Variable Speed Limit Control at Fixed Freeway Bottlenecks Using Connected Vehicles

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ABSTRACT

The connected vehicle (CV) technology is applied to develop VSL strategies to improve bottleneck discharge rates and reduce system delays. Three VSL control strategies are developed with different levels of complexity and capabilities to enhance traffic stability using: (i) only one CV (per lane) (Strategy 1), (ii) one CV (per lane) coupled with variable message signs (Strategy 2), and (iii) multiple CVs (Strategy 3). We further develop adaptive schemes for the three strategies to remedy potential control failures in real time. These strategies are designed to accommodate different queue detection schemes (by CVs or different sensors) and CV penetration rates. Finally, probability of control failure is formulated for each strategy based on the stochastic features of traffic instability to develop a general framework to (i) estimate expected delay savings, (ii) assess the stability of different VSL control strategies, and (iii) determine optimal control speeds under uncertainty. Compared to VMS-only strategies, the CV-based strategies can effectively impose dynamic control over continuous time and space, enabling (i) faster queue clearance around a bottleneck, (ii) less restrictive control with higher control speed (thus smoother transition), and (iii) simpler control via only one or a small number of CVs.

Keywords: Variable speed limit, Connected vehicle, Freeway bottlenecks, Traffic breakdown, Capacity drop

1. INTRODUCTION

Freeway performance is often hindered by the ‘capacity drop’ phenomenon, where discharge rates from fixed bottlenecks (or moving jams) diminish significantly (10-30%) upon onset of queue (Banks, 1991; Bertini and Leal, 2005; Cassidy and Bertini, 1999; Hall and Hall, 1990) presumably due to lane changing (Cassidy and Rudjanakanoknad, 2005; Laval and Daganzo, 2006; Leclercq et al., 2011; Zhao et al., 2013), time-dependent driver car-following characteristics (Chen et al., 2014b; Goñi Ros et al., 2014), or rubbernecking induced by incidents (Knoop et al., 2008). Variable speed limit (VSL) control, developed originally to improve safety, has emerged as a promising method to mitigate capacity drop. Earlier strategies sought to harmonize the speed among vehicles using VSL control to create a more homogenous, stable one-pipe flow (Smulders, 1990; Zackor, 1991), thereby achieving higher capacity and critical density (Papageorgiou et al., 2008). Several empirical studies support that VSL control is able to reduce speed differences among vehicles and encourage more uniform lane utilization (Duret et al., 2012; Knoop et al., 2010; Smulders, 1990; Zackor, 1991).

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In another group of studies, VSL control algorithms were formulated explicitly to achieve higher discharge rates from moving jams (Hegyi and Hoogendoorn, 2010; Hegyi et al., 2005, 2008, 2009; Popov et al., 2008), fixed recurrent bottlenecks (Carlson et al., 2010a; Chen et al., 2014a) and non-recurrent bottlenecks (Chen and Ahn, 2015). For example, the pioneering SPECIALIST algorithm (Hegyi et al., 2008) is specifically designed to resolve a moving jam and maximize the discharge rate by limiting the speed and density of inflow to the moving jam, assuming that traffic behaves according to the Kinematic Wave (KW) model (Lighthill and Whitham, 1955; Richards, 1956). This algorithm was implemented for a real-world field testing on a Dutch freeway and shown to be effective in resolving moving jams and increasing the discharge rates (Hegyi et al., 2009; Hegyi and Hoogendoorn, 2010).

Likewise, several studies attempted to improve efficiency at fixed freeway bottlenecks. Specifically, Carson et al. (2010a) proposed the mainstream traffic flow control (MTFC) based on the optimal control concept. This method proactively limits the inflow from upstream free-flow traffic to a fixed recurrent bottleneck by VSL or ramp metering to prevent bottleneck activation and ensuing capacity drop. The algorithm was tested on a Dutch network using simulation (Carlson et al., 2010a). Later, local feedback control was incorporated to improve control efficiency (Carlson et al., 2011), and the benefit was confirmed by microscopic simulation. (Muller et al., 2015). Chen et al. (Chen et al., 2014a; Chen and Ahn, 2015) developed a general theoretical framework based on the KW model for comprehensive VSL control to increase discharge rates at fixed bottlenecks and provide smoother speed transition upstream (for safety). They addressed different types of queues: steady and oscillatory queues at recurrent bottlenecks (Chen et al., 2014a) and severe queues due to non-recurrent road events (Chen and Ahn, 2015).

The VSL strategies cited above use variable message signs (VMSs) to deliver control, which introduce some major disadvantages. Because they are placed at discrete locations, it is difficult to detect dynamic traffic conditions and deliver control in time. Notably, the analysis for the SPECIALIST field test revealed that control failure likely resulted from dynamic traffic conditions such as increases in demand or on-ramp inflow (Han et al., 2015). Moreover, deployment of VMSs can be very costly. A system of successive VMSs, including expensive structures such as gantries, is typically required over extended areas. In addition, VMSs should be placed close enough to detect and respond to dynamic conditions effectively. The compliance of drivers to VMS and compulsory enforcement are also critical issues in reality.

On the other hand, the connected vehicle (CV) technology presents a novel method for traffic detection and VSL control. As CVs can share information with each other (V2V) or with infrastructure (V2I), highly efficient and adaptive control is possible. For example, Van de Weg et al. (2014) presented a cooperative system of ramp metering and VSL, where vehicles are controlled, assuming 100% penetration rate, to resolve a moving jam and achieve a stable flow. Lu et al. (2015) developed a VSL (or variable speed advisory) algorithm for a two-bottleneck (in series) freeway segment. Their control principle was simply to reduce speed upstream of the downstream bottleneck when it is congested. However, no explicit schemes were proposed to clear a queue at the bottleneck and regulate inflow to the bottleneck to prevent further breakdown. They conducted field tests using three CVs (for a three-lane roadway) to demonstrate that the concept of VSL control via CVs is practically possible (e.g., sensors can detect real-time traffic; CVs can receive and respond to the VSL; and regular vehicles behind CVs follow CVs' speed). However, the field tests were not designed to actually test the

performance of their VSL algorithm in real traffic. Instead, they used simulations to show that the overall system performance can be improved with the proposed VSL if the compliance rate is 10% or higher. Grumert et al. (2015) demonstrated potential benefits of a cooperative VSL system (C-VSLs), in which vehicles receive speed limit information (via I2V) in advance at an upstream VMS. This early information allows drivers to decelerate more smoothly, thereby stabilizing traffic flow and reducing exhaust emissions. Furthermore, the CV technology can improve the quality and resolution of traffic data. Notably, Netten et al. (2013) showed that V2I communication can enable faster and more accurate detection of a moving jam, and thus more proactive control.

While the CV technology can clearly benefit traffic management, one important consideration is its penetration rate. If the penetration rate is low, the behavior/response of regular vehicles (without the CV technology) can impose significant constraints for control: a small proportion of vehicles (CVs) must be controlled in ways to optimally change the behavior of vaster traffic. For example, in Nishi et al. (2013), only a single vehicle, not necessarily a CV, is controlled to intentionally create a void ahead to resolve a moving jam while other vehicles upstream follow their natural response. This basic concept provides a valuable theoretical basis for the present study, where VSL control using a single or a few CVs is explored to maximize discharge rates from fixed bottlenecks.

Another important consideration for VSL control is the stochastic nature of bottleneck capacity. Scores of empirical studies suggest that the breakdown/maximum flow for a freeway section can vary (Athol and Bullen, 1973; Banks, 1991; Elefteriadou et al., 1995; Hall and Agyemang-Duah, 1991; Lorenz and Elefteriadou, 2001; Persaud et al., 1998). Accordingly, stochastic bottleneck capacity has been modeled in various frameworks, such as Markov chains (Evans et al., 2001), Kaplan-Meier estimation (Brilon et al., 2005), nonparametric Bayesian estimation (Ozguven and Ozbay, 2008), platoon-based traffic flow model (Shiomi et al., 2011), and Bayesian hierarchical Weibull model (Xie et al., 2014). These studies suggest that the distribution of capacity depends on flow or speed. Nevertheless, most existing VSL strategies assume constant and/or deterministic bottleneck capacity, resulting in suboptimal control. For more effective control, it should be recognized that the probability of traffic breakdown (and thus control failure) is associated with the control speed and the resulting traffic state around a bottleneck. Thus, the control speed should be optimized such that the *expected* bottleneck discharge rate is maximized. This concept is proposed and thoroughly explored in the present paper.

