Magnetic Field Devices for Neutron Spin Transport and Manipulation in Precise Neutron Spin Rotation Measurements

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Abstract

The neutron spin is a critical degree of freedom for many precision measurements using low-energy neutrons. Fundamental symmetries and interactions can be studied using polarized neutrons. Parity-violation (PV) in the hadronic weak interaction and the search for exotic forces that depend on the relative spin and velocity, are two questions of fundamental physics that can be studied via the neutron spin rotations that arise from the interaction of polarized cold neutrons and unpolarized matter. The Neutron Spin Rotation (NSR) collaboration developed a neutron polarimeter, capable of determining neutron spin rotations of the order of 10^{-7} rad per meter of traversed material. This paper describes two key components of the NSR apparatus, responsible for the transport and manipulation of the spin of the neutrons before and after the target region, which is surrounded by magnetic shielding and where residual magnetic fields need to be below 100 μ G. These magnetic field devices, called input and output coils, provide the magnetic field for adiabatic transport of the neutron spin in the regions outside the magnetic shielding while producing a sharp nonadiabatic transition of the neutron spin when entering/exiting the low-magnetic-field region. In addition, the coils are self contained, forcing the return magnetic flux into a compact region of space to minimize fringe fields outside. The design of the input and output coil is based on the magnetic scalar potential method.

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33 1. Introduction

34 The determination of very small neutron spin rotations ($d\phi/dz \approx 1 \times 10^{-7}$
35 rad/m) in the interaction of polarized low-energy neutrons and unpolarized
36 matter is of relevance for different reasons. Corkscrew neutron spin rotations
37 of transversely polarized neutrons can come from the weak interaction between
38 the neutrons and the atoms of the material they interact with. The nucleon-
39 nucleon (NN) weak interaction remains poorly understood due to the dominance
40 in intensity of the strong interaction and the non-perturbative nature at low en-
41 ergies of the theory that describes it, quantum chromodynamics (QCD). The
42 measurement of the weak coupling constants that appear in the theories that
43 aim to describe the interaction and that cannot yet be calculated accurately [1–
44 3] is underway by several experimental collaborations. Experiments that involve
45 few-nucleon systems and low-energy polarized neutrons have been developed to
46 measure PV observables that arise from the NN weak interaction [4–6]. These
47 observables can be related to the weak coupling constants and a lot of effort
48 has been dedicated to determine enough linearly independent combinations of
49 them, in systems where the uncertainties related to nuclear wavefunctions are
50 smaller than in the case of many-nucleon systems [7, 8].

51 A different aspect of neutron spin rotations as a result of the interaction of neu-
52 trons with matter is the possibility to search for exotic forces. In particular for
53 the case of forces with relatively long range (μm to cm) that depend on the spin
54 of one (or both) of the interacting particles, experimental constraints are scarce
55 due to the technical challenges that macroscopic amounts of polarized matter
56 impose on precision experiments. However, setting limits on these type of forces
57 is important since a number of theories predict the existence of new interactions
58 mediated by light particles with weak couplings to matter and long ranges, ei-
59 ther as a result of broken symmetries in theories beyond the Standard Model,

