A coordinated location-inventory problem in closed-loop supply chain

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Abstract

This paper considers a coordinated location-inventory model under uncertain demands for a closed loop supply chain comprising of one plant, forward and reverse distribution centers, and retailers. The inventory of new and returned products is managed at forward and reverse distribution centers respectively through a periodic review policy. The proposed model determines the location of forward and reverse distribution centers and the associated capacities, the review intervals of the inventory policy at distribution centers, and the assignments of retailers to the distribution centers. We model six different coordination strategies. All the models are formulated as nonlinear integer programs with chance constraints and transformed to conic quadratic mixed-integer programs that can be efficiently solved by CPLEX. An outer approximation based solution algorithm is developed to solve the conic quadratic mixed-integer program. The benefit of different types of coordination strategies is shown through extensive computational testing.

Keywords: Supply chain coordination, closed-loop supply chain, conic quadratic mixed-integer program, location-inventory model, periodic review inventory

1. Introduction

Successful supply chain management strategies are critical for reducing costs and providing competitive advantage in an increasingly fierce business environment (Friesz et al., 2011). Traditionally, researchers have approached the strategic (such as location of facilities) and tactical (such as inventory control) supply chain management decisions in a sequential manner. Over the last decade, several researchers have shown that such an approach leads to sub-optimal decision making and developed joint location-inventory models which use a single optimization model to determine both strategic and tactical strategies (Daskin et al., 2002; Shen et al., 2003; Miranda and Garrido, 2004).

This paper considers a three level closed loop supply chain comprising of a single plant, forward and reverse distribution centers, and retailers. The plant manufactures new products which are stored at forward distribution centers and then transported to retailers. Increasingly businesses are focusing on optimizing the forward and reverse supply chain for cost reduction, sustainability implications, and brand protection. In this paper we assume that used products from retailers are transported and stored at reverse distribution centers which are then shipped back to the plant.

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Inventories are assumed to be present at the forward and reverse distribution centers and are managed by a periodic review policy (Hadley and Whitin, 1963). At every forward distribution center, an order is placed at the end of every review period to bring the inventory up to the order-up-to level. Every reverse distribution center stores the returned products which are shipped back to the plant at the end of the review interval. Retailers demand for new products is assumed to be multi-variate normally distributed with known means and covariance matrix. The volume of returned product at each retailer is assumed to be uncertain with known means and standard deviations. The mathematical model developed in this paper simultaneously determines the location of forward and reverse distribution centers, the allocation of retailers to forward and reverse distribution centers, the order-up-to level at each forward distribution center (which serves as a proxy for capacity), the capacity at each reverse distribution center, and the review interval at each distribution center so that the total cost is minimized.

A majority of the research in joint location-inventory models assume a continuous review inventory policy where orders are placed when the inventory falls below a pre-specified level (Daskin et al., 2002; Shen et al., 2003; Miranda and Garrido, 2004; Shahabi et al., 2014). This creates difficulties in operational decision making (transportation of goods, routing, and fleet management) as each distribution center may potentially have a different order cycle. When all order cycles are restricted to one value or a limited number of values, a supply chain manager can better manage the delivery schedules. This can be captured using a periodic review policy when the review intervals are either restricted to one value or chosen from a pre-specified limited set. Berman et al. (2012) showed that full coordination (where all distribution centers have the same resupply cycle) results only in a minor cost increase when compared to the optimal uncoordinated scenario. In this paper, we further extend Berman et al. (2012), by considering the impact of different coordination scenarios in a closed loop supply chain. Specifically, we consider five different scenarios: (i) Full coordination - where all forward and reverse distribution centers have the same review interval, (ii) Partial coordination scenario 1 - where all forward distribution center share the same periodic review interval while all reverse distribution centers share another periodic review interval, (iii) Partial coordination scenario 2 - where all forward distribution centers share the same periodic review interval while all reverse distribution centers have different cycles, (iv) Partial coordination scenario 3 - where all reverse distribution center share the same periodic review interval while forward distribution centers may have different cycles, (v) Partial coordination scenario 4 - where the new and returned products share the same periodic review at a joint distribution center. A distribution center will be frequently referred to as DC in the rest of the paper.

In this paper we model the strategic facility location and allocation and tactical inventory management decisions only. We do not explicitly model the details of the transportation routing which is an operational decision. This is consistent with almost all of the joint location-inventory papers cited in this work. Our premise is that when the replenishment cycles of the inventory management is coordinated or made similar, then it is easier for the supply chain manager to make day-to-day operational routing decisions to multiple locations which save transportation costs. One potential way to reduce this costs is by minimizing empty runs. We do not explicitly model the cost reduction as this will involve detailed modeling of truck routing scheduling which is beyond the scope of this paper. Such an approach

is used in Berman et al. (2012) in the context of a forward supply chain with no correlations in demand. According to Silver et al. (1998), “the coordination afforded by a periodic review system can provide significant savings”. Several other studies in the literature have noted the value of coordinating various aspects of the supply chain (Thomas and Griffin, 1996; Büyükkaramikli et al., 2014; Kaya et al., 2013; Guide Jr and Van Wassenhove, 2009).

The coordinated joint location-inventory model is formulated as a nonlinear integer program with chance constraints. The mixed-integer formulation is transformed into a conic quadratic mixed-integer program to exploit the advances made by solvers such as CPLEX in solving second order conic integer programs. An outer approximation based solution algorithm is developed to solve the conic quadratic mixed-integer program. The impact of coordination is studied through extensive computational experiments.

The remainder of the paper is organized as follows. Section 2 presents a literature review on integrated inventory and location models. Section 3 presents the coordinated closed-loop supply chain network design model. Section 4 transforms the proposed formulation to a conic quadratic mixed-integer program. Section 5 presents the outer approximation based solution method used in this work. The performance of the solution approach is reported in section 6. The benefits of coordinated inventory control at the DCs are explored in Section 7. Conclusions are drawn and future research directions are outlined in Section 8.

2. Literature review

Integrated supply chain design leads to significant cost savings and considerable efforts have been made in the related research fields. For comprehensive reviews we refer the reader to Shen (2007) and Kanda et al. (2008). The focus of this literature review is on analytical models for the joint location-inventory setting.

Daskin et al. (2002) was among the first to propose the uncapacitated joint location-inventory formulation with risk pooling for a two level supply chain network comprising of distribution centers supplying goods to multiple retailers. An economic order quantity (EOQ) model with continuous review (r, Q) policy was used to manage the inventories at located DCs. A lagrangean relaxation based solution heuristic was developed for the special case where the ratio of variance to the mean of demands at each retailer was assumed to be the same. Shen et al. (2003) reformulated the joint location-inventory model studied in Daskin et al. (2002) as a set covering integer programming formulation. A column generation based algorithm was proposed where the pricing sub-problem was solved efficiently for two cases: (i) the ratio of variance to the mean of demands at each retailer is the same, and (ii) the retailer demands have zero variance. Shen (2005) studied the multi-commodity variant of Daskin et al. (2002) and Shen et al. (2003). Shu et al. (2005) further improved the set covering formulation of Shen et al. (2003) by developing an efficient algorithm which solves the nonlinear pricing subproblem of the column generation for the general case. Snyder et al. (2007) studied the stochastic variant of Daskin et al. (2002) where the mean and variance of retailer demands are different for each uncertain scenario. A lagrangean relaxation algorithm embedded in a branch and bound scheme was developed for the special case where the ratio of variance to the mean of demands at each retailer was assumed to be the same for

each scenario. Ozsen et al. (2008) and Miranda and Garrido (2004) studied the capacitated variant of Daskin et al. (2002) and Shen et al. (2003). While Miranda and Garrido (2004) only considered the throughput handling capacity at each warehouse, Ozsen et al. (2008) modeled a more conservative worst case scenario by enforcing the sum of order quantity, safety stock, and demand during lead time to be less than the capacity of the warehouse. Ozsen et al. (2009) further developed Ozsen et al. (2008) by incorporating multi-sourcing where one retailer could be served by multiple warehouses. All the three capacitated models - Ozsen et al. (2008), Miranda and Garrido (2004), and Ozsen et al. (2009) - use a lagrangean relaxation based solution methodology. Atamtürk et al. (2012) demonstrated the efficiency of using conic quadratic reformulations in solving generalized variants of joint location-inventory problems including the presence of retailer demand correlations.

Other studies have extended the two level joint location-inventory problems to multiple echelon inventory management and facility location settings. Vidyarthi et al. (2007) considered a multi-product three level production-inventory-distribution system design model under stochastic demands. A nonlinear mixed-integer program was proposed to determine plant and DC locations, shipment levels from plants to the DCs, safety-stock levels at DCs, and the assignment of retailers to DCs. Park et al. (2010) considered a single sourcing network design problem for a three level supply chain in which DCs manage inventory by using continuous review (r, Q) policy. Shahabi et al. (2014) further extended Park et al. (2010) by considering demand correlations among retailers. Benyoucef et al. (2013) modeled random demands, supplier lead times, and supplier reliability issues in a three level supply chain location-inventory problem. While Vidyarthi et al. (2007), Park et al. (2010), and Benyoucef et al. (2013) used a lagrangean relaxation based approach, Shahabi et al. (2014) used an outer approximation solution algorithm to efficiently solve the conic reformulation of the model. Tancrez et al. (2012) proposed a continuous formulation and an approximation heuristic for a location-inventory problem in a three level supply chain to determine facility locations and allocations, and shipment sizes. The inventory costs are modeled using the classic EOQ formulation. However, Tancrez et al. (2012) assume perfect coordination and deterministic demands.

All the aforementioned models use a continuous review (r, Q) policy. In such cases each DC (or inventory location) will have different order cycles making it difficult to manage coordination issues in supply chain networks. This leads to the loss of many opportunities for cost savings especially when load deliveries are being scheduled. This paper adopts a periodic review inventory policy with coordinated replenishment in the integrated joint location-inventory setting. Unlike Tancrez et al. (2012), this paper studies different levels of coordination and considers uncertainty in demand. The main differences of this work from Shahabi et al. (2014) are as follows: (i) we consider an integrated forward and reverse supply chain setting; (ii) periodic review inventory policies are adopted to control the inventory at warehouse; and (iii) the benefits of coordinated inventory control are investigated.