The objective of this paper is to develop efficient VSL strategies using CVs for fixed recurrent bottlenecks where congestion has inevitably occurred. We assume partial penetration of CVs. The main principle of the developed strategies is to control the speed of a CV upstream of an existing queue (with capacity drop) at the speed corresponding to the maximum stable flow (without capacity drop) that the bottleneck can sustain. This creates a void ahead of the CV that can efficiently resolve the existing queue. Behind the CV, inflow to the bottleneck is regulated at the maximum stable flow, thereby preventing traffic breakdown (and ensuing capacity drop), and achieving a higher bottleneck discharge rate. Three VSL strategies are developed with different levels of complexity and capabilities to enhance traffic stability using: (i) only one CV (per lane) (Strategy 1), (ii) one CV (per lane) coupled with a few variable message signs (Strategy 2), and (iii) multiple CVs (Strategy 3). These strategies are designed to accommodate different queue detection schemes (by CVs or different sensors) and CV penetration rates. Considering the

stochastic nature of traffic instability, we formulate probability of control failure for each strategy to determine the optimal control speed that maximizes the *expected* bottleneck discharge rate and thus delay savings.

The remaining paper is organized as follows. In section 2, three (non-adaptive) VSL control strategies (strategies 1-3) using a CV(s) are presented. Section 3 describes the framework for adaptive control to remedy potential control failure that can result from strategies 1-3. Section 4 presents the formulation for probabilistic control failure and the analysis of optimum control speed. Section 5 compares different strategies based on relative orders of instability probabilities in different traffic states. Concluding remarks and limitations of this study are provided in the final section.

2. VSL STRATEGIES USING CONNECTED VEHICLE(S)

We consider a congested freeway section due to high demand exceeding the capacity of a fixed recurrent bottleneck. We assume for the purpose of developing a theoretical framework that (i) traffic is composed of CVs and regular vehicles (with no CV technology), (ii) CVs behave the same as regular vehicles when uncontrolled, (iii) traffic evolves according to the Kinematic Wave (KW) theory (Lighthill and Whitham, 1955; Richards, P, 1956), and (iv) CVs comply 100% with speed control. We also assume that (v) a bottleneck is able to temporarily tolerate high flow exceeding long-term average capacity, which is implied by the finding of Lorenz and Elefteriadou (2001). This assumption will be a key basis for developing a strategy with multiple CVs (Strategy 3). Finally, we assume (vi) single lane traffic for the purpose of developing a theoretical framework. Some issues related to these assumptions, particularly the application for multi-lane facilities, and potential remedies are discussed in section 6.

In this section, we first present a baseline case without any traffic control. Then we develop three VSL strategies considering availability of other devices (e.g. detectors or VMS) and CV penetration rates: Strategy 1 using a CV, Strategy 2 using a combination of a CV and few VMSs, and Strategy 3 using multiple CVs.

2.1. Baseline case

Fig. 1 illustrates our baseline case with possible traffic states (Fig. 1a) and queue evolution (Fig. 1b) near a hypothetical active bottleneck without any traffic control. For illustration purpose, we assume a triangular fundamental diagram (FD) throughout this paper, defined by free-flow speed u , congestion wave speed w , and jam density k_j . However, the main principle of the proposed control strategies does not depend on the shape of FD, though some details, such as shock wave speed, would change. On the FD in Fig. 1a, state a represents the demand, and e and g correspond to the traffic states upstream and downstream of the bottleneck, respectively, if the bottleneck were to operate at its capacity, q_{BN} . As in Chen et al. (2014a), q_{BN} is assumed to present the stable, maximum bottleneck discharge rate that can be sustained for an extended period without breakdown. After the onset of queue, the actual discharge rate may be lower than bottleneck capacity due to the capacity drop phenomenon. This reduced discharge rate is denoted by q'_{BN} , which is smaller than q_{BN} , and the resulting traffic states upstream and downstream of bottleneck are h and i , respectively. Then at the onset of queue, a backward-

moving shock wave, s_{ah} , is instigated. (Hereafter, s_{-} refers to the shock wave dividing the traffic states denoted in the subscript. In equations and logical expressions, it represents shock wave speed.) When the demand decreases to state f , a forward shock, s_{af} , is generated. The s_{af} eventually collides with s_{ah} and forms a slower forward shock, s_{fh} . The q'_{BN} persists until the queue dissipates with the arrival of s_{fh} at the bottleneck.

Below we develop three strategies, each with added sophistication, to prevent capacity drop by regulating inflow to the bottleneck using CVs and to maximize the likelihood of sustaining q_{BN} as long as possible. For these strategies, we assume a deterministic, fixed bottleneck capacity q_{BN} for simplicity. This assumption will be relaxed later, and stochastic capacity will be considered in section 4.

FIGURE 1 Traffic evolution near bottleneck without traffic control: (a) Fundamental diagram; (b) Traffic evolution in the time-space domain.

2.2. Strategy 1: control using single connected vehicle

Strategy 1 is the simplest one, requiring only one CV. The main idea is to limit the speed of a CV approaching the queue, such that the flow behind it is regulated at q_{BN} when arriving at the bottleneck. We first assume that CV penetration is relatively low ($\sim 5\text{-}10\%$) and that a queue could be detected by another type of sensors (e.g., loop detectors); referred to as Strategy 1a. The case of queue detection by a CV will be discussed later as Strategy 1b.

Control procedure for Strategy 1a

Fig. 2a and 2b show the FD and traffic evolution by Strategy 1a in the time-space domain respectively. In Fig. 2b, the red bold line represents the trajectory of the CV controlled by this strategy. Specifically, when the queue is detected² at time t_d , a CV approaching the queue in free-flow state a is directed to reduce its speed to v_e . (Hereafter, q_{-} and v_{-} respectively refer to the flow and speed of the traffic state denoted in the subscript unless otherwise noted.) Ahead of the CV, this strategy generates a void (state o), which begins to dissipate the queue at the rate of

² Queue detection is usually done by checking speed difference of speed (Chan et al., 2003; Chen et al., 2004; Wiecek et al., 2010) or occupancy (Lin and Daganzo, 1997; Zhang and Levinson, 2004) from free-flow state, depending on the traffic management system and site characteristics. For recurrent bottlenecks on highways equipped with fixed-location sensors, the detection location can be assumed fixed (e.g., the nearest detector upstream of the bottleneck) while the detection time could vary. However, if moving sensors (e.g., CVs) are used, the detection time and location will depend on the distribution of moving sensors.

v_h . This is marked by forward shock s_{oh} in Fig. 2b. Behind the CV, this speed limit generates congested state e . The CV is directed to resume u after passing the bottleneck. Notice that after the CV arrives at the bottleneck, the inflow to the bottleneck is limited at $q_e (=q_{BN})$, the maximum level possible without instigating capacity drop.

FIGURE 2 Strategy 1a: (a) Fundamental diagram; (b) Traffic evolution in the time-space domain.

Parameter calculation

To maximize the benefit, the controlled CV should arrive at the bottleneck at t_c , just as the queue in state h dissipates, as depicted in Fig. 2b, so that the bottleneck can discharge at q_e at the earliest. The corresponding optimum control location in Strategy 1a measured from the tail of the queue at t_d , x_1^* , can be derived based on the queue length at t_d , denoted by L_q , v_e , u , and the queue forming and clearing shock waves, s_{ah} and $s_{oh} (=v_h)$:

$$x_1^* = \frac{(s_{ah}+u)(v_e-v_h)}{v_h \cdot (u-v_e) - s_{ah} \cdot (v_e-v_h)} L_q = \alpha L_q, \quad (1)$$

where α captures the constant term. In reality, however, it is unlikely that a CV is just present at the optimum location, x_1^* , especially if the CV penetration rate is low. Therefore, we address the cases when the free-flowing CV that is closest to the x_1^* is upstream and downstream of x_1^* : i.e., $x_{CV} > x_1^*$ and $x_{CV} < x_1^*$, respectively, where x_{CV} represents the location of the closest CV at t_d measured from the tail of queue (see Fig. 3).