60 or from theoretical attempts to explain dark matter and dark energy [9–11].
 61 Polarized low-energy neutrons have emerged as an effective tool to search for
 62 these type of forces since intense beams of polarized neutrons are available, their
 63 momentum transfer corresponds to the mesoscopic scale and, because of their
 64 lack of electric charge, they can pass through macroscopic amounts of matter
 65 suffering very small attenuations.
 66 The Neutron Spin Rotation (NSR) collaboration has built a slow neutron po-
 67 larimeter to measure neutron spin rotations that are of the order of tenths of
 68 micro radians per meter of traversed target material. The experimental appa-
 69 ratus, described in detail in [12], has several purposes: It was first developed
 70 to measure the PV neutron spin rotation angle for polarized neutrons passing
 71 through unpolarized ^4He , however it was found that the results of the first stage
 72 of this experiment [6] could set the most stringent constraints, in the sub-cm
 73 range, for a possible PV exotic force between the neutron and matter, mediated
 74 by the exchange of a light spin 1 boson and the first upper bound on parity-
 75 odd components of possible in-matter gravitational torsion coupled to neutrons
 76 [13]. The upgraded version of the NSR apparatus, to be used to improve the
 77 measurement of the PV NSR in $n\text{-}^4\text{He}$ (weak interaction as well as PV exotic
 78 forces) is also being used in an experiment to search for a possible parity-even
 79 exotic force between neutrons and matter that depends on the neutron spin and
 80 velocity and that is mediated by the exchange of spin 1 light bosons (meV mass
 81 range) with axial couplings [14]. This paper describes two key pieces in the
 82 NSR apparatus: the input and output coils that transport and manipulate the
 83 neutron spin before and after the target low-magnetic-field region. In section
 84 2 we describe the physics of adiabatic and nonadiabatic neutron spin transport
 85 as well as the magnetic field requirements for the NSR apparatus. Section 3
 86 describes the technique, based on the magnetic scalar potential, used in the
 87 design of the coils. The characterization of the magnetic fields produced by the
 88 coils is described in section 4. Finally, the conclusions are presented in section
 89 5.

90 2. Neutron spin transport and magnetic field requirements for the 91 NSR apparatus

92 Neutrons are fermions with spin $s = \hbar/2$. They also possess a magnetic
93 moment that is antiparallel to the spin ($\vec{\mu} = -\gamma_n \vec{s}$, $\gamma_n = 1.83247172 \times 10^4$
94 $\text{s}^{-1}\text{G}^{-1}$). In the presence of an external magnetic field $\vec{B}$, from a semiclassical
95 point of view, a torque is exerted on the neutron spin ($\vec{\tau} = \vec{\mu} \times \vec{B}$), causing
96 it to precess about $\vec{B}$ with a frequency given by $\omega_L = \gamma_n |\vec{B}|$, known as the
97 Larmor frequency. If the external field is homogeneous, the projection of the
98 precession axis of the rotating neutron spin will maintain its orientation relative
99 to the external field. In the case of an inhomogeneous external magnetic field,
100 the relative orientation of the neutron spin projection and the field can be
101 maintained if the fractional rate of change of the field as seen in the rest frame of
102 the neutron ($1/\tau = 1/|\vec{B}||d\vec{B}/dt|$) is small compared to the precession frequency
103 of the neutron spin (ω_L), or in terms of the adiabaticity parameter η :

$$\eta = \frac{1}{|\vec{B}|} \left| \frac{d\vec{B}}{dz} \right| \frac{v_n}{\omega_L} = \frac{1}{B^2} \left| \frac{d\vec{B}}{dz} \right| \frac{v_n}{\gamma_n} \ll 1 \quad (1)$$

104 with z the direction of propagation of the neutron [15]. On the other hand, if
105 the adiabaticity condition (1) is not met, the neutron spin projection will not
106 maintain its relative orientation with the external magnetic field. The ability to
107 establish both adiabatic and nonadiabatic conditions is important for neutron
108 spin transport.

109 The NSR apparatus, described in [12], is a neutron polarimeter that is capable
110 of isolating neutron rotary powers that are in the 10^{-7} rad/m range. The
111 magnetic field of the earth as well as other background magnetic fields present
112 in an experiment can produce much larger neutron spin rotations over distances
113 of order a meter than the neutron-matter interactions of interest. For example,
114 a magnetic field of only 0.5 G (the earth's magnetic field ranges between 0.25
115 and 0.65 G) can produce a rotation by an angle of 90° over a distance of only 5
116 cm for neutrons with energy of 5 meV (978 m/s of velocity). The determination
117 of very small neutron spin rotations requires, among other things, that the