Under stochastic demands, Berman et al. (2012) adopted a periodic-review (T, S) inventory policy to control the inventory at warehouse because the (T, S) system is easier to coordinate than a continuous review system (Silver et al., 1998). The replenishment lead time at a DC was assumed to be a DC dependent constant. They proposed a coordinated location-inventory model to address coordination issue in a supply chain system consisting of a single

supplier, multiple DCs, and multiple retailers. The supplier and the DCs were uncapacitated and the coordination was achieved by choosing the review intervals at the DCs from a menu of permissible values. This paper differentiates itself from Berman et al. (2012) in the following aspects. We assume that the supplier and distribution centers have capacities. We consider both the forward as well as reverse flow of materials in the supply chain. We provide a conic integer reformulation which can be solved efficiently using solvers such as CPLEX as well as an outer approximation based solution algorithm. Naseraldin and Herer (2011) proposed a location-inventory model to study an infinite horizon inventory system at DCs with periodic review and stochastic retailer demand. However, the paper modeled the special case where DCs (referred to as retail outlets in the paper) and customers are located on a homogeneous straight line.

Several researchers have considered the impact of inventory management and control decisions at multiple levels of the supply chain in a joint location-inventory context. Teo and Shu (2004) studied the multiple warehouse retailer location-inventory problem where inventory costs were considered at both warehouses and retailers. The infinite horizon system wide inventory replenishment costs was approximated using a convex program. A set partitioning formulation was developed which was solved using a column generation scheme. Shu (2010) developed an efficient greedy algorithm to solve the formulation studied in Teo and Shu (2004). Romeijn et al. (2007) further extend Teo and Shu (2004) by considering safety stocks and throughput capacities. Note that the current paper only considers inventory management and coordination control decisions at a single level in the supply chain (forward and reverse DC's).

Üster et al. (2008) developed a mathematical model to determine the warehouse location in a plane and coordinated replenishment policy parameters for a three level distribution network which consists of a single supplier, a single intermediate warehouse, and multiple retailers. Each retailer has a constant deterministic demand. Inventories were modeled at both warehouses and retailers. A power of two policy was followed for reorder intervals, i.e., the reorder interval at the warehouse and the retailers were considered to be a power of two multiple of a base reorder time period. Keskin and Üster (2012) extended Üster et al. (2008) to a problem setting of multiple capacitated suppliers and multiple warehouses. Diabat et al. (2013a) also modeled inventory decisions at warehouses and retailers using a power-of-two policy in a joint inventory-location framework. However, Diabat et al. (2013a) focused on the uncapacitated case and used a lagrangean relaxation based solution scheme whereas Keskin and Üster (2012) considered capacities and used a construction heuristic. All the three papers considered deterministic demand and did not model a closed loop supply chain and study the associated coordination issues.

While this paper models the uncertainty in demands, we do not account for the impact of disruptions and associated reliability related issues (Masih-Tehrani et al., 2011). Friesz et al. (2011) studied the impact of disruption on supply chain flows in an urban competitive supply chain. Schmitt (2011) emphasize the importance of strategic inventory management to maintain customer service levels in a multi-echelon supply chain network design under disruptions. Wang and Ouyang (2013), Chen et al. (2011), Peng et al. (2011), and Li and Ouyang (2010) develop models to study the impact of probabilistic disruptions on joint location-inventory problem. While the above mentioned papers

consider facility disruption related issues they do not consider issues associated with coordination and a closed loop supply chain.

This paper also does not model the stability issues and the associated bullwhip effect in multi-echelon supply chain networks. In the bullwhip effect, the focus is on time varying inventories and states of the supply chain. A systems dynamics or optimal control approach is used to study the evolution of the states of the supply chain across time (Ouyang and Daganzo, 2006a,b; Ouyang, 2007; Ouyang and Daganzo, 2008; Ouyang and Li, 2010). In this paper we are using an optimization approach and the goal is to focus on long term strategic and medium tactical operational decisions. We do not explicitly model the time varying inventories or the states of the system at different points in time. Our goal is to adopt a more static approach to model the average effect of a temporal phenomenon. Hence, the stability related issues are not captured in this work. In this paper in addition to inventory management we are also concerned with facility location which is a longer time scale issue when compared to the time scale commonly used to studying stability issues. Note that the papers in the literature focusing on the bullwhip effect do not model the facility location decision (Ouyang and Daganzo, 2006a,b; Ouyang, 2007; Ouyang and Daganzo, 2008; Ouyang and Li, 2010).

The research on integrated location-inventory model in closed loop supply chain is relatively limited. Abdallah et al. (2012) and Diabat et al. (2013b) studied a location-inventory model for an uncapacitated closed loop supply chain with forward distribution centers, reverse remanufacturing centers, and retailers. Returned products are collected and sorted by the retailers and then shipped to reverse remanufacturing centers who process them as spare parts. Single sourcing policies were adopted to assign the retailers to the DCs. Zhang et al. (2014) proposed a capacitated location-inventory model with bidirectional flows. Forward, reverse, and joint distribution centers were considered. However, all of the three papers incorporated continuous review (r, Q) inventory policies to control the inventory of new and returned products and did not focus on coordination related issues.

3. Problem statement and formulation

This section describes the mathematical programming formulation for a coordinated joint location-inventory problem for a supply chain with a single plant that not only produces new products but also reprocesses returned products for remanufacturing or recycling. The formulation involves the following decisions: (i) locating forward and reverse distribution centers from a set of candidate sites, (ii) allocation decisions assigning retailers to forward and reverse distribution centers, and (iii) inventory control decisions at each distribution center. The objective is to minimize the facility location, transportation, and inventory management costs. Figure 1 shows the supply chain considered in this work.

The model considered in this work makes the following assumptions:

- There is a site dependent fixed setup costs (per unit time) for locating distribution centers.
- There is a site dependent variable construction cost (per unit time and unit shipment) at distribution centers.

Figure 1: Structure of the supply chain considered

- The reverse DCs can only be opened near the forward DCs - a common assumption in the closed-loop supply chain literature (Jayaraman et al., 1999; Abdallah et al., 2012; Diabat et al., 2013b).
- There is a unit shipment transportation cost between plant and distribution centers, distribution centers and retailers, retailers and distribution centers, and distribution centers and plants which is proportional to the Euclidean distance.
- Transportation coordination is not considered in the model and transportation cost associated with empty run is neglected.
- We consider a single product.
- We consider a single sourcing strategy. Each retailer gets new products from a single forward distribution center. And, each retailer collects and sorts returned products and keeps them at one reverse distribution center.
- Demand at retailers for new products is multi-variate normally distributed with known means and covariance matrix.
- Volume of returned products at retailers is uncertain with known means and standard deviations.
- Inventory control is considered only at distribution centers. At each forward DC, periodic-review (T, S) policy is adopted to control the stock of new products. Back order cases are considered. In a (T, S) inventory policy, a DC places an order every T units of time to bring the inventory position up to the order-up-to-level S . Inventory position is on-hand inventory plus on-order inventory minus back orders (Hadley and Whitin, 1963).
- The cost of making a review does not depend on the review interval and the order-up-to-level (Hadley and Whitin, 1963).
- Back orders are small and such that they can be met when an order arrives (Hadley and Whitin, 1963).
- The penalty of each back order does not depend on the length of time for which the back order exists (Hadley and Whitin, 1963).

- The replenishment lead time is a distribution center dependent constant.
- There are no capacity restrictions at plants and forward distribution centers. The order-up-to-level is assumed to be a proxy for capacity at forward distribution centers. The reverse distribution centers are assumed to be capacitated.

Table 1 defines the sets, parameters, and variables used in the formulation. To identify the benefits of coordinated inventory control, six nonlinear integer programs are proposed. One is for an uncoordinated scenario, in which the review interval at each DC is independent. The others are for five coordination scenarios, in which the review intervals are partially or fully shared among the DCs.

3.1. Model with uncoordinated scenario

The mathematical programming formulation for the joint location-inventory model for the uncoordinated scenario is shown below:

$$P : \min \sum_{j \in J} f_j^F X_j^F + \sum_{j \in J} \sum_{i \in I} d_{ij}^F \mu_i^F Y_{ij}^F + \sum_{j \in J} \mathbb{K}_j^F(S_j^F, T_j^F) + \sum_{j \in J} \sum_{i \in I} d_{ij} \mu_i^R Y_{ij}^R + \sum_{j \in J} f_j^R X_j^R + \sum_{j \in J} \sum_{i \in I} d_{ij}^R \mu_i^R Y_{ij}^R + \sum_{j \in J} \mathbb{K}_j^R(C_j^R, T_j^R) + \sum_{j \in J} \sum_{i \in I} d_{ij} \mu_i^R Y_{ij}^R, \quad (1)$$

$$s.t. \sum_{j \in J} Y_{ij}^F = 1, \sum_{j \in J} Y_{ij}^R = 1, \forall i \in I, \quad (2)$$

$$Y_{ij}^F \leq X_j^F, Y_{ij}^R \leq X_j^R, \forall i \in I, j \in J, \quad (3)$$

$$\sum_{k \in K} Z_{jk}^F = X_j^F, \sum_{k \in K} Z_{jk}^R = X_j^R, \forall j \in J, \quad (4)$$

$$X_j^R \leq \sum_{i \in I} Y_{ij}^R, \forall j \in J, \quad (5)$$

$$X_j^F, X_j^R, Y_{ij}^F, Y_{ij}^R \in \{0, 1\}, \forall i \in I, j \in J, \quad (6)$$

$$S_j^F, C_j^R \geq 0, \forall j \in J. \quad (7)$$

where, $\mathbb{K}^F(S_j^F, T_j^F)$ represents the variable construction cost, the average working inventory, and shortage penalty costs per unit time at forward DC j and $\mathbb{K}^R(C_j^R, T_j^R)$ represents the variable construction cost and the average working inventory costs per unit time at reverse DC j .