If $x_{CV} > x_1^*$, controlling at v_e from t_d would result in a persisting void at the bottleneck until the arrival of the CV; i.e., it would be over controlled. A remedy would be to simply wait to control the CV; see Fig. 3a. The waiting time, Δt_w , depends on x_{CV} and other parameters defined above and can be derived based on simple geometry:

$$\Delta t_w = \frac{x_{CV} - x_1^*}{(1+\alpha)s_{ah}+u}. \quad (2)$$

On the other hand, if $x_{CV} < x_1^*$, the CV would eventually encounter the queue before it arrives at the bottleneck if it is controlled at v_e from t_d . To remedy this, the CV should be controlled at a lower speed, v_l , until a sufficient void is created ahead and then at v_e later; see

Fig. 3b. This generates two backward shocks, s_{al} and s_{le} , and the following vehicles evolve to state l and then e . State l is resolved when s_{al} and s_{le} meet. The duration of v_l , Δt_l , depends on x_{CV} , v_l , and other parameters defined above:

$$\Delta t_l = (v_e - v_l)^{-1} \left(x_{CV} \left(\frac{v_e(v_h + s_{ah})}{v_h(s_{ah} + u)} - 1 \right) + L_q \left(\frac{v_e}{v_h} - 1 \right) \right). \quad (3)$$

One can see that the higher the v_l is, the longer the Δt_l is. However, for a given x_{CV} there is an upper bound for v_l , which is the speed corresponding to the arrival of the CV at the bottleneck just as the queue dissipates. The range of v_l is then derived as:

$$0 \leq v_l < \frac{v_h(L_q + x_{CV})(s_{ah} + u)}{x_{CV}(v_h + s_{ah}) + L_q(s_{ah} + u)}. \quad (4)$$

Obviously, the theoretical lower bound of v_l is 0 (a complete stop) though it may not be desired in practice for safety reasons. Notice that the control for $x_{CV} < x_1^*$ generates congested state l upstream (Fig. 3b), which is not ideal for speed transition. Moreover, if the CV is close to the tail of queue (i.e., $x_{CV} \approx 0$), either a stricter value of v_l or longer duration of state l (i.e. larger Δt_l) would be necessary to create a sufficient void ahead. Thus, one may control another CV upstream if it is detected near x_1^* .

FIGURE 3 Extensions of Strategy 1a: (a) CV located upstream of the optimum location; (b) CV located downstream of the optimum location

Control procedure for Strategy 1b

We now consider the case where the queue is detected by a CV (i.e., a CV detects the queue when it joins the tail of the queue), which is likely if the CV penetration rate is sufficiently high. Fig. 4 shows the traffic evolution by this alternative method (see Fig. 2a for the FD). Upon queue detection, the same CV is immediately controlled at v_l ($< v_h$) to create a void ahead and resolve state h . Soon after, the CV is controlled at the higher speed of v_e to achieve q_{BN} when it arrives at the bottleneck. (This method is conceptually similar to Strategy 1a presented in Fig. 3b.) The duration of control at v_l , Δt_l , in this method is derived by equation (3) with $x_{CV} = 0$. This method also generates state l upstream of the CV, which can persist for a while. Notice that

while the prompt control is more effective in resolving the queue and will result in more delay saving, it has a trade-off. Particularly, state l is more congested than state h and is not desirable in terms of speed transition and safety. Thus, one may resort to Strategy 1a presented in Fig. 3a by controlling a CV upstream of x_1^* , albeit with less delay savings. Thus, selection of a proper strategy should consider this trade-off as well as the queue detection method and CV penetration rate.

FIGURE 4 The traffic evolution of Strategy 1b

Features

Strategy 1 is very simple in that only one CV needs to be controlled at two or three points: (deceleration to v_l), deceleration (or acceleration) to v_e , and then recovery to u after passing the bottleneck. Traffic evolution behind the CV is a result of vehicles' natural response to the behavior of CV (according to the KW theory) and additional control is not necessary. Note that, if the control speed corresponding to q_e is significantly lower than the prevailing speed, a gradual speed reduction of CV would be desired for better stability and safety. In that case, the main control mechanism remains the same but some parameters, such as x_1^* , should be changed for earlier control activation. As a trade-off, this would result in a larger area of controlled congestion. Furthermore, in the early stage of implementation, the control parameters should be tuned carefully to minimize the void state (state o) while ensuring queue clearance and stability thereafter.

The deceleration of a CV results in congested state e behind it. Though this controlled-congestion is better than state h with no-control, it may not be desirable, particularly around the bottleneck, since congested traffic is often unstable and prone to persisting speed disturbances. In the following sections, we will present more advanced strategies, building on Strategy 1, to remedy this issue. Specifically, we aim to create a free-flow state with the flow, q_e , around the bottleneck to achieve better stability while maintaining the same maximum stable flow. For illustration purposes, we will build on Strategy 1a presented in Fig. 2b; however, the main control logic applies to other strategies, although some details of traffic evolution would change.

2.3. Strategy 2: control using a single connected vehicle and VMSs

Similar to Strategy 1, a CV in free-flow state a reduces its speed to v_e after queue detection. Later, this controlled CV can create an acceleration region (of 0.35-1 km in length as in Chen et al. (2014a)) by accelerating to u ; see Fig. 5. Without additional traffic control, however, vehicles following the CV will form the maximum flow state when approaching the bottleneck (state c in Fig. 5a), in which flow exceeds the bottleneck capacity. Thus, traffic will break down and revert to state h . Therefore, in Strategy 2, we impose v_e until the entrance to the acceleration region using a VMS to regulate the flow at q_e . Then, as vehicles accelerate to u , traffic will evolve to state g when passing the bottleneck; see Fig. 5b.

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Considering the length of the acceleration region, L_a , the optimum location of CV control (from tail of queue at the detection time), x_2^* , is derived based on the geometry as :

$$x_2^* = \frac{(s_{ah}+u)(v_e-v_h)}{v_h(u-v_e)-s_{ah}(v_e-v_h)} L_q + \frac{(s_{ah}+u)(u-v_e)}{v_h(u-v_e)-s_{ah}(v_e-v_h)} \cdot \frac{v_h}{u} L_a = \alpha L_q + \beta L_a. \quad (5)$$

The first term is the same as in equation (1) and the second term captures the additional length due to the acceleration zone. Similar modifications can be made to accommodate various extensions of Strategy 1 described in section 2.2. For conciseness, we omit those modifications and related derivations.

Features

Strategy 2 appears similar to existing strategies that use only VMSs (Carlson et al., 2011, 2010a, 2010b; Chen et al., 2014a; Chen and Ahn, 2015). However, the combined usage of a CV and VMSs offers significant advantages over the existing strategies, in that it is (i) much more efficient in achieving q_{BN} because it resolves an existing queue by creating a void using a CV, rather than a low flow state using VMSs, (ii) less restrictive in the speed limit, (iii) simpler (e.g., in terms of upper queue management for smoother speed transition), particularly if CV penetration is reasonable, and (iv) potentially more cost-effective: two VMSs will suffice in this strategy, whereas multiple VMSs along control areas are necessary in the previous strategies for comprehensive control (including upper queue management). Moreover, infrastructure for CV communications can serve multiple purposes and facilitate integration with other CV-enabled controls (e.g., safety enhancement and congestion pricing), which can save overall implementation costs.

To see the advantage of (i) and (ii) more clearly, we briefly introduce the existing VMS-only strategy by Chen et al. (2014a) and Chen and Ahn (2015) and explain the difference from Strategy 2. Firstly, the VMS-only control creates a low-density state, state $\tilde{h}$, to resolve the existing queue around the bottleneck (See Chen et al., 2014a for the details); see Fig. 5(c). Clearly, the queue resolution speed with the CV-based control is higher than with the VMS-only control (i.e., $s_{oh} > s_{bh}$), suggesting that the discharge rate can be recovered earlier with the CV-based control. Additionally, the VMS-only control creates states $\tilde{h}$ and $\tilde{h}'$ upstream of the original queue (in state h), which is more restrictive (i.e., with lower speed) and probably less stable than state e created by Strategy 2.