118 neutron-matter interaction region is surrounded by a magnetic shielding, so
 119 that the magnetic fields are below 100 μG . Polarized neutrons, however, need to
 120 be transported in and out of this low-magnetic-field region. Entering neutrons
 121 require a vertical homogeneous static field to preserve their neutron polarization
 122 direction during transport from a super-mirror (SM) neutron polarizer, while
 123 exiting neutrons require a static non-homogeneous field to rotate the neutron
 124 spin transverse component by 90° . The transverse component of the neutron
 125 spin carries the information of the neutron spin rotation angle and needs to be
 126 oriented in the vertical direction to be analyzed by a SM analyzer. The direction
 127 of this rotation needs to alternate between counterclockwise and clockwise to
 128 reduce systematic effects. The neutron spin transport before and after the
 129 target region is adiabatic, however the transitions with the magnetic shielding
 130 region has to be nonadiabatic to ensure that neutrons maintain their direction
 131 of polarization when no guiding field is present as well as to avoid leakage of
 132 magnetic fields into the magnetic shielding region. Two electromagnetic devices,
 133 the input and output coils (ic and oc), were developed to produce the magnetic
 134 fields for neutron spin transport before and after the magnetic shielding. Figure
 135 1 shows a schematic representation of the magnetic fields.
 136 The magnetic field produced by the input coil is

$$\vec{B}_{ic} = B_o \hat{x}, \quad (2)$$

137 while the rotating magnetic field of the output coil can be achieved by the
 138 combination of a decreasing transverse field and an increasing vertical field,

$$\vec{B}_{oc} = B_o [\pm \cos(\pi z/2L) \hat{y} + \sin(\pi z/2L) \hat{x}], \quad (3)$$

139 along L the length of the coils. The plus sign corresponds to the counterclock-
 140 wise rotations of the transverse component of neutron spin while the negative
 141 sign corresponds to clockwise rotations. The amplitude of these fields, B_o , needs
 142 to be ~ 4 G in order to comfortably meet the adiabatic condition for cold neu-
 143 trons. Small misalignments in the total magnetic field of the input and output
 144 coils due to the Earth's magnetic field are expected, however this is not an is-

Figure 1: Scheme of the magnetic fields for neutron spin transport at the entrance and exit of the magnetic shielding region. The top figure illustrates the magnetic field for counterclockwise rotations of the transverse neutron spin component after the magnetic shielding, while the bottom figure illustrates the field for clockwise rotations.

145 sue for the relative spin rotation measurement performed with this polarimeter,
 146 where target properties are changed in the magnetic shielding region to isolate
 147 an effect [6, 12, 14]. Both coils need to include in their interior the input/output
 148 SM neutron guide, radiological shielding and its corresponding holding system,
 149 which has a total cross sectional area of $16.5 \times 16.5 \text{ cm}^2$. Additionally, the input
 150 and output coils need to be self contained, meaning that they have to control
 151 the return magnetic flux and keep it in a compact region of space. Finally their
 152 cross section needs to be as small as possible to prevent leakage of magnetic
 153 fields into the magnetic shielding.

3. The magnetic scalar potential in the design of coils and the NSR I/O coils

The physical significance of the magnetic scalar potential as a ‘source potential’ has practical applications in designing coils to produce precision magnetic fields in situations where the geometry is complicated or the field requirements are difficult to meet using other methods [16]. Design techniques based on this physical principle have been successfully used in the past few years in the development of various electromagnetic devices for spin transport in experiments in fundamental neutron physics.

In a region of space free of currents $\nabla \times \vec{H} = 0$, so we can express the magnetic field strength in terms of the magnetic scalar potential φ as $\vec{H} = -\nabla\varphi$. From Gauss’s law, $\nabla \cdot \mu\vec{H} = 0$, and therefore the magnetic scalar potential must satisfy the Laplace’s equation $\nabla^2\varphi = 0$. This PDE can be solved as a boundary value problem for the given geometry with appropriate flux boundary conditions on the magnetic field. The magnetic scalar potential solution forms equipotential surfaces which are perpendicular to the magnetic lines of flux everywhere in space. The physical interpretation of the magnetic scalar potential is that its isocontours on the boundaries of the given geometry represent the paths of electrical current required to produce the corresponding field in that region. The current in each path equals the difference in potential between two adjacent isocontours. For this property to hold, each region of nonzero field must be wound along its scalar potential isocontours. This geometric interpretation provides a powerful practical method of calculating the windings of a coil based on the desired field and geometry, in contrast with the usual technique of calculating the magnetic field of approximate windings and iterating to a solution.

