Solving simultaneous route guidance and traffic signal optimization problem using space-phase-time hypernetwork

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Abstract

This paper addresses the problem of simultaneous route guidance and traffic signal optimization problem (RGTSO) where each vehicle in a traffic network is guided on a path and the traffic signals servicing these vehicles are set to minimize their travel times. The network is modeled as a space-phase-time (SPT) hyper-network to explicitly represent the traffic signal control phases and time-dependent vehicle paths. A Lagrangian-relaxation-based optimization framework is proposed to decouple the RGTSO problem into two subproblems: the Route Guidance (RG) problem for multiple vehicles with given origins and destinations and the Traffic Signal Optimization (TSO) problem. In the RG subproblem, the route of each vehicle is provided subject to time-dependent link capacities imposed by the solution of the TSO problem, while the traffic signal timings are optimized according to the respective link travel demands aggregated from the vehicle trajectories. The dual prices of the RG subproblem indicate search directions for optimization of the traffic signal phase sequences and durations in the TSO subproblem. Both RG and TSO subproblems can be solved using a computationally efficient finite-horizon dynamic programming framework, enhanced by parallel computing techniques. Two numerical experiments demonstrated that the system optimum of the RGTSO problem can be quickly reached with relatively small duality gap for medium-size urban networks.

Keywords: dynamic traffic assignment; network traffic signal control; route guidance; Lagrangian relaxation; dynamic programming; **optimization; network flow modeling**

1. Introduction

Most traffic signal optimization (TSO) methods, whether offline or online, assume link flows and turning ratios, also referred to as turning proportions, of the flows at intersections. Implicit assumption in most of the underlying optimization models is that the flows for the signal timing planning horizon are stationary and, thus, signal timings are determined for the given flows and turning proportions. In the consideration of the larger picture of estimating vehicle flow from origins to destinations, researchers have developed Dynamic Traffic Assignment model (DTA, further discussed in literature review in Section 2, to predict individual route of each vehicle from its origin to its destination at “equilibrium” which effectively defines stationary traffic flows. This equilibrium is generally called “user equilibrium” originally defined as the first principal of Wardrop, and in some applications “system optimal” originally defined as the second principal of Wardrop (1952). Since route travel times are essential to model travelers’ route choices, the underlying network model assumes link travel times and some approximate travel times through the intersections specific to straight-through, right-turning and left-turning routes. Although rarely done in practice, it is possible to iteratively solve a DTA model to come up with link flows and

turning proportions and then solve a TSO model to come up with signal timings for those flows and proportions. Unfortunately the approximate travel times through intersections are poorly defined in DTA models since individual vehicles may reach the intersection during various signal phases and thus travel times for the same route could have large variations.

Furthermore, with the advent of advanced computing, and significant penetration of navigation systems, it should be possible to provide both route guidance and signal service by setting appropriate signal timings for vehicles being provided route guidance. The application of the model/method developed in this paper also apply to (i) connected vehicles systems which would require simultaneous route guidance and signal service, (ii) multimodal traffic signal systems where different modes (private automobiles, transit, etc.) may be provided with differentiated routes and signal services, and (iii) automated vehicle operation where such vehicles may be required to travel in specific lanes and require specific signalization.

Briefly, in the combined Route Guidance and Traffic Signal Optimization (RGTSO) model described in this paper, the phase status of each traffic signal is modeled with variables describing the movement allowed at the scheduled phase. Link travel times depend on the number of vehicles routed through each link as per traffic flow theory. Each vehicle traveling from its origin to its destination travels each link in a candidate route with travel time depending upon the resultant vehicle flow on the link; the waiting, if any, at the intersection, explicitly depends on the queues and the signal status. The space-time trajectory of each vehicle through the network will give its total travel time and the optimization objective value becomes the total travel times of all guided vehicles which can be iteratively decreased by changing guided routes and signal phase schedules. Other optimization criteria could be considered such as minimizing number of stops to improve coordination, minimizing average queue sizes, minimizing fuel consumption, etc. Decomposition of the space-phase-time (SPT) hyper-network results in two subnetworks: namely, space-time network and phase-time network; while decomposition of the overall optimization problem results in two subproblems: TSO and route guidance (RG), where dual prices from the route guidance problem indicate search directions for the TSO subproblem. Details of the model and solution methods are provided in Sections 3-5. Some illustrative computational results are provided in Section 6. Section 7 gives some concluding remarks and points to some promising future research directions.

2. Related previous work

Traffic flow models are fundamental underlying bases for both RG and TSO problems. Specifically, vehicle routing and traffic signal performance are dependent on traffic flow attributes and traffic states of

the links which, in turn, are dependent on vehicle flow propagation within networks, modeled through fundamental relationships between flow and density. As an example, Daganzo (1994, 1995) developed the cell transmission model (CTM) where network links are divided into many cells and vehicles move from cell to cell. The core part of the CTM model is a discrete approximation of the continuous Lighthill-Whitham-Richards traffic flow model proposed by Lighthill and Whitham (1955) and Richards (1956). Further review of related literature below is divided into four parts: (1) TSO models; (2) dynamic traffic assignment (DTA) models; (3) relationship between DTA and TSO models, and (4) solving combined DTA and TSO problems.

2.1 Traffic signal optimization models

The models for TSO can be further divided into two categories: off-line signal optimization and on-line (real-time) methods. Among the off-line signal optimization models, TRANSYT model (or its version in North America, TRANSYT-7F) is most used; it aims to optimize traffic signals timings along corridors or within networks. Wallace et al. (1984) describe the underlying traffic models in TRANSYT as estimating the flow propagation and resultant control delays for given link flows, turning ratios and signal timings based on macroscopic nonlinear delay models. TRANSYT uses a steepest decent algorithm for optimizing cycle lengths, splits and offsets. In PASSER-II, the coordination along corridors is optimized, referred to as bandwidth maximization, and the underlying delay model is a modified nonlinear delay model proposed by Webster (1958). As described by Chang and Messer (1991), PASSER-II uses first-order optimization methods, such as gradient descent, to obtain the optimal signal timings in terms of cycle length, phase sequencing, splits, and offsets. Macroscopic nonlinear control delay models have been defined in the Highway Capacity Manual published by National Research Council (2000) and they result in the underlying control delay model in Synchro (Husch and Albeck, 2003) for setting signal timings in a network, where the optimization approach is in essence an enumeration method to seek the optimal signal timings. MAXBAND is another off-line optimization method proposed by Little et al. (1981) to maximize the bandwidth where the model was formulated as a mixed integer linear program (MILP) problem. Later, MAXBAND was extended by Gartner et al. (1991) to a so-called multi-band approach to optimize multiple bandwidths, applied on multiple road segments and recently further extended by Zhang et al. (2015) to an asymmetrical version. In other literature, the TSO problem on networks has been formulated as: (a) other MILPs, e.g., Gartner and Stamatiadis (2002) and Han et al. (2014), (b) linear programming problems, e.g., Li and Zhang (2014), (c) microscopic-simulation-based optimization: such as the genetic-algorithm-based signal optimization by Park et al. (2001) and Stevanovic et al. (2008), “retrospective approximation” by Li et al. (2010), and emission reduction signal optimization by Osorio and Nanduri (2015) (d) reinforcement-learning-based signal optimization by Zain et al. (2009), (e)

subgradient-based optimization approaches by Rinaldi and Tampère (2015) and Li et al. (2015); (f) store-and forward queues of vehicles, e.g., D'Ans and Gazis (1976), and (e) space-time networks such as of Carey and Srinivasan (1994).

As an on-line TSO approach, the SCOOT (Split Cycle Offset Optimisation Technique) system is an on-line version of TRANSYT model. Compared with TRANSYT, SCOOT optimizes cycles, splits and offsets according to the newly collected vehicle counts via detectors instead of prior collected link flows. The Optimized Policies for Adaptive Control (OPAC) system by Gartner (1983) estimates the resultant delay for a sequence of green phases by calculating the number of vehicles in the queue multiplied by their waiting time under different traffic signal control options. The optimization in OPAC is based on dynamic programming to minimize the total delay at a single intersection. Later, Gartner et al. (2001) have applied OPAC to network of intersections.

RHODES is another on-line adaptive signal control system that uses the hierarchical control structure proposed by Mirchandani and Head (2001). The lower layer focuses on individual intersections in which vehicle arrivals at stop bars are estimated according to the upstream detectors. The optimization is formulated as an on-line knapsack problem and solved using a dynamic programming framework. Prediction of arrivals and resultants queues from upstream detectors to downstream allows RHODES to provide arterial signal control and progression, as described by Shelby (2003). For network traffic control, simulation-based traffic optimization in the upper layer of RHODES was proposed by Dell'Olmo and Mirchandani (1995), referred to as REALBAND, in which platoons were identified through upstream detectors and the platoon splits and movements were estimated using given turning ratios. The target of REALBAND was to provide an appropriate signal timings through decision tree analysis so that the total delay and stops of multiple arriving platoons can be minimized. The resultant signal timing of REALBAND may be either directly deployed at local intersections or superimpose additional constraints to intersection signal timing optimization at the lower layer. Recently, He et al. (2012) have used identified platoons to extend RHODES-type logic for signal timings at intersections to allow for treatment to platoons, buses etc. He et al. (2014) proposed a request-based MILP model to consider multiple modes of vehicles as well as pedestrians. Xie et al. (2012) proposed adaptive signal control strategy based on the concept of single machine scheduling which shares some similarity with the RHODES system in predicting vehicle arrivals. Sun et al. (2015) proposed a “quasi-optimal” (i.e., sub-optimal) signal control strategy at oversaturated intersections based on queue length estimation and store-and-forward network modeling. There are several other on-line TSO models including: UTOPIA by Mauro and Taranto (1989),

PRODYN by Khoudour et al. (1991) and MOTION by Bielefeldt and Busch (1994). Readers are referred to recent review by Stavanovic (2010) for the state of practice of adaptive signal control.