The objective of the model (1) is to minimize the long-run average cost per unit time associated with the forward and reverse flows. The objective includes the fixed and variable costs of locating the DCs, transportation costs between the plant and the DCs for both forward and reverse flows, the working inventory costs for both forward and reverse flows, shortage penalty costs for forward flows only, and transportation costs between the DCs and the retailers for both forward and reverse flows. Constraint (2) ensures that one retailer is served by only one forward DC and one reverse DC. Constraint (3) ensures that each retailer is assigned to open DCs only. Constraint (4) stipulates that only one review interval is selected at each open DC. Constraint (5) enforces the restriction that a reverse DC cannot be

Table 1: Notations

Sets:

- I set of retailers indexed by i
- J set of candidate DC sites indexed by j
- K set of review intervals indexed by k , $K = \{t_1, t_2, \dots, t_k, \dots\}$

DC-related parameters:

- f_j^F, f_j^R fixed construction cost per unit time at forward/reverse DC j
- p_j^F, p_j^R unit variable construction cost per unit time at forward/reverse DC j
- h_j^F, h_j^R inventory holding cost per unit time at forward/reverse DC j
- π_j^F shortage cost at forward DC j
- g_j^F, g_j^R fixed cost of a review and shipment of new/returned products between the plant and DC j
- a_j^F, a_j^R cost per unit of new/returned products to ship between the plant to DC j
- l_j replenishment lead time when forward DC j places an order to the plant
- α service level, which is a probability that the quantity of returned products stored at a reverse DC does not exceed the capacity of that reverse DC

Retailer-related parameters:

- μ_i^F, μ_i^R mean demand of new/returned products per unit time at retailer i
- σ_i^F, σ_i^R standard deviation of demand of new/returned products per unit time at retailer i
- $\phi_{ii'}^F, \phi_{ii'}^R$ correlation coefficients of new/returned products between retailers i and i'

DC-to-Retailer-related parameter:

- d_{ij} cost per unit to ship between retailer i and DC j

Decision variables:

- X_j^F, X_j^R 1, if forward/reverse DC j is opened; 0 otherwise
- Y_{ij}^F, Y_{ij}^R 1, if retailer i is assigned to forward/reverse DC j ; 0 otherwise
- Z_{jk}^F, Z_{jk}^R 1, if review interval t_k is selected at forward/reverse DC j ; 0 otherwise
- T_j^F, T_j^R review interval at forward/reverse DC j , i.e., $T_j^F = \sum_{k \in K} t_k Z_{jk}^F$, $T_j^R = \sum_{k \in K} t_k Z_{jk}^R$
- S_j^F order-up-to-level at forward DC j
- C_j^R storage capacity at reverse DC j

opened unless a forward DC has been opened at the same site and retailers are assigned to this forward DC. Constraints (6) and (7) are standard binary and nonnegative constraints.

Derivation of $\mathbb{K}^F$

Periodic review (T, S) policy is adopted at the forward DCs to control the stock of new products (Hadley and Whitin, 1963). The average working inventory and shortage cost per unit time at a forward DC j , $\mathbb{K}_j^F(S_j^F, T_j^F)$ is obtained as follows:

$$\mathbb{K}_j(S_j^F, T_j^F) = p_j^F S_j^F + \frac{g_j^F}{T_j^F} + h_j^F \left[\frac{\mu_j^{F,DC} T_j^F}{2} + S_j^F - \mu_j^{F,DC} (T_j^F + l_j) \right] + \frac{\pi_j^F}{T_j^F} E \left[D_j^F(T_j^F + l_j) - S_j^F \right]^+,$$

where, D_j^F represents demand per unit time at forward DC j , which also follows a normal distribution with mean $\mu_j^{F,DC} = \sum_{i \in I} \mu_i^F Y_{ij}^F$ and standard deviation $\sigma_j^{F,DC} = \sqrt{\sum_{i \in I} \sum_{i' \in I} \phi_{ii'}^F \sigma_i^F \sigma_{i'}^F Y_{ij}^F Y_{i'j}^F}$. $D_j^F(\tau)$ is the demand during an interval of length τ at DC j . The first term is the variable construction cost that is proportional to the capacity of forward DCs (approximated by the order-up-to level), the second term is the average fixed ordering and plant to DC transportation cost of new products, the third term is the average inventory holding cost, and the last term is the average shortage penalty cost within one review interval.

The optimal order-up-to-level at each forward DC, S_j^{F*} , can be obtained by taking the first order derivative of $\mathbb{K}_j$ with respect to S_j^F and setting the resulting equation to zero (Berman et al., 2012):

$$\frac{\partial \mathbb{K}_j}{\partial S_j^F} = p_j^F + h_j^F + \frac{\pi_j^F}{T_j^F} \frac{\partial E[D_j^F(T_j^F + l_j) - S_j^F]^+}{\partial S_j^F} = 0,$$

Solving the above equation, we obtain

$$S_j^{F*} = \mu_j^{F,DC} (T_j^F + l_j) + z_j(T_j^F) \sigma_j^{F,DC} \sqrt{T_j^F + l_j},$$

where $z_j(T_j^F) = \Phi^{-1} \left(1 - \frac{(p_j^F + h_j^F) T_j^F}{\pi_j^F} \right)$, and Φ denotes the standard normal cumulative distribution function. Substituting this into $\mathbb{K}_j$, we obtain:

$$\mathbb{K}_j(S_j^{F*}, T_j^F) = \frac{g_j^F}{T_j^F} + \left(p_j^F + \frac{h_j^F}{2} \right) \mu_j^{F,DC} T_j^F + p_j^F \mu_j^{F,DC} l_j + \frac{\pi_j^F}{T_j^F} \phi(z_j(T_j^F)) \sigma_j^{F,DC} \sqrt{T_j^F + l_j}.$$

where $\phi(\cdot)$ represents the standard normal probability density function.

Note that

$$\begin{aligned}
T_j^F &= \sum_{k \in K} t_k Z_{jk}^F, \\
\frac{1}{T_j^F} &= \sum_{k \in K} \frac{1}{t_k} Z_{jk}^F, \\
\frac{\pi_j^F}{T_j^F} \phi(z_j(T_j^F)) &= \sum_{k \in K} \hat{t}_{jk} Z_{jk}^F, \\
\text{where } \hat{t}_{jk} &= \frac{\pi_j}{t_k} \phi(z_j(t_k)).
\end{aligned}$$

Then, we obtain

$$\begin{aligned}
\mathbb{K}_j(S_j^{F*}, T_j^F) &= \sum_{k \in K} \frac{g_j^F}{t_k} Z_{jk}^F + \sum_{i \in I} \sum_{k \in K} \left(p_j^F + \frac{h_j^F}{2} \right) \mu_i^F t_k Z_{jk}^F Y_{ij}^F + \sum_{i \in I} p_j^F l_j \mu_i^F Y_{ij}^F \\
&\quad + \sum_{k \in K} \hat{t}_{jk} \sqrt{\sum_{i \in I} \sum_{i' \in I} \sum_{k' \in K} t_{k'} \phi_{ii'}^F \sigma_i^F \sigma_{i'}^F Z_{jk'}^F Z_{jk}^F Y_{ij}^F Y_{i'j}^F + \sum_{i \in I} \sum_{i' \in I} l_j \phi_{ii'}^F \sigma_i^F \sigma_{i'}^F Z_{jk}^F Y_{ij}^F Y_{i'j}^F} \\
&= \sum_{k \in K} \frac{g_j^F}{t_k} Z_{jk}^F + \sum_{i \in I} \sum_{k \in K} \left(p_j^F + \frac{h_j^F}{2} \right) \mu_i^F t_k Z_{jk}^F Y_{ij}^F + \sum_{i \in I} p_j^F l_j \mu_i^F Y_{ij}^F \\
&\quad + \sum_{k \in K} \hat{t}_{jk} \sqrt{\sum_{i \in I} \sum_{i' \in I} (t_k + l_j) \phi_{ii'}^F \sigma_i^F \sigma_{i'}^F Z_{jk}^F Y_{ij}^F Y_{i'j}^F}.
\end{aligned}$$

Derivation of $\mathbb{K}^R$

A periodic review like inventory policy is also adopted at the reverse DCs to control the stock of returned products. All of the returned products collected by the distribution centers in one review period are shipped to the plant at the end of that period. For any reverse distribution center j , $\mathbb{K}_j^R$ consists of average variable construction cost for the reverse DC which is assumed to be proportional to the capacity of the reverse DC, average fixed transportation costs for shipping returned products to the plant ($\frac{g_j^R}{T_j^R}$), and average inventory holding costs ($\frac{h_j^R T_j^R \mu_i^R Y_{ij}^R}{2}$) within one review interval. We propose a mean-risk model to calculate $\mathbb{K}_j^R(C_j^R, T_j^R)$, which is expressed as follows:

$$\mathbb{K}_j^R(C_j^R, T_j^R) = \begin{cases} \min & p_j^R C_j^R + \frac{g_j^R}{T_j^R} + \sum_{i \in I} \frac{h_j^R \mu_i^R Y_{ij}^R T_j^R}{2}, \\ \text{s.t.} & Pr \left\{ \sum_{i \in I} \mu_i^R Y_{ij}^R T_j^R \leq C_j^R \right\} \geq \alpha, \\ & C_j^R \geq 0. \end{cases} \quad (8)$$

Substituting T_j^R and $\frac{1}{T_j^R}$ with $\sum_{k \in K} t_k Z_{jk}^R$ and $\sum_{k \in K} \frac{1}{t_k} Z_{jk}^R$, respectively, $\mathbb{K}_j^R$ is rewritten as follows:

$$\mathbb{K}_j^R(C_j^T, T_j^R) = \begin{cases} \min & p_j^R C_j^R + \sum_{k \in K} \frac{g_j^R}{t_k} Z_{jk}^R + \sum_{i \in I} \sum_{k \in K} \frac{h_j^R \mu_i^R t_k}{2} Z_{jk}^R Y_{ij}^R, \\ \text{s.t.} & \Pr \left\{ \sum_{i \in I} \sum_{k \in K} \mu_i^R t_k Z_{jk}^R Y_{ij}^R \leq C_j^R \right\} \geq \alpha, \\ & C_j^R \geq 0. \end{cases} \quad (9)$$

Because generally returned and new products utilize different amounts of storage space, and have separate storage facilities or areas (Jayaraman et al., 1999), storage capacity chance constraint (8) for the inventory of returned products is introduced, which stipulates that the probability of the quantity of returned products stored at a reverse DC not exceeding the capacity of that reverse DC is larger than a predetermined value (α).