The input and output coils for the NSR apparatus, as described in the previous section, need to produce static magnetic fields along the path of the neutron beam. At the interface with the magnetic shielding region, the fields need to abruptly decrease to zero so that a nonadiabatic transition is produced. The coils also need to have an external region to confine the return magnetic flux;

184 this region needs to have approximately the same volume as the internal region
185 (neutron path region) to avoid large fields and achieve good confinement. The
186 geometry of the input and output coils is shown in figure 2: they are octagonal
187 prisms with a hollow region at the center (neutron path) with $16.5 \times 16.5 \text{ cm}^2$
188 of cross section. The total width and height of the coils is 37.5 cm and their
189 length along the neutron beam direction is 127 cm.

190 Using the finite element analysis software COMSOL Multiphysics [17] to solve
191 the Laplace equation with the magnetic flux boundary conditions described in
192 section 2, we calculated the magnetic scalar potential for the internal (φ_1) and
193 external (φ_2) regions of the coil, corresponding to a given magnetic flux den-
194 sity $\vec{B}$ and its return flux $-\vec{B}$. Nominally, these potentials specify two sets of
195 contours, one in each region, which must be wound in order to generate the
196 required fields. However the inner region shares almost its entire boundary
197 with the outer region, except for the squared surface at each end. Therefore
198 it is more practical to wind both regions as a single coil around the external
199 region, with a small flat endcap coil at one end to correct for current in the
200 missing square (only the end near the magnetic shielding needs a correction
201 endcap since the magnetic field at the other end is continuous with the adjoin-
202 ing guide field from the SM polarizer and analyzer). In order to shift φ_1 to the
203 surrounding region φ_2 and consolidate the two coils, we need to reroute the coil
204 from the inner (beam path) region to the outer region. We do this by solving
205 an auxiliary Laplace equation for φ_3 , constrained to $\varphi_3 = \varphi_1$ at the interface
206 of the two coils, $\varphi_3 = 0$ on the outer surface, and $\partial\varphi_3/\partial n = 0$ on the end-
207 caps (see figure 2). Now φ_2 and φ_3 occupy the same region, and may be added
208 algebraically to produce the isocontours $\varphi_2 - \varphi_3$ of a single inner/outer winding.

209

210 Figure 3 shows, for the vertical homogeneous field of the input coil and the
211 decreasing transverse field of the output coil, the equal magnetic scalar poten-
212 tial surfaces ($\varphi_2 - \varphi_3$), the contours at the intersection of the equal potential
213 surfaces and the geometry (winding pattern) and the magnetic fields that are
214 generated when electrical current circulates through the contours. As can be

Figure 2: Geometry of the input and output coils and their endcap. The different magnetic scalar potential regions are marked.

215 seen, the return magnetic flux is contained in the outer region of each coil; by
 216 design no magnetic flux crosses the external boundary of the coils. Figures 4
 217 and 5 show COMSOL calculations of the magnetic fields produced along the
 218 axis of the coils. As can be seen, the generated magnetic field does not have
 219 an abrupt step-like profile at the end of the input coil and the beginning of the
 220 output coil. This abrupt transition to zero field is important to ensure nonadia-
 221 batic spin transport at the entrance and the exit of the magnetic shielding, and
 222 was lost as a result of rerouting currents from the inner φ_1 region to the outer
 223 φ_3 region. To restore this transition profile of the magnetic fields, an endcap
 224 for each coil was designed with the contours of φ_1 in the inside square and φ_3
 225 in the surrounding octagon. Figure 2 shows the magnetic scalar potentials used
 226 for the two regions of the endcap, so that when combined with the main coil we
 227 recover the magnetic scalar potentials φ_1 and φ_2 in the internal and external
 228 regions of the coil at the interface with the target region. The equipotential
 229 levels, the contours (winding pattern), and the magnetic fields for the endcaps
 230 are shown in figure 6. The magnetic field produced by the endcaps as well as

Figure 3: a) and c) show the equal magnetic scalar potential surfaces and the contours at the intersection of the isosurfaces and the geometry for the vertical field of the input coil and the decreasing transverse field of the output coil; b) and d) show the corresponding magnetic flux.