2.2 Dynamic traffic assignment

The dynamic traffic assignment (DTA) problem aims to describe the path choice and/or departure time decisions of travelers under time-dependent demand and road capacity constraints. Since the formulation of the problem by Merchant and Nemhauser (1978a, 1978b) there has been much research on this problem; early works include the analytical models of Ran and Boyce (1994) and Smith (1993), and simulation models of Peeta and Mahmassani (1995). The network representation for typical DTA models include links and nodes with simplified traffic control models. The traffic flow propagation in DTA networks is carried out from link to link, following some constructs of traffic flow models. As categorized in a comprehensive survey paper by Peeta and Ziliaskopoulos (2001), there are two types of DTA models and solution algorithms: analytical DTA model and simulation-based DTA model. In typical analytical DTA models, a set of link/node exit constraints is used to represent traffic flow propagation, and the calculation of path travel costs typically assumes convex, continuous and strictly monotonic link performance functions, such as those of Smith (1993) and Lo and Szeto (2002). On the other hand, simulation-based DTA models aim to utilize mesoscopic traffic simulators to iteratively find time-dependent dynamic user equilibrium or system optimal solutions while realistically capturing traffic dynamics, such as given by Peeta and Mahmassani (1995a) and Ben-Akiva et al. (1997). **Xie et al. (2014) proposed an agent-based probabilistic sequential-decision-making process to determine route choice and the underlying signal-controlled link travel times were estimated via a microscopic traffic simulator containing high-fidelity signal control mechanism.** A wide range of dynamic traffic management scenarios, such as providing pre-trip information (e.g., Mahmassani and Liu, 1999), can be further embedded in a simulation framework. Simulation-based DTA models can also be used for on-line traffic state estimation and route guidance framework, see Mahmassani (2001), Ben-Akiva et al. (2001), Mirchandani et al. (2004), Zhou and Mahmassani (2007), as well as Chiu and Mirchandani (2008).

2.3 Modeling traffic signals within DTA

Allsop (1974) and Allsop and Charlesworth (1977) first proposed an analytical relationship between traffic signal control and traffic assignment. Later, Heydecker (1983, 1987) provided more detailed analysis to identify the impact of traffic signal control on the overall uncertainty and variability of traffic systems. Yang and Yagar (1995) studied the relationship between DTA and traffic signal control problem in saturated road networks as a bi-level network design problem; equilibrium flows and control delays

were iteratively optimized through sensitivity analysis. Gartner and Stamatiadis (1998) proposed a framework for the interaction between dynamic traffic assignment and adaptive signal control.

Traffic signal effects are sometimes included within simulation based DTA models with the use of simple multi-phase delays at intersections; for example, in the study by Sun et al. (2006), signal timing plans are coded and optimized by a Genetic Algorithm (GA) by adjusting cycle, offset and green split in a bi-level programming formulation framework. With a simple reactive traffic control representation, Hu and Mahmassani (1997) studied day-to-day traffic patterns from DTA model to assess the impact of pre-timed, daily responsive and local coordination signals. In the linear programming model proposed by Gazis (1974) and D'Ans and Gazis (1976) for optimal control of signals at intersections, vehicles in queues at intersections are propagated through a “store-and-forward” mechanism, with exogenously given routings from a static traffic assignment model; explicit phases were not modeled. Papageorgiou (1990) extended the store-and-forward model with a tight integration with DTA. Messner and Papageorgiou (1990) developed METANET simulation model for traffic routing and control using the store-and-forward network representation. In summary, although many DTA models include the representation of traffic signal control, most DTA models use relatively simplified assumptions (without explicitly considering complex but realistic phase sequence and duration restrictions) to calculate traffic control delays at intersections by introducing estimated travel time penalties when vehicles cross intersections.

2.4 Solving the combined traffic assignment and traffic signal optimization (CTATSO) problem

Carey and Srinivasan (1994) formulated the CTATSO problem as a space-time network expanded from the store-and-forward network and proposed an iterative solution procedure through Lagrangian decomposition. Gartner and Stamatiadis (1998) proposed the possibility of combining the DTA model of Ben-Akiva et al. (1994) with the OPAC model to conduct path-flow-based traffic signal optimization. Chen and Ben-Akiva (1998) considered the combined traffic assignment and traffic signal optimization problem (CTATSO) problem as two separate variational-inequalities problems and addressed it through a framework to solve three games: Cournot, Stackelberg and Monopoly. Lo (1999, 2001) transformed the original nonlinear CTM model into a linear model by replacing the original minimization constraints with three linear constraints and formulated the traffic signal control problems as a MILP model. Since the inflow and outflow of each cell need to be calculated at each time interval in the CTM model, traffic signal control strategies can be implemented by superposing additional constraints to inflows and outflows at each time interval and the effects of traffic control can propagate through the entire network. Along this line, Lin and Wang (2004) also developed a CTM-based MILP model with certain enhanced constraints to ensure the realism of signal control logics. Wu (2008) designed a heuristic solution algorithm to conduct path-flow-based traffic signal control. Closely related to the CTM model that aims

to capture traffic evolution over time, there are also other dynamic traffic flow models based on a discrete approximation of the kinematic wave theory at a link level, such as the study by Han et al. (2014) which solved the developed optimization model with CPLEX. Smith and Mounce (2011) proposed a splitting rate model with additional signal control variables to determine the flow splitting rates for traffic re-routing and traffic control problem implicitly. Liu and Smith (2015) developed a route-flow swapping approach in conjunction with a dynamic signal control policy to reach approximate equilibrium. Aziz and Ukkusuri (2012) formulated the CTATSO problem as a MILP problem based on the CTM model. Ukkusuri et al. (2013) formulated the CTATSO problem as a Nash-Cournot game and a Stackelberg game and proposed an iterative procedure to solve the resulting mixed integer linear programming (MILP) problem based on the CTM model. Srivastava et al. (2014) modified the CTM model with known features of the headway distribution to describe signalized intersection. Ramezani et al. (2015) solved the CTATSO problem based on a macroscopic fundamental diagram (MFD) and model predictive control approach.

In telecommunications network design, the problem referred to as the “simultaneous data routing and resource allocation problem” has a similar structure to CTATSO. Lin et al. (2004) proposed an iterative procedure to solve the data routing and switch resource allocation problem via a general Lagrangian dual decomposition optimization framework. In the context of computer networks problems, Lin et al. (2004) formulated a flow-based, nonlinear optimization model for resource allocation subject to nonlinear constraints. In contrast, we present an agent-based route guidance model with linear objective functions and a more specific representation for dynamic road resource allocation problem (i.e. the traffic signal optimization problem) in the context of RGTSO applications.

2.5 Preview of contributions of the paper

The paper addresses the above mentioned modeling challenges to combine network route guidance and explicit traffic signal control mechanism in the optimization model. Based on the aforementioned literature review, we can see that most existing traffic models to solve CTATSO-type problems, which are related to RGTSO, are based on aggregate flows in the network, and aggregate flows through intersections with approximate representations of signal timings through increase or decrease of capacities. In these models TSO solution is provided to traffic assignment algorithms and the latter’s solution is provided to TSO and the iteration is repeated; the coupling between the two problems is generally ad-hoc. Furthermore, with an exception of one or two models, signal phases and phase sequencing are not considered. The model presented explicitly represents signal phasing and durations, whether flexible or cyclic, and the signal timing decisions as well as route guidance decisions are solved concurrently. The coupling between the TSO and route guidance is explicit through Lagrangian multipliers or dual variables.

The proposed RGTSO formulation in this paper is based on route guidance for individual vehicles with specific origins, destinations, and preferred departure/arrival times in space-time networks which is related to system optimal version of the DTA model. The optimal routing and signal phasing/timing solution to the RGTSO problem includes a set of waiting times at signals in the prescribed routes for each vehicle and the scheduled signal timings to accommodate the resultant flows on the routes provided by the RGTSO solution. The approach described in this paper is naturally suitable for a wide range of emerging on-line proactive traffic management applications, such as connected vehicles or autonomous vehicles.

In the solution approach, the RGTSO problem is decomposed into two less computationally complex subproblems through Lagrangian relaxation: the route-guidance (RG) and the traffic signal optimization (TSO) subproblems. After the problem relaxation and decomposition in to two separate subproblems coupled only by the Lagrangian variables, the constraints associated with each of the subproblems contain only the flow balance constraints used in minimum cost flow problems and least-cost path problems. Thus we can use a wide range of network flow algorithms can generate integral optimal solutions. In this paper, we develop specialized dynamic programming based algorithms to handle a large number of binary constraints for vehicle route guidance and signal optimization problems. More specifically, TSO subproblem is cast as finding a time-dependent least-cost path problem for green times within a new time-dependent network, referred to as a “*phase-time network*”. In the construction of the phase-network, most naturally nonlinear constraints for signal control logic and configurations, such as minimum/maximum green duration requirements for signal phases, can be explicitly embodied in the network model by including only allowable phase-time arcs. In doing so, the linearity of both TSO and the route guidance subproblems is strictly maintained without loss of signal control or routing flexibility.

This decomposition allows for solving both subproblems using repeatedly a polynomial least-cost path finding algorithm. The Lagrangian multipliers (the dual variables) are adjusted by a dual ascent approach using approximate subgradients during each iteration. These iterations are repeated until the gap between the current upper and lower bounds are satisfactory or until a specified iteration number is reached. This algorithm is also designed in such a way that parallel-or distributed-computing techniques can be used so solve the subproblems thereby effectively facilitating large-scale applications.

3. Modeling constructs and components

First, Table 1 shows the notation system to be used the mathematical formulation.

(Insert Table 1)

The RGTSO problem has two interconnected components: RG and TSO problems. The RG problem aims to provide system optimal least-cost paths under given signal timings for individual vehicles whose origins, destinations and departure time windows are all known; the TSO problem aims to schedule signal timings at intersections to allow guided vehicles to cross efficiently and safely. As illustrated in Fig.1, two hypothetical vehicles are provided with departure times, route guidance and suggested waiting, if necessary, at intersections to minimize their total travel time and guarantee safety at intersections. To have a minimum total travel time, those intersections crossed by vehicle 1 (intersections 7, 8, 5, 2, 3) and vehicle 2 (intersections 1, 4, 5, 6) should be well scheduled to guarantee two vehicles can cross without stopping or stopping only for a minimum time. If the two vehicles arrives at intersection 5 at the same time, it is necessary to hold one vehicle from entering intersection 5 until the other has crossed to avoid collision. When there are many such guided vehicles on the roads, scheduling vehicle routes and signal timings at all intersections becomes challenging because it is necessary to consider waiting at intersections and link capacity constraints at the same time. To reflect these challenges in models, we formulated the RGTSO problem using coupled space-time and phase-time networks, namely space-phase-time hypernetwork.

(Insert Figure 1)

Space-time network representation for RG problem: A widely used model in route guidance problems is the space-time network that discretizes the planning horizon to small time intervals. The space-time network was first proposed by Cooke and Halsey (1966) in finding the time-dependent shortest path by including waiting links in physical networks in order to get vehicle time-dependent shortest path trajectories. **The concept of space-time networks has been adapted in many applications of the network design problem, for example, transportation road network design for maximizing space-time accessibility (Tong et al., 2015).** In a space-time network, travelers can be modeled as individual agents with their own departure time and the least-cost path from given origins to destinations. Now the agent-based vehicle flow model is expanded to a five-index integer linear programming model: variable $x_{(i,j,t,s)}^f = 0$ if vehicle f enters link (i, j) at t and leaves at time s , otherwise $x_{(i,j,t,s)}^f = 0$. When traffic signal control is superimposed into the space-time network, it can be viewed as temporarily removing the capacity of certain link(s) when a signal phase is red and adding the link capacity back when the signal phase switches to green. The advantage of the space-time network over the physical network is that the space-time network representation is flexible in holding vehicles by explicitly allowing waiting links.

Building the above space-time network starts from expanding the physical network. Consider a physical network which has a set of nodes $N = \{1, 2, \dots, i, \dots, n\}$ and the nodes are linked by a set of directed links

$A = \{(i, j)\}$. In the most common node-link network representation, each intersection is represented simply with only one node, as shown on the left part of Fig. 1; however here it is difficult to represent vehicle movements within intersections. Therefore, we expand this standard intersection into a sub-network with twelve movements (left turn, through and right turn on each approach) as shown on the right side of Fig. 2, used before in traffic control modeling (see Ben-Akiva et al. (1998) and Mahmassani et al. (2004)). Now in the network model the links can be categorized into two types, namely travel links such as road segments and signal links that represent the vehicle movements within intersections. The signal link status variable $y_{(i,j,t)}^k$ is associated with the signal link(s) within intersection k where i is the origin approach and j is the destination departure lane ($i, j \in j_1, \dots, j_8$) in Fig. 2-b). When a signal link is open (green signal) or closed (red signal) at time t , $y_{(i,j,t)}^k$ is set as one or zero. There are also special nodes in the new physical network, referred to as signal nodes that are used to build space-time waiting links.

(Insert Figure 2)

Let $STG = (V, E)$ represent a space-time network, where V is the set of space-time nodes and E is the set of space-time links (including travel links and signal links). This research uses one second as the time resolution assuming that guided vehicles report locations every second, while one second resolution is typically sufficient for the solution to traffic signal optimization problems,. Each physical node i is extended to a set of space-time nodes (i, t) at each time interval $t, t=1, 2, \dots, T$, where T is the length of the decision horizon. We consider four types of space-time links in the proposed space-time network.

1. Travel links extended from a physical link (i, j) and each travel link traverses from space-time node (i, t) to space-time node (j, s) , where $s = t + FTT(i, j, t)$; where FTT stands for free-flow travel time;
2. Waiting space-time links at the origin node of the guided vehicle, $o(f)$, of vehicle f from $(o(f), t)$ to $(o(f), t+1)$, $t=1, 2, \dots, T$;
3. Waiting space-time links at inbound signal nodes (connecting from a travel link to a signal link) from (i, t) to $(i, t+1)$, $i \in V$, $t=1, 2, \dots, T$. By introducing the waiting space-time links at inbound signal nodes, vehicles are able to wait before entering an intersection;
4. Waiting space-time links at the destination node, $d(f)$, of guided vehicle f from $(d(f), t)$ to $(d(f), t+1)$, $t=1, 2, \dots, T$;

The initial space-time link costs for vehicle f is denoted as:

- $c(f, i, j, t, s) = FTT(i, j, t)$ on travelling space-time links;
- $c(f, o(f), o(f), t, t+1) = 1$ s on waiting space-time links at origin node $o(f)$;
- $c(f, i, i, t, t+1) = 1$ s, ($i \in N_c$) on waiting space-time links at inbound signal nodes; and
- $c(f, d(f), d(f), t, t+1) = 0$ s on waiting space-time links at destination node $d(f)$;

The left portion of Fig. 3 shows a simple two-phase intersection with one-way flows and the right portion represents the corresponding space-time network. In this illustration, link travel times are uniformly set as 1 s and increments of waiting times as 1 s. In the space-time network, a vehicle can determine when to leave its origin node, how long it should wait at inbound signal nodes, and when it will reach its destination node to achieve its minimum travel cost.

(Insert Figure 3)

Phase-Time network representation for TSO problem: We propose a new type of network representation at intersection k , referred to as phase-time network, so as to cast a TSO problem into a least-cost path finding problem. The term “signal phases” are used slightly differently in models and from place to place. For instance, in North America, a signal phase is usually associated with one possible traffic movement at an intersection, where only specified non-conflicting movements are allowed at any time. In Europe, the term signal stage is often used, where each stage is associated with a combination of non-conflicting traffic movements. Both definitions have the same flexibility for traffic signal control. The definition of signal phase in this paper is similar with the “stage” concept. Readers with interests in traffic signal control mechanism are suggested to read Traffic Signal Timing Manual by Koonce et al. (2008).

Without loss of generality, a standard four-phase signal control strategy is shown in the left part of Fig. 4. Each vertex in Fig. 4 represents a signal phase. The phasing sequence can be either flexible, meaning that one signal phase to move to any other signal phase, or cyclic, meaning the signal phases have a fixed sequence. Holding on to any phase can result in vehicle delays, because vehicles on all other conflict approaches must stop. The longer a phase is held, the higher the cost will be. Following this rationale, signal optimization essentially is to find appropriate vertex sequences and the corresponding holding times on each vertex within time horizon T so as to minimize the total cost. In the meantime, since there are only one signal phase active at any time, at most one vertex can be selected when moving from the current vertex.

Since signal phases are mutually exclusive, we can view traversing from vertex to vertex as finding a minimum cost route for one vehicle in a network. As a result, it is possible to view signal optimization as finding a least-cost path traversing all phases with allowed revisits within T .

(Insert Figure 4)

Let $PTG = (PV, PA)$ represent a phase-time network, where PV is the set of phase nodes and PA is the set of transition arcs. A signal phase node m_k is extended to a set of vertices (m_k, t) at each time interval t in the time horizon T , $t=1, 2 \dots T$. A super sink node Z is also added; see Fig. 5 illustration.

A transition arc in PTG is indexed as (m_k, m'_k, τ, h) , indicating that signal phase m_k stays green starting from time τ to time $h-1$ and then turns over green to m'_k at h . The duration of each signal phase, $(h - \tau)$, should also satisfy the requirements for minimum green and maximum green or $g_{\min} \leq h - \tau \leq g_{\max}$. In the illustration of Fig. 5 the minimum green time $g_{\min} = 2$ and maximum green time $g_{\max} = 3$ is used.

In order to guarantee all phasing sequence candidates will reach the same destination after T , it is necessary to add some dummy arcs with zero costs. For any vertex (m_k, τ) , if $\tau < T$ and $\tau + g_{\min} > T$, then a dummy arc $(m_k, Z, \tau, T+1)$ is added. Fig. 5 illustrates how the phase-time network for flexible four-phase intersection with $T=7$ s. The transition arc costs in the phase-time network can be given exogenously determined by projected delays, queue lengths or other measures of effectiveness (MOEs).

(Insert Figure 5)

Together the coupled $STG = (V, E)$ and $PTG = (PV, PA)$ networks will be referred to as a *space-phase-time hypernetwork* in the paper. Above we have defined the costs for a vehicle f traversing the links of this space-phase-time hypernetwork.

4. Optimization model

We decompose the RGTSO problem into two sub-problems via Lagrangian decomposition by relaxing network capacity constraints for vehicles and traffic signal systems. As discussed before, without loss of generality, the time resolution is assumed to be one second. As a result, the two sub-problems are both discrete and viewed as integer programming (IP) problems. These IP problems are NP-hard and therefore large problems cannot be solved quickly using general purpose commercial solvers. Hence, we partition

the original problem into a set of easy-to-solve problems and develop an iterative algorithm that approaches the system optimum.

Direct decision variables are classified into two types: route choice variable set $X = \{x_{(i,j,t,s)}^f\}$, $(i, j) \in A$ for vehicle f and traffic signal status variable set at intersection k , $Y = \{y_{(i,j,t)}^k\}$, $(i, j) \in A_s$. Such classification is important because it can result in the natural problem decomposition discussed later.

Objective function: The objective function is formulated as:

$$\text{Min} \sum_{t=1}^T \sum_{(i,j) \in A} \sum_{f \in F} \left[x_{(i,j,t,s)}^f \times c(f, i, j, t, s) \right] \quad (1)$$

The costs $c(f, i, j, t, s)$ on four types of links in the space-phase-time hypernetwork were discussed above.

Constraints: We include four types of constraints: flow conservation constraints, link capacity constraints, queue-spillback constraints, and traffic signal control logic constraints.

Network flow conservation constraints:

For vehicle f , the flow conservation constraint at space-time vertex (j, s) is:

For those nodes where waiting is not allowed:

$$\sum_{(i,j,t,s) \in E} x_{(i,j,t,s)}^f - \sum_{(j,i',s,l) \in E} x_{(j,i',s,l)}^f = \begin{cases} 1 & (j = o(f), s = r(f)) \\ -1 & (j = d(f), s = T) \\ 0 & \text{Otherwise} \end{cases} \quad (2-a)$$

For those nodes where waiting is allowed:

$$\sum_{(i,j,t,s) \in E} x_{(i,j,t,s)}^f - \sum_{(j,i',s,l) \in E} x_{(j,i',s,l)}^f + x_{(j,j,s-1,s)}^f - x_{(j,j,s,s+1)}^f = \begin{cases} 1 & (j = o(f), s = r(f)) \\ -1 & (j = d(f), s = T) \\ 0 & \text{Otherwise} \end{cases} \quad (2-b)$$

Constraints (2) impose that vehicle f uses exactly one space-time link to enter a node in the hypernetwork and one to leave each except at the origin node where it only leaves and the destination node where it only arrives. Constraints (2) are also illustrated in Fig. 6.

