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Improved neutron-gamma discrimination for a ^3He neutron detector using subspace learning methods

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Abstract. ^3He gas based neutron Linear-Position-Sensitive Detectors (LPSDs) have been used for many neutron scattering instruments. Traditional Pulse-height Analysis (PHA) for Neutron-Gamma Discrimination (NGD) resulted in the neutron-gamma efficiency ratio (NGD ratio) on the order of 10^5 - 10^6 . The NGD ratios of ^3He detectors need to be improved for even better scientific results from neutron scattering. Digital Signal Processing (DSP) analyses of waveforms were proposed for obtaining better NGD ratios, based on features extracted from rise-time, pulse amplitude, charge integration, a simplified Wiener filter, and the cross-correlation between individual and template waveforms of neutron and gamma events. Fisher Linear Discriminant Analysis (FLDA) and three Multivariate Analyses (MVAs) of the features were performed. The NGD ratios are improved by about 10^2 - 10^3 times compared with the traditional PHA method. Our results indicate the NGD capabilities of ^3He tube detectors can be significantly improved with subspace-learning based methods, which may result in a reduced data-collection time and better data quality for further data reduction.

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I. INTRODUCTION

^3He gaseous detectors are the workhorses for neutron scattering [1, 2], homeland security [3], and high-energy physics. ^3He position-sensitive detectors used for neutron scattering have high thermal-neutron detection efficiency, excellent Neutron-Gamma Discrimination (NGD), reasonable spatial resolution (1-10 mm), and high count rate capability [1, 2, 4, 5]. Neutron reflectometers, small-angle neutron scattering, and inelastic scattering spectrometers require the NGD ratio (the detection efficiency ratio between thermal neutrons and gamma-rays) on the order of 10^6 - 10^8 or even higher for studying many advanced materials [1,2].

The 8-pack ^3He neutron detector [6] with a sensitive area of 0.1-0.3 m² per module developed at Oak Ridge National Laboratory (ORNL) has been deployed for time-of-flight (TOF) and reactor-based scattering instruments [1, 5-7]. Traditionally, Pulse-Height Analysis (PHA) [8] has been used for the NGD of these ^3He detectors. However, the PHA only leads to NGD ratios of 10^5 - 10^6 . This limits the signal-to-noise ratio (SNR) of neutron scattering data and therefore compromises the applications of the instruments for thin-film materials, samples with low scattering cross-sections, or small-size samples. The potential benefits of increased NGD ratio (or SNR) include i) reducing the total number of counts (or the counting time) needed for a neutron scattering measurement [9], ii) increasing the accuracy of the position of a peak in a neutron scattering spectrum [10], iii) revealing weak peaks otherwise buried in the high background in a spectrum, and iv) improving data quality for data reduction. Increasing NGD ratios is possible by typically increasing the discrimination threshold channel in PHA. However, this will reduce the neutron detection efficiency significantly.

Pulse-shape analysis methods have been used for NGD of ^3He -based neutron detectors previously. For example, Rise-time analysis [11] was applied for NGD in combination with the pulse amplitude or energy [12, 13], resulting in the NGD ratio on the order of 10^6 [13]. Similarly, Duxbury *et al.* used time derivative at a low level discrimination threshold (5 mV) versus pulse height and obtained a NGD ratio of about 10^6 [14]. Though charge integration method was used to identify the daughter particle (triton or proton) that arrives at the anode earlier for a ^3He tube detector [15], its applicability for the NGD of ^3He tube detectors is unknown. Recently, DSP methods such as Wiener filter and correlation analysis between the individual and neutron template waveforms have shown promise in NGD in a scintillator detector [16]. It would be interesting to investigate their applicability for a ^3He gas detector. There were reports on the utilization of machine learning methods including artificial neural networks [17], support vector machine [18], and fuzzy c-means [19] for NGD of liquid-scintillator based fast-neutron detectors, however, these methods typically have high computational burdens and may not be well suitable for real-time applications for ^3He detectors under high fluxes of thermal neutrons.

In this work, we report novel NGD algorithms using pattern recognition methods [20] and the extracted features (or parameters) [11-16] from pulse shapes of neutron and gamma events. We utilize pulse shape differences for ^3He tubes between neutrons and gammas, due to different geometries (vertices, lengths, densities) of the ionization tracks generated by triton-proton pairs on the one hand, and by Compton- or photo-electrons on the other hand. These differences affect the subsequent charge collection processes in the ^3He tubes. There is no straight dependence of pulse shape on the type of particle, nevertheless, the statistical differences of feature parameters extracted from pulse decay curves of neutrons and gammas allow reducing

the contamination of neutron signals with gamma events. In this work, four subspace learning methods are proposed to reduce the dimensionality of the feature space and produce significantly improved NGD, which is well suited for online-classification using fast digital electronics with field-programmable gated arrays (FPGA) or post computer processing. Some of these new methods may increase the energy resolution for secondary ions or electrons generated in the ^3He tube. Furthermore, the proposed methods represent general approaches suitable not only for ^3He -based gaseous detectors, but also for other types of neutron detectors and for particle (or event) identification by other radiation detectors.

II. EXPERIMENTS

A ^3He tube detector assembled in Institutut Laue-Langevin (ILL) was used in this work. Each of the four stainless tubes has a length of 46.2 cm, an external diameter of 7.88 mm and a 0.25 mm-thick wall. The tubes contain a mixture of ^3He , Ar and CO_2 gases with a pressure ratio of 5:7:0.1 and a total pressure of 1.21×10^6 Pa. The electronics, used for 8-pack ^3He tube detectors developed at Spallation Neutron Source (SNS), was connected to the tubes [6]. Charge-division method was implemented to locate the neutron position along the tube [6, 7]. The output of a first-stage preamplifier in the front-end board was connected to an oscilloscope (Agilent MSO9254A) and its signals were digitized and recorded in a 10- μs time-window and in 0.5 ns bins with a sampling resolution of 8 bits. The trigger level was carefully chosen for the oscilloscope to reject the electronic-noise induced background as much as possible while allowing high recording efficiencies of the neutron or gamma waveforms.

A 3-mm-wide slit of a boron-aluminum composite plate was installed in the front of a single detector tube. The installation of the slit allows easy calculations of incident neutron rate and neutron detection efficiency of the ^3He tube by mitigating the non-uniformity of a neutron

beam from either a moderated ^{252}Cf thermal neutron source or the CG1A beam line ($\lambda=4.26 \text{ \AA}$) at High Flux Isotope Reactor (HFIR), ORNL. It also limits the apparent pulse-height distribution due to the broad spatial-profiles of neutrons from the two neutron sources. Without a slit, the non-uniform large-area neutron beam may distort the histograms of pulse-heights and other energy-related features, and subsequently the projection-vectors obtained in the subspace learning algorithms may not be used for the NGD when neutrons hit at different positions along the tube. The slit was positioned 16.2 cm away (along the tube) from the end B of Tube 2. Only the digital waveforms from the end B of Tube 2 were recorded. The pulse-height spectra were recorded using the data acