pollard and Fletcher, 2005 pp. 213-216). Tension is positive.

Stresses near a crack tip control inelastic behavior such as crack tip blunting and crack propagation. The two elastic moduli, E and ν , do not influence the stress field of a material subjected to stress (traction) boundary conditions (Timoshenko and Goodier, 1970 p. 19), as used in this study. For example, the analytical solution for the circumferential stress field near the tip of a pressurized crack in a 2-D, homogeneous, isotropic elastic material subject to uniform, biaxial remote principal stresses is

$$\sigma_{\theta\theta} \simeq \sqrt{\frac{c}{2r}} \left[\Delta\sigma_n \cos^3\left(\frac{\theta}{2}\right) - 3\Delta\sigma_t \sin\left(\frac{\theta}{2}\right) \cos^2\left(\frac{\theta}{2}\right) \right]. \quad (5)$$

The crack tip stress field is a function of the driving stresses, crack half-length, and position. Here r and θ are radial and angular polar coordinates, with the origin at the crack tip. Analytical solutions for the stresses at a crack tip can be found in the literature on fracture

mechanics (e.g. Anderson, 2005 pp. 81-82, 95; Erdogan and Sih, 1963; Jaeger et al., 2007 p. 240; Lawn and Wilshaw, 1993 p. 25; Zehnder, 2012 pp. 19, 21).

3.2 Dimensional Analysis

A dimensional analysis is used to gain insight about the relationships between physical variables, and is used to design efficient numerical experiments. The Buckingham Theorem is a way to relate the number of physical variables, n , and the number of independent dimensions occurring in those variables, k , to give the dimensionless quantities or scale factors, p , for the physical process; such that $p = n - k$ (Anderson, 2005 pp. 18-21; Buckingham, 1915; Pollard and Fletcher, 2005 p. 140).

From the given solutions for the displacements on crack surfaces and the stresses at crack tips (Eqs. 1, 2, 5), c , σ_1^∞ , σ_3^∞ , P , E , and ν are relevant independent quantities. For multiple, parallel, equal-length cracks in an echelon array, crack spacing, s , the angle, α , of the crack traces to the array trend, and the angle, γ , of the array trend to σ_3^∞ are additional geometric quantities. Varying the number of cracks, N , in an echelon array from 5 to 15 produces negligible effects on relative crack shear displacement, and minor effects on crack opening (Supplementary Figure 1a) (Seyum, 2015). Veins are associated with pressure solution seams, so relative closing of a seam, Δu_n^s , is evaluated for its effect on crack surface relative displacements and crack tip stresses.

Quantities x , y , r , c , s , and Δu_n^s have dimension length, L . Quantities σ_1^∞ , σ_3^∞ , P , and E have derived dimension $MT^{-2}L^{-1}$ for stress; where M is mass and T is time. Quantities ν , θ , α , and γ are dimensionless. Since M and T are always grouped as $MT^{-2}L^{-1}$ in this analysis, we consider two independent dimensions ($k = 2$): fundamental dimension, length, and derived dimension, stress. We choose c and σ_1^∞ to normalize variables with dimensions carrying length

and stress, respectively. Note that $\beta = \alpha - \gamma$, so the measurable quantities α and γ are substituted in for β in Eqs. (3) and (4).

Twelve physical quantities ($n = 12$) influence displacements along the echelon crack surfaces, and ten quantities ($n = 10$) influence the stress field near the crack tips. Therefore, the displacement components are functions of 10 dimensionless quantities (p). Crack opening is, for example,

$$\frac{\Delta u_n^c}{c} \simeq F_1 \left(\frac{\sigma_3^\infty}{\sigma_1^\infty}, \frac{E}{\sigma_1^\infty}, \nu, \frac{x}{c}, \frac{y}{c}, \frac{P}{\sigma_1^\infty}, \frac{s}{c}, \alpha, \gamma, \frac{\Delta u_n^s}{c} \right).$$

Normalized stress components are functions of 8 dimensionless quantities (p);

$$\frac{\sigma_{\theta\theta}}{\sigma_1^\infty} \simeq F_2 \left(\frac{\sigma_3^\infty}{\sigma_1^\infty}, \frac{r}{c}, \theta, \frac{P}{\sigma_1^\infty}, \frac{s}{c}, \alpha, \gamma, \frac{\Delta u_n^s}{c} \right).$$

We demonstrate in the following subsections why some of the variables in Eqs. (6) and (7) can be held constant based on interpretations of the geology and based on the relative contribution of each variable on crack surface displacements and crack tip stresses.

3.3 Key Physical Quantities

We infer vein and seam arrays to be associated with folding of Raplee Anticline and Comb Monocline during the Laramide orogenic event between 70 Ma and 40 Ma (Gregory and Moore, 1931; Kelley, 1955; Mynatt et al., 2009; Tweto, 1975). Using published depth and temperature history profiles of the stratigraphy (Nuccio and Condon, 1996), the stratigraphic units containing echelon veins and seams were between 3 km and 4 km depth during the Laramide Orogeny, and as shallow as 1.5 km at the onset of rapid burial earlier than 70 Ma. The overburden pressure and fluid pressure is assumed to have been between 37.5 MPa and

100 MPa, and 15 MPa and 40 MPa, respectively, given a 25 MPa/km lithostatic gradient and a 10 MPa/km hydrostatic gradient.

To ensure that all cracks open in the following models within the range of inferred overburden and fluid pressures, we test conditions of opening for a single crack oriented 35° to σ_3^∞ . This is the maximum angle calculated from the maximum α (50°) and minimum γ (15°) values measured in the field. Choosing a uniform fluid pressure, P , of 30 MPa, this crack will open for $\sigma_3^\infty = -65$ MPa and for $\sigma_1^\infty = -10$ MPa (Figure 4); compression is negative.

Triaxial compressive test results from saturated Solnhofen Limestone (Heard, 1960; Rutter, 1972a) guide our choices of material properties and constitutive relationships needed to represent the limestones at Raplee and Comb folds. Figure 5 shows the differential stress versus axial strain results for samples tested at temperatures and confining pressures spanning the range of temperatures and pressures inferred from the burial history of strata at Raplee Ridge. Deformation is approximately elastic up to 1% to 1.5% strain before inelastic deformation ensues between 250 MPa and 490 MPa differential stresses. The elastic material stiffness, E , measured from the graphs in Figure 5, is between 26 GPa and 38 GPa, and is relatively insensitive to changes in temperature, fluid pressure, and confining pressures. When the yield point in compression is reached (Figure 5), the Solnhofen Limestone either approximates perfectly plastic behavior (strain increases with nearly constant applied stress), or softens with increased strain (strain increases with decreasing applied stress). Model results show that plastic deformation only occurs near the crack tips (Supplementary Figure 2). Aside from blunting of the crack tips, inelastic deformation of the limestone, outside of the seam area, is a minor contributor to the overall shape of echelon cracks (Supplementary Figure 3) (Seyum, 2015).