(Insert Figure 6)

Link capacity constraints: At any time, the inflow traffic on links can only be less than or equal to the road capacity volume. This constraint can be formulated as:

The total inflow traffic on signal link (i, j) of intersection k :

$$\sum_{f \in F} \left(x_{(i,j,t,s)}^f \right) \leq y_{(i,j,t)}^k \times SR(i, j, t), \forall (i, j) \in A_s, 1 \leq t \leq T \quad (3)$$

The total inflow traffic on travel link (i, j) :

$$\sum_{f \in F} \left(x_{(i,j,t,s)}^f \right) \leq SR(i, j, t), \forall (i, j) = A \setminus A_s, 1 \leq t \leq T \quad (4)$$

(3) and (4) impose that the traffic volume on a space-time link is less than capacity at any time t .

Vehicle spatial capacity constraints: If a link is fully occupied by a queue and it is at jam density, referred to having a “queue spillback”, new vehicles cannot enter that link. The queue length on a physical link (i, j) can be estimated based on cumulative arrivals at the entering node i , cumulative departures at exiting node j , of a link (i, j) and physical length $L_{(i,j)}$:

$$\sum_{0 \leq \xi \leq t} \sum_{f \in F} x_{(i,j,\xi,\xi+FFTT(i,j))}^f - \sum_{(j,j') \in A} \sum_{0 \leq \xi \leq t} \sum_{f \in F} x_{(j,j',\xi,\xi+FFTT(j,j'))}^f \leq L_{(i,j)} \times k_{jam}, \forall (i, j), t \quad (5)$$

where ξ is a integer time index between 0 and t . According to (5), the queue length at t is the difference between the cumulative vehicle arrivals $A(i, j, t)$ and the total cumulative vehicle departures $D(i, j, t)$ at time t . (5) can also be expressed as $A(i, j, t) - D(i, j, t) \leq L_{(i,j)} \times k_{jam}$. To further handle congestion and delays caused by the propagation of backward shock waves such as those predicted by the LWR, one can adopt a simplified Kinematic-wave-based modeling approach,

$$A(i, j, t) - D(i, j, t - BWTT(i, j)) \leq L_{(i,j)} \times k_{jam},$$

where $BWTT(i, j)$ is the travel time for the backward wave to propagate through the link assuming a simplified triangular Q-K model. Interested readers can obtain further details in the original paper by Newell (1993) and in a recent implementation effort documented by Zhou and Taylor (2014).

It should be noted that, in a general optimization model, additional constraints can be added to represent the traffic flow dynamics. For instance, on any link at any time, $A(i, j, t - FWTT(i, j)) \geq D(i, j, t)$; this free-flow travel time constraint can be enforced automatically in our network building process. Moreover, the key to the kinematic wave model lies in the transition of link status. That is, when the strict inequality '<' in Eq. (5) changes to equality '=', this implies that a queue spillback has occurred and the queue will begin

to propagate backward to upstream links. Such a transition further influences the flow demand and supply on related links in a network. A recent study by Tampère et al. (2011) provides systematic discussions of the traffic flow/capacity allocation models at the node level.

Traffic signal control logic constraints: Signal control logic constraints at intersections include minimum green, maximum green and mutual exclusiveness of conflicting movements. Without loss of generality, we use a four- phase signal control at intersections, where only protective left turns are allowed right-turns are permitted all the time.

Signal phase m_k and signal link variables $y_{(i,j,t)}^k, (i, j) \in A_S$ at any intersection k can be represented using a 0-1 matrix. Fig. 7 illustrates a mapping matrix between signal status variables and signal phases.

(Insert Figure 7)

Once the mapping matrix $\Phi_{(i,j,m,k)}$ is established, we can derive the exact relationship between Y and W as:

$$y_{(i,j,t)}^k = \sum_{m_k} \sum_{m'_k \neq m_k} \sum_{\tau \leq t} \sum_{h > t} \varphi(i, j, m_k, k) \times w_{(m_k, m'_k, \tau, h)}^k, \forall (i, j) \in A_S, \forall k \quad (6)$$

The mathematical modeling approach of (6) is also referred to as “variable substitution” or “variable splitting”, e.g., by Xing and Zhou (2011). As in Fig. 6, each signal phase usually maps to multiple signal links. Through (6), the status of signal link control variables can be expressed in a form of summation of signal phase variables, keeping the constraint linear. For instance, signal link (j_3, j_6) at intersection k in Fig. 7 corresponds to signal phase m_k^1 which can be exactly described by (6):

$$y_{(j_3, j_6, t)}^k = \varphi(j_3, j_6, m_k^1, k) \times \sum_{\tau \leq t} \sum_{h > t} \sum_{m'_k = 2, 3, 4} w_{(m_k^1, m'_k, \tau, h)}^k, \forall t, k, \text{ meaning } y_{(j_3, j_6, t)}^k \text{ is 1 if the arc}$$

$w_{(m_k^1, m'_k, \tau, h)}^k$ in the phase-time network is selected ($m'_k = 2, 3 \text{ or } 4$).

Additionally, constraint (7) for intersection k represents phase mutual exclusiveness and it is illustrated in Fig. 7.

$$\sum_{(m_k, m'_k, \tau, h) \in PA} w_{(m_k, m'_k, \tau, h)}^k - \sum_{(m'_k, m''_k, h, l) \in PA} w_{(m'_k, m''_k, h, l)}^k = \begin{cases} 1, (m'_k = o(m), h = 1) \\ -1, (m'_k = Z, h = T + 1) \\ 0, \text{ Otherwise} \end{cases} \quad (7)$$

According to (7), at any vertex (m'_k, h) , there is only one inbound arc and outbound arc except the origin and destination vertices, i.e., current phase $o(m)$ and super ending phase Z as illustrated in Fig. 8. In other words, for each signal phase, the number of its predecessor and successor phases can only be one and this constraint guarantees signal phases in conflict will not turn green at the same time.

Although (6) needs summation over 5 indices, the number of variables on the right side will be much fewer because, for any given signal status variable $y_{(i,j,t)}^k$, most $\varphi(i, j, m_k, k)$ will be zero and $w_{(m_k, m'_k, \tau, h)}^k$ is only relevant when $\tau \leq t < h$. We can also prove that (6) is a valid representation to map from signal phase to signal link variable.

Lemma: Value of on-off signal link control variable $y_{(i,j,t)}^k$ in (6) can be exactly determined by the summation of related signal phase variables $w_{(m_k, m'_k, \tau, h)}^k$

Proof: In (6), if signal phase variables $w_{(m_k, m'_k, \tau, h)}^k$ are associated with the signal link variable $y_{(i,j,t)}^k$, then $\varphi(i, j, m_k, k)=1$; otherwise $\varphi(i, j, m_k, k)=0$. There are only two possibilities regarding the signal phase status at any time: (1) the signal phase m_k is red at t and therefore all corresponding signal phase variables $w_{(m_k, m'_k, \tau, h)}^k$ are zero. Then from (6), $y_{(i,j,t)}^k=0$; (2) the signal phase m_k is green at time t and, from (7), then associated signal phase variables $w_{(m_k, m'_k, \tau, h)}^k$ equal to 1. As a result, corresponding $y_{(i,j,t)}^k=1$. The proof ends.

Minimum green and maximum green requirements of signal phases can be met without introducing new constraints. These two constraints can be realized by including only selective phase-time arcs $\forall(m_k, m'_k, \tau, h), g_{\min} \leq h - \tau \leq g_{\max}, g_{\min} \leq l - h \leq g_{\max}$. The minimum green and maximum green constraints can also be expressed using a group of “IF-THEN” and “big M” conditions, such as in He et al. (2012) and Meng and Zhou (2014).

(Insert Figure 8)

Substituting $y_{(i,j,t)}^k$ of (3) in (6) to get constraint (8) for intersection k ,

$$\sum_f (x_{(i,j,t,s)}^f) \leq \sum_{m_k} \sum_{m'_k \neq m_k} \sum_{\tau \leq t} \sum_{h > t} [\varphi(i, j, m_k, k) \times w_{(m_k, m'_k, \tau, h)}^k \times SR(i, j, t)], \forall t, \forall (i, j) \in A_s \quad (8)$$

5. Overall model, problem decomposition, and a solution method

The overall problem can be written as the following problem:

P1: Objective function (1); subject to constraints (2) - (5);

In essence it is a 0-1 integer programming problem with linear constraints. We can decompose it using Lagrangian relaxation.

Let us recall all the constraints defined in. (2) through (8) and they can be divided into four groups according to their forms:

Group I: In a form of “ $\mathcal{AX}=\mathcal{B}$ ” for flow balance constraints (2) for space-time vehicle flows

Group IV: In a form of “ $\mathcal{AW}=\mathcal{B}$ ” for flow balance constraints (7) for phase-time network

Group II: In a form of “ $\mathcal{AX}\leq\mathcal{BW}$ ” for outflow capacity constraints (8) on signal links;

Group III: In a form of “ $\mathcal{AX}\leq\mathcal{B}$ ” for inflow capacity and spatial capacity constraints (4) and (5) on all links.

There are multiple approaches to relax the primal problem P1 using Lagrangian multipliers, depending on the constraints relaxed. Our approach is to introduce Lagrangian multipliers into Group II and Group III constraints to create Problem **P2**.

P2: Lagrangian relaxation.

