1 Reply to comment by Chen et al. on “Controls on the size and geometry
2 of landslides: Insights from discrete element numerical simulations.”

3 Einat Aharonov^{1,*}; Oded Katz²; Julia Morgan³; Brandon Dugan³

4 1) The Institute of Earth Sciences, The Hebrew University, Givat Ram, Jerusalem 91904, Israel

5 einatah@cc.huji.ac.il

6 2) Geological Survey of Israel, 30 Malkhe Israel St., Jerusalem 95501, Israel

7 3) Department of Earth Science, Rice University, 6100 Main St., Houston, TX 77005, USA

8 *Corresponding author. Tel.: +972 2 6584670; E-mail: einatah@cc.huji.ac.il

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10 **Abstract**

11 Chen et al.'s comment presents Limit Equilibrium (LE) calculations of slope stability,
12 which yield different landslide geometries than those obtained by Katz et al. (2014) using the
13 Discrete Element Method (DEM). Previous work, however, has demonstrated excellent
14 agreement in the slide geometries and sizes obtained by DEM vs. those obtained by limit
15 analysis, thereby lending confidence to DEM and to limit analysis as methods to study slope
16 instability and geometry. We suggest three reasons why the LE results may differ from DEM:
17 (1) LE is a static method, which seeks a single failure surface to predict slope stability.
18 Although it captures well the average slope conditions, the details of the stress distribution
19 may be inaccurate. (2) DEM is a dynamic method that holistically simulates the evolution of
20 stress and strain. Thus it is better suited to simulate far from equilibrium situations, such as
21 overly steep slopes with $FS < 1$, which have strong dynamic responses. (3) The geometries of
22 the slides presented by Chen et al. appear to be constrained by the domain size. We expect
23 that a larger numerical domain may allow exploration of additional slide geometries,
24 potentially with better correspondence to those of the DEM simulations.

25 *Keywords:* landslide, slope failure, limit equilibrium, Discrete Element Method

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28 **Reply**

29 It is important to compare the results from different analytical methods for slope
30 stability and to contrast failure geometry obtained by the various methods, as was done by
31 Chen et al. in their comment. Previous comparisons between the Discrete Element Method
32 (DEM) and limit analysis were made by Utili and Nova (2008) and Utili and Crosta
33 (2011a,b) for assessing failure of slopes and cliffs. These studies found excellent agreement
34 between the two numerical methods, lending confidence to the DEM and to the limit analysis
35 as reliable methods for studying geometry of slope instability.

36 In Katz et al. (2014), we used the DEM method to study slope failures and found that
37 slide size increases as slopes become steeper. Utili (2005) (his Fig. 23) and Utili and Crosta
38 (2011b) (their Fig. 19) showed a similar relation when simulating retrogressive failure of
39 cliffs using limit analysis and DEM methods (Utili and Nova, 2008). To further this
40 comparison, we present a reanalysis of the data from Fig. 23 from Utili (2005) where the
41 slope fails in a retrogressive manner. This enables us to measure collapsed volume in each
42 failure event, as a function of average slope angle at the time of collapse (Fig. 1); as the slope
43 is not planar it is necessary to define average slope angles. The combination of our reanalysis
44 (Fig. 1), Utili and Crosta's (2011b) Fig. 19, and our numerical experiments (Katz et al., 2014)
45 all confirm that that slide volume increases as slope steepens, whether modeled using DEM or
46 limit analysis methods.

47 In light of benchmarking of DEM to assess slope stability, and in light of similar
48 results obtained by Utili (2005) and Utili and Crosta (2011b), it warrants asking why do the
49 results of Chen et al. differ when using the limit equilibrium (LE) method?

50 Reviewing the previous work, our work, and the presentation of Chen et al., we
51 suggest three possible reasons for the divergence of results.

(1) The LE method predicts simultaneous failure along a single failure surface, where the problem is statically determinate with no considerations of internal force distributions.

The LE static formulation does not give realistic forces, as presented by Krahn's 2001 Hardy Lecture (Krahn, 2003):

The fundamental shortcoming of limit equilibrium methods, which only satisfy equations of statics, is that they do not consider strain and displacement compatibility... the stress conditions as computed from a limit equilibrium analysis may be vastly different from finite element computed stresses. The finite element stresses are more realistic and are much closer to the actual conditions in the ground. The implication is that the limit equilibrium computed stresses are not representative of actual field conditions... It is interesting that the early developers of the method of slices recognized the limitations of computing realistic stresses on the slip surface. ... Morgenstern and Sangrey (1978) state that 'one of the uses ... of the factor of safety is to provide a measure of the average shear stress mobilized in the slope'. They go on to state that 'This should not be confused with the actual stresses'. Unfortunately, these fundamental issues are sometimes forgotten as use of a method is gradually adopted in routine practice.