Poisson's ratio, ν , of the Solnhofen Limestone ranges from 0.12 to 0.36 (Birch, 1966; Deere and Miller, 1966 p. 78; Renner and Rummel, 1996). The Poisson's ratio has a negligible effect on crack shape (Supplementary Figure 1b), so it is kept constant at 0.25 in the following models.

The magnitudes of σ_1^∞ , σ_3^∞ , and P are held constant in the following models since their effect on crack surface displacements is predictable from linear elastic analytical solutions (Eqns. 1, 2; Supplementary Figure 1c). Crack opening is negligible when varying γ over the range of field measurements (Supplementary Figure 1d), and crack shearing is approximately equal and opposite in magnitude for γ values that give equal and opposite crack angles from the direction of σ_3^∞ (Supplementary Figure 4) (Seyum, 2015). Additionally, orienting a Cartesian coordinate axis parallel with the crack allows crack-normal and crack-shear relative displacements to be recorded in terms of a single coordinate; x in this case. Polar coordinate r is held constant when evaluating stresses in the near tip field of a crack. The remaining values, s , α , E and Δu_n^s , are chosen as the key physical quantities of this problem. Therefore, the crack-normal relative displacement and the circumferential component of stress near the crack tip, Eqs. (6) and (7) are reduced to

$$\frac{\Delta u_n^c}{c} \simeq F_3 \left(\frac{x}{c}, \frac{s}{c}, \alpha, \frac{E}{\Delta \sigma_n}, \frac{\Delta u_n^s}{c} \right), \quad (8)$$

$$\frac{\sigma_{\theta\theta}}{\Delta \sigma_n} \simeq F_4 \left(\theta, \frac{s}{c}, \alpha, \frac{\Delta u_n^s}{c} \right). \quad (9)$$

The relative shear displacement of a crack's surfaces, $\Delta u_t^c/c$, is a function of the quantities in Eqn. 8. Since the remote stresses and fluid pressure are held constant, the crack normal driving stress, $\Delta\sigma_n$ (Eqn. 3), is chosen to normalize quantities with dimension $MT^{-2}L^{-1}$ for stress.

3.4 Design of the Mechanical Model

We use a commercial finite element software, Abaqus FEA (v. 6.11-1 © Dassault Systèmes, 2011), for modeling static, 2-D plane strain deformation of isotropic, homogeneous limestone containing echelon, opening cracks and orthogonal, closing seams. A finite element method is used because it is capable of solving nonlinear constitutive equations that compute inelastic deformation and admit larger strains. Cracks are not allowed to propagate in an effort to demonstrate the effects of simpler, static physical deformation. Limestone surrounding veins and seams (Figure 3) is considered homogeneous because the composition and texture vary little within tenths of a millimeter to meter distances. Material isotropy is assumed since there are no apparent mineral lineations or cleavage planes in the limestone strata that would imply directional variations in stiffness.

The exterior boundaries of the model are placed at least 20 times the half-length of an array from the array center to minimize boundary effects (Figure 6a; Supplementary Figure 5) (Seyum, 2015). Uniform, remote, compressive stresses, σ_1^∞ and σ_3^∞ , are applied normal to the four external, equal-length boundaries. Uniform pressure, P , is applied normal to the crack surfaces. Displacement boundary conditions are prescribed at the center nodes of the four exterior boundaries so that rigid rotation and translation of the body is prohibited (Figure 6a).

Echelon cracks are modeled as parallel discontinuities in the limestone material continuum. Each crack is designed with two, equal-length, coincident surfaces that are connected at their ends to create sharp tips and zero apertures. The middle crack of each modeled array is

evaluated, and the results of which closely approximates the shapes of all cracks between the two cracks at each end of an array when $N \geq 5$. The base model geometry consists of a single array with five, left-stepping echelon cracks, for which $s/c = 3.15$, $\alpha = 25^\circ$, and $\gamma = 25^\circ$ (Supplementary Figure 6a; Supplementary Table 2, experiment 18). Since the observed array sets are approximately symmetric, the insight gained from the model results of left-stepping echelon cracks can be used to understand symmetrically similar right-stepping echelon cracks.

A model seam is designed to facilitate the inward displacement kinematics of a natural pressure solution seam. It does not include dissolution and transportation processes of soluble minerals, nor does it reproduce the aggregation process of insoluble minerals that characterizes the final seam shape and composition (Figure 3). A kinematic representation of seam closing serves as a first step toward building our intuition of their contribution to the shapes of veins, before including additional attributes, not studied here, that are more representative of pressure solution seam processes.

Model seams are thin, tapered zones of homogeneous, isotropic, soft material with their long axes bisecting the crack lengths (Figure 6b). The seam boundaries are in bonded contact with the surrounding model limestone, and the displacement field across those boundaries is continuous. A Young's modulus of the seams, E_s , is assigned, and is less than the surrounding limestone stiffness. This will promote inward displacement of the seam boundaries to give seam closing, Δu_n^s . The quantity, Δu_n^s , is a measurement of the relative displacement of two points, where the two seam surfaces intersect one of the vein surfaces. Poisson's ratio for the seam, ν_s , is 0 to restrict extension perpendicular to its length when the seam is closing. The seam half-length, l_s , is chosen to be $2.5c$. The length is within the range of measured lengths, and minimizes interaction of the seam tips with crack tips. The maximum seam width, w_s , is at the

crack-seam intersection (Supplementary Figure 7a) and is $0.25c$ initially to admit as much as 1.5 mm of inward displacement, Δu_n^s . The effect of l_s , w_s , v_s , and E_s on seam closing, Δu_n^s , is illustrated in Supplementary Figure 8 (Seyum, 2015). Crack fluid pressure, P , is not thought to act on the seam, and so, is not applied on the portion of the crack surface in the model that is coincident with the seam. A crack and seam pair in the model does not intersect neighboring crack-seam pairs. Supplementary Figure 9 illustrates the model geometry limitations due to orthogonal seams.

The crack-normal and crack-tangential components of displacement are recorded at approximately 120 nodes on the two surfaces of the middle crack to quantify the distribution of opening and shearing along the crack (Supplementary Figure 7c). The circumferential component of stress, $\sigma_{\theta\theta}$, with the origin of the polar coordinate system centered at a tip of the middle crack, is recorded at 361 angular positions, θ , at radius, r , $0.005c$ (Supplementary Figure 7b) to predict the direction of infinitesimal crack propagation. The coordinates are within the near-field of the crack tip where the values for stress are within 15% of the exact solution (Pollard and Segall, 1987), and the stresses in the following finite element models increase as a function of $1/\sqrt{r}$ toward the crack tips (Supplementary Figure 10a) (Seyum, 2015), consistent with the analytical solutions (Eqn. 5). We employ the idea that the initial propagation direction will be in a radial direction from the crack tips in the plane perpendicular to the greatest tension and coincides with the position of zero shear stress, as described by (Cotterell, 1965; Cotterell and Rice, 1980; Erdogan and Sih, 1963; Lawn and Wilshaw, 1993 p. 27; Thomas and Pollard, 1993), where the mode II stress intensity factor, $K_{II} = \Delta\sigma_t\sqrt{\pi c}$, is zero. Since many echelon veins at Raplee Ridge are relatively straight, we seek conditions that would cause a symmetric

distribution of $\sigma_{\theta\theta}$ near the crack tip, and the maximum value of $\sigma_{\theta\theta}$ is at $\theta = 000^\circ$ (ahead of the crack tip in line with the crack trace). Where $\sigma_{\theta\theta}$ is at a maximum, $\theta = \theta_m$.

4. RESULTS

Crack surface displacements and the crack tip stress field are evaluated for a range of s , α , E , and Δu_n^s . Elastic stiffness, E , is not normalized in the following results to readily compare model stiffness values to published, experimental data. Note that the relative displacement profiles presented here are only nearly symmetric because the model mesh element size distributions are different at either tip of the evaluated crack (Supplementary Figure 12) (Seyum, 2015).

4.1 Echelon cracks without seams in a linear elastic model limestone

Quantities s , α , and E are varied for an array of left-stepping echelon cracks without seams in a linear elastic model limestone (Supplementary Table 2, experiments 17-25). A decrease in normalized crack spacing, s/c , from 4.73 to 1.28 (Figure 7a), increases the magnitude of normalized crack opening, $\Delta u_n^c/c$, at the middle of the crack by a factor of 1.9 (Figure 7b). Echelon cracks with an s/c value of 4.73 or greater form a shape similar to an isolated crack (Supplementary Figure 1a) (Seyum, 2015). The crack opening distribution is approximately elliptical for an s/c value equal to or greater than 1.96. For $s/c = 1.28$ and smaller, the $\Delta u_n^c/c$ distribution diverges from an elliptical shape, bulging at the crack center and having approximately linear opening distributions toward the crack tips. Crack shearing, $\Delta u_t^c/c$, decreases from negligible amounts to a minimum of -7.1×10^{-4} (a negative value indicates a left-lateral shear sense) for $s/c = 4.73$ to 1.28 (Figure 7c). The shear distribution is bimodal with extreme left-lateral shear magnitudes at $x/c = \pm 0.68$ for $s/c = 1.96$ and $x/c = \pm 0.35$ for $s/c = 1.28$.

Varying α causes minor changes in the opening distribution of echelon cracks (Supplementary Figure 15a) (Seyum, 2015). A predictable relative shear profile across echelon cracks occurs with changes in α . Right-lateral shearing occurs when α is reduced, rotating cracks counterclockwise from the σ_3^∞ direction, and left-lateral shearing occurs when α is increased (Supplementary Figure 15b) (Seyum, 2015).

Elastic stiffness, E , has a linear inverse relationship with displacement (Eqns. 1, 2). Decreasing E from 60 GPa to 5 GPa increases crack opening by a factor of 12 (Figure 7d). The crack opening distribution remains approximately elliptical. Shearing across the crack is negligible when $E = 60$ GPa; s/c is 3.15 and α and γ are 25° (Figure 7e). Decreasing E to 5 GPa increases $\Delta u_t^c/c$ by a factor of 12. The sense of shear is right-lateral at the crack center and left-lateral near the crack tips, with $\Delta u_t^c/c = 0$ at $x/c = \pm 0.6$. Varying E does not affect the crack tip stress field, consistent with Eqn. (5).

Changes in s and α affect the stress distributions at the crack tips, accompanied by changes in the $\Delta u_t^c/c$ distribution. The distribution of the circumferential component of stress, $\sigma_{\theta\theta}$, at a radius $0.005c$ from the crack tip is sinusoidal and symmetric for $s/c = 4.73$ (Figure 8a). Values for $\sigma_{\theta\theta}/\Delta\sigma_n$ are positive (tensile) in the region ahead of the crack tip. A decrease in s results in left-lateral shearing of the crack (Figure 7c), and an asymmetric distribution of $\sigma_{\theta\theta}/\Delta\sigma_n$ about the crack tip. The position of maximum circumferential stress, θ_m , changes from being 000° (in line with the crack plane) to -035° ; angled toward the neighboring crack. The maximum values of $\sigma_{\theta\theta}$ range from 190 MPa to 280 MPa ($\Delta\sigma_n = 20$ MPa).

Decreasing α to 15° causes right-lateral shearing of the crack and shifts θ_m away from the neighboring crack ($\theta_m = 028^\circ$) (Figure 8b). For $\alpha = 35^\circ$, the $\sigma_{\theta\theta}$ distribution shifts to the left toward the neighboring crack ($\theta_m = -029^\circ$).

4.2 Single crack with a seam

Normalized seam closing, $\Delta u_n^s/c$, is inversely related to the model seam stiffness, E_s (Supplementary Figure 6b) (Seyum, 2015). Decreasing E_s to 0.1 GPa (E is 60 GPa) results in a maximum of 0.013 for $\Delta u_n^s/c$. Crack opening, $\Delta u_n^c/c$, increases by a factor of 4.1 compared to a single crack without a seam in a linear elastic model limestone (Figure 9a). The crack opening distribution diverges from an elliptical shape, and has an approximately linear profile toward the crack-seam intersection. Reducing E from 60 GPa to 5 GPa (E_s is 0.1 GPa) causes $\Delta u_n^s/c$ to be 0.089. With the inclusion of a seam, reducing E by a factor of 12 increases crack opening by a factor of only 7.7 (Figure 9b), relative to opening of a single crack without a seam (Supplementary Figure 1; Figure 7d; Eqn. 1).

See Supplementary Table 2, experiments 8-14, for model attributes referred to in this section.