2.4. Strategy 3: control using multiple CVs

Although Strategy 2 is relatively simple, it would be preferable to rely only on CVs for control. To this end, we develop Strategy 3 only using CVs. We assume that the CV penetration rate is relatively high and that CVs are randomly distributed in time and space.

Control procedure

Fig. 6a presents a FD with traffic states resulting from Strategy 3 and Fig. 6b the corresponding traffic evolution. The first CV (CV₁) creates a void to dissipate the existing queue by decelerating to v_e . The following vehicles, including other CVs, naturally decelerate to v_e along the shock wave, s_{ae} . Thereafter, all CVs are directed to accelerate to $v_{e'}$, a design speed to pass the bottleneck, at the entrance of the acceleration region. Then, behind each CV, regular vehicles will naturally accelerate to state e' along $s_{ee'}$. This process creates a void ahead of each CV and a platoon headed by each CV. Note that $q_{e'}$ is higher than q_{BN} but low enough to be sustained in the short term until the next void arrives (per assumption (v) stated in the beginning of Section 2). We will discuss the value setting of $q_{e'}$ later. Notice that for each platoon, a higher $q_{e'}$ (resulting from higher $v_{e'}$) is offset by a longer void, so that regardless of the value of $v_{e'}$, the average discharge rate is q_e .

Parameter calculation

The control speed of $v_{e'}$ is a design parameter that must be tuned carefully to minimize traffic breakdown. However, thanks to the void between platoons, some breakdown is local; i.e., it can be resolved before the arrival of the next platoon. Fig. 6c describes this traffic evolution. If traffic breaks down, state h will ensue and propagate backward along $s_{e'h}$ until it encounters the void. Thereafter, the queue starts to dissipate along s_{oh} . Clearly, state h can be resolved within the platoon if the breakdown occurs *after* some critical number of vehicles (or duration), which can be derived as:

$$n_k^c = \left\lceil n_k \left(\frac{1}{q_{BN}} - \frac{1}{q_e} \right) \left(\frac{1}{q_{BN}} - \frac{1}{q_{e'}} \right)^{-1} \right\rceil, \quad (6a)$$

$$\frac{n_k^c}{n_k} = \left(\frac{1}{q_{BN}} - \frac{1}{q_e} \right) \left(\frac{1}{q_{BN}} - \frac{1}{q_{e'}} \right)^{-1}, \quad (6b)$$

where, n_k^c is the critical number of vehicle for the k^{th} platoon, and n_k is the number of vehicles in the k^{th} platoon. (The detailed derivation is provided in the Appendix A.) The ratio, n_k^c/n_k , denotes the proportional position of the critical vehicle in the platoon (or the proportional duration with respect to the headway of the next CV). Other parameters, such as the optimum control location of CV₁ at v_e , is similarly derived from equation (5), replace u to $v_{e'}$ in the last term.

Features

If breakdown occurs earlier than the threshold vehicle, n_k^c , there will be a residual queue when the next CV arrives; see Fig. 6d. Once a residual queue arises, it will persist through subsequent platoons, and the strategy will fail. To prevent this, $v_{e'}$ must be chosen carefully. While a higher $v_{e'}$, and $q_{e'}$ as a result, would decrease n_k^c/n_k as desired (see equation (6)), it would also increase the probability of traffic instability and capacity drop. This trade-off is carefully examined in a probabilistic framework in Section 4, including the determination of an optimal value of $v_{e'}$. Nevertheless, since control failure is possible, we next present adaptive control to remedy possible failure.

FIGURE 6 Strategy 3: (a) Fundamental diagram; (b) Traffic evolution in the time-space domain; (c) Control success case; (d) Control failure case.

3. ADAPTIVE CONTROL TO REMEDY CONTROL FAILURE

In this section, we develop adaptive schemes for strategies 1-3 to resolve a queue and restore the maximum stable discharge rate in the event of a control failure.

Procedure for adaptive control of Strategy 1a

To illustrate the concept, we present an adaptive control for Strategy 1a (see Fig. 7b, the FD is presented Fig. 7a). When a new queue (in state h) is detected at t_{d2} due to a control failure, the first unaffected CV in state a will be directed to decelerate to v_e before it encounters state e , creating a void ahead; see Fig. 7b. The void will resolve the new queue after some period, Δt_R . Thereafter, the existing traffic control can resume. Similar concepts apply for other strategies.

Parameter calculation

The optimum location of CV control at t_{d2} , x_R^* , measured from the end of the queue in state e is derived as:

$$x_R^* = \left(\frac{L_{q2} + L_e}{v_e} - \frac{L_{q2}}{v_h} \right) \left(\left(1 + \frac{s_{eh}w}{v_h} \right) \left(\frac{v_e + s_{ae} + L_e}{s_{eh}w + v_e} \right) - \frac{1}{v_e} \right)^{-1}, \quad (7)$$

A more prompt adaptive control is possible to achieve faster recovery of q_e . The concept is similar to Strategy 1b: rather than controlling a CV far upstream in free-flow traffic, the closest CV upstream of the new queue, even if it is already in queue, is immediately controlled at $v_l (< v_h)$ at t_{d2} ; see Fig. 7c. Then the void is created ahead of the CV and resolves the new queue. Later the CV is controlled at v_e to achieve the inflow of q_e to the bottleneck. This method has an advantage of faster recovery of the maximum sustainable discharge rate, thus a greater delay saving. However, the trade-off is that it introduces a low-speed state of l upstream, which is not ideal for speed transition.

For Strategy 3, late acceleration to v_e' is also possible to extend a void longer to resolve the new queue; see Fig. 7d. This method is able to recover q_e faster without introducing a low-speed state (state l). However, the trade-off is that the acceleration region will shrink gradually with successive adaptive control. Thus, if the control fails often, the acceleration region may quickly diminish. Thus, selection of an adaptive control should consider these trade-offs and frequency of control failure.

FIGURE 7 Adaptive control: (a) Fundamental diagram; (b) Traffic evolution under adaptive control; (c) Alternative adaptive control; (d) Alternative adaptive control for Strategy 3.

4. PROBABILISTIC CONTROL FAILURE

The adaptive control presented above can remedy control failures, yet in a reactive fashion. In this section, we present a framework to proactively minimize control failures and maximize delay savings. Specifically, we determine optimal v_e and $v_{e'}$ incorporating the probabilistic nature of traffic instability. We recognize that the bottleneck capacity is stochastic (e.g. (Lorenz and Eleftheriadou, 2001)), and thus, instabilities can arise and trigger capacity drop in states e , g , and e' with certain probabilities. Thus, traffic instability that eventually leads to a control failure will be modeled as a probabilistic event that depends on the traffic state and its duration. With this stochastic premise, we will provide some insight into how the control parameters should be determined to maximize the system delay savings.

4.1 Probability of control failure

The probability of a control failure due to instability³ is modeled with a microscopic approach. Specifically, we assume for Strategy 1 and 2 that:

³ In this paper, instability is defined as a significant speed reduction regardless of the traffic state.

- (a) Each vehicle in traffic state i has an equal probability of causing instability (and subsequent capacity drop), p_i ;
- (b) The probability of a driver causing instability is independent of his/her leader.

For assumption (a), a CV might have a lower instability probability than a regular vehicle due to better awareness of surrounding conditions; however, in favor of simplicity, we assume an equal probability. While not accurate, it leads to a more conservative parameter setting. With these assumptions, vehicles arriving at the bottleneck can be considered as independent Bernoulli trials with two possible outcomes: stable vs. unstable. Then, the first vehicle that causes instability and ensuing capacity drop follows the geometric distribution.