Min $L =$

$$\begin{aligned} & \sum_{(i,j) \in A} \sum_{t=1}^T \sum_{f \in F} \left[x_{(i,j,t,s)}^f \times c(f,i,j,t,s) \right] + \\ & \sum_{(i,j) \in A_s} \sum_{t=1}^T \left\{ \lambda(i,j,t) \times \left[\sum_{f \in F} \left(x_{(i,j,t,s)}^f \right) - \sum_k \sum_{m_k} \sum_{m'_k \neq m_k} \sum_{\tau \leq t} \sum_{h > t} \varphi(i,j,m_k,k) \times w_{(m_k,m'_k,\tau,h)}^k \times SR(i,j,t) \right] \right\} \quad (9) \\ & + \sum_{\substack{(i,j) \in \\ A \setminus A_s}} \sum_{t=1}^T \left\{ \mu_t(i,j,t) \left[\sum_{f \in F} \left(x_{(i,j,t,s)}^f \right) - SR(i,j,t) \right] \right\} \\ & + \sum_{(i,j) \in A} \sum_{t=1}^T \left\{ \mu_s(i,j,t) \left[A(i,j,t) - D(i,j,t) - L_{(i,j)} \times k_{jam} \right] \right\} \end{aligned}$$

Subject to constraints (2) - (7).

Where:

- $\lambda(i,j,t)$: Vectors of Lagrangian multipliers (≥ 0) representing the penalties for relaxed inflow capacity constraints on signal links (A_s);

- $\mu_t(i, j, t)$: Vectors of Lagrangian multipliers (≥ 0) representing the penalties for relaxed inflow capacity constraints on travel links ($A \setminus A_s$);
- $\mu_s(i, j, t)$: Vectors of Lagrangian multipliers representing the penalties for relaxed jam density constraints on all links (A).

Effectively, given the current link costs, the decision variables in the objective of Problem P2 are the travel link flow variables $\mathbf{x}$ and signal link variables $\mathbf{w}$, coupled by the Lagrangian dual variables $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$. Given $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$, the objective becomes separable, resulting into two subproblems, $\mathbf{P}_x$ with respect to decision variables $\mathbf{x}$, and $\mathbf{P}_w$, with respect to decision variables $\mathbf{w}$. The corresponding dual problem for $\mathbf{P2}$ then is a maximization with respect to $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ as will be developed below.

Consider first the objective for $\mathbf{P}_x$:

$$\begin{aligned}
 & \text{Min:} \\
 & \sum_{\substack{(i,j) \in \\ \{A \setminus A_s\}}} \sum_{t=1}^T \sum_{f \in F} \left\{ [\mu_t(i, j, t) + c(f, i, j, t, s)] \times x_{(i,j,t,s)}^f \right\} \\
 \mathbf{P}_x: & + \sum_{(i,j) \in A_s} \sum_{t=1}^T \sum_{f \in F} \left\{ [(\lambda(i, j, t) + c(f, i, j, t, s)) \times x_{(i,j,t,s)}^f] \right\} \\
 & + \sum_{(i,j) \in \{A\}} \sum_{t=1}^T \mu_s(i, j, t) (A(i, j, t) - D(i, j, t) - L_{(i,j)} \times k_{jam}) - \sum_{\substack{(i,j) \in \\ \{A \setminus A_s\}}} \sum_{t=1}^T \mu_t(i, j, t) \times SR(i, j, t)
 \end{aligned} \tag{10}$$

From (10), it can be seen that introducing Lagrangian multipliers for queue spillback constraints (the 3rd term) into $\mathbf{P}_x$ by relaxing capacity and spillback constraints can create an unrealistic solution under highly congested traffic conditions. However, during the later network loading process, which considers the queue spillback at each iteration, certain vehicles may have to wait longer in the queue given the spatial capacities at that time. Subsequent iterations will always result in a feasible solutions even though the initial solution to $\mathbf{P}_x$ is unrealistic.

In order to develop efficient optimization algorithms to solve $\mathbf{P}_x$ later, we further simplify. (10) to (11) by dropping the 3rd term. Such a simplification will not substantially lower the validity of the overall solution because most of elements in the 3rd term of (10) in essence cancel out each other. However, since the 3rd term of (10) is non-positive, the simplification will make the objective value of (11) smaller than that of (10) given the same solutions. As a result, the objective value of simplified $\mathbf{P2}$, which is the lower bound of the system optimal (SO) solution to the primal problem $\mathbf{P1}$, will decrease as well. In other words, such a simplification will increase the gap value between the upper bound and lower bound of the SO solution over the LR iterations.

Min :

$$\begin{aligned}
& \sum_{\substack{(i,j) \in \\ \{A \setminus A_S\}}} \sum_{t=1}^T \sum_{f \in F} \left\{ [\mu_t(i, j, t) + c(f, i, j, t, s)] \times x_{(i,j,t,s)}^f \right\} \\
\mathbf{Px}: & + \sum_{(i,j) \in A_S} \sum_{t=1}^T \sum_{f \in F} \left\{ [\lambda(i, j, t) + c(f, i, j, t, s)] \times x_{(i,j,t,s)}^f \right\} \\
& - \sum_{\substack{(i,j) \in \\ \{A \setminus A_S\}}} \sum_{t=1}^T \mu_t(i, j, t) \times SR(i, j, t)
\end{aligned} \tag{11}$$

subject to constraint (2).

The other subproblem **Pw** is

$$\begin{aligned}
& \mathbf{Min}: \\
\mathbf{Pw}: & - \sum_k \sum_{m_k} \sum_{m'_k \neq m_k} \sum_{\tau} \sum_h \left\{ \left[\sum_{(i,j) \in A_S} \sum_{t=\tau}^{h-1} (\lambda(i, j, t) \times \varphi(i, j, m_k, k) \times SR(i, j, t)) \right] \times w_{(m_k, m'_k, \tau, h)}^k \right\}
\end{aligned} \tag{12}$$

Subject to constraint (7);

5.2 Solve **Px** using dynamic programming

Px can be viewed as a time-dependent least-cost path problem in a space-time network. Compared with the classic least-cost path finding algorithm, time index is introduced in the construction of space-time networks and algorithms, such as the approach by Yang and Zhou (2014). In this paper, first, a space-network is built according to the topology of the physical network, link travel times, origin ($o(f)$) and destination ($d(f)$) for each vehicle f and time horizon T . From (11), the space-time link travel costs are calculated as the default link travel costs (e.g., link travel times) plus the corresponding Lagrangian multipliers (that is, the Lagrangian multipliers are penalties for choosing the links). Then, for each vehicle f , its least-cost path from $o(k)$ to $d(k)$ can be obtained using a customized dynamic programming algorithm. In a least-cost path for vehicle f , if a space-time link (i, j, t, s) is selected, then the corresponding variable $x_{(i,j,t,s)}^f = 1$, otherwise $x_{(i,j,t,s)}^f = 0$. The optimal solution to **Px** is obtained after the least-cost paths are obtained for all vehicles. (See Appendix A for details).

5.3 Solve **Pw** using dynamic programming

Similarly, a phase-time network is built according to the phasing sequence, minimum green, maximum green and time horizon. The phase-time arc costs are calculated as the product of the corresponding

Lagrangian multipliers and the saturation capacities of signal links. After selecting a starting phase and starting time, a least-cost path from starting vertex to the super sink vertex (Z, T+1) can be found using the dynamic-programming-based algorithm. In the least-cost path, if a phase-time arc $w_{(m_k, m'_k, \tau, h)}^k$ is selected, it means that the signal phase m_k stays green from time τ to $h-1$ and then m'_k turns green at h . Otherwise, signal phase m_k stays red from time τ to $h-1$. (See Appendix B for details).

5.4 Optimization algorithm to solve P1 and P2

Now we are ready to solve the primal problem **P1** through optimization based on the Lagrangian-relaxation. Problem **P2** provides a lower bound to **P1** for given feasible Lagrangian multipliers. On the other hand, a feasible solution from **P2** applied to **P1** gives an upper bound of **P1**. We developed an iterative algorithm to reduce the gap between the upper bound and lower bound values by adjusting the Lagrangian multipliers at each iteration using a dual accent approach. The optimization algorithm is as follows:

Step 1: Initialization

- 1.1 Set iteration number $n=0$;
- 1.2 Establish the mapping matrix $\Phi(i, j, m, k)$ for intersection k ($k \in K$) to exactly map signal phases and signal links;
- 1.3 Build the space-time network according to given time-dependent physical network
- 1.4 Build the phase-time network according to signal control logic at intersection k ($k \in K$)
- 1.5 Choose initial nonnegative values, such as 0, for all Lagrangian multipliers in $\{\{\lambda(i, j, t), \mu_t(i, j, t)\}\}$;

Step 2: Solve **Px** and **Pw**

- 2.1 Given Lagrangian multipliers $\{\{\lambda(i, j, t), \mu_t(i, j, t), \mu_s(i, j, t)\}\}$, Call the modified least-cost algorithm in Appendix A to solve **Px** to determine the values for all route choice variables $x_{(i, j, t, s)}^f$.
- 2.2 Solve **Pw** at all intersections to determine the values of $w_{(m, m', \tau, h)}^k$
 - 2.2.1 Calculate the arc costs for the phase-time network in **Pw**: obtain costs
$$c(m_k, m'_k, \tau, h) = \sum_{(i, j) \in A_S} \sum_{t=\tau}^{h-1} (\lambda(i, j, t) \times \varphi(i, j, m_k, k) \times SR(i, j, t))$$
 from latest Lagrangian multipliers. The pseudo code of calculation at intersection k is as follows:

For $\tau = 1$ to T

For $h = \tau + g_{\min}$ to $\tau + g_{\max}$

For each signal phase m_k at intersection k

For each signal phase m'_k at intersection k

$$c(m_k, m'_k, \tau, h) = \sum_{t=\tau}^{h-1} [\lambda(i, j, t) \times \varphi(i, j, m_k, k) \times SR(i, j, t)]$$

End

End

End

End

2.2.2 Call DP algorithm in Appendix B.

2.3 Calculate the objective function values of dual problem (i.e., $\mathbf{Px} + \mathbf{Pw}$) using new values for 0-1 decision variables $\{x_{(i,j,t,s)}^f\}$ and $\{w_{(m,m',\tau,h)}^k\}$; this give a lower bound of system optimum;

2.4 Obtain a feasible solution by converting flow with hard capacity constraints using network loading tools, such as DTALite (Zhou and Taylor, 2014), to get a feasible solution to P1; this gives a upper bound of system optimum;

2.5. Calculate the gap value between upper bound (Eq. (1)) and dual lower bound (Eq. (9)); terminate the iterative process if n is greater than the maximum iteration or the relative gap is smaller then a specified threshold.

Step 3: Update Lagrangian multipliers using an approximate subgradient method

3.1 Calculate subgradients for all $\lambda(i, j, t)$ and $\mu_i(i, j, t)$ as:

$$\nabla L_{\lambda(i,j,t)} = \left(\sum_f x_{(i,j,t,s)}^f \right) - SR(i, j, t), \forall (i, j) \in A_s, \forall t ;$$

$$\nabla L_{\mu_i(i,j,t)} = \left(\sum_f x_{(i,j,t,s)}^f \right) - SR(i, j, t), \forall (i, j) \in \{A \setminus A_s\}, \forall t ;$$

3.2 Updates all $\lambda(i, j, t)$ and $\mu_i(i, j, t)$ as:

$$\lambda(i, j, t) = \text{Max}\left(0, \lambda(i, j, t) + \text{stepsize} \times \nabla L_{\lambda(i, j, t)}\right);$$

$$\mu_t(i, j, t) = \text{Max}\left(0, \mu_t(i, j, t) + \text{stepsize} \times \nabla L_{\mu_t(i, j, t)}\right);$$

where step size is: $\frac{1}{n+1}$;

Go back to step 2.1 increasing iteration count to $n+1$.

The rationale of calculating subgradients this way is that, if over congestions or queue spillbacks occur on some links, it is possible to reduce the allocated traffic in the next iteration by increasing the Lagrangian multipliers (i.e., penalties). If some links can accommodate more vehicles during certain periods, then the corresponding Lagrangian multipliers can be lowered in the next iteration to attract more vehicles on those under-used links.

Step 4: Termination Examination

If the number of iterations n reaches a predefined maximum iteration or the gap values between **P1** and **P2** has been smaller than a specified tolerance threshold then STOP; otherwise, replace n by $n+1$ and go back to Step 2.

5.5 Summary of the proposed optimization approach

After the problem formulation and decomposition, the solution procedure can be summarized via the flowchart shown in Fig. 9.

(Insert Figure 9)

5.6 Discussions

Cyclic vs. flexible signal phasing sequence: Optimal signal timing may or may not exactly follow any pre-defined phasing sequence because the least-cost phasing sequence may skip certain phases to achieve the overall minimum cost. In practice, if local regulations require a fixed phasing sequence, the transition arcs should be set in such a way that a signal phase can only turn over green to the next signal phase for at least a minimum green.

Even though the simplified four-phase signal phasing schema cannot represent all compatible traffic movements at intersections, it can be easily extended. For instance, all eight possible signal phases at a four-approach intersection are illustrated in Fig. 10 (It is usually assumed that vehicle right turns are always allowed as long as acceptable gaps can be found). It is possible to choose all or part of those signal phases to construct a traffic signal phase set for a specific intersection. The selected phases also need to

meet the three signal logic requirements: minimum green, maximum green and phase mutual exclusiveness.

(Insert Figure 10)

Including the interphase loss into the space-phase-time hypernetwork

In Section 3, although we simplified the phase duration as the green time in the phase-time network, the interphase loss (yellow and all-red clearance) can be easily reflected in the phase-time network by slightly transforming the phase-time network, as shown in Fig. 11 for including the interphase loss in the phase-time network. Obviously, introducing the interphase loss will prevent very frequent phase transitions within a given time horizon.

(Insert Figure 11)

Modeling multi-modal traffic signal control mechanism using the phase-time network

Frequent traffic signal priority requests by special vehicles are increasingly common in urban areas. Developing traffic signal timing with the consideration of priorities for special vehicles is also known as “multi-modal traffic signal timing”. Developing traffic signal timings for both regular vehicles and special vehicles can be naturally formulated and solved using our proposed space-phase-time hypernetwork. In essence, granting very high-level signal preemptions (e.g., for fire trucks and ambulances) is to schedule the traffic signal timing in such a way that a particular signal phase is guaranteed green at certain time point(s) while granting low-level signal priority requests (e.g., buses) is to schedule the traffic signal timing in such a way that a particular signal phase is encouraged to turn green at certain time point(s) by providing certain arcs with additional benefits (i.e., negative costs) which may be dynamically adjusted for various scenarios. As illustrated in Fig. 12, for a low-level bus priority request, additional benefit is introduced to an inbound arc and the least-cost path finding algorithms may possibly select $(3, t_1)$, depending on how much the additional arc benefit is. On the other hand, for a high-level fire truck preemption request, vertex $(2, t_2)$ must be visited and this requirement is an additional constraint for the least-cost path finding algorithm. The final optimal signal timing will be required to pass the vertex $(2, t_2)$ to reach the end of horizon. When multiple high priority requests are exogenously scheduled within the time horizon, a least-cost path required to passing several vertexes can be computed in the space-phase-time hypernetwork as well. However, it should be noted that it may not be possible to find a feasible route to visit the mandatory vertices (high-level preemptions) from starting vertices to the end of time horizon. In that case, some high-level preemptions must be declined or postponed to have feasible signal timings.

(Insert Figure 12)

Multi-objective traffic signal optimization through seeking K-least-cost paths in the phase-time networks

In many cases, the objectives of interests in traffic signal optimization are more than one. For instance, it is often desired to achieve a balance between traffic mobility and safety under appropriate signal timings. Such problems are referred to as “multi-objective traffic signal optimization problems” and they are often solved to reach the Pareto optimality (Censor 1977). Based on the proposed phase-time network, it is possible to perform multi-objective traffic signal optimization. As shown in Fig. 13, while performing the multi-objective traffic signal optimization, instead of finding one least-cost path from starting vertex to the end of horizon, k least-cost paths are using certain polynomial-time search algorithms, such as the K-shortest path finding algorithm by Martins (Martins 1984). It is obvious that each feasible path represents a feasible signal timing which in turn results in a direct MOE (the path cost) and other derivative MOE(s) based on their relationships with the direct MOE. If all feasible timings are plotted in a multi-dimension diagram according to their resulting MOEs, a Pareto frontier can be drawn and the most appropriate signal timing solutions can be selected according to the balance between two MOEs, such as travel delay vs. traffic emissions, which can be generated from an integrated traffic flow and emission estimation model (e.g., Zhou et al., 2015).

(Insert Figure 13)

Time resolution in constructing the space-phase-time hypernetwork

Although we uniformly assume the one-second time resolution in this paper, the one-second time resolution may not be necessary in some traffic control applications, such as ramp metering. In that case, one can only keep those links and arcs compatible with the lowest time resolution in the space-phase-time hypernetwork.

Model’s suitability for distributed computing: since **P2** is solved by searching the least-cost path for each vehicle and searching optimal signal timing at each intersection, from a computing perspective, the proposed optimization algorithm can be naturally computed on multiple computers when the scope of problem becomes large. Fig. 14 illustrates this hierarchical structure.

(Insert Figure 14)

6. Computational results

6.1 Experiment One: single simplified intersection

The network illustrated in Fig. 15 represents a simplified one-way intersection with two signal phases. Using this simple example, we aim to illustrate how to update vehicle routes and signal timing during the iterative optimization. The time horizon is 2 s and link travel times in second are shown in Fig. 15. The minimum and maximum greens are set as 1 s and 2 s for both signal phases. One vehicle moves from node 1 to node 2, the other vehicle moves from node 3 to 4, and both vehicles are required to reach their destinations at no later than the 2 s. In addition, assume that the initial timing is: phase one is green at $t=1$ and phase two is green at $t=2$. The phase-time network is built as in Fig. 16.

(Insert Figure 15)

First iteration (n=1):

Step 1: Search least-cost paths for two vehicles in space-time network. The upper portion of Fig. 16 shows the initial optimal routes for two vehicles under uninterrupted traffic condition and the total travel time for two vehicles is 2 s. Therefore the value of primal problem (P1) is equal to 2 and the value of P_x is equal to 2 as well.

Step 2: Calculate subgradients and update multipliers according to Section 5.4 and the results are shown in Table 2. According to the current timing, the time-dependent link capacity is: $SR(1, 2, 1) = 1$; $SR(1, 2, 2) = 0$; $SR(3, 4, 1) = 0$; $SR(3, 4, 2) = 1$.

Step 3: Signal optimization: The arc-costs for the phase-time network are listed in Table 2 and it is found that the least-cost path in the phase-time network is $(1, 1) \rightarrow (2, 2) \rightarrow (1, 3) \rightarrow (Z, 4)$ (the same as the current timing) and $P_w = -0.5$. The gap value between the primal and dual problems is 0.5.

Second iteration (n=2)

Step 1: Update the least-cost paths for two vehicles according to the updated Lagrangian multipliers, shown in Fig. 14. Therefore $P1=3.0$ s and $P_x=3.5$ s; the corresponding timing is unchanged and $P_w=-0.5$;

Step 2: Update subgradients and Lagrangian multipliers: The updated subgradients, Lagrangian multipliers and arc-costs in phase-time networks are updated as shown in Table 2;

Step 3: Get feasible solution to P1: The solution to P2 is also a feasible solution to P1 in this case, therefore there is no need for network loading to get the feasible solution to P1;

After the 2nd iteration, $P1=3.0$; $P2=P_x+P_w=3.0$. The gap value between the primal and dual problems is 0. Therefore, after the 2nd iteration, all subgradients are zero and the optimal solution is reached.

(Insert Figure 16)

(Insert Table 2)

6.2 Experiment Two: network grid with 9 signalized intersections

In the second example, we solve the RGTSO problem on a three-by-three road network as in Fig. 1. Some inputs are:

- Time resolution: 1 s;
- Time Horizon: 400 s;
- Link capacity (saturation rate): one vehicle per second;
- Link travel time: 3 s on signal links; 10 s on travel links;
- Signal phases at each intersection: four phases as in Fig. 3 with fixed phasing sequence: $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1 \dots$; min and max greens are 3 s and 20 s respectively;
- Signal phase at all intersections: Start with east bound and west bound, cyclic and the green duration of each phase is at least the minimum green (3 s); and
- Initial values of $\lambda(i, j, t)$ and $\mu_t(i, j, t)$: zero;

In addition, drivers are supposed to provide their origins and destinations before their trips and therefore each agent's origin and destination nodes are known. The O-D matrix within the time horizon is summarized in Table 3. The total number of vehicles is 960 and they are required to reach their destination before the time horizon is reached.