4.3 Echelon cracks with seams in a linear elastic model limestone

For an array of cracks with orthogonal seams in a stiff, linear elastic model limestone ($E = 60$ GPa), a normalized seam closing, $\Delta u_n^s/c$, of 0.0087 (E_s is 0.1 GPa) causes an increase of crack opening by a factor of 3.1, compared to a similar array of cracks without seams (Figure 10a). Crack opening decreases by 0.93 in comparison to a single crack with a seam (Figures 9a, 10a). Crack shearing is right-lateral and on the order of 10^{-4} mm (Figure 10b). Shearing occurs despite the parallel orientation of cracks with the direction of greatest compression, σ_3^∞ (Figure 10c).

Shearing of echelon cracks due to seam closing affects the crack tip stress field (Figure 11). For $s/c = 1.28$, increasing $\Delta u_n^s/c$ to 0.0085 ($E_s = 0.1$ GPa), shifts θ_m from -025° to

055° (Figure 11a). Softening the surrounding model limestone by decreasing E from 60 GPa to 2.6 GPa shifts θ_m from 055° to 000° (Figure 11b), increasing $\Delta u_n^s/c$ to 0.10.

See Supplementary Table 2, experiments 11, 18, 31, 45-52, for model attributes referred to in this section.

4.4 Conditions for predicting straight crack traces

For a particular geometry of echelon cracks and seams in these models, Figure 11 and Supplementary Figure 16 illustrate that there are two scenarios in which the maximum circumferential stress would occur ahead of the crack tip in line with the crack plane, promoting straight crack propagation. One is by decreasing E_s and keeping E constant (Figure 11a), and the other is by keeping E_s constant at the assigned minimum of 0.1 GPa and decreasing E (Figures 11b). Here, we emphasize the latter because E is a physical quantity that can be compared to the elastic properties of limestone, whereas E_s is an unnatural quantity in the model to facilitate kinematic closing of seams. See Supplementary Table 2, experiments 18, 38, 47, 54-59, for model attributes referred to here.

Certain combinations of s/c , α , E , and $\Delta u_n^s/c$ values will promote straight propagation of echelon cracks. At $E_s = 0.1$ GPa, decreasing E increases seam closing and shifts θ_m from a positive value to 000° (Figures 11b). Figure 12a is a plot of symmetric stress distributions near the crack tip for a set of s/c , α , and E value combinations ($E_s = 0.1$ GPa) (Supplementary Table 2, experiments 35, 44, 53, 60-62). When α is 25°, the model limestone stiffness required to promote straight crack propagation increases from 0.2 GPa to 2.6 GPa with decreasing crack spacing. Increasing α to 35° requires greater limestone stiffness to promote straight cracks, and less seam closing (Figure 12a). For a crack-array angle of 35°, an E of 3.0 GPa will cause echelon cracks with s/c of 2.32 to propagate in a straight path, and an E of 18 GPa is required

for echelon cracks with s/c of 1.45. A summary of the crack spacing, crack-array angle, and model seam closing conditions necessary for straight crack propagation is shown in Figure 12b.

Further increasing α to 45° requires stiffer model limestone ($E = 60$ GPa) to encourage straight crack propagation. The crack opening and shearing profiles for an echelon array of cracks and seams with $\alpha = 45^\circ$, $s/c = 1.77$, $E = 60$ GPa, and $\Delta u_n^s/c = 0.014$ are shown in Figures 13a and 13b (Supplementary Table 2, experiment 62). The two-dimensional crack and seam shapes, with the surrounding displacement field, are shown in Figure 13c. The crack opening gradient is linear from the tips to the seam intersection (Figure 13a), and the maximum opening is 1.5 to 8 times larger than echelon crack apertures in an array with seams ($E = 60$ GPa) and smaller crack-array angles (e.g. Figures 9a, 10a). Maximum crack opening is 0.0094c (Figure 13a), and is an order-of-magnitude smaller than the average vein aperture to length ratios measured (Table 1). Figure 12a shows that the crack tip stress distribution is slightly asymmetric when α is 45° , and θ_m is -008° ; consistent with left-lateral shearing of the crack (Figure 13b).

5. DISCUSSION

Vein half-length, vein spacing, and vein-array angle are essential physical quantities that affect the shape of echelon veins, and are key for understanding the physical interaction between veins. Similar to findings presented in previous studies (Chau and Wang, 2001; Fleck, 1991; Mandal, 1995; Olson and Pollard, 1991; Pollard et al., 1982), stresses at the tips of overlapping veins are perturbed and consequently affect the propagation path of veins. Small spacing between echelon veins causes shearing of veins, in a left-lateral sense for left-stepping veins (Figure 7c), and causes an asymmetric vein tip stress distribution with the greatest tension skewed to promote propagation toward adjacent veins (Figure 8a). Greater overlap of veins increases opening near their midpoints, but decreases opening toward the vein tips (Figure 7b).

This mechanical interaction between neighboring veins causes a linear vein opening distribution near the vein tips, contributing to triangular vein shapes (Figure 1).

Where arrays of each set intersect at Raplee Ridge (Figure 1), veins are commonly oriented approximately parallel to the adjacent array orientation. The veins are oblique to the axis bisecting the arrays such that the vein-array angle is greater than the angle of the array to the bisecting axis: $\alpha > \gamma$. If the bisecting axis is assumed to be coincident with the remote greatest compressive stress orientation, left-lateral shearing across left-stepping veins is expected to occur when closing seams are not present. This is shown in Figure 7c for angles $\alpha = 35^\circ$ and $\gamma = 25^\circ$. As a result, the greatest tension near the vein tip is toward the neighboring vein (Figure 8a). Smaller vein spacing and greater vein-array angles increase vein overlap and vein interaction, and may explain sigmoidal vein shapes (Fleck, 1991; Mandal, 1995; Olson and Pollard, 1991).

Pressure solution seams are a critical component to the formation of the echelon veins observed at Raplee and Comb folds, affecting vein shapes and the stresses near vein tips. Seam closing increases vein opening and causes the vein opening profile to be approximately linear, particularly toward the vein-seam intersections (Figure 9 and Supplementary Figures 3c, d); also contributing to the triangular shape of veins (Figure 1). Small amounts of seam closing shift the angle, θ_m , of the greatest tensile stress near the vein tips away from the adjacent veins (Figures 11a, c), countering the mechanical interaction between overlapping veins. Additional closing of seams by reducing the stiffness, E , of the surrounding limestone results in shifting θ_m at the vein tips toward the neighboring veins again, such that θ_m is directly ahead of the vein at 000° (Figures 11b, d); i.e. suggesting straight vein propagation. Veins parallel to the orientation of the greatest remote compressive stress, σ_3^∞ , require seam closing and a soft limestone ($E < 3$ GPa) to promote straight vein propagation. Echelon veins that are 10° and 20° from the orientation of σ_3^∞

(greater α) can propagate in their own plane in stiffer limestone with orthogonal, closing seams; $E = 18$ GPa with $\Delta u_n^s/c = 0.032$, and $E = 60$ GPa with $\Delta u_n^s/c = 0.015$, respectively for those angles.