We derive the probability mass function (PMF, denoted by f_s for Strategy s), cumulative distribution function (CDFs, denoted by F_s for Strategy s) and the expected value for each strategy. For Strategy 1, the PMF is simply derived as the probability that the n^{th} vehicle in state e causes instability and control failure:

$$f_1(n) = p_e(1 - p_e)^{n-1}. \quad (8)$$

Then, the CDF for Strategy 1 is obtained as:

$$F_1(n) = \sum_{j=1}^n f_1(j) = \sum_{j=1}^n p_e(1 - p_e)^{j-1} = 1 - (1 - p_e)^n. \quad (9)$$

Similarly, the PMF and CDF for Strategy 2 can be expressed based on the probability of instability in state g :

$$f_2(n) = p_g(1 - p_g)^{n-1}. \quad (10)$$

$$F_2(n) = 1 - (1 - p_g)^n. \quad (11)$$

Note that free-flow state g is likely more stable than state e ; thus, we expect $p_g < p_e$.

For Strategy 3, we make additional assumptions that:

- (c) A CV has a lower probability of causing instability, $p_{v_{e'}}^{CV}$, when controlled at $v_{e'}$ than a regular vehicle in state e' due to a void ahead; i.e., $p_{v_{e'}}^{CV} < p_{e'}$.
- (d) Platoon $(k - 1)$ and platoon k are independent as long as state h is resolved within the platoon (see Fig. 6(c)).
- (e) The headway of CVs follows some distribution, and thus the platoon size varies.

In Strategy 3, the process is considered at the platoon level rather than a single vehicle: control fails if state h arises due to instability in state e' and is not resolved within the platoon (see Fig. 6(d)). Notice that the process does not exactly follow the Bernoulli trials because the platoon size and thus the probability of instability for each platoon vary. However, the process is similar. Each platoon has two possible outcomes: no residual state h vs. residual state h (control failure).

The probability of the latter case, denoted by P_k , equals to the probability that instability arises within the first n_k^c vehicles in platoon k , given as:

$$P_k = p_{v_{e'}}^{CV} + \sum_{n=2}^{n_k^c} (1 - p_{v_{e'}}^{CV})(1 - p_{e'})^{n-2} p_{e'}, \quad (12)$$

where the first term captures instability by a CV, the first vehicle in the platoon and the second term by a non-CV within n_k^c . The expression can be further simplified to:

$$P_k = p_{v_{e'}}^{CV} + (1 - p_{v_{e'}}^{CV})(1 - (1 - p_{e'})^{n_k^c - 1}). \quad (13)$$

Let k be the first platoon in which the control fails. Then its PMF can be expressed similarly to the geometric distribution:

$$f_3(k) = \begin{cases} P_1, & k = 1 \\ P_k \prod_{j=2}^k (1 - P_{k-1}), & k > 1 \end{cases} \quad (14)$$

The product term in the second expression represents the joint probability that all state h 's that may arise in the first $(k - 1)$ platoons are resolved within each platoon (i.e., no failure). Finally, the CDF for control failure is derived as:

$$F_3(k) = 1 - \prod_{j=1}^k (1 - P_j). \quad (15)$$

4.2 Optimal design to maximize delay saving

4.2.1 Delay saving and optimum control speed for Strategy 1-3

The stochastic properties explored in the previous section imply a trade-off between control speed limit (thus the discharge rate) and probability of instability. Thus, the objective is to find the optimal speed limit, v_e^* (and $v_{e'}^*$ for Strategy 3), that maximizes the expected delay saving. This concept is illustrated in Fig. 8 with queuing diagrams, where $A(t)$ and $D(t)$ respectively represent the cumulative arrival and departure curves at the bottleneck. (The subscript in $D(t)$ denotes different scenarios: the baseline (B), control success (S), and control failure (F).) Note that t_f' represents the time of demand change at the bottleneck. For the baseline case without any traffic control (Fig. 8a), the slope of $D_B(t)$ is q'_{BN} ($< q_{BN}$) during the entire duration of queue, and the total delay is equal to the area between $A(t)$ and $D_B(t)$.

FIGURE 8 Queueing diagram at the bottleneck: (a) Maximum delay saving; (b) Delay saving with control failure at the n^{th} vehicle.

With VSL control, the discharge rate increases to q_e at t_c , the time at which the existing queue is resolved. In the absence of instability (i.e., fully successful control), the discharge rate remains at q_e (see the departure curve $D_S(t)$), and the delay is saved as much as the shaded triangular area in Fig. 8a. Thus, this area represents the maximum delay saving by VSL control, ΔW_{Max} , derived as:

$$\Delta W_{Max} = \int (D_S(t) - D_B(t)) dt = \frac{1}{2} (q_e - q'_{BN}) (t_{end}^S - t_c) (t_{end}^B - t_c), \quad (16)$$

where t_{end}^S and t_{end}^B are the queue clearance times for the cases with fully successful control and for the baseline case, respectively (see Fig. 8a):

$$t_{end}^S = \frac{(q_a - q_f)t_{f'} + (q_e - q'_{BN})t_c}{q_e - q_f}. \quad (17)$$

$$t_{end}^B = \frac{(q_a - q_f)t_{f'}}{q'_{BN} - q_f}. \quad (18)$$

On the other hand, if the control fails, the discharge rate will revert to q'_{BN} as shown by $D_F(t)$, resulting in a partial delay saving as depicted by the shaded area in Fig. 8b. If the control fails with the n^{th} vehicle after t_c , the partial delay saving, $\Delta W(n)$, can be derived simply as a fraction of ΔW_{Max} according to the properties of similar triangles:

$$\Delta W(n) = \int (D_F(t) - D_B(t)) dt = \left(1 - \left(1 - \frac{n}{N}\right)^2\right) \Delta W_{Max}, \quad (19)$$

where N is the total number of vehicles that pass the bottleneck by t_{end}^S :

$$N = \lfloor q_e(t_{end}^S - t_c) \rfloor, \quad (20)$$

Then, the expected delay saving for the control of Strategy s , $E_s(\Delta W)$, is derived as:

$$E_s(\Delta W) = (1 - F_s(N))\Delta W_{Max} + \sum_{n=1}^N f_s(n)\Delta W(n). \quad (21a)$$

The first term captures the delay saving if the control fully succeeds, and the second term if it fails. The equation can be further simplified to:

$$E_s(\Delta W) = \Delta W_{Max} \left(1 - \sum_{n=1}^N f_s(n) \left(1 - \frac{n}{N} \right)^2 \right), \quad (21b)$$

where the second component (the term inside the first parenthesis) can be interpreted as the overall success probability. Equation (21b) highlights the trade-off in setting v_e : a higher v_e (and thus q_e) leads to a larger ΔW_{Max} (see equation (16)), but a lower overall success probability by increasing f_s . This trade-off is illustrated with a numerical example in Fig. 9a. Here we assume implementation of Strategy 1 with the FD of the following parameter values: $u = 60$ mph, $w = 12$ mph, and $k_j = 200$ veh/mile. The upstream demand is initially at 2,000 vph ($= q_a$) and drops to 1,000 vph ($= q_f$) after 20 minutes. q'_{BN} is assumed constant at 1,500 vph. Note that p_e is assumed to be 0.01250% when $v_e = 20$ mph and double for every 5 mph increase of v_e ; for example, for $v_e = 45$ mph, $p_e = 0.40\%$ ($= 0.01250\% \times 2^{(45-20)/5}$). This assumption is made based on the empirical observation that the probability of traffic breakdown at a bottleneck has a concave relationship with flow (Persaud et al., 1998; Lorenz and Elefteriadou, 2001; Brilon et al., 2005; Ozguven and Ozbay, 2008; Xie et al., 2014).

The optimal control speed, balancing the trade-off, is determined numerically since an analytical solution is difficult to obtain for the discrete random variable. Fig. 9b illustrates the expected delay saving with respect to the optimal control speed for the example above. The lower (red) and upper (blue) bars represent the delay savings without and with control failure (i.e., the first and the second terms of equation (21a)), respectively. (Note that the blue bars are drawn on top of the red bars rather from the x -axis to show the overall expected delay saving.) Each component (and the overall delay saving) exhibits a concave relationship, and thus, there exists a unique solution for optimal control speed. Nonetheless, the optimal control speed is different for the two components (30 mph and 43 mph), as highlighted in the figure. Balancing these components, the optimal control speed, v_e^* , that maximizes the overall expected delay saving is achieved at 35 mph for this example (denoted by the green bar).