231 their combination with the magnetic fields of the coils along the axis are also
 232 shown in figures 4 and 5. The effective field at the position of the endcap (exit
 233 of the input coil and entrance of the output coil) varies between the 10% and
 234 the 90% of its full amplitude over a distance of 0.75 cm for the input coil and
 235 0.5 cm for the output coil along the axis. Off axis rise/fall distances differ by
 236 less than 0.25% from those on axis.

237

238 The output coil needs to have two layers of windings: one to produce the trans-
 239 verse decreasing field, which also needs to alternate polarity to produce clockwise
 240 or counterclockwise rotations of the neutron spin; and the second to produce a
 241 vertical increasing field. The input coil also needs to produce a second magnetic
 242 field which is homogeneous and perpendicular to the transport field. This field
 243 is used to generate controlled neutron spin rotations that are required to cali-

Figure 4: Calculated magnetic fields along the axis of the input coil.

244 brate the apparatus. For the construction of the coils it is very important that
 245 nonmagnetic materials are used. Because of the low intensity of the magnetic
 246 fields (~ 4 G), the dissipated power for each coil is of the order of 40 W, there-
 247 fore we were able to use acrylic plates, 5 mm thick, for the structure of the coils.
 248 Channels of approximately 1.3×1.3 mm² cross section were machined into the
 249 surface of the acrylic following the trajectories of the magnetic scalar potential
 250 isocontours. Since the endcaps have current paths crossing the neutron beam
 251 trajectory, it was important to use a material that has a low neutron absorption
 252 cross section. Aluminum enameled magnet wire (2 coatings of insulation, modi-
 253 fied polyester and polyamideimide), gauge 18, was used to wind the coils placing
 254 the wire inside the channels. The required electrical current was ~ 2.5 A, well
 255 below the maximum current of 10 A for this type of wire. For the endcaps,
 256 in order to provide a frame for the wires crossing the beam path, 5 mm-thick
 257 aluminum plates were used instead of acrylic. Photos of the machined channels

Figure 5: Calculated magnetic fields along the axis of the output coil.

Figure 6: a) Isolevels of scalar magnetic potential on the encap; b) contours or winding patterns and magnetic flux amplitude and direction.

258 on the acrylic, the structure as well as of the wound coils are shown in figure 7.
 259 Both coils have two layers of acrylic, one for each field component (vertical and

Figure 7: Photos of the coils at different stages of construction; the top row shows images of the machined channels on the surface of the acrylic and aluminum plates used for the structure of the coils and endcaps, as well as the assembly of the inner layer of one of the coils; the lower row shows different views of the of the two layers of winding of the coils.

transverse).

4. Characterization of the coils

Magnetic field maps of the coils were performed using a 3-channel gaussmeter (Lakeshore 460) and a 3-axis probe (Lakeshore MMZ-2518-UH) mounted on a XYZ system consisting of three non-magnetic slides (Velmex MT10 and MN10). The spatial resolution of each slide is $6.35 \mu\text{m}$ (400 step/rev, equivalent to 0.1 inch advance). To eliminate the background magnetic fields in the field map, a pulse generator was used to energize the coils, using a sinusoidal wave with amplitude of 10 V peak-to-peak and frequency of 200 Hz. Using the RMS setting, the gaussmeter can measure time oscillating magnetic fields with frequencies up to 400 Hz with resolution of 0.001 G. Figures 8 and 9 show a comparison of the magnetic fields calculated with COMSOL Multiphysics from the design and the measured magnetic fields along the axis of the coil. The only

273 significant differences between calculated and measured magnetic fields occur
 274 when the magnetic field amplitudes are close to zero; these differences can be
 275 due to winding accuracies.

Figure 8: Calculated and measured input coil magnetic fields along the axis. The vertical magnetic field for neutron spin transport and the magnetic field of the corresponding endcap (downstream end of the input coil), as well as the transverse magnetic field for calibration of the apparatus, are presented.