These results suggest that a narrow range of orientations for σ_3^∞ can be inferred from the orientation of a single array of straight echelon veins with orthogonal seams. The angle, γ , between the orientation of σ_3^∞ and array trace orientations is likely to be oblique to echelon veins, such that $\alpha > \gamma$. For smaller ratios of α/γ , limestone would need to be softer to produce straight veins (Figure 12b). Deformation of stiffer limestone, within the range of inferred limestone stiffness at Raplee Ridge (26 GPa and 38 GPa), would encourage straight veins for larger α/γ ratios. Echelon veins with a 45° angle for α and a 1.54 to 1.58 spacing-to-length ratio, s/c , which are the mean values from measurements made at Raplee Ridge (Table 1), will propagate straight if γ is 35° or less. Calculated γ values from conjugate arrays at Raplee Ridge are between 8° and 26° .

In Figure 12b, we show that you can conclude a range of limestone stiffness values and pressure solution seam closing amounts associated with the formation of straight, echelon veins and intersecting pressure solution seams. It requires that one of those two values can be estimated, and measurements of echelon vein spacing and vein-array angle are made. Limestone stiffness, as an independent quantity, is inferred from experimental results produced using the rock types and physical conditions within the range of inferred burial and deformation conditions. Pressure solution seam closing may be determined independently using, for example, a one-dimensional, mass conservation estimate from thin-sections (Seyum, 2015).

In addition to providing a tool for inferring limestone stiffness, pressure solution seam closing, and the orientation of the remote principal stresses during formation of echelon veins

and seams, the model results and assumptions introduce future research topics. Additional controls on echelon vein shape may be realized with further analyses of vein-seam intersection relationships, for example, such as veins with multiple intersecting seams, seams truncating veins, and oblique vein-seam intersections. Since closing of pressure solution seams is a major factor to the deformation in carbonate rock, a more realistic representation of the mechanisms responsible for seam closing is important, and may include dissolution of calcite and the concentration of insoluble minerals at the seam. One more potential research topic is a mechanical description of the stepwise evolution of echelon vein formation, which may provide insight about the deformation history of the rock. To form sigmoidal vein shapes during vein growth, or to maintain straight veins, the physical quantities (E , s , α , and Δu_n^s , for example) will change with each increment of vein growth. The results presented here provide some insight for how the physical quantities may change during growth of straight echelon veins. The rate at which the quantities change depends on the rate of geological processes, such as diagenetic processes that effect rock stiffness and seam closing.

6. CONCLUSIONS

Four independent quantities are significant for investigating the opening and shearing of model echelon veins in a 2-D, linear elastic, homogeneous, isotropic material. Those four quantities are echelon vein spacing, vein-array angle, limestone elastic stiffness, and closing of vein-intersecting pressure solution seams. Except for stiffness, these dimensionless quantities are needed to investigate the near-tip stress field to describe echelon vein propagation behavior.

The following are main conclusions of the presented mechanical model results.

- The formation of echelon veins is the result of predominantly elastic deformation of limestone. Plasticity in limestone, outside of pressure solution seams, is a minor contributor to the shape of veins.
- Echelon vein spacing and vein-array angle contribute to the mechanical interaction between neighboring veins and small vein spacing causes a linear opening distribution near vein tips (Figure 7b).
- Closing of vein-intersecting pressure solution seams increases vein apertures by a factor of at least three (Figure 10a).
- At vein-seam intersections, the vein opening distribution is linear (Figures 9, 10a), and together with small echelon vein spacing, causes triangular vein shapes (Figure 13a); similar to echelon vein shapes observed in the field (Figure 1).
- Small amounts of seam closing can cause shearing of veins (Figure 10b), resulting in an asymmetric distribution of stress near vein tips (Figure 11), potentially causing veins to propagate at an angle to their traces.
- Reducing limestone stiffness enhances seam closing.
- Echelon veins oblique to the remote maximum compressive stress direction preferentially propagate in their own plane in model limestone when stiffness values are similar to inferred values for limestone at Raplee Ridge, when pressure solution seams intersect the veins (Figure 12).
- The orientations of the remote principal stresses can be inferred from only a single array of straight echelon veins with pressure solution seams, instead of two conjugate arrays.

- Limestone stiffness and pressure solution seam closing can be inferred from echelon vein spacing and vein-array angle measurements of straight echelon veins.

Given the heterogeneous nature of the deformation associated with echelon veins and pressure solution seams, and the many physical quantities necessary to describe them, words used to classify these structures should be based on observational qualities. *Echelon fractures* or *echelon veins* are words, for example, that identify the geometry and components of the structures. *Extension veins* (e.g. Fossen, 2010 pp. 305) or *tension gashes* (e.g. Allmendinger et al., 2012 pp. 145; Ramsay and Lisle, 2000 pp. 780) are example terms that make presumptions about the deformation mechanism with implications about the strain and stress states, respectively.

ACKNOWLEDGMENTS

Funding was provided by a National Science Foundation Collaborations in Mathematical Geosciences grant (award number EAR 0417521), a National Science Foundation Tectonics grant (award number EAR 1250447), the Stanford University Rock Fracture Project, and a Stanford University, School of Earth Sciences McGee Research grant.

Akhtar Zaman and Brad Nesemeier from the Navajo Division of Natural Resources, Minerals department granted permission in 2009 and 2013 to collect rock samples within the Navajo Nation land borders at Raplee Ridge. Shang Yip contributed to the preliminary mapping, measurements, and calculations of pressure solution seam closing from thin-section images during his 2011 summer internship supported by the School of Earth Sciences' outreach educational program at Stanford University. The authors are grateful to Christopher Zahasky, Nathaniel Levine, Meredith Townsend, and Emily Pope for their field assistance at Raplee

621 Ridge. The 2011 Stanford Rock Fracture Project members also contributed thoughtful insight
622 about the geology during a field excursion to Raplee Ridge. Discussions with Johanna Nevitt
623 regarding finite element modeling using Abaqus FEA improved our understanding of modeling
624 echelon veins and seams. The suggestions from reviewers David J. Sanderson and John
625 Cosgrove helped to improve the contents of this paper.
626

FIGURE CAPTIONS

Figure 1. a) Conjugate arrays of left-stepping and right-stepping echelon veins and pressure solution seams on the top surface of the McKim Limestone, north Raplee Anticline. UTM coordinates are 12S 606477m 4120593m, northern hemisphere. b) A map of the veins and seams that includes array trace orientations, orientation of the line that bisects the array traces, vein-array angles, and an inferred remote stress state. The remote least and greatest compressive stresses are σ_1^∞ and σ_3^∞ , respectively. Compression is negative.