In this paper, we assume that the probability distributions of returned products is unknown. For new products, it is commonly assumed that the demand follows a normal or poisson distribution in the joint location-inventory literature. However, it is often hard to estimate the distribution followed by the returned products collected because of the absence of historical data of returned product in most applications. Also, the returned products often show stronger randomness. Therefore, it limits the application of the model if we also assume that the quantities of returned products collected follow a specific distribution such as normal distribution. In summary, the model is formulated as follows:

$$\begin{aligned} P : \quad \min \quad & \sum_{j \in J} f_j^F X_j^F + \sum_{j \in J} \sum_{i \in I} \bar{a}_{ij}^F \mu_i^F Y_{ij}^F \\ & + \sum_{j \in J} \left\{ \sum_{k \in K} \bar{g}_{jk}^F Z_{jk}^F + \sum_{i \in I} \sum_{k \in K} \bar{h}_{jk}^F \mu_i^F Z_{jk}^F Y_{ij}^F + \sum_{k \in K} \hat{t}_{jk} \sqrt{\sum_{i \in I} \sum_{i' \in I} (t_k + l_j) \phi_{ii'}^F \sigma_i^F \sigma_{i'}^R Z_{jk}^F Y_{ij}^F Y_{i'j}^F} \right\} \\ & + \sum_{j \in J} f_j^R X_j^R + \sum_{j \in J} \sum_{i \in I} \bar{a}_{ij}^R \mu_i^R Y_{ij}^R + \sum_{j \in J} \left\{ p_j^R C_j^R + \sum_{k \in K} \bar{g}_{jk}^R Z_{jk}^R + \sum_{i \in I} \sum_{k \in K} \bar{h}_{jk}^R \mu_i^R Z_{jk}^R Y_{ij}^R \right\}, \\ \text{s.t.} \quad & (2) \sim (6), (9). \end{aligned}$$

where

$$\begin{aligned} \bar{a}_{ij}^F &= a_j^F + d_{ij} + p_j^F l_j, \bar{a}_{ij}^R = a_j^R + d_{ij}, \\ \bar{g}_{jk}^F &= \frac{g_j^F}{t_k}, \bar{g}_{jk}^R = \frac{g_j^R}{t_k}, \\ \bar{h}_{jk}^F &= \left(p_j^F + \frac{h_j^F}{2} \right) t_k, \bar{h}_{jk}^R = \frac{h_j^R t_k}{2}. \end{aligned}$$

4. Model reformulation and approximation

Model P is a nonlinear integer program with chance constraints. Therefore it is difficult to find the optimal solution to P in a reasonable amount of time. However, the above formulation can be transformed into an approximated conic

quadratic mixed-integer program (QMIP) that can be solved efficiently using standard optimization softwares such as CPLEX (Atamtürk et al., 2012). The first step in the transformation is the linearization of the quadratic binary terms. Then, conic quadratic approximations are derived for the chance constraints for different assumptions on the distributions of returned products.

4.1. Linearization of the quadratic binary terms

To linearize the model, two auxiliary binary decision variables, V_{ijk}^F and V_{ijk}^R , are introduced:

$$V_{ijk}^F = \begin{cases} 1, & \text{if retailer } i \text{ is assigned to forward DC } j \text{ whose review interval is } t_k, \\ 0, & \text{otherwise,} \end{cases}$$

$$V_{ijk}^R = \begin{cases} 1, & \text{if retailer } i \text{ is assigned to reverse DC } j \text{ whose review interval is } t_k, \\ 0, & \text{otherwise.} \end{cases}$$

which satisfy the following constraints.

$$V_{ijk}^F \geq Y_{ij}^F + Z_{jk}^F - 1, \forall i \in I, j \in J, k \in K, \quad (10)$$

$$V_{ijk}^F \leq Y_{ij}^F, V_{ijk}^F \leq Z_{jk}^F, \forall i \in I, j \in J, k \in K, \quad (11)$$

$$V_{ijk}^R \geq Y_{ij}^R + Z_{jk}^R - 1, \forall i \in I, j \in J, k \in K, \quad (12)$$

$$V_{ijk}^R \leq Y_{ij}^R, V_{ijk}^R \leq Z_{jk}^R, \forall i \in I, j \in J, k \in K. \quad (13)$$

Note that V_{ijk}^F and V_{ijk}^R can be relaxed to two continuous non-negative variables and they still find exact integer solutions. Shahabi et al. (2014) show that constraints (10) to (13) can be replaced by the following constraints to obtain more efficient formulations:

$$\sum_k V_{ijk}^F = Y_{ij}^F, \sum_k V_{ijk}^R = Y_{ij}^R, \forall i \in I, j \in J,$$

$$V_{ijk}^F \leq Z_{jk}^F, V_{ijk}^R \leq Z_{jk}^R, \forall j \in J, k \in K,$$

$$V_{ijk}^F, V_{ijk}^R \geq 0, \forall i \in I, j \in J, k \in K.$$

4.2. Approximation of the chance constraint

As the nonconvexity of the chance constraint (9) causes computational difficulties, we approximate the chance constraint to a conic quadratic constraint by extending the approximation methods proposed by Bonami and Lejeune (2009).

Lemma 1. Capacity chance constraints (9) can be approximated as the following conic quadratic constraints

$$\sum_{i \in I} \sum_{k \in K} \mu_i^R t_k V_{ijk}^R + \rho_\alpha \sqrt{\sum_{i \in I} \sum_{i' \in I} \sum_{k \in K} t_k \phi_{ii'}^R \sigma_i^R \sigma_{i'}^R V_{ijk}^R V_{i'jk}^R} \leq C_j^R,$$

where

$$\rho_\alpha = \begin{cases} \sqrt{\frac{\alpha}{1-\alpha}}, & \text{if the quantity of returned products at each retailer is a arbitrary random variable,} \\ \sqrt{\frac{1}{2(1-\alpha)}}, & \text{if the quantity of returned products at each retailer is a symmetric random variable, } \alpha \in [0.5, 1) \\ \sqrt{\frac{2}{9(1-\alpha)}}, & \text{if the quantity of returned products at each retailer is a unimodal symmetric random variable, } \alpha \in [0.5, 1). \end{cases}$$

PROOF. See Appendix A. $\square$

Finally, by introducing non-negative variables U_{jk}^F and U_j^R , the model is reformulated as the following conic quadratic mixed-integer program(P^L):

$$P^L : \min \sum_{j \in J} f_j^F X_j^F + \sum_{j \in J} \sum_{i \in I} \bar{a}_{ij}^F \mu_i^F Y_{ij}^F + \sum_{j \in J} \left\{ \sum_{k \in K} \bar{g}_{jk}^F Z_{jk}^F + \sum_{i \in I} \sum_{k \in K} \bar{h}_{ijk}^F \mu_i^F V_{ijk}^F + \sum_{k \in K} \hat{t}_{jk} U_{jk}^F \right\} \\ + \sum_{j \in J} f_j^R X_j^R + \sum_{j \in J} \sum_{i \in I} \bar{a}_{ij}^R \mu_i^R Y_{ij}^R + \sum_{j \in J} \left\{ p_j^R C_j^R + \sum_{k \in K} \bar{g}_{jk}^R Z_{jk}^R + \sum_{i \in I} \sum_{k \in K} \bar{h}_{ijk}^R \mu_i^R V_{ijk}^R \right\}, \quad (14)$$

$$s.t. \quad U_{jk}^F \geq \sqrt{\sum_{i \in I} \sum_{i' \in I} (t_k + l_j) \phi_{ii'}^F \sigma_i^F \sigma_{i'}^F V_{ijk}^F V_{i'jk}^F}, \forall j \in J, k \in K, \quad (15)$$

$$U_j^R \geq \sqrt{\sum_{i \in I} \sum_{i' \in I} \sum_{k \in K} t_k \phi_{ii'}^R \sigma_i^R \sigma_{i'}^R V_{ijk}^R V_{i'jk}^R}, \forall j \in J, \quad (16)$$

$$\sum_{i \in I} \sum_{k \in K} \mu_i^R t_k V_{ijk}^R + \rho_\alpha U_j^R \leq C_j^R, \forall j \in J, \quad (17)$$

$$\sum_{j \in J} Y_{ij}^F = 1, \sum_{j \in J} Y_{ij}^R = 1, \forall i \in I, \quad (18)$$

$$Y_{ij}^F \leq X_j^F, Y_{ij}^R \leq X_j^R, \forall i \in I, j \in J, \quad (19)$$

$$\sum_{k \in K} Z_{jk}^F = X_j^F, \sum_{k \in K} Z_{jk}^R = X_j^R, \forall j \in J, \quad (20)$$

$$X_j^R \leq \sum_{i \in I} Y_{ij}^F, \forall j \in J, \quad (21)$$

$$\sum_k V_{ijk}^F = Y_{ij}^F, \sum_k V_{ijk}^R = Y_{ij}^R, \forall i \in I, j \in J, \quad (22)$$

$$V_{ijk}^F \leq Z_{jk}^F, V_{ijk}^R \leq Z_{jk}^R, \forall j \in J, k \in K, \quad (23)$$

$$C_j^R \geq 0, U_{jk}^F, U_j^R, V_{ijk}^F, V_{ijk}^R \geq 0, \forall j \in J, k \in K, \quad (24)$$

$$X_j^F, X_j^R, Y_{ij}^F, Y_{ij}^R, Z_{jk}^F, Z_{jk}^R \in \{0, 1\}, \forall i \in I, j \in J, k \in K. \quad (25)$$

4.3. Models with coordination scenarios

Models P^L corresponds to the uncoordinated scenario where the review intervals at each forward and reverse DC can be different. The following section describes the various coordination scenarios considered in this work.

1. Full coordination (model P^{Full}), where all DCs (both forward and reverse DCs) share the same periodic review interval. This can be modeled by replacing Z_{jk}^F and Z_{jk}^R by one variable Z_k in the formulation P^L .

2. Partial coordination 1 (model P^{FR}), where all forward DCs share the same periodic review interval while all reverse DCs share another periodic review interval. This can be modeled by replacing Z_{jk}^F with Z_k^F and Z_{jk}^R with Z_k^R in the formulation P^L .
3. Partial coordination 2 (model $P^{Forward}$), where all forward DCs share the same periodic review interval while reverse DCs may have different cycles. This can be modeled by replacing Z_{jk}^F with Z_k^F in the formulation P^L .
4. Partial coordination 3 (model $P^{Reverse}$), where all reverse DCs share the same periodic review interval while forward DCs may have different cycles. This can be modeled by replacing Z_{jk}^R with Z_k^R in the formulation P^L .
5. Partial coordination 4 (model P^{DC}), where the new and returned products share the same periodic review interval at a joint DC. This can be modeled by replacing Z_{jk}^F and Z_{jk}^R with Z_{jk} in the formulation P^L .

5. Solution approach

Although the proposed models are conic quadratic mixed-integer programs that can be solved directly by CPLEX, the performance of CPLEX become worse when increasing the coefficient of variation (CV) of new and returned products. Therefore, an outer approximation (OA) algorithm is developed to solve the proposed nonlinear program when the CVs are larger than one.

OA was presented originally by Duran and Grossmann (1986) for solving mixed-integer nonlinear program with linear integer variables and convex nonlinear functions involving continuous variables. OA is based on the cutting plane algorithm and decomposes the original problem into a linear master problem (MP) and a non-linear subproblem (SP). The MP and SP are solved in an iterative manner. At each iteration, the solution of the SP is feasible for the original problem and provides an upper bound. The MP is a linearization of the original problem and gives a lower bound. Moreover, Bonami et al. (2008) prove that OA can find a global optimal solution for convex mixed integer nonlinear programs. More detailed introduction of the OA algorithm is provided by Duran and Grossmann (1986) and Fletcher and Leyffer (1994).

Before introducing the OA algorithm, we show that the two continuous nonlinear functions associated with constraints (15) and (16) are convex, which ensures that the OA algorithm can converge to a global optimal solution.