276 The field maps and field uniformity in the internal region of the coils (volume
 277 of $14 \times 14 \times 127 \text{ cm}^3$) are presented in figure 10 for the input coil and figure 11
 278 for the output coil; a) and c) show the field map of the vertical and transverse
 279 components of the field, while b) and d) show the field uniformity as the residuals
 280 of the field and the field in the axis of the coil. The calculated residuals of the
 281 magnetic field components have average values over the mapped volume of 0.03
 282 (transverse) and 0.04 (vertical) for the input coil and of 0.02 G for the two
 283 components of the output coil. These averages have contributions of larger
 284 deviations (up to 0.7 G) from the on axis field that occur near the winding

Figure 9: Calculated and measured output coil magnetic fields along the axis. The transverse and vertical components of the output coil are measured and compared to calculated fields separately (see equation 3); the horizontal magnetic field produced by the endcap on the upstream side of the output coil is also shown.

285 wires, at the edges of the internal region of the coil, however for most of the
 286 volume the residuals are below 0.01 G for the two components in both coils.
 287 Additionally the inner cross section of the SM input and output guides inside
 288 the coils is $10 \times 10 \text{ cm}^2$ and therefore the neutron trajectories are away from
 289 the edges of the coil. The z component of the fields in all cases is below 0.01 G.
 290 Finally the adiabaticity parameter η , described in section 2, is calculated for
 291 the measured magnetic fields along the axis of the coils; figure 12 shows η for
 292 5 meV neutrons for both input and output coils. As required, the neutron spin
 293 transport before (input coil) and after (output coil) the magnetic shielding is
 294 adiabatic ($\eta \ll 1$), however the transition with the low-magnetic-field region
 295 (exit of the input coil and entrance of the output coil) is nonadiabatic. The
 296 observed noise (logarithmic scale) is consistent with field gradients $|d\vec{B}/dz|$ of

Figure 10: Input coil field map and field uniformity. a) and c) show the field map of the vertical field (holding field for polarized neutron beam) and the transverse field (field for calibration of the apparatus) in the volume inside the coil; b) and d) show the residuals of the magnetic field respect to the magnetic field along the axis of the coil for both components. Neutron beam propagates along the z-axis.

Figure 11: Output coil field map and field uniformity. a) and c) show the field map of the two components of the effective magnetic field in the volume inside the coil (transverse and vertical correspondingly); b) and d) show the residuals of the magnetic field and the magnetic field along the axis of the coil for both components. Neutron beam propagates along the z-axis.

the order of 0.008 G/cm for the input coil and 0.005 G/cm for the output coil,
which corresponds to the gradients (along the axis) measured in the field maps.

5. Conclusions

The magnetic scalar potential method has been successfully applied in the
design of precision coils to produce magnetic fields for slow neutron spin trans-

Figure 12: Calculated adiabaticity parameter η (see equation 1) for 5 meV neutrons along the axis of the coils using measured fields. Neutron spin transport is adiabatic along the length of the coils ($\eta \ll 1$), however a nonadiabatic transition can be observed at the exit of the input coil and the entrance of the output coil, where the coils interface with the magnetic shielding region.

port in the Neutron Spin Rotation apparatus. The input and output coils, developed with this method, are two electromagnetic devices that provide static and extremely uniform magnetic fields (calculated residuals below 0.01 G) to adiabatically transport the neutron spin between the polarizer and the target, and between the target and the analyzer. The input coil produces a homogeneous magnetic field in the vertical direction (neutron polarization direction) while the output coil produces a field that in the rest frame of the neutrons rotates by 90° about the beam direction. Both coils have an abrupt step like transition with the virtually zero magnetic field region where the target is immersed. The coils also confine the return magnetic flux into a small region in space, reducing the leakage of magnetic fields into the target region. These devices were recently

used with success in an experiment to search for a possible parity-even exotic force between the neutrons and matter, and will be used in the future in an experiment to measure the PV rotary power of polarized neutrons in ^4He .

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