Figure 2. a) Illustration of two-dimensional, homogeneous simple shear. Displacement component u_x varies linearly across a narrow, infinitely long zone of half-height, h . The shear angle is ψ . b) Orientation of veins is inferred to be perpendicular to the greatest extension in the initial increment of shearing, which is uniform throughout the zone and at 45° to the shear planes. Crack positions, lengths, spacing, and aperture are arbitrary. c) The distribution of the displacement magnitudes (color contours) and directions (arrows) are shown for simple shear of a narrow zone without cracks in a finite element model; $\psi = 1^\circ \approx 0.0175$ radians. Arrow lengths reflect relative magnitudes. d) Echelon cracks are added to the shear zone within the area marked by the dashed box in c). The displacement magnitudes and orientations near the cracks vary markedly from the simple shear case shown in c), and the strain field is heterogeneous.

Figure 3. Thin-section images of the McKim Limestone with veins and pressure solution seams. a) Image captured in plane polarized light shows the micritic carbonate matrix in darker colors, and calcite, quartz, and alkali feldspars in lighter colors. A set of calcite veins truncate and traverse a pressure solution seam containing mostly quartz and alkali feldspar grains. Two veins

are observed to intersect fossils. b) Image captured in plane polarized light showing closely spaced, relatively straight veins bound by approximately orthogonal pressure solution seams. Veins crosscut and coalesce. c) Image captured in cross-polarized light through a gypsum, wave-retardation plate. Darker crystal interference colors are the matrix material. Veins are thin, relatively straight, and sub-parallel. Some veins are truncated by the pressure solution seam. d) Image captured in cross-polarized light through a gypsum plate, and shows a vein tip. The vein has intersected a fossil near its tip. e) and f) Images captured in cross polarized light through a gypsum plate, and shows pressure solution seam geometries, and relative distribution of constituents in veins, seams, and matrix. Sample locations are (a) 12S 606478m 4120561m, (b) and (c) 12S 606849m 4120808m, (d) and (e) 12S 606475m 4120556m, and (f) 12S 606455m 4120529m.

Figure 4. The remote stress boundary conditions in Abaqus FEA for opening of a crack inclined at an angle, β , clockwise from the remote least compressive stress, σ_1^∞ , is shown for a range of uniform pressures, P , applied to the crack surfaces. The material is linear elastic with stiffness, E , and Poisson's ratio, ν . A crack is considered closed if the crack surfaces are touching at any position along the crack length. The dot indicates the combination of stress conditions selected for this study.

Figure 5. Applied differential stress (axial stress less radial stress) versus axial strain results from conventional triaxial compression tests on saturated Solnhofen Limestone. a) Deformation results for various internal fluid pressures, P ; with constant temperature, T , confining pressure, σ_{radial} , and strain rate, $\dot{\epsilon}$ (from Heard, 1960 fig. 11). b) Deformation results for various internal

fluid pressures, with constants T , σ_{radial} , and $\dot{\epsilon}$ (from Heard, 1960 fig. 11). c) Deformation results for various strain rates, with constants T , σ_{radial} , and P (from Rutter, 1972b fig. 1c).

Figure 6. a) Initial configuration of the mechanical model size, material properties (stiffness, E , and Poisson's ratio, ν), and external, principal stresses, σ_1^∞ and σ_3^∞ . A 2D global coordinate system (X, Y) is used to describe the displacement boundary conditions (u_X and u_Y) at the boundary midpoints. b) An illustration of parallel echelon crack and seam geometries and boundary conditions prescribed near the center of the model in (a). The geometries are crack half-length, c , crack spacing, s , angle, β , of cracks to the σ_3^∞ direction, angle, γ , of the array to the σ_3^∞ direction, and crack-array angle, α . The boundary conditions are uniform pressure, P , on the crack surfaces and closing, Δu_n^S , of seams.

Figure 7. a) An illustration of the spacing s/c between echelon cracks with crack-array angle, α , of 25° . b) Opening and c) shearing of the middle crack in an echelon array of five cracks without seams for various s/c values. The crack middle is at $x/c = 0$, and the crack tips are at $x/c = \pm 1$. Positive $\Delta u_n^C/c$ values represent crack opening. A positive value of $\Delta u_t^C/c$ refers to a right-lateral sense of shear across the crack, and negative is left-lateral shear. d) Opening and e) shearing of the middle crack in an echelon array of five cracks without seams for various limestone stiffness, E , values.

Figure 8. Circumferential stress, $\sigma_{\theta\theta}$, distribution at $r = 0.005c$ from the top tip of the middle crack, as part of an array of five echelon cracks without seams. a) Crack spacing, s/c , is varied. b) Crack-array angle, α , is varied. Positive values represent tension, and negative values

represent compression. The circumferential position, θ_m , of maximum $\sigma_{\theta\theta}$ for each model result is indicated. Stress is scaled by the crack-normal driving stress, $\Delta\sigma_n$, which is a function of α .

Figure 9. Opening of a single crack with an orthogonal seam. a) Crack opening is plotted for a range of seam stiffness, E_s , values and a limestone stiffness, E , equal to 60 GPa. Reducing E_s facilitates model seam closing. b) For E_s equal to 0.1 GPa, crack opening is plotted for a range of E values.

Figure 10. a) Opening of the middle crack in a five-crack echelon array with seams (seam stiffness, E_s) and without seams. The model limestone stiffness, E , is the same for both models. Crack spacing, s/c , is 3.15, and crack-array angle, α , is 25° . Cracks are parallel to the greatest compressive stress direction. b) Complimentary shearing of the middle crack in the two models. c) An illustration of the deformed crack and seam shapes plotted in (a) and (b).

Figure 11. Circumferential stress, $\sigma_{\theta\theta}$, distribution at $r = 0.005c$ from the top tip of the middle crack in a five-crack echelon array. For crack spacing, s/c , equal to 1.28, a) seam stiffness, E_s , is decreased to 0.1 GPa with constant model limestone stiffness, E , and b) E is decreased to 2.6 GPa with E_s constant at 0.1 GPa. The circumferential position, θ_m , of maximum $\sigma_{\theta\theta}$ for each model result is indicated. Crack-array angle, α , in each model is 25° . Stress is scaled by the crack-normal driving stress, $\Delta\sigma_n$.