Lemma 2. *Define*

$$\Phi(V_{ijk}^F, U_{jk}^F) = \sqrt{\sum_{i \in I} \sum_{i' \in I} (t_k + l_j) \phi_{ii'}^F \sigma_i^F \sigma_{i'}^F V_{ijk}^F V_{i'jk}^F} - U_{jk}^F,$$

and

$$\Psi(V_{ijk}^R, U_j^R) = \sqrt{\sum_{i \in I} \sum_{i' \in I} \sum_{k \in K} t_k \phi_{ii'}^R \sigma_i^R \sigma_{i'}^R V_{ijk}^R V_{i'jk}^R} - U_j^R.$$

Both of them are convex.

PROOF. Similar to the proof of Proposition 2 in Shahabi et al. (2014).

5.1. The subproblem

The subproblem is a nonlinear program, which gives an upper bound of the original problem. At iteration h , denote the solutions of the binary decision variables obtained from the master problem as $\tilde{X}_j^F$, $\tilde{X}_j^R$, $\tilde{Y}_{ij}^F$, $\tilde{Y}_{ij}^R$, $\tilde{Z}_{jk}^F$, and $\tilde{Z}_{jk}^R$. Then, the upper bound at iteration h is obtained as follows:

$$\begin{aligned} UB^h = & \sum_{j \in J} f_j^F \tilde{X}_j^F + \sum_{j \in J} \sum_{i \in I} \tilde{a}_{ij}^F \mu_i^F \tilde{Y}_{ij}^F + \sum_{j \in J} \left\{ \sum_{k \in K} \tilde{g}_{jk}^F \tilde{Z}_{jk}^F + \sum_{i \in I} \sum_{k \in K} \tilde{h}_{ijk}^F \mu_i^F \tilde{V}_{ijk}^F + \sum_{k \in K} \tilde{t}_k^F \tilde{U}_{jk}^F \right\} \\ & + \sum_{j \in J} f_j^R \tilde{X}_j^R + \sum_{j \in J} \sum_{i \in I} \tilde{a}_{ij}^R \mu_i^R \tilde{Y}_{ij}^R + \sum_{j \in J} \left\{ p_j^R \tilde{C}_j^R + \sum_{k \in K} \tilde{g}_{jk}^R \tilde{Z}_{jk}^R + \sum_{i \in I} \sum_{k \in K} \tilde{h}_{ijk}^R \mu_i^R \tilde{V}_{ijk}^R \right\}, \end{aligned} \quad (26)$$

where

$$\tilde{U}_{jk}^F = \sqrt{\sum_{i \in I} \sum_{i' \in I} (t_k + l_j) \phi_{ii'}^F \sigma_i^F \sigma_{i'}^F \tilde{V}_{ijk}^F \tilde{V}_{i'jk}^F}, \forall j \in J, k \in K, \quad (27)$$

$$\tilde{U}_j^R = \sqrt{\sum_{i \in I} \sum_{i' \in I} \sum_{k \in K} t_k \phi_{ii'}^R \sigma_i^R \sigma_{i'}^R \tilde{V}_{ijk}^R \tilde{V}_{i'jk}^R}, \forall j \in J, \quad (28)$$

$$\tilde{C}_j^R = \sum_{i \in I} \sum_{k \in K} \mu_i^R t_k \tilde{V}_{ijk}^R + \rho_\alpha \tilde{U}_j^R, \forall j \in J, \quad (29)$$

$$\tilde{V}_{ijk}^F = \tilde{Y}_{ij}^F \tilde{Z}_{jk}^F, \forall i \in I, j \in J, k \in K, \quad (30)$$

$$\tilde{V}_{ijk}^R = \tilde{Y}_{ij}^R \tilde{Z}_{jk}^R, \forall i \in I, j \in J, k \in K. \quad (31)$$

5.2. The master problem

The master problem of the OA algorithm is modelled by substituting constraints (15) and (16) with the two OA cuts shown below:

Property 1. OA cuts associated with constraints (15) and (16) are, respectively, as follows

$$\sum_{i \in I} \sum_{i' \in I} (t_k + l_j) \phi_{ii'}^F \sigma_i^F \sigma_{i'}^F \tilde{V}_{ijk}^F \tilde{V}_{i'jk}^F - U_{jk}^F \tilde{U}_{jk}^F \leq 0, \quad (32)$$

$$\sum_{i \in I} \sum_{i' \in I} \sum_{k \in K} t_k \phi_{ii'}^R \sigma_i^R \sigma_{i'}^R \tilde{V}_{ijk}^R \tilde{V}_{i'jk}^R - U_j^R \tilde{U}_j^R \leq 0, \quad (33)$$

where $\tilde{V}_{i'jk}^F$, $\tilde{V}_{i'jk}^R$, $\tilde{U}_{jk}^F$, and $\tilde{U}_j^R$ are an optimal solution for the nonlinear subproblem of the OA algorithm.

PROOF. OA cut (32) is obtained by taking the first-order Taylor series expansion of function Φ around $\tilde{V}_{ijk}^F$ and $\tilde{U}_{jk}^F$. Due to the convexity of Φ , the following inequality holds:

$$\Phi(\tilde{V}_{ijk}^F, \tilde{U}_{jk}^F) + \nabla \Phi(\tilde{V}_{ijk}^F, \tilde{U}_{jk}^F)^T \begin{bmatrix} V_{ijk}^F - \tilde{V}_{ijk}^F \\ U_{jk}^F - \tilde{U}_{jk}^F \end{bmatrix} \leq \Phi \leq 0,$$

And $\Phi(\tilde{V}_{ijk}^F, \tilde{U}_{jk}^F) = \sqrt{\sum_{i \in I} \sum_{i' \in I} (t_k + l_j) \phi_{ii'}^F \sigma_i^F \sigma_{i'}^F \tilde{V}_{ijk}^F \tilde{V}_{i'jk}^F} - \tilde{U}_{jk}^F$, we obtain

$$\sqrt{\sum_{i \in I} \sum_{i' \in I} (t_k + l_j) \phi_{ii'}^F \sigma_i^F \sigma_{i'}^F \tilde{V}_{ijk}^F \tilde{V}_{i'jk}^F} - \tilde{U}_{jk}^F + \frac{\sum_{i \in I} \sum_{i' \in I} (t_k + l_j) \phi_{ii'}^F \sigma_i^F \sigma_{i'}^F \tilde{V}_{ijk}^F (V_{ijk}^F - \tilde{V}_{ijk}^F)}{\sqrt{\sum_{i \in I} \sum_{i' \in I} (t_k + l_j) \phi_{ii'}^F \sigma_i^F \sigma_{i'}^F \tilde{V}_{ijk}^F \tilde{V}_{i'jk}^F}} - (U_{jk}^F - \tilde{U}_{jk}^F) \leq 0.$$

Then, OA cut (32) is obtained as follows:

$$\sum_{i \in I} \sum_{i' \in I} (t_k + l_j) \phi_{ii'}^F \sigma_i^F \sigma_{i'}^F \tilde{V}_{ijk}^F V_{ijk}^F - U_{jk}^F \tilde{U}_{jk}^F \leq 0.$$

In the similar way, we obtain OA cut (33):

$$\sum_{i \in I} \sum_{i' \in I} \sum_{k \in K} t_k \phi_{ii'}^R \sigma_i^R \sigma_{i'}^R \tilde{V}_{ijk}^R V_{ijk}^R - U_j^R \tilde{U}_j^R \leq 0.$$

The master problem at iteration h is formulated as follows:

$$\min \quad \eta^h, \quad (34)$$

$$\begin{aligned} s.t. \quad \eta^h \geq & \sum_{j \in J} f_j^F X_j^F + \sum_{j \in J} \sum_{i \in I} \bar{a}_{ij}^F \mu_i^F Y_{ij}^F + \sum_{j \in J} \left\{ \sum_{k \in K} \bar{g}_{jk}^F Z_{jk}^F + \sum_{i \in I} \sum_{k \in K} \bar{h}_{jk}^F \mu_i^F V_{ijk}^F + \sum_{k \in K} \hat{t}_k^F U_{jk}^F \right\} \\ & + \sum_{j \in J} f_j^R X_j^R + \sum_{j \in J} \sum_{i \in I} \bar{a}_{ij}^R \mu_i^R Y_{ij}^R + \sum_{j \in J} \left\{ p_j^R C_j^R + \sum_{k \in K} \bar{g}_{jk}^R Z_{jk}^R + \sum_{i \in I} \sum_{k \in K} \bar{h}_{jk}^R \mu_i^R V_{ijk}^R \right\}, \end{aligned} \quad (35)$$

$$\sum_{i \in I} \sum_{i' \in I} (t_k + l_j) \phi_{ii'}^F \sigma_i^F \sigma_{i'}^F \tilde{V}_{ijk}^F V_{ijk}^F - U_{jk}^F \tilde{U}_{jk}^F \leq 0, \forall j \in J, k \in K, \quad (36)$$

$$\sum_{i \in I} \sum_{i' \in I} \sum_{k \in K} t_k \phi_{ii'}^R \sigma_i^R \sigma_{i'}^R \tilde{V}_{ijk}^R V_{ijk}^R - U_j^R \tilde{U}_j^R \leq 0, \forall j \in J, \quad (37)$$

$$\eta^h \leq UB^h - \varepsilon, \forall h, \quad (38)$$

$$\eta^h \geq 0, \quad (39)$$

$$(17) \sim (25).$$

The MP gives the lower bound of the original problem and optimal solutions of the binary decision variables $\tilde{X}_j^F$, $\tilde{X}_j^R$, $\tilde{Y}_{ij}^F$, $\tilde{Y}_{ij}^R$, $\tilde{Z}_{jk}^F$, and $\tilde{Z}_{jk}^R$. Constraints (36) and (37) are the OA cuts. Constraint (38) ensures that the solution of the MP is less than the upper bound at every iteration h . We employ the OA framework proposed by Fletcher and Leyffer (1994) that terminates the algorithm whenever the MP is infeasible and the solution is ε -optimal.