(Insert Table 3)

Evolution of System optimum: the initial signal timing is determined according to the vehicle routes under uninterrupted traffic conditions. In each iteration, each vehicle's least-cost route is searched according to the latest arc costs, link travel times and signal timings. Then the arc costs are updated according to the link usages and link capacities using the subgradient method. This optimization iterates totally 50 times. To evaluate the scalability of the proposed optimization approach, two scenarios are created: 960 vehicles with full routing options; 480 vehicles (50% reduction) with restricted routes (see Fig. 17). From Fig. 17, the smallest relative gaps for both scenarios are approximately 30%, similar with the reported values in other literature Meng and Zhou (2014). We think a possible reason for the gap is that the traffic signal control logic is in nature discrete and therefore link flow capacity may become stepwise while a signal phase turns over greens to the next phase. Subgradients at those time moments will be less effective in determining new search directions. To solve this problem, Han et al. (2014, 2015a, 2015b) proposed the continuum approximation for such type of on-and-off signal control so as to have a differentiable attribute in searching optimal solutions. Lo (1999) also applied the continuum

approximation in modeling the adaptive signal control. Another reason is that we dropped the Lagrangian multipliers for queue spillback constraints in P_x and it represents a relatively loose lower bound of SO solution to P1.

From the two scenarios of Experiment Two, we consider the proposed optimization approach in this paper is stable and has the similar performance under different scenarios.

The signal timings and route guidance options output from the 48th iteration and the 43rd iteration are selected as the optimal solutions. Using a 32-core computing server and OpenMP[®] parallel computing technique, the computing time of 50 iterations was approximately 90 s. 67 percent of computing time was for finding least-cost paths for vehicles and 33 percent of computing time was for network loading to get feasible solutions to the primal P1.

(Insert Figure 17)

Selected route guidance solution: To demonstrate the selected route guidance solution, we selected three vehicles to compare the difference of their initial and optimized routing policies. The origins and destinations of those three vehicles are shown in Fig. 18. It can be seen that, under the optimized routing policies (solution to P2), vehicles may hold at their origins until they can find least-cost paths to their destinations to minimize the Lagrangian penalties. However, the initial least-cost path finding algorithms (for P1) can only prevent vehicles from violating traffic signals but cannot prevent violations of link capacities because the least-cost path of each vehicle is calculated separately. So the initial shortest routes for vehicles are not realistic. After the dynamic network loading, it is observable that vehicles form queues due to the capacity constraints if they try to cross an intersection from the same approach at the same time. Comparing the final feasible solution of three vehicles' paths with the initial feasible solution, vehicle 1's travel time is longer than its initial path, vehicle 2's travel time is unchanged even though it is held at the origin node, and vehicle 3's travel time is significantly reduced. The total travel time of 3 vehicles is reduced.

(Insert Figure 18)

6.3 Experiment Three: solve the RGTSO problem within an mixed traffic environment

In Experiment Three, we solve the RGTSO problem in a more realistic condition where some guided vehicles share the road network with regular (i.e., unguided) vehicles. The road network in Fig.1 and O-D demands in Table 3 are used in the third experiment. When the guided vehicles and the unguided vehicles co-exist in the road network, the capacity constraints defined in (3) and (4) should be modified as:

The total inflow traffic on signal link (i, j) for intersection k :

$$\sum_{f \in G} (x_{(i,j,t,s)}^f) + \sum_{f' \in NG} (x_{(i,j,t,s)}^{f'}) \leq y_{(i,j,t)}^k \times SR(i,j,t), \forall (i,j) \in A_s, 1 \leq t \leq T \quad (13)$$

The total inflow traffic on travel link (i, j) :

$$\sum_{f \in G} (x_{(i,j,t,s)}^f) + \sum_{f' \in NG} (x_{(i,j,t,s)}^{f'}) \leq SR(i,j,t), \forall (i,j) = A \setminus A_s, 1 \leq t \leq T \quad (14)$$

where G and NG are sets of guided vehicles and non-guided vehicles, respectively. The spatial constraint of any link (i, j) can be also derived accordingly,

Since the unguided vehicles likely follow the UE principle and the signal timings are impacted by both guided and unguided vehicles, Experiment Three in essence leads to a DTA model for the multiple user classes. Interested readers are suggested to read the literature by Peeta and Mahmassani (1995b). For simplicity, we use a simulation-based two-stage approach in this experiment to evaluate the solutions to the guided vehicles with the background traffic:

Stage 1: following the approach proposed by Lu et al. (2009), all vehicles are assigned with routes in such a way that traffic reaches the dynamic user equilibrium (UE) condition under fixed signal timings. **The uniform phasing sequence is $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1, \dots$ and the phase durations are uniformly 20 s (maximum green).**

Stage 2: after the traffic reaches the UE condition, the routes of guided vehicles are adjusted with the proposed Lagrangian-relaxation-based optimization framework while the unguided vehicles keep their original UE routes from the first stage. In the meantime, the signal timings become adaptive and adjusted according to both updated routes of guided vehicles and the original UE routes of non-guided vehicles.

It should be pointed out that the above UE condition is approximate because the unguided vehicles likely further adjust their paths for updated signal timings to reach a new UE condition. **One possible solution to resolve this issue is to conduct multiple iterations for non-guided vehicles to re-route based on the latest approximate UE condition. This is different from the current “one-shot” approach to reaching the approximate UE condition where the non-guided vehicles are assumed not to change their original routes. Further examination on how to reach a more realistic UE condition is planned in our future study.**

Fig. 19 shows the evolutions of upper bound, lower bound and gap values under the 20%, 40%, 60% and 80% penetration rates of guided vehicles, respectively. From Fig. 19 we can conclude that, if we only focus on the guided vehicles, the minimal relative gaps are similarly around 30% under different penetration rates. However, from Fig. 20, it is clear that the average travel time of all vehicles (guided and non-guided) will significantly reduce with the increase of penetrate rates.

(Insert Figure 19 and Figure 20)

7. Conclusions and future research

This paper is inspired by emerging research needs for modeling simultaneous route guidance and proactive traffic signal control within an environment of connected and autonomous vehicles. A feature of connected or autonomous vehicles is highly reliable two-way communications between vehicles and infrastructure which makes it possible to achieve a system optimum (i.e., minimum total travel time) through synergistic route guidance and traffic signal control.

The simultaneous *Route Guidance and Traffic Signal Optimization* (RGTSO) has been formally described in the paper and formulated in a newly defined *Space-Phase-Time hypernetwork* to allow the explicit joint consideration of traffic dynamics in an urban network and signals at intersections. Each vehicle is explicitly treated and guided on minimum cost paths, which includes costs through signalized intersections, in a system optimal network loading. Link costs are obtained by aggregating individual vehicles' over space-time trajectories through the links. A set of capacity constraints were superimposed to avoid traffic signal phase violations with over-congestion or queue spillbacks.

The crux of the model development is the Space-Phase-Time hypernetwork which (a) explicitly models allowable movements at intersections given phase decisions, (b) allows flexible phase sequencing within the network structure, (c) allows vehicles to travel from physical node to adjacent node in a discrete event fashion, and (d) effectively moves green durations to provide capacities for intersection movements. The Space-Phase-Time hypernetwork has an effective pre-processing step that implicitly discards all inapplicable decisions (for examples., a phase duration has to be within specified min-green and max-green, finite capacity of spill back queue, and restrictions in phase sequencing if required by a jurisdiction, any imposed capacities of vehicles on a link, etc.) in the network construction. It should however be noted that time is treated as a discrete variable, for example at a resolution of one second in this paper, to end up with a Space-Phase-Time hypernetwork.

The RGTSO problem was formulated as a nonlinear integer program. A Lagrangian-relaxation-based optimization framework was used to solve the RGTSO problem. This results in a separable objective function for given Lagrangian multipliers (i.e., the dual variables). The resulting separable subproblems have the well-known structure of shortest path and minimum-cost flow problems which can be solved using pseudo-polynomial finite-horizon dynamic programming approaches. The dual problem of adjusting the Lagrangian multipliers was solved by an ascent method using approximate subgradients. The structure of the relaxed problem, with well-defined subproblems is naturally suitable for parallel computing and distributed computing techniques.

In numerical experiments, the RGTSO problem was solved on a network containing 9 intersections and 960 vehicles was solved with a sufficiently small relative gap of in system optimal objective value of total travel time. We also consider a more realistic traffic condition with both guided and regular (non-guided) vehicles under an initial user equilibrium condition. The results show that the optimizing process can always reduce the total system-wide travel times, especially the penetrate rate of guided vehicles is large. , It would be useful to perform sensitivity analysis of the optimization framework with respect to different parameter settings such as the length of time horizon and modeling resolution.

The Space-Phase-Time hypernetwork modeling allows us to include a whole range of practical considerations and real-time advanced applications such as:

- (a) In real-time applications, signal preemptions will proactively make some signal links “on” that is the corresponding 0-1 decision variable will be 1.
- (b) In real-time applications, provide signal priority to special vehicles such as buses and trucks by using more “weight” for the vehicle in that it would represent a demand of many vehicles and thereby favoring the corresponding green phase.
- (c) In real-time applications, provide integrated estimation and control models for proactive traffic signal control systems using estimated traffic states from multiple data sources, e.g., based on a stochastic three-detector approach proposed by Deng et al. (2013) and a shockwave-based queue estimation via high-resolution detector data proposed by Liu et al. (2009).
- (d) The formulation of the RGTSO problem allows the consideration of multiple weighted objectives that could minimize energy, minimize emissions, etc.
- (e) Including “pricing” mechanisms through Lagrangian multipliers to consider congestion tolls, and incentives for taking more sustainability-friendly alternate routes, and
- (f) Including addition preprocessing of the Space-Phase-Time hypernetwork by adding space-time links that consider platooning of automated vehicles for more efficient passenger transportation.