Figure 12. Conditions that would cause the position, θ_m , of maximum circumferential stress, max. $\sigma_{\theta\theta}$, to be positioned ahead of the crack tip. a) Symmetric distributions of $\sigma_{\theta\theta}$ around the

tips of cracks that are within arrays of echelon cracks and seams with various values for crack-array angle, α , crack spacing, s/c , and model limestone stiffness, E (seam stiffness, E_s , is 0.1 GPa). Numbered labels refer to the model attributes listed in Supplementary Table 2. Stress is scaled by the crack-normal driving stress, $\Delta\sigma_n$, which is a function of α . b) A relationship of the amount of seam closing, Δu_n^s , and E needed to promote straight crack propagation ($\theta_m = 000^\circ$) for various values of s/c and α ; given the stress boundary conditions. Positive θ_m promotes crack propagation away from the neighboring echelon crack.

Figure 13. a) Crack opening profile of the middle crack in an array where $\alpha = 45^\circ$ and $s/c = 1.77$, and the material properties and boundary conditions result in nearly pure crack opening (Figure 12b). b) Profile of the relative shear across the crack. c) The displacement field around an array of deformed echelon cracks and seams. Arrows represent displacement directions, and relative magnitudes, of material points in the model limestone. Actual displacement magnitudes are contoured. See text for description of the listed variables. Rough contours are the result of coarse element meshes at those locations.

Table 1. Mean values of vein and pressure solution seam geometries measured from the outcrop image shown in Figure 1. Range of values are in parentheses. Veins and pressure solution seams near the intersection of the arrays were not measured. Raw measurements are presented in Supplementary Table 1. The variables are crack half-length, c , crack-array angle, α , crack spacing, s , seam half-length, l_s , and seam spacing, s_s .

REFERENCES

- Allmendinger, R.W., Cardozo, N., Fisher, D.M., 2012. Structural Geology Algorithms: Vectors and Tensors. Cambridge University Press, Cambridge, UK.
- Anderson, T.L., 2005. Fracture Mechanics: Fundamentals and Applications, 3rd ed. Taylor & Francis Group, Boca Raton, FL.
- Bathurst, R.G.C., 1975. Carbonate Sediments and their Diagenesis, 2nd enlarged ed. Elsevier Science B.V., London.
- Beach, A., 1975. The geometry of en-echelon vein arrays. Tectonophysics 28, 245-263.
- Birch, F., 1966. Compressibility; elastic constants, in: Clark, S.P., Jr. (Ed.), Handbook of Physical Constants: Revised Edition. The Geological Society of America Memoir, New York, NY, p. 587.
- Bons, P.D., Elburg, M.A., Gomez-Rivas, E., 2012. A review of the formation of tectonic veins and their microstructures. Journal of Structural Geology 43, 33-62.
- Buckingham, E., 1915. Model Experiments and the Forms of Empirical Equations. Transactions of the American Society of Mechanical Engineers 37, 263-292.
- Chau, K.T., Wang, G.S., 2001. Condition for the onset of bifurcation in en echelon crack arrays. International Journal for Numerical and Analytical Methods in Geomechanics 25, 289-306.
- Cotterell, B., 1965. On brittle fracture paths. Int J Fract 1, 96-103.
- Cotterell, B., Rice, J.R., 1980. Slightly curved or kinked cracks. International Journal of Fracture 16, 155-169.
- Deere, D.U., Miller, R.P., 1966. Engineering classification and index properties for intact rock, Technical Report No. AFWL-TR-65-116. Air Force Weapons Laboratory, New Mexico, p. 299.

- 764 Dey, T.N., Wang, C.-Y., 1981. Some mechanisms of microcrack growth and interaction in
765 compressive rock failure. *International Journal of Rock Mechanics and Mining Sciences*
766 & Geomechanics Abstracts 18, 199-209.
- 767 Du, Y., Aydin, A., 1991. Interaction of multiple cracks and formation of echelon crack arrays.
768 *International Journal for Numerical and Analytical Methods in Geomechanics* 15, 205-
769 218.
- 770 Erdogan, F., Sih, G.C., 1963. On the Crack Extension in Plates Under Plane Loading and
771 Transverse Shear. *Journal of Basic Engineering* 85, 519-525.
- 772 Fleck, N.A., 1991. Brittle Fracture Due to an Array of Microcracks. *Proceedings of the Royal*
773 *Society of London. Series A: Mathematical and Physical Sciences* 432, 55-76.
- 774 Fletcher, R.C., Pollard, D.D., 1981. Anticrack model for pressure solution surfaces. *Geology* 9,
775 419-424.
- 776 Fossen, H., 2010. *Structural Geology*. Cambridge University Press, Cambridge, UK.
- 777 Gregory, H.E., Moore, R.C., 1931. The Kaiparowits Region; a geographic and geologic
778 reconnaissance of parts of Utah and Arizona, U.S. Geological Survey Professional Paper,
779 p. 161.
- 780 Heard, H.C., 1960. Transition from brittle fracture to ductile flow in Solenhofen Limestone as a
781 function of temperature, confining pressure, and interstitial fluid pressure, in: Griggs, D.,
782 Handin, J. (Eds.), *Rock Deformation (A Symposium)*. The Geological Society of
783 America Memoir, Baltimore, M.D., p. 382.
- 784 Jackson, R.R., 1991. Vein arrays and their relationship to transpression during fold development
785 in the Culm Basin, central south-west England. *Proceedings of the Ussher Society* 7, 356-
786 362.

- 787 Jaeger, J.C., Cook, N.G.W., Zimmerman, R.W., 2007. Fundamentals of Rock Mechanics, 4th ed.
788 Blackwell Publishing Ltd., Oxford, UK.
- 789 Katsman, R., 2010. Extensional veins induced by self-similar dissolution at stylolites: analytical
790 modeling. *Earth and Planetary Science Letters* 299, 33-41.
- 791 Kelley, V.C., 1955. Monoclines of the Colorado Plateau. *Geological Society of America Bulletin*
792 66, 789-804.
- 793 Kelly, P.G., Sanderson, D.J., Peacock, D.C.P., 1998. Linkage and evolution of conjugate strike-
794 slip fault zones in limestones of Somerset and Northumbria. *Journal of Structural*
795 *Geology* 20, 1477-1493.
- 796 Lawn, B.R., Wilshaw, T.R., 1993. *Fracture of Brittle Solids*, 2nd ed. Cambridge University
797 Press, New York, NY.
- 798 Lisle, R.J., 2013. Shear zone deformation determined from sigmoidal tension gashes. *Journal of*
799 *Structural Geology* 50, 35-43.
- 800 Mandal, N., 1995. Mode of development of sigmoidal en echelon fractures. *Journal of Earth*
801 *System Science* 104, 453-464.
- 802 Mynatt, I., Seyum, S., Pollard, D.D., 2009. Fracture initiation, development, and reactivation in
803 folded sedimentary rocks at Raplee Ridge, UT. *Journal of Structural Geology* 31, 1100-
804 1113.
- 805 Nenna, F., Aydin, A., 2011. The formation and growth of pressure solution seams in clastic
806 rocks: A field and analytical study. *Journal of Structural Geology* 33, 633-643.
- 807 Nuccio, V.F., Condon, S.M., 1996. Burial and thermal history of the Paradox Basin, Utah and
808 Colorado, and petroleum potential of the Middle Pennsylvanian Paradox Formation. U.S.
809 Geological Survey Bulletin 2000-O, 41.