The procedure of the OA algorithm is summarized as follows:

- Step 1: Initialization. Set $h = 1$ and an incumbent value of the upper bound of the original problem, UB_{min} , to be $+\infty$. Solve problem (34), (35), and (17) $\sim$ (25) to obtain the initial values of $\tilde{X}_j^F$, $\tilde{X}_j^R$, $\tilde{Y}_{ij}^F$, $\tilde{Y}_{ij}^R$, and $\tilde{Z}_{jk}^F$. The initial values of $\tilde{U}_{jk}^F$, $\tilde{U}_{jk}^R$, $\tilde{C}_j^R$, $\tilde{V}_{ijk}^F$, and $\tilde{V}_{ijk}^R$ are obtained by Equations (27) to (31);
- Step 2: Solve the SP. Obtain the upper bound UB^h according to Equation (26). If $UB^h < UB_{min}$, $UB_{min} = UB^h$ and the corresponding value of the decision variables are saved.
- Step 3: Solve the MP. Construct the OA cuts and update the MP model. Solving the MP and obtaining the values of the binary decision variables $\tilde{X}_j^F$, $\tilde{X}_j^R$, $\tilde{Y}_{ij}^F$, $\tilde{Y}_{ij}^R$, $\tilde{Z}_{jk}^F$. If the problem is infeasible, stop and return the incumbent value; else $h = h + 1$. If h is larger than a maximum iteration number, stop; else goto Step 2;

6. Computational results

Extensive computational testing is carried out to evaluate the performance of the solution approach. All the computational experiments were conducted on a HP 380 G7 server with 2.80 GHz Intel Xeon X5660 CPU running the CentOS 5.7 operating system. The MIQCP solver of CPLEX 12.4 was used to solve the conic formulation and it is terminated if the CPLEX gap is less than 0.5%. The OA algorithm was coded in C++ and used CPLEX12.4 to solve the linear MP with $\varepsilon = 1$. The optimality gap of CPLEX for solving the MP is set to be 10%. Fletcher and Leyffer (1994) show that it is not essential to solve the MP to optimality as long as it generates feasible integer solutions.

6.1. Generation of test data

The test data are generated according to procedures similar to the one used by Vidyarthi et al. (2007) and Berman et al. (2012). The default parameter values for generating test data are listed in Table 2. The unit time is one year. The Euclidean distance between DC j and retailer i is calculated from the coordinates for the retailer and the DC that are generated randomly in a square of length 3.

6.2. Performance of the solution approach

For each different number of the retailers and candidate DCs, five instances were generated and solved to test the solution approach's performance that was evaluated in terms of the average CPU times and the optimality gaps. Table 3 shows the average CPU times and gaps obtained by solving model P^L using CPLEX when CV^F and CV^R are less than or equal to one. In the table, "Arbitrary random variable", "Symmetric random variable", and "Unimodal symmetric random variable" means that the quantity of returned products is an arbitrary, symmetric, and unimodal symmetric random variable respectively. For convenience, we use the same notations in the rest of the paper. We find that with the exception of the instances with $CV = 1$ (i.e., $CV^F = CV^R = 1$), optimal solutions were found for all the instances within the time limit of 7200 seconds. The CPU time was found to increase with CV^F and CV^R .

We solve the instances with larger CV (≥ 1) by the proposed OA approach and directly using CPLEX. Table 4 reports the comparison results in terms of CPU times and gaps for the model associated with arbitrary random variable. The proposed OA approach outperforms CPLEX for the instances with larger CV ($CV^F \geq 1.5$ and $CV^R \geq 1.5$). For the models associated with symmetric and unimodal symmetric random variables, the comparisons of the proposed OA approach and CPLEX are reported in Table B.5 and B.6 and the same conclusion is attained. Due to space limitations, we do not report the corresponding computational results of the other coordinated models because the same conclusions are drawn.

7. Benefits of coordinated inventory control at the DCs

This section studies the benefits of coordination. The CV^F and CV^R is assumed to be 1 (except in Section 7.6) and time limit is set to be 7200s. We first study the impact of coordination on the long-run average costs and the corresponding cost components. Then, we compare the long-run average costs of the coordinated and the uncoordinated

Parameters	Value	Description
I	{10, 20, 50, 100}	set of retailers
J	{10, 20, 50}	set of candidate DCs
K	{1/12, 1/6, 1/4, 1/3, 1/2}	set of review intervals
<i>DC-related parameters:</i>		
f_j^F	$U[500, 1000]$	fixed construction cost (per unit time) at forward DC j
f_j^R	$U[500, 1000]$	fixed construction cost (per unit time) at reverse DC j
p_j^F	$U[0, 10]$	variable construction cost (per unit time) at forward DC j
p_j^R	$U[0, 10]$	variable construction cost (per unit time) at reverse DC j
h_j^F, h_j^R	1	inventory holding cost per unit time at forward/reverse DC j
π_j^F, π_R	10	shortage cost per unit time at forward/reverse DC j
g_j^F, g_j^R	300	fixed cost of a review and shipment of new/returned products between the plant and DC j
a_j^F, a_j^R	1	cost per unit of new/returned products to ship between the plant to DC j
l_j	$U[0, 1]$	replenishment lead time when forward/reverse DC j places an order to the plant
α	97.5%	
<i>Retailer-related parameters:</i>		
μ_i^F	$U[50, 300]$	mean demand of new products per unit time at retailer i
μ_i^R	return rate $\times \mu_i^F$	mean demand of returned products per unit time at retailer i
σ_i^F	$CV^F \times \mu_i^F$	standard deviation of demand of new products per unit time at retailer i
σ_i^R	$CV^R \times \mu_i^R$	standard deviation of demand of returned products per unit time at retailer i
$\phi_{ii'}^F, \phi_{ii'}^R$	0.5, $i \neq i'$ and 1, $i = i'$	correlation coefficients of new/returned products between retailers i and i'
return rate	0.8	

Table 2: Default parameter values for generating test data

I	J	CV						
		0	0.1	0.2	0.5	1		
		Arbitrary random variable						
		10	10	0.05	0.40	0.22	1.39	2.48
		20	10	0.05	1.01	0.43	49.19	22.75
		20	20	0.23	3.52	4.78	3.68	46.46
		50	10	0.20	15.77	3.18	52.78	167.20
		50	20	0.64	5.28	129.08	13.54	2128.15
		50	50	3.78	27.58	88.41	74.29	5140.63 (35.72)
		100	10	0.82	149.27	150.32	1022.53	1818.19
		100	20	3.47	31.32	698.95	111.36	2791.00 (0.51)
		100	50	16.87	84.11	1243.53	232.85	4961.88 (0.51)
		Symmetric random variable						
		10	10	0.03	0.21	0.14	1.61	1.44
		20	10	0.04	0.79	0.39	7.52	11.30
		20	20	0.23	2.75	2.41	4.15	26.05
		50	10	0.21	8.20	2.85	60.61	141.72
		50	20	0.69	4.55	49.75	9.41	802.67
		50	50	3.59	26.61	92.08	73.74	-
		100	10	0.77	81.13	71.83	435.01	1810.91
		100	20	3.46	31.47	745.64	69.68	3131.95 (0.01)
		100	50	17.76	80.54	898.05	206.53	4948.95 (0.68)
		Unimodal symmetric random variable						
		10	10	0.02	0.19	0.12	1.25	0.80
		20	10	0.03	0.71	0.33	5.37	5.55
		20	20	0.23	2.18	1.86	4.83	20.13
		50	10	0.22	3.61	2.16	40.35	110.42
		50	20	0.62	5.78	34.91	7.01	645.15
		50	50	3.69	26.98	85.85	53.28	5280.91 (38.59)
		100	10	0.79	34.99	34.59	414.31	1044.35
		100	20	3.14	33.45	587.85	59.08	3685.88 (0.33)
		100	50	16.86	72.83	619.73	200.98	-

Number in bracket is the average gap (%) obtained in CPLEX within the time limit; the CPU times without gap means that all the instances found their optimal solutions within the time limit; '-': feasible solutions could not be found for some instances within the time limit.

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Table 3: Average CPU time (s) of model P^L solved by CPLEX, arbitrary random variable, time limit = 7200s

		OA	CPLEX	OA	CPLEX	OA	CPLEX
$ I $	$ J $	$CV^F=CV^R=1.0$		$CV^F=CV^R=1.5$		$CV^F=CV^R=2.0$	
10	10	11.32 (0.00)	4.99 (0.00)*	22.33 (0.00)	10.57 (0.00)*	18.70 (0.00)	26.98 (0.00)
20	10	35.53 (0.00)	73.57 (0.00)	74.91 (0.00)	209.41 (0.00)	82.82 (0.00)	339.53 (0.00)
50	10	283.36 (0.00)	180.25 (0.00)*	182.87 (0.00)	1068.07 (0.01)	210.46 (0.00)	1836.33 (0.01)
100	10	285.54 (0.00)	2021.61 (0.01)	958.25 (0.00)	5480.04 (1.71)	1993.05 (0.00)	7204.20 (3.74)
20	20	80.95 (0.00)	54.57 (0.00)*	99.40 (0.00)	52.12 (0.00)*	130.12 (0.00)	2319.28 (0.00)
50	20	688.75 (0.00)	1772.64 (0.00)	912.41 (0.00)	1876.21 (0.00)	2767.20 (0.67)	5343.04 (1.05)
100	20	2201.56 (0.00)	2898.31 (0.39)	3414.79 (0.42)	3340.18 (0.42)*	4165.77 (0.67)	-
50	50	4188.79 (0.00)	4207.36 (31.51)	6815.88 (3.46)	7217.78 (49.11)	7204.56 (20.63)	7223.46 (57.52)
100	50	7210.75 (3.06)	4930.97 (0.28)*#	7206.74 (32.00)	-	7204.84 (48.15)	-
$ I $	$ J $	$CV^F=CV^R=3.0$		$CV^F=CV^R=5.0$		$CV^F=CV^R=10.0$	
10	10	24.28 (0.00)	49.85 (0.00)	33.54 (0.00)	131.40 (0.00)	302.14 (0.00)	4838.60 (4.62)
20	10	86.22 (0.00)	2546.62 (0.08)	2432.56 (3.03)	2468.24 (3.03)	1679.67 (0.00)	4914.42 (7.20)
50	10	700.50 (0.00)	2106.59 (0.01)	2536.33 (2.26)	5054.08 (3.21)	4818.95 (7.33)	7205.68 (11.06)
100	10	4933.04 (4.23)	7096.92 (4.21)#	4965.10 (4.74)	6890.39 (4.90)	4866.80 (5.68)	7205.35 (5.74)
20	20	156.13 (0.00)	707.08 (0.00)	280.15 (0.00)	2015.62 (0.00)	391.29 (0.00)	3431.10 (0.00)
50	20	857.96 (0.00)	3761.19 (0.44)	2523.43 (0.00)	7212.80 (1.45)	3251.06 (0.48)	5786.78 (0.53)
100	20	4178.57 (0.00)	7211.32 (16.61)	6502.77 (2.19)	-	7207.06 (2.32)	-
50	50	7210.08 (27.52)	7218.94 (53.59)	7204.84 (30.69)	7219.80 (27.22)#	7202.63 (57.95)	-
100	50	7208.35 (51.92)	-	7210.58 (68.15)	-	7209.10 (81.54)	-

Number in bracket is the average gap (%) obtained in CPLEX within the time limit; *: CPLEX is better than OA in CPU time;

#: CPLEX is better than OA in Gap; -: Any feasible solution cannot be found within the time limit.