Figure 9 Example of expected delay savings with respect to control speed (Strategy 1): (a) Maximum delay saving and probability of control success with respect to the control speed; (b) Relationship between delay savings and the control speed.

We further examine the sensitivity of v_e^* with respect to the duration of high demand, q_a , and probability of instability, p_e , for the above example. The former is examined because it affects both components in equation (21b): the longer the q_a , (i) the greater the ΔW_{Max} (per equations (16) and (17)); but (ii) the lower the overall success probability due to an increased probability of control failure (per equations (9), (11), and (15)). Thus, v_e^* depends on the relative magnitudes of these effects. Since the precise relation is not straightforward, we illustrate it through numerical examples. Fig. 10(a) – 10(c) show the contour plots of expected delay savings with respect to v_e (y-axis) and the duration of q_a (x-axis), with q_a ranging from 1 to 30 minutes for three levels of p_e : (a) 0.00125%, (b) 0.01250%, and (c) 0.12500% when v_e is 20 mph. Note that, as in the previous example, the value of p_e is doubled every 5 mph increase of v_e . The blue line in each plot represents the locus of optimal control speeds, v_e^* 's, that maximize the expected delay savings with respect to the duration of q_a . The result shows that v_e^* decreases with the duration of q_a , indicating that the effect of overall success probability has a greater effect on the expected delay saving. Moreover, as expected, v_e^* decreases as the failure probability increases; see Fig. 10d. These results indicate that a lower control speed is desired for a higher probability of instability. Similar trends and insight are obtained for Strategy 2.

FIGURE 10 Relationship between optimum control speed and duration of high demand:
(a) $p_e=0.00125\%$ for $v_c=20\text{mph}$; (b) $p_e=0.0125\%$; (c) $p_e=0.125\%$; (d) Sensitivity analysis
of the optimum speed by p_e .

The analysis for Strategy 3, however, is much more complex because it should be done at the platoon level and consider both speed parameters, v_e and $v_{e'}$. Recall that the expected delay saving depends on two components, ΔW_{Max} and the overall success probability according to equation (21b). While ΔW_{Max} only depends on and increases with v_e (equation (16)), the overall success probability for Strategy 3 is determined by the platoon instability, P_k , which has a complex relationship with v_e and $v_{e'}$. Specifically, P_k depends on two factors (see equation (13)): $p_{e'}$ and n_k^c . When v_e increases, n_k^c and thus P_k increase because a larger q_e makes it more challenging to resolve any instabilities. On the other hand, $v_{e'}$ has more complex effects on P_k : when $v_{e'}$ increases, $p_{e'}$ increases as expected, but n_k^c decreases because a larger $v_{e'}$ can create a larger void.

Integrating these effects, we obtain overall expected delay savings as exemplified in Fig. 11 (the assumptions are the same as the previous example, except that duration of $q_a=30$ minutes,

$p_{e'}$ of 20mph = 0.0125%, and $p_{v_{e'}}^{CV} = 0.01\%$ regardless of $v_{e'}$). For simplicity, we assume that the penetration rate of CV is 1/3, and the platoon size, n_k , is constant at 3. Note that the upper diagonal area ($v_e > v_{e'}$) is not viable for Strategy 3. Different regions are observed in the figure, depending on the value of n_k^c : the dark bottom region corresponds to $n_k^c = 0$ and 1, and discrete jumps in delay savings are observed where n_k^c increases to the next integer.

Details aside, the largest delay saving for this example (the brightest area in the figure) is found toward the top right side of the bottom region, where $v_e \ll v_{e'}$ ($v_e = 35$ mph, $v_{e'} = 57\sim 60$ mph). However, another local optimum is observed near the diagonal boarder, where $v_e \approx v_{e'}$. In fact, depending on the parameters assumed, optimal solutions are often found in this area (near the diagonal boarder), in which case, the unique advantage of Strategy 3 (i.e., periodic voids to resolve local instabilities) is not well exploited. This typically happens with (i) a short duration of high demand, (ii) a low probability of instability, or (iii) a low CV penetration rate. In this case, simpler Strategy 1 or 2 is favored. A further discussion on the preferred control strategy is provided in Section 5.

FIGURE 11 Numerical example of Strategy 3: (a) Probability of platoon instability with respect to the control speed; (b) Overall expected delay saving.

4.2.2 Delay saving and optimal control speed for the adaptive control

The adaptive control proposed earlier can recover q_e after remedying a control failure and realize a larger delay saving as illustrated in Fig. 12. In the figure, it is assumed that a control failure occurs at the n^{th} vehicle after t_C , and the discharge rate reduces to q'_{BN} . With the adaptive control, however, the discharge rate recovers to q_e , as shown by $D_R(t)$, after Δt_R , the time needed to resolve state h . Then, the blue shaded area in Fig. 12 represents the maximum additional delay saving, $\Delta W_{Max}^R(n)$, assuming no further failures. Considering multiple cycles of control failure and recovery, we can estimate the expected additional delay saving by adaptive control (the derivation is presented in Appendix C).

1

2

3 **Figure 12 Queueing diagram considering adaptive control**

4

5 With the adaptive control, we would expect that (a) v_e^* (and thus q_e) is higher since a
6 control failure can be remedied (i.e., the system is more forgiving) and that (b) the quicker the
7 remedy, the higher the control speed (vice versa).⁴ These premises are examined in Fig. 13,
8 building on the example in Fig. 9. To examine premise (a), the duration of recovery from a
9 control failure, Δt_R , is assumed to be constant at 450 sec. In reality, Δt_R depends on several
10 factors, including the queue length, failure detection time (i.e., CV penetration rate), and v_e ;
11 however, it is assumed constant for simplicity without losing any main insights. Fig.13a shows
12 the expected delay savings with respect to the control speed, consisting of three components:
13 expected delay saving without control failure (lower, red bars), with control failure (middle, blue
14 bars), and additional delay saving by the adaptive control (upper, grey bars). For the third
15 component, the maximum delay saving is achieved at 50 mph, which is higher than the optimal
16 speeds for the other two components. Overall, v_e^* is achieved at 40 mph, as opposed to 35 mph
17 without the adaptive control.

18 Premise (b) is verified by Fig. 13b that shows v_e^* with respect to the duration of high
19 demand for three levels of Δt_R : 1800 sec, 900 sec, and 450 sec. It is clear that the smaller the
20 Δt_R , the higher the v_e^* . This result suggests that if an effective adaptive control is available, a
21 higher control speed can be implemented despite a higher probability of control failure.

22

⁴ We assume a single control speed for the entire period of queue. It may be possible to implement time-dependent control speed, but this is beyond the scope of this work.

(a)

(b)

Figure 13 Example of determining optimal control speed, v_e^* (Strategy 1): (a) Relationship between the expected delay saving and the control speed; (b) Optimum control speed for three levels of Δt_R .

5. DISCUSSION ON TRAFFIC INSTABILITY

We turn our attention to the probabilities of instabilities that can arise in different traffic states (p_i 's). These are important parameters that significantly impact the optimal control speed and expected delay savings. Unfortunately, estimates of these probabilities, particularly for congested states, are not readily available in the literature. Nevertheless, existing empirical studies help us infer the relative orders of instability probabilities. Specifically, previous studies suggest that (i) the probability of instability increases rapidly as the flow in the free-flow regime approaches/exceeds capacity (e.g. Elefteriadou et al., 1995; Lorenz and Elefteriadou, 2001; Persaud et al., 1998) and that (ii) it increases with density for a given flow (Chow et al., 2010). Based on (i), we further presume that the probability of instability increases with the flow in congested traffic. Thus, we can infer that $p_g < p_a$ and $p_g < p_e < p_{e'}$. (However, the relationships between p_a and p_e and between p_a and $p_{e'}$ are not clear.) Following these relationships, we expect Strategy 2 (with p_g around the bottleneck) to be more stable than Strategy 1 (p_e). However, stability of Strategy 3 relative to Strategy 1 or 2 is not straightforward. While we expect $p_{e'}$ to be relatively high since the corresponding flow likely exceeds q_{BN} ; however, a void created deliberately between platoons would significantly reduce the probability of instabilities propagating persistently upstream.