- 810 Olson, J., Pollard, D.D., 1989. Inferring paleostresses from natural fracture patterns: A new
811 method. *Geology* 17, 345-348.
- 812 Olson, J.E., Pollard, D.D., 1991. The initiation and growth of en échelon veins. *Journal of*
813 *Structural Geology* 13, 595-608.
- 814 Peacock, D.C.P., Sanderson, D.J., 1995. Pull-aparts, shear fractures and pressure solution.
815 *Tectonophysics* 241, 1-13.
- 816 Pollard, D.D., Aydin, A., 1988. Progress in understanding jointing over the past century.
817 *Geological Society of America Bulletin* 100, 1181-1204.
- 818 Pollard, D.D., Fletcher, R.C., 2005. *Fundamentals of Structural Geology*. Cambridge University
819 Press, New York, NY.
- 820 Pollard, D.D., Segall, P., 1987. Theoretical displacements and stresses near fractures in rock:
821 with applications to faults, joints, veins, dikes, and solution surfaces, in: Atkinson, B.K.
822 (Ed.), *Fracture Mechanics of Rock*. Academic Press Inc. Ltd., London, pp. 277-349.
- 823 Pollard, D.D., Segall, P., Delaney, P.T., 1982. Formation and interpretation of dilatant echelon
824 cracks. *Geological Society of America Bulletin* 93, 1291-1303.
- 825 Ramsay, J.G., 1967. *Folding and Fracturing of Rocks*. McGraw-Hill Book Company, New York.
- 826 Ramsay, J.G., Lisle, R.J., 2000. *The Techniques of Modern Structural Geology*. Academic Press,
827 San Diego, CA.
- 828 Renner, J., Rummel, F., 1996. The effect of experimental and microstructural parameters on the
829 transition from brittle failure to cataclastic flow of carbonate rocks. *Tectonophysics* 258,
830 151-169.
- 831 Roering, C., 1968. The geometrical significance of natural en-echelon crack-arrays.
832 *Tectonophysics* 5, 107-123.

- 833 Rogers, R.D., Bird, D.K., 1987. Fracture propagation associated with dike emplacement at the
834 Skaergaard intrusion, East Greenland. *Journal of Structural Geology* 9, 71-86.
- 835 Rutter, E.H., 1972a. The effects of strain-rate changes on the strength and ductility of Solenhofen
836 limestone at low temperatures and confining pressures. *International Journal of Rock*
837 *Mechanics and Mining Sciences & Geomechanics Abstracts* 9, 183-189.
- 838 Rutter, E.H., 1972b. The influence of interstitial water on the rheological behaviour of calcite
839 rocks. *Tectonophysics* 14, 13-33.
- 840 Secor, D.T., 1965. Role of fluid pressure in jointing. *Am J Sci* 263, 633-646.
- 841 Seyum, S., 2015. Deformation at Raplee Ridge, Utah, inferred from joints, veins, and pressure
842 solution seams. Thesis. Stanford University, p. 290.
- 843 Smith, J.V., 1999. Inter-array and intra-array kinematics of en échelon sigmoidal veins in cross-
844 bedded sandstone, Merimbula, southeastern Australia. *Journal of Structural Geology* 21,
845 387-397.
- 846 Thomas, A.L., Pollard, D.D., 1993. The geometry of echelon fractures in rock: implications from
847 laboratory and numerical experiments. *Journal of Structural Geology* 15, 323-334.
- 848 Timoshenko, S., Goodier, J.N., 1970. *Theory of Elasticity*, 3rd ed. McGraw Hill Education, New
849 York, NY.
- 850 Tweto, O., 1975. Laramide (Late Cretaceous - Early Tertiary) Orogeny in the Southern Rocky
851 Mountains, in: Curtis, B.F. (Ed.), *Cenozoic History of the Southern Rocky Mountains*.
852 Geological Society of America, Inc., Boulder, CO, pp. 1-44.
- 853 Weyl, P.K., 1959. Pressure solution and the force of crystallization: a phenomenological theory.
854 *Journal of Geophysical Research* 64, 2001-2025.

- 855 Willemse, E.J.M., Peacock, D.C.P., Aydin, A., 1997. Nucleation and growth of strike-slip faults
856 in limestones from Somerset, U.K. *Journal of Structural Geology* 19, 1461-1477.
- 857 Wiltschko, D.V., Lambert, G.R., Lamb, W., 2009. Conditions during syntectonic vein formation
858 in the footwall of the Absaroka Thrust Fault, Idaho-Wyoming-Utah fold and thrust belt.
859 *Journal of Structural Geology* 31, 1039-1057.
- 860 Zehnder, A.T., 2012. *Fracture Mechanics*. Springer, Netherlands.
- 861 Zhou, X., Aydin, A., 2010. Mechanics of pressure solution seam growth and evolution. *Journal*
862 *of Geophysical Research: Solid Earth* 115, B12207.
- 863
- 864

866

867

868

869

870

871

872

873

874

875

876

877

878

879

880

881

882

883

884

885

886

889

890

	veins				
	c (mm)	α (°)	max. aperture (mm)	max. aperture / c	s (mm)
array with right-stepping veins	15.1 (3.6 - 28.6)	44 (37 - 54)	1.3 (0.4 - 2.3)	0.09 (0.04 - 0.13)	23.9 (1.8 - 60.4)
array with left-stepping veins	9.3 (2.3 - 26.4)	49 (26 - 68)	1.7 (0.4 - 5.6)	0.20 (0.03 - 1.04)	14.4 (2.4 - 70.0)

	seams			vein-seam angle	
	$2l_s$ (mm)	seam-array angle (°)	s_s (mm)	mean seam-array angle less mean α (°)	vein-seam angle (°)
array with right-stepping veins	23.1 (5.6 - 76.4)	131 (78 - 155)	27.9 (8.4 - 49.0)	87	91 (72 - 143)
array with left-stepping veins	17.6 (3.8 - 78.7)	145 (94 - 187)	33.9 (4.2 - 137.0)	96	104 (51 - 138)

891