Table 4: The comparison in average CPU time (s) and average gap (%) between the OA approach and CPLEX, time limit = 7200s, arbitrary random variable, return rate=0.8

scenarios by varying four factors - the scale of the problem, the service level of the returned products, and the mean and variance of new and returned products.

For all the instances, feasible solutions with small gaps ($\leq 1.5\%$) were found within the time limit, that is, we obtain near-optimal solutions of the instances instead of their optimal solution. Therefore, we make comparisons between the various coordinated scenarios and the uncoordinated scenario through parameter $err_{scenario}^{max}$, which is defined as

$$err_{scenario}^{max} = \frac{UB_{scenario} - LB}{UB_{scenario} + LB} \times 200,$$

where $UB_{scenario}$ is upper bound of the coordination scenario mentioned in Section 4.3, LB represents lower bound of model P^L . The parameter defines an incremental percentage of the long-run average cost of the coordination scenarios to that of the uncoordinated scenario. Note that $err_{scenario}^{max}$ is an overestimation of the difference of the long-run average costs between the coordinated and uncoordinated scenarios. This definition is used instead of $\frac{UB_{scenario} - LB}{LB} \times 100$, which underestimates the comparison quality, and $\frac{UB_{scenario} - LB}{UB_{scenario}} \times 100$, which overestimates the comparison quality (Brahimi et al., 2006).

7.1. The impact of coordination on the long-run average costs

Figure 2 illustrates the comparison between the five coordination scenarios and the uncoordinated scenario. In the x-axis, “DC”, “FR”, “Full”, “Forward”, and “Reverse” represent models P^{DC} , P^{FR} , P^{Full} , $P^{Forward}$, and $P^{Reverse}$, respectively. For each combination of I and J five instances are solved. Each bar in the figure reports the average values of $err_{scenario}^{max}$ of the forty five instances associated with each coordination scenario.

Although coordination normally leads to increase in the long-run average costs, the numerical results demonstrate that the increase is marginal and the maximum average increase is less than 2%. The full coordination scenario, which have identical review interval at all open DCs, leads to the highest increase of the costs while the partial coordination scenario 4, where new and returned products share an identical review interval at joint DCs, has the lowest increase. The models where returned products have an unimodal symmetric random variable have the smallest cost increase. This implies that more we know about the uncertainty of the return products (symmetric and unimodal versus arbitrary) the lesser the long run average cost increase due to coordination.

7.2. The impact of coordination on the cost components in the long-run average cost

The impact of the coordination inventory on the cost components in objective function (1), which include the location costs, plant-to-DC and DC-to-retailer transportation costs, and working inventory costs, are illustrated in Figure 3. They are labeled as “Location cost”, “Transportation cost”, and “Inventory cost”, respectively. The y-axis is average difference percentage of the cost component between the costs of the coordinated scenario and that of the uncoordinated scenario, and the difference percentage is defined as:

$$diff.(%) = \left(\frac{\text{the cost component of the coordinated scenario}}{\text{the cost component of the uncoordinated scenario}} - 1 \right) \times 100\%.$$

Figure 2: Comparison between the coordination scenarios and the uncoordinated model

As shown in Figure 3, the impact of coordination on the three cost components show different trends. Coordination leads to increase in inventory management costs (all the coordinated scenario have positive average differences of inventory costs). The increase is higher for higher levels of coordination. And the transportation cost also increases because there are more shipments occurred under coordinated scenarios. While, the location costs decrease under some scenarios (there are negative average differences of location costs under scenario DC, FR, and Forward associated with unimodal symmetric random variable), which imply that the number of open forward/reverse DCs under the coordinated scenarios is less than that under the uncoordinated scenario.

7.3. The impact of the scale of the instances

Figures 4 and 5 illustrate the impact of $|J|$ and $|I|$ on the average values of $err_{scenario}^{max}$ obtained from five instances generated randomly. We find that increase of the long-run average costs of the coordination scenarios is relatively small and the maximum average $err_{scenario}^{max}$ does not exceed 2%. The average values of $err_{scenario}^{max}$ increases with increasing number of the candidate DCs (see Figure 4) because of the consideration of coordinated inventory control in the DCs. However, it is interesting to note that $err_{scenario}^{max}$ decreases when increasing number of the retailers (see Figure 5). Thus coordinated inventory control is an attractive option for the supply chain with more retailers considered because the benefit of coordination can be obtained and its corresponding negative impact on the long-run average cost is negligible.

Figure 3: Comparison of each cost component between the coordination scenarios and the uncoordinated model

Figure 4: Comparison between the coordination scenarios and the uncoordinated model as $|J|$ varies, $|I|=100$

Figure 5: Comparison between the coordination scenarios and the uncoordinated model as $|J|$ varies, $|J|=20$

7.4. The impact of the service level (α)

We also test the impact of the service level (α) on the long-run average costs. As shown in Figure 6, each coordination scenario under arbitrary random variables results in higher long-run average costs compared to the uncoordinated scenario when increasing the service level. The impact of coordination on the long-run average cost is lesser when we have more information on the random quantity of returned products (symmetric and unimodal symmetric).

7.5. The impact of the mean of new and returned products

The impacts of the quantity of new and returned products are illustrated in Figures 7 and 8. The return rate corresponds to the percentage of new products which is returned. The "ratio" is a scaling factor used to scale up the mean of the new products. For each value of "Ratio" or "Return rate", five instances are generated and their average $err_{scenario}^{max}$ are reported in the figures. No specific trends are observed on the cost increases due to coordination. However we find that the average cost increase is low (less than 1.5% for all but one scenario where it is less than 2 %).

7.6. The impact of the variances of new and returned products

The impact of variances of new and returned products is studied by varying the coefficients of variation of new and returned products from zero (deterministic) to ten (high variability), respectively. The comparison results associated with arbitrary random variable are illustrated in Figure 9. We observe that the average values of $err_{scenario}^{max}$ do not

Figure 6: Comparison between the coordination scenarios and the uncoordinated model under different service levels, $|I|=100$, $|J|=50$

Figure 7: Comparison between the coordination scenarios and the uncoordinated model under different quantity of new products, $|I|=100$, $|J|=50$

Figure 8: Comparison between the coordination scenarios and the uncoordinated model under different return rate of returned products, $|I|=100$, $|J|=50$

exceed 2% for all the instances. No significant changing trend of $err_{scenario}^{max}$ can be found. We do not report the comparison results associated with the other two random variable because they have the same conclusions.

8. Conclusions

The paper considers a coordinated location-inventory problem in a closed-loop supply chain, which consists of one plant, multiple DC's, and multiple retailers. New and returned products are stored at the corresponding DCs. Periodic review inventory policies are adopted to manage the stock of new and returned products. Beside the location and assignment decision variables, the review intervals at the DCs are also determined to minimize the long-run average total cost. Through the selection of the review intervals at the DCs, we achieve the coordinated inventory control of the supply chain. The fully coordinated system is far easier to implement as the supply chain manager will be able to better control the pickup and delivery schedules to multiple locations across plants, forward/reverse distribution centers, and retailers (Berman et al., 2012). Note that in this paper we do not model the potential savings obtained from transportation as it will involve detailed modeling of the transportation routing. The uncoordinated strategy will be the most unconstrained strategy and therefore will provide lower costs. Introducing coordination will obviously result in higher costs as we are adding constraints to the model. However, as we show in the numerical experiments, the increase in cost is marginal for all cases we tested. We believe that the minor cost increase can be offset by easier and better day to day operational routing which can be done under coordination.

Figure 9: Comparison between the coordination scenarios and the uncoordinated model under different CV of new and returned products, $|I|=100$, $|J|=50$, arbitrary random variable

The closed loop joint inventory-location problem is formulated as nonlinear mixed-integer programs and then reformulated as a conic quadratic mixed-integer program, which can be solved using commercial optimization package. An outer approximation based solution algorithm is developed for solving the conic formulation which is found to be more efficient for higher coefficient of variation of demands.

Although the coordinated inventory management results in a higher long-run average cost, the increases of the costs are relatively small. Compared to the long-run average cost of the uncoordinated scenario, we also observe that:

1. On average, the maximum incremental percentage of the long-run average cost of the coordination scenarios does not exceed 2%.
2. The increases of costs become larger slightly when increasing the number of candidate DCs in most of the instances while it becomes smaller when increasing the number of the retailers.
3. The increases of costs can be reduced further if the quantity of returned products can be estimated more exactly.
4. The increases of the long-run average costs of coordination scenarios become larger when increasing the service level (α) of returned products.

This research can be extended in multiple directions. These models can naturally be extended to incorporate multiple products. It will also be interesting to study a coordinated location-inventory model with random demands in a multi-echelon inventory system. Another potential direction of research is to integrate the routing aspect creating a location-inventory-routing model which can more accurately model the cost reduction from transportation

coordination also.

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Appendix A. Proof of Lemma 1

PROOF. Let D_j^R be the quantity of returned products at retailer j and $\mathbb{D}_j^R = \sum_{i \in I} D_j^R Y_{ij}^R$. $\mathbb{D}_j^R$ follows probability distribution with mean of $\sum_{i \in I} T_j^R \mu_i^R Y_{ij}^R$ and variance of $\sum_{i \in I} T_j^R (\sigma_i^R)^2 Y_{ij}^R$. According the work of Popescu (2005), if D_j^R is an unimodal symmetric random variable, then the following inequality holds:

$$Pr\{\mathbb{D}_j^R > C_j^R\} \leq \begin{cases} \frac{1}{2} \min \left[1, \frac{4}{9} \frac{\sum_{i \in I} \sum_{i' \in I} T_j^R \sigma_{ii'}^R \sigma_i^R \sigma_{i'}^R Y_{ij}^R Y_{i'j}^R}{(C_j^R - \sum_{i \in I} T_j^R \mu_i^R Y_{ij}^R)^2} \right], & C_j^R > \sum_{i \in I} T_j^R \mu_i^R Y_{ij}^R, \\ 1, & \text{otherwise.} \end{cases}$$