A well-known example where LE produces unrealistic failure surfaces occurs when $\phi = 0$ (undrained conditions). In that case, the most unstable failure surface is predicted to be an arc with a radius much deeper than the slope height (see Fig. 25a of Uti, 2005). These unrealistic results were already known to Taylor (1937, 1948) and render LE methods unsuitable to study evolution of slopes under undrained conditions (Uti, 2005). Thus, it is critical to emphasize that the LE gives on average correct predictions for the Factor of Safety (FS), but the stresses are not accurate. The stresses are important for failure mobility and also

for the potential for retrogressive failure where stresses are dynamic. In such scenarios it is more advisable to use DEM or other dynamic methods for details such as geometry of failure and its evolution under deformation.

(2) It has been observed and calculated that slope failure often initiates at the lowermost part of the slope where stresses are highest. This has been modeled (see Fig. 3, Katz et al., 2014) and has been observed in natural slopes: (<https://www.youtube.com/watch?v=ZVjr4mii3cE&index=1&list=PLCK0UVoRSk1Vpt4lYwDmRWbc3MOMgfgqQS>). In this video, the sliding plane is clearly seen climbing from the foot to the crest of the cliff before failure. This progression of slope failure migrating upslope from the toe is not addressed with LE models, which assume a continuous failure surface and simultaneous failure. The fact that LE deals with statics does not allow it to predict failure evolution spatially or temporally, i.e., the process by which failure occurs and changing mechanical conditions that enable it. The slopes that Chen et al. analyzed, which are similar to the slopes that Katz et al. analyzed, are actually not near 'limit equilibrium'. These slopes all actually have FS far below 1, and they represent situations that are far from stability and thus may develop strong dynamic effects. Limit equilibrium is not suited for such slopes or for understanding dynamic effects. The DEM has the advantage of being a holistic approach enabling analysis of the development of the failure surface in time and space, simulating force changes during failure and deformation of the sliding mass that may impact its motion, as well as simulating the resulting deposit and its effects on slope stability.

(3) It is predicted that slope failure surfaces in the case of planar slopes (as modeled by Katz et al. (2014) and Chen et al. above) will pass through the toe of the slope or, for a very low friction angle, below the toe (see also Klar et al., 2011). The fact that Chen et al.

predicted situations where failure does not reach the toe of the slope (their Figs. 2d,e) is unexpected for a planar slope, and may be attributed to model-size affect, where their numerical box is too small to develop the failure surface that will pass through the toe. We believe that the size of their simulation box may have impacted the failure size and shape that they obtained. In addition to the partial slope failures seen in Chen et al. Figs. 2d,e, model size effects are also suggested by comparing the five presented slides in their Figs. 2a-e. All these slides are seen to daylight at the very top left edge of their numerical box. For any numerical modeling approach for slope failure, it is advisable to use model domain much larger than the predicted failure surface so as to avoid numerical or boundary constraints controlling the results. A larger solution domain allows a more robust investigation of true mechanical constraints on the system. We believe that using the same LE but with a larger simulation box may change their results, as it would allow their LE model to explore a broader range of landslide sizes and shapes, yielding results that might be more comparable to DEM simulations.

To conclude, LE approaches are well suited for assessing slope stability in many cases, especially when $FS \geq 1$. Indeed, LE provides a good approximation of average conditions on the slope. However, LE does not provide the real stresses and cannot simulate slopes that undergo dynamic failures and evolving failure surfaces. These situations, especially in the strong dynamic regime of $FS < 1$, are better resolved and understood with numerical approaches that accommodate failure dynamics, such as DEM, or other dynamic methods that allow modeling of the full process of slope failure and drawing of inferences about resulting landslide sizes and deposits.

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Figures

Fig. 1. Analysis of the landslide geometries plotted in Utili (2005), Fig. 23, as derived from his limit analysis simulations. Theta (θ), the average slope angle at time of collapse, is plotted vs. collapsed slide volume normalized to the first collapse volume. The analysis was done for two friction angles, $\phi = 15$ and $\phi = 35$.