To gain further insight, we examine the stability of the three strategies through a numerical example presented in Table 1. Demand is assumed higher than the bottleneck capacity (q_{BN}), and $v_{e'}$ is assumed as 55 mph, which is a little lower than the free flow speed. Each probability of instability is assumed according to the relative orders stated above, and other parameters are derived from the assumed FD. Based on these values, the CDFs for different strategies are obtained from equations (8), (10) and (14); see Fig. 14b. Note that this approach is different from the previous research on stochastic capacity (cited earlier), in that it captures the temporal evolution of the likelihood of instability rather than a static relationship between flow and instability probability. As expected, the CDFs for all strategies lie below the CDF for the

uncontrolled case, indicating that stability is improved with any of the proposed strategies. Moreover, Strategy 2, which creates a free-flow state around the bottleneck, achieves much better stability than Strategy 1 that creates a congested state with the same flow. For Strategy 3, the stability improves with the CV penetration rate, which is expected because (i) the probability of control failure in a platoon, P_k , is lower due to a higher proportion of CVs with a lower instability probability, and (ii) instabilities are more likely to be resolved with a smaller platoon size. The stability of Strategy 3 compared to Strategy 1 and 2, however, is complicated because that depends on p_e , $p_{e'}$ and $p_{e'}^{CV}$, in addition to the CV penetration rate. A better estimation of these parameters based on empirical tests is needed to make reasonable comparisons. Nevertheless, the numerical example here suggests that if the CV penetration rate is low, Strategy 3 could be less stable than Strategy 1 and 2 because the proportion of controllable vehicles (i.e., CVs) is too small, but the stability significantly improves as the penetration rate increases.

Table 1 Numerical example to examine stability: Assumed parameter values for the three strategies

Control Parameters	Base case	Strategy 1	Strategy 2	Strategy 3
Traffic state at bottleneck	a	e	g	e' and o (void)
Speed at the bottleneck (mph)	60	24	60	55
Flow rate (vph)	1800	1600	1600	1970 / 0
p_i	0.50 %	0.25%	0.05%	1.00% ($p_e^{CV} = 0.01\%$)
[Assumption of fundamental diagram]				
<ul style="list-style-type: none"> $u = 60\text{mph}$, $w = 12\text{mph}$, $k_j = 200\text{ vpm}$ Capacity = 2000 vph, $q_{BN} = 1600\text{ vph}$, $q'_{BN} = 1440\text{ vph}$ (-10%)

(a)

(b)

FIGURE 14 Example for the instability probability distribution for each strategy

6. CONCLUSIONS AND DISCUSSIONS

This study applied connected vehicle technology to develop VSL control strategies to improve discharge rate at recurrent freeway bottlenecks and reduce system delay with the

assumption of partial penetration of CVs. We developed three different VSL strategies using different number of CVs for control: (i) a single CV (per lane) for control (Strategy 1), (ii) a single CV (per lane) coupled with variable message signs (Strategy 2), and (iii) multiple CVs (Strategy 3). We further developed adaptive schemes to complement the three strategies to address potential control failures. The VSL control strategies developed in this study offer significant benefits over conventional control in three perspectives: (i) they deliver more efficient control by creating a void ahead of CVs to dissipate the dense queue, which is less restrictive and simpler; (ii) they could be more cost-effective since control can be done using only CVs or a single CV complemented by a few VMSs; (iii) CVs can serve two key functions simultaneously, traffic monitoring (e.g., queue detection) and control action, which simplifies the implementation and improves the implementation effectiveness as they reduce the potential delay caused by system coordination.

We also formulated probabilistic control failure based on the stochastic features of traffic instability to develop a framework to assess the stability of VSL control strategies and enable optimal control under uncertainty. The framework was built up from the probability of instability by individual vehicles in different traffic states to obtain temporal evolution (via CDF) of probability of traffic instability and ensuing control failure. The formulation facilitates stability assessment of different control strategies and the effect of the CV penetration rate (for Strategy 3). Based on the formulation, an optimal control speed was determined to maximize the expected delay saving that incorporates probabilistic control failures over time. Finally, a sensitivity analysis was performed to obtain insight into the behavior of the optimal control speed with respect to the duration of high demand and probability of instability by individual vehicles in different traffic states. Among the three control strategies, we found that with the complement of VMSs, Strategy 2 achieves better stability and thus delay savings than Strategy 1. The performance of Strategy 3 depends on various factors, but in general it is better than the other two strategies if the probability of traffic instability is high and the CV penetration rate is sufficiently high.

Overall, the CV-based control has notable advantages over the VMS-only control in terms of efficiency and simplicity due to its ability to effectively impose dynamic control over continuous time and space, enabling: (i) faster queue clearance around a bottleneck using a void rather than a lower flow state in VMS-only control, (ii) less restrictive control with higher controlled speed, and (iii) simpler control via only a small number of CVs. Particularly, with (ii) and (iii), sophisticated upper queue management (e.g., as in Chen et al. (2014) and Chen and Ahn (2015)) is not necessary to achieve smooth speed transitions; i.e., gradual speed reductions, if necessary, can be imposed on control CVs. Additionally, we expect that CVs will be widely used in the near future for multiple purposes, including safety enhancement and data collection. Therefore, the CV-based traffic control to improve traffic efficiency, such as the ones developed in this study, could be a part of an integrated system, which would be more streamlined and cost-effective.

Nevertheless, there are several issues, especially for real-world implementations, that merit future investigations. One key issue is application of the proposed VSL control for multilane roadways. In this paper, the effect of lane changing was not considered for simplicity to develop a theoretical framework. However, it is likely that vehicles from adjacent lanes insert into voids, which could undermine the queue clearance process. Thus, VSL control via a combination of CVs and road side units (e.g., VMSs) is desired in early CV adoption with a low

CV penetration rate to apply the control across lanes simultaneously. Once the technology matures, (near) simultaneous control would be possible with CVs alone by controlling nearby CVs in all lanes. In that case, our proposed strategies can be applied with some simple modifications.

Another issue for implementation is traffic instability. With (sudden) deceleration of controlled CVs, instabilities may arise upstream of the CVs. To remedy this, a gradual speed reduction of CV would be desired with some modifications of the control parameters at the expense of a larger area of controlled congestion, as discussed in Section 2. In addition, in low CV penetration, VMSs may be accompanied to alert drivers of low speed ahead.

The third issue is queue detection, which depends on many factors such as the definition of queue, speed threshold, buffer time, etc., which are subjective or implementation-specific. Thus, to be general, we did not assume any queue detection schemes but instead used the general parameter, L_q (queue length at the time of detection/confirmation), to capture the potential effects. In practice, sensor location and accuracy, and queue detection/confirmation algorithm (e.g., speed threshold) will significantly affect the control effectiveness and the success of implementation. Thus, future research in this regard including field tests is needed.

Furthermore, fundamental diagram and other traffic parameters (e.g., capacity drop level and probability distribution of traffic instability in different traffic states) should be calibrated carefully using historical empirical data for successful implementation. We also expect that pilot implementations would be necessary to fine-tune the control parameters as well as the control schemes based on real driver behavior under VSL control (e.g., complied speed).

Last but not the least, the probability of traffic instability is a critical factor to determine the optimal control speed. A comprehensive investigation of historical data is needed to obtain the instability probability distribution. Particularly, we expect that daily traffic stochasticity can be captured by the same calibrated probability distribution function of instability, but considering different traffic states as shown in Section 4. However, the distribution itself may change with exogenous factors such as weather and seasons. Thus, an analysis of long-term longitudinal data is desired to understand the changes in probability distribution and their impact on the control schemes.

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APPENDIX A

Here we show the derivation of equation (6) in Section 2.

1. Eq. (6a) and (6b): derived from the relation of inflow and outflow at the BN shown on Fig. A

Figure A. Critical number of vehicle, n_k^c , for Strategy 3.

In, Fig. A,

$$T_k = \frac{n_k}{q_e} = \frac{n_k^c}{q_{e'}} + \frac{n_k - n_k^c}{q_{BN}}.$$

Then, we have:

$$n_k^c = n_k \left(\frac{1}{q_{BN}} - \frac{1}{q_e} \right) \left(\frac{1}{q_{BN}} - \frac{1}{q_{e'}} \right)^{-1},$$

and

$$\frac{n_k^c}{n_k} = \left(\frac{1}{q_{BN}} - \frac{1}{q_e} \right) \left(\frac{1}{q_{BN}} - \frac{1}{q_{e'}} \right)^{-1}.$$

APPENDIX B

Here we show the optimum location of CV for adaptive control for each strategy in Section 3.

1. Strategy 1(Eq. (7)): derived from the geometric relationship shown on Fig. 7b
2. Strategy 2: derived from the geometric relationship shown on Fig. B.

Figure B. Optimum location of CV for adaptive control in Strategy 2.

From Fig. B,

$$\Delta t_1 + \Delta t_2 + \frac{s_{eh}(\Delta t_1 + \Delta t_2) + L_{q2}}{v_h} = \frac{x_R^* + L_{q2} + L_e - L_a}{v_e} + \frac{L_a}{u} \quad (a)$$

$$\Delta t_1 = \frac{x_R^*}{s_{ae} + u} = Ax_R^*, \quad \left(A = \frac{1}{s_{ae} + u} \right) \quad (b)$$

$$\Delta t_2 = \frac{L_e - (s_{eh} - s_{ae})\Delta t_1}{s_{eh} + v_e} = \frac{L_e - (s_{eh} - s_{ae})}{s_{eh} + v_e} \cdot \frac{x_R^*}{s_{ae} + u} = Bx_R^*, \quad \left(B = \frac{L_e - (s_{eh} - s_{ae})}{(s_{eh} + v_e)(s_{ae} + u)} \right) \quad (c)$$

from (b), (c) $\rightarrow$ (a), we have:

$$\begin{aligned} x_R^* &= \left(\frac{L_{q2} + L_e - L_a}{v_e} + \frac{L_a}{u} - \frac{L_{q2}}{v_h} \right) \left((A + B) \left(1 + \frac{s_{eh}}{v_h} \right) - \frac{1}{v_e} \right)^{-1} \\ &= \left(\frac{L_{q2} + L_e - L_a}{v_e} + \frac{L_a}{u} - \frac{L_{q2}}{v_h} \right) \left(\frac{1}{s_{ae} + u} \left(\frac{s_{ae} + v_e + L_e}{s_{eh} + v_e} \right) \left(1 + \frac{s_{eh}}{v_h} \right) - \frac{1}{v_e} \right)^{-1}. \end{aligned}$$

3. Strategy 3: replace u to v_e' in equation (a), then, we have:

$$x_R^* = \left(\frac{L_{q2} + L_e - L_a}{v_e} + \frac{L_a}{v_e'} - \frac{L_{q2}}{v_h} \right) \left(\frac{1}{s_{ae} + u} \left(\frac{s_{ae} + v_e + L_e}{s_{eh} + v_e} \right) \left(1 + \frac{s_{eh}}{v_h} \right) - \frac{1}{v_e} \right)^{-1}$$

APPENDIX C

Here we show the derivation of the equations and specific process for additional delay saving by the adaptive control in Section 4.

1. Eq. (19): derived from the geometry of queuing diagram in Fig. C1 (adopted from Fig. 8b),

Figure C1. Delay saving with control failure at n^{th} vehicle.

As the slopes of $\overline{BC}$ and $\overline{DE}$ are equal to q'_{BN} , $\triangle ABC$ and $\triangle ADE$ are similar triangles.

Then, from the geometry relation, $\overline{AB} : \overline{AD} = N : (N - n)$. Thus,

$\Delta W(n) = \text{area of } \triangle ABC - \text{area of } \triangle ADE$

$$= \Delta W_{Max} - \left(\frac{N-n}{N}\right)^2 \Delta W_{Max} = \left(1 - \left(\frac{N-n}{N}\right)^2\right) \Delta W_{Max} = \left(1 - \left(1 - \frac{n}{N}\right)^2\right) \Delta W_{Max}$$

2. Derivation of the expected delay saving with the adaptive control

Figure C2. Queuing diagram with adaptive control.

With the adaptive control, it is possible that multiple cycles of control failure and recovery can occur, and the delay saving depends on when the control fails. Thus, the expected delay saving with the adaptive control, $E_R(\Delta W)$, can be expressed in terms of the conditional expected delay saving with respect to n :

$$E_R(\Delta W) = (1 - F(N))\Delta W_{Max} + \sum_{n=1}^N f(n) E_R(\Delta W(n)), \quad (d)$$

where, F and f are CDF and PMF for control failure of the applied strategy, respectively (equation (8)-(11), (14)-(15)), and $E_R(\Delta W(n))$ is the conditional expected delay saving with the adaptive control when the control fails with the n th vehicle after t_c . Comparing equation (d) to (21a), the first term is the same, which represents the delay saving with control success. The second term in equation (d) represents the partial delay saving with the remedy, which is larger than $\Delta W(n)$ in (21a) as;

$$E_R(\Delta W(n)) = \Delta W(n) + (1 - F(N_R(n))) \cdot \Delta W_{Max}^R(n) + \sum_{n_1=1}^{N_R(n)} f(n_1) \cdot E_R(\Delta W(n_1; n)) \quad (e)$$

where, $N_R(n)$ is the total number of vehicles that pass the bottleneck by t_{end}^R when the control fails with the n th vehicle after t_c (see Fig. C2), which is derived as;

$$N_R(n) = N - n - \frac{\Delta t_R}{3600} \left(\frac{(q'_{BN} - q_f)q_e}{q_e - q_f} \right).$$

and t_{end}^R is the queue clearance time after the recovery by adaptive control, derived as:

$$t_{end}^R = t_{end}^S + \frac{(q_e - q'_{BN})}{(q_e - q_f)} \Delta t_R.$$

and $\Delta W_{Max}^R(n)$ can be derived using similar triangles encompassing ΔW_{Max} and $\Delta W_{Max}^R(n)$ as:

$$1 \quad \Delta W_{Max}^R(n) = \left(1 - \frac{n}{N} - \frac{t_{end}^R - t_{end}^S}{t_{end}^B - t_{end}^S}\right)^2 \Delta W_{Max}.$$

2 The third term of equation (e), $E_R(\Delta W(n_1; n))$, represents the conditional expected delay saving
 3 with respect to n_1 and n . Similar to $E_R(\Delta W(n))$, $E_R(\Delta W(n_1; n))$ represents the partial delay
 4 saving with the remedy, which is larger than $\Delta W(n_1; n)$ in Fig. C2b as:

$$5 \quad E_R(\Delta W(n_1; n)) = \Delta W(n_1; n) + (1 - F(N_{R2}(n_1; n)))\Delta W_{Max}^{R2}(n_1; n) + \sum_{n_2=1}^{N_{R2}(n_1; n)} f(n_2) E_R(\Delta W(n_2; n_1, n)).$$

6 This derived expression also include another conditional expected delay saving,
 7 $E_R(\Delta W(n_2; n_1, n))$, and finally, equation (d) can be expanded as below until there is no added
 8 delay saving by the adaptive control:

$$\begin{aligned} E_R(\Delta W) = & \left[(1 - F(N))\Delta W_{Max} + \sum_{n=1}^N f(n) \Delta W(n) \right] \\ & + \sum_{n=1}^N f(n) \left[(1 - F(N_R(n)))\Delta W_{Max}^R(n) + \sum_{n_1=1}^{N_R(n)} f(n_1) \Delta W(n_1; n) \right] \\ & + \sum_{n=1}^N f(n) \sum_{n_1=1}^{N_R(n)} f(n_1) \left[(1 - F(N_{R2}(n_1; n)))\Delta W_{Max}^{R2}(n_1; n) + \sum_{n_2=1}^{N_{R2}(n_1; n)} f(n_2) \Delta W(n_2; n_1, n) \right] \\ & + \dots. \end{aligned}$$

9
 10 Notably, each term in square brackets represents the expected delay saving without considering
 11 further remedy.

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