Because $\alpha \in [0.5, 1)$, we obtain

$$\begin{aligned} 1 - Pr\{\mathbb{D}_j^R \leq C_j^R\} &\leq \frac{2}{9} \frac{\sum_{i \in I} \sum_{i' \in I} T_j^R \phi_{ii'}^R \sigma_i^R \sigma_{i'}^R Y_{ij}^R Y_{i'j}^R}{(C_j^R - \sum_{i \in I} T_j^R \mu_i^R Y_{ij}^R)^2}, \\ Pr\{\mathbb{D}_j^R \leq C_j^R\} &\geq 1 - \frac{2}{9} \frac{\sum_{i \in I} \sum_{i' \in I} T_j^R \phi_{ii'}^R \sigma_i^R \sigma_{i'}^R Y_{ij}^R Y_{i'j}^R}{(C_j^R - \sum_{i \in I} T_j^R \mu_i^R Y_{ij}^R)^2}. \end{aligned}$$

Therefore,

$$1 - \frac{2}{9} \frac{\sum_{i \in I} \sum_{i' \in I} T_j^R \phi_{ii'}^R \sigma_i^R \sigma_{i'}^R Y_{ij}^R Y_{i'j}^R}{(C_j^R - \sum_{i \in I} T_j^R \mu_i^R Y_{ij}^R)^2} \geq \alpha$$

is sufficient for constraint (9) to hold. The above inequality can be rewritten as

$$\sum_{i \in I} T_j^R \mu_i^R Y_{ij}^R + \rho_\alpha \sqrt{\sum_{i \in I} \sum_{i' \in I} T_j^R \phi_{ii'}^R \sigma_i^R \sigma_{i'}^R Y_{ij}^R Y_{i'j}^R} \leq C_j^R,$$

where $\rho_\alpha = \sqrt{\frac{2}{9(1-\alpha)}}$. Replace T_j^R with $T_j^R = \sum_{k \in K} t_k Z_{jk}^R$ and $Z_{jk}^R Y_{ij}^R$ with V_{ijk}^R , the conic quadratic constraint is obtained:

$$\sum_{i \in I} \sum_{k \in K} \mu_i^R t_k V_{ijk}^R + \rho_\alpha \sqrt{\sum_{i \in I} \sum_{i' \in I} \sum_{k \in K} t_k \phi_{ii'}^R \sigma_i^R \sigma_{i'}^R V_{ijk}^R V_{i'jk}^R} \leq C_j^R.$$

According the following inequalities (Popescu, 2005), the approximation associated with arbitrary random variable and symmetric random variable can be shown in the same way, respectively.

For arbitrary random variables D_j^R :

$$Pr\{\mathbb{D}_j^R > C_j^R\} \leq \begin{cases} \frac{\sum_{i \in I} \sum_{i' \in I} T_j^R \phi_{ii'}^R \sigma_i^R \sigma_{i'}^R 2Y_{ij}^R Y_{i'j}^R}{\sum_{i \in I} T_j^R (\sigma_i^R)^2 Y_{ij}^R + (C_j - \sum_{i \in I} T_j^R \mu_i^R Y_{ij}^R)^2}, & C_j^R > \sum_{i \in I} T_j^R \mu_i^R Y_{ij}^R, \\ 1, & otherwise. \end{cases}$$

For symmetric random variables D_j^R :

$$Pr\{\mathbb{D}_j^R > C_j^R\} \leq \begin{cases} \frac{1}{2} \min \left[1, \frac{\sum_{i \in I} \sum_{i' \in I} T_j^R \phi_{ii'}^R \sigma_i^R \sigma_{i'}^R Y_{ij}^R Y_{i'j}^R}{(C_j - \sum_{i \in I} T_j^R \mu_i^R Y_{ij}^R)^2} \right], & C_j^R > \sum_{i \in I} T_j^R \mu_i^R Y_{ij}^R, \\ 1, & otherwise. \end{cases}$$

This completes the proof. $\square$

Appendix B. The comparison of the OA approach and CPLEX for symmetric and unimodal symmetric random variables

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		OA	CPLEX	OA	CPLEX	OA	CPLEX
$ I $	$ J $	$CV^F=CV^R=1.0$		$CV^F=CV^R=1.5$		$CV^F=CV^R=2.0$	
10	10	9.76 (0.00)	2.68 (0.00)*	19.91 (0.00)	9.38 (0.00)*	19.10 (0.00)	15.38 (0.00)*
20	10	25.52 (0.00)	18.34 (0.00)*	47.95 (0.00)	168.94 (0.00)	98.95 (0.00)	622.71 (0.01)
50	10	286.53 (0.00)	207.67 (0.00)*	152.59 (0.00)	1064.12 (0.00)	157.06 (0.00)	1028.71 (0.01)
100	10	392.45 (0.00)	1539.81 (0.01)	1071.46 (0.00)	5161.91 (0.20)	1447.32 (0.00)	7203.55 (3.41)
20	20	73.00 (0.00)	32.95 (0.00)*	80.83 (0.00)	94.36 (0.00)	115.96 (0.00)	2794.86 (0.25)
50	20	693.28 (0.00)	908.01 (0.00)	741.38 (0.00)	349.05 (0.00)*	2165.15 (0.00)	5243.79 (0.58)
100	20	1915.27 (0.00)	2563.86 (0.01)	3179.18 (0.34)	5372.08 (0.75)	4026.71 (0.63)	6693.76 (17.21)
50	50	4302.98 (0.00)	-	6900.52 (2.87)	7215.90 (58.77)	7209.93 (16.65)	7222.25 (57.30)
100	50	7146.38 (5.11)	3830.41 (0.28)*#	7207.43 (34.48)	-	7202.94 (46.05)	-
$ I $	$ J $	$CV^F=CV^R=3.0$		$CV^F=CV^R=5.0$		$CV^F=CV^R=10.0$	
10	10	24.61 (0.00)	28.81 (0.00)	40.82 (0.00)	180.77 (0.00)	419.65 (0.00)	3650.22 (2.30)
20	10	100.29 (0.00)	1663.64 (0.01)	1593.14 (0.00)	2517.03 (0.93)	3022.86 (2.79)	4902.59 (4.04)
50	10	261.66 (0.00)	3069.08 (0.54)	2584.12 (2.04)	5095.18 (3.47)	4818.76 (4.10)	5613.01 (4.13)
100	10	4901.41 (3.05)	6116.20 (3.05)	4969.45 (3.64)	6844.17 (3.67)	4840.12 (4.27)	7204.99 (4.34)
20	20	106.18 (0.00)	586.67 (0.00)	241.06 (0.00)	3401.98 (0.00)	285.19 (0.00)	3399.88 (0.00)
50	20	632.18 (0.00)	3444.16 (0.24)	1938.01 (0.00)	7210.45 (1.33)	3704.79 (0.42)	7209.80 (0.81)
100	20	4656.80 (0.00)	7207.22 (14.01)	6283.01 (1.09)	-	6942.68 (1.34)	-
50	50	7210.08 (23.14)	7228.92 (51.21)	7208.84 (26.59)	7218.96 (22.84)#	7202.92 (40.40)	-
100	50	7208.35 (58.73)	-	7208.97 (66.99)	-	7207.54 (81.59)	-

Number in bracket is the average gap (%) obtained in CPLEX within the time limit; *: CPLEX is better than OA in CPU time;

#: CPLEX is better than OA in Gap; -: Any feasible solution cannot be found within the time limit.

Table B.5: The comparison in average CPU time (s) and average gap (%) between the OA approach and CPLEX, time limit = 7200s, symmetric random variable, return rate=0.8

		OA CPLEX		OA CPLEX		OA CPLEX	
$ I $	$ J $	$CV^F=CV^R=1.0$		$CV^F=CV^R=1.5$		$CV^F=CV^R=2.0$	
10	10	10.23 (0.00)	1.51 (0.00)*	18.44 (0.00)	24.54 (0.00)	16.87 (0.00)	38.99 (0.00)
20	10	21.50 (0.00)	8.81 (0.00)*	54.68 (0.00)	248.26 (0.01)	99.09 (0.00)	490.88 (0.00)
50	10	237.73 (0.00)	123.35 (0.00)*	195.86 (0.00)	499.50 (0.01)	124.20 (0.00)	857.00 (0.01)
100	10	362.03 (0.00)	1300.03 (0.01)	935.22 (0.00)	6517.49 (0.34)	900.93 (0.00)	5426.90 (1.03)
20	20	59.93 (0.00)	9.45 (0.00)*	128.04 (0.00)	2423.96 (0.34)	77.96 (0.00)	56.08 (0.00)*
50	20	193.18 (0.00)	101.27 (0.00)*	1326.08 (0.00)	2543.78 (0.13)	556.23 (0.00)	3080.32 (0.20)
100	20	683.03 (0.00)	1766.04 (0.00)	3418.85 (0.15)	5329.99 (13.90)	2495.48 (0.00)	6334.76 (14.37)
50	50	4728.69 (0.00)	7211.96 (54.07)	6905.80 (1.42)	-	7208.18 (3.20)	7214.71 (60.99)
100	50	7172.07 (8.59)	-	7207.43 (31.89)	-	7206.70 (45.20)	-
$ I $	$ J $	$CV^F=CV^R=3.0$		$CV^F=CV^R=5.0$		$CV^F=CV^R=10.0$	
10	10	17.77 (0.00)	54.98 (0.00)	16.93 (0.00)	346.50 (0.00)	68.84 (0.00)	2594.41 (1.27)
20	10	59.94 (0.00)	3096.52 (0.43)	1244.73 (0.00)	4845.23 (3.06)	2351.39 (0.00)	4854.92 (3.15)
50	10	538.86 (0.00)	1466.44 (0.01)	2818.65 (1.50)	5321.15 (1.86)	4064.52 (0.68)	6008.77 (2.82)
100	10	4605.51 (1.23)	7205.12 (2.50)	4948.18 (2.48)	6567.92 (2.48)	4951.45 (3.03)	7231.33 (4.50)
20	20	259.69 (0.00)	2812.30 (0.00)	142.42 (0.00)	211.59 (0.00)	232.03 (0.00)	1880.46 (0.00)
50	20	2868.91 (0.22)	5382.73 (0.44)	1135.99 (0.00)	3277.78 (0.29)	1511.66 (0.00)	7208.57 (0.75)
100	20	4375.63 (0.55)	7212.42 (34.24)	4118.66 (0.00)	7209.97 (14.84)	6992.85 (0.52)	-
50	50	7209.38 (17.93)	7222.35 (41.61)	7209.24 (25.87)	7218.79 (30.10)	7206.17 (23.40)	7222.67 (17.85) [#]
100	50	7206.74 (55.53)	-	7214.48 (68.30)	-	7203.76 (79.14)	-

Number in bracket is the average gap (%) obtained in CPLEX within the time limit; *: CPLEX is better than OA in CPU time;

[#]: CPLEX is better than OA in Gap; -: Any feasible solution cannot be found within the time limit.

Table B.6: The comparison in average CPU time (s) and average gap (%) between the OA approach and CPLEX, time limit = 7200s, unimodal symmetric random variable, return rate=0.8

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