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Appendix A: Dynamic programming approach to search waiting-allowable least-cost path in space-Time networks

In a space-time network $STG=(V, E)$, denote $lc(i, t)$ as the label cost of node (i, t) ; $Pred(i, t)$ as the predecessor node of node (i, t) ; $FFTT(i, j)$ as free flow travel time of physical link $l=(i, j)$; $c(f, i, j, t, s)$ as travel cost from (i, t) to (j, s) , $s=t+FFTT(i, j)$ and it is associated with the corresponding coefficient of $x_{(i, j, t, s)}^f$ in $\mathbf{Px}$ and n_l^U, n_l^D as the upstream node and downstream node of link l ; For vehicle f , its least-cost path from $o(f)$ to $d(f)$ can be searched using the following waiting-allowable least-cost finding algorithm based on dynamic programming:

Step 1: Initialization

$lc(o(f), r(f)):=0$; $Pred(o(f), r(f)):=0$; $lc(j, t):=\infty$ for each node $(j, t) \in N - \{(o(f), r(f))\}$; and $LIST:=\{(o(f), r(f))\}$;

Step 2: Recursion

For $t = r(f)$ to $T-1$

For each physical link $l \in E$

If $lc(n_l^U, t) + c(f, n_l^U, n_l^D, t, t+FFTT(i, j)) < lc(n_l^D, t+FFTT(i, j))$

Then $lc(n_l^U, t) + c(f, n_l^U, n_l^D, t, t+FFTT(i, j)) = lc(n_l^D, t+FFTT(i, j))$

$Pred(n_l^D, t+FFTT(i, j)) = (n_l^U, t)$

End if

If n_l^U is a waiting node or an origin node

If $lc(n_l^U, t+1) > lc(n_l^U, t) + c(f, n_l^U, n_l^U, t, t+1)$

Then $lc(n_l^U, t+1) = lc(n_l^U, t) + c(f, n_l^U, n_l^U, t, t+1)$

$Pred(n_l^U, t+1) = (n_l^U, t)$

End if

End if

End//for each link

End // for each stage

Step 3: Trace back to get the least-cost path

From the destination node $(d(f), T)$, trace back using $Pred(j, t)$ until reaching the origin node $(i, 0)$;

According to the algorithm structure, the upper bound of computing complexity for each vehicle f is:

$$T \times (\text{No. of links} + \text{No. of waiting nodes})$$

Appendix B: Dynamic programming approach to optimize traffic signal timings in Phase-Time network at intersection k

In a phase-time network $PTG = (PV, PA)$ for intersection k , denote Ψ as the set of signal phases; signal phase $m_k, m'_k \in \Psi$; τ, h as the time index; $lc(m_k, \tau)$ as the label cost of vertex (m_k, τ) with the initial value of 0 if m_k = starting phase and $\tau = 0$, and positive infinity otherwise; $Pred(m_k, \tau)$ as the predecessor of vertex (m_k, τ) ; $c(m_k, m'_k, \tau, h)$ as arc cost from (m_k, τ) to (m'_k, h) . T is the time horizon. If this problem is viewed as a T-stage problem and each stage stands for one second, the DP-based optimization can be formulated as below:

For $\tau = 0$ to T

For each signal phase $m_k \in \Psi$

For each signal phase $m'_k \in \Psi$ ($m'_k \neq m_k$)

For $\Delta t = g_{\min}$ to $g_{\max}$

If $(lc(m'_k, \tau + \Delta t) > lc(m_k, \tau) + c(m_k, m'_k, \tau, \tau + \Delta t))$

Then $lc(m'_k, \tau + \Delta t) = lc(m_k, \tau) + c(m_k, m'_k, \tau, \tau + \Delta t)$

$Pred(m'_k, \tau + \Delta t) = (m_k, \tau)$

End If

End// for each second between $\tau + g_{\min}$ and $\tau + g_{\max}$

End //for each next signal phase

End //for each current signal phase

End //for each stage

$\tau + \Delta t = h$ and h is defined in (6) and (8). The optimal phasing sequence and phase durations can be traced back after DP-based search is finished.

Computing complexity: according to the algorithm structure, the computing complexity is polynomial for each intersection as below:

$$T \times (\text{No. of Phases}) \times (\text{No. of phases} - 1) \times (g_{\max} - g_{\min})$$

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Table 1. Set, subscripts, parameters and variables used in mathematical formulations

Physical Network to Represent Road Network	
N	Set of nodes in physical network
A	Set of links in physical network
N_s	Set of signal nodes in physical network. $N_s \subset N$. A signal node is to connect a travel link and a signal link
A_s	Set of signal links in physical network. $A_s \subset A$. A signal link is open if the corresponding movement is allowed (green traffic light); Otherwise, closed (red traffic light).
f	Vehicle index, $f \in F$, F is the set of vehicles
$o(f), d(f)$	Origin and destination of vehicle f
$r(f)$	Initial departure time (i.e. release time) of vehicle f
i, j, i'	Node index in physical network, $i, j, i' \in N$
$(i, j), (j, i')$	Link index in physical network, $(i, j), (j, i') \in A$
k_{jam}	Jam density (number of vehicles per mile per lane)
$L_{(i, j)}$	Length of link (i, j) in physical network in terms of lane-mile
Space-time Network to Represent Vehicle Flow Network	
V	Set of space-time vertexes in space-time network
E	Set of space-time arcs in space-time network
(i, t)	Space-time node index in space-time network; $i \in N$; t : index of time interval, $t = 1, \dots, T$, T is the length of planning horizon
$FFTT(i, j, t)$	free-flow travel time on physical link (i, j) at time t
(i, j, t, s)	Space-time link index in space-time network associated with a vehicle entering link (i, j) at time t leaving from link (i, j) at time s
$c(f, i, j, t, s)$	Cost for vehicle f to travel on space-time link (i, j, t, s)
$A(i, j, t)$	Cumulative arrival flow count on link (i, j) from time 0 to t
$D(i, j, t)$	Cumulative departure count on link (i, j) from time 0 to t
$SR(i, j, t)$	Saturation capacity or maximum capacity per unit time interval on physical link (i, j) at time t

Phase-Time Network to Represent Traffic Signal Control Logic	
PV	Set of phase-time vertexes in phase-time network
PA	Set of phase-time arcs in phase-time network
k	Intersection index, $k \in K$, K is the set of intersections
$g_{\min}, g_{\max}$	Minimum green and maximum green
m_k, m'_k, m''_k	Signal phase index at intersection k
τ, h	Starting time (τ) and ending time (h) of a signal phase, serve as the variable indices for signal control variables
$(m_k, \tau), (m'_k, h)$	Phase-time vertex index at intersection k
(m_k, m'_k, τ, h)	Phase-time arc index in phase-time network
$c(m_k, m'_k, \tau, h)$	Cost for signal phase m_k is green from time τ to $h-1$ and then turns green to signal phase m'_k at time h at intersection k
$\Phi(i, j, m_k, k)$	Mapping matrix from a signal phase m_k to its corresponding signal link (i, j) at intersection k : $\varphi(i, j, m_k, k) = 1$ if the mapping exists; otherwise 0

Decision Variables

$x_{(i,j,t,s)}^f$	Route guidance decision variable, =1 if a space-time arc (i, j, t, s) is chosen in the route for vehicle f ; 0 otherwise
$y_{(i,j,t)}^k$	Signal link status variable at intersection k : =1 if the signal link (i, j) is open from t to $t+I$; 0 otherwise
$w_{(m_k, m'_k, \tau, h)}^k$	Signal phase variable: = 1 if signal phase m_k is green from τ to $h-1$ and then turns green to signal phase m'_k at time h ; otherwise 0

Lagrangian Multipliers

$\{\lambda(i, j, t)\}$	Set of Lagrangian multipliers as penalties for dualized inflow capacity constraints on signal links in physical network, $(i, j) \in A_s$, $t = \{1, 2, \dots, T\}$
$\{\mu_t(i, j, t)\}$	Set of Lagrangian multipliers as penalties for dualized inflow capacity

	constraints on travel links in physical network, $(i, j) \in \{A \setminus A_s\}$,
	$t = \{1, 2, \dots, T\}$
$\{\mu_s(i, j, t)\}$	Set of Lagrangian multipliers corresponding to queue spillback constraints on all links in physical network, $(i, j) \in A$, $t = \{1, 2, \dots, T\}$

Table 2 Lagrangian multipliers and updated signal timing after two iterations

Subgradients and Multipliers	$\nabla_{\lambda(1,2,1)}$	$\nabla_{\lambda(1,2,2)}$	$\nabla_{\lambda(3,4,1)}$	$\nabla_{\lambda(3,4,2)}$	$\lambda(1,2,1)$	$\lambda(1,2,2)$	$\lambda(3,4,1)$	$\lambda(3,4,2)$
1 st Iteration	0	0	1	0	0	0	1/2	0
2 nd Iteration	1	0	0	0	1/3	0	1/2	0

Arc-cost	$c(1, 2, 1, 2)$	$c(1, 2, 1, 3)$	$c(1, 2, 2, 3)$	$c(2, 1, 1, 2)$	$c(2, 1, 1, 3)$	$c(2, 1, 2, 3)$
$c(m_k, m'_k, \tau, h)$						
1 st Iteration	0	0	0	-1/2	-1/2	0
2 nd Iteration	-1/3	-1/3	0	-1/2	-1/2	0

Table 3 Origin-Destination Matrix in Experiment Two and Experiment Three

	(D)1	3	5	33	60	86	94	92	90	88	62	32
(O)2	0	6	6	6	6	6	6	6	6	12	6	6
4	6	0	12	6	12	12	6	12	6	12	6	6
6	6	6	0	6	6	6	6	6	6	12	6	12
34	6	12	12	0	6	6	6	6	6	12	12	6
61	6	6	6	6	0	6	6	12	6	12	6	6
87	6	6	6	6	12	0	6	6	6	12	6	6
93	6	12	6	6	6	6	0	6	6	6	6	6
91	6	6	6	6	6	6	12	0	6	12	12	6
89	6	6	6	6	6	6	6	6	0	12	6	6
63	6	6	6	6	6	6	6	6	6	0	12	12
35	6	6	6	6	12	6	6	6	6	12	0	12
7	12	6	6	6	6	6	6	6	6	6	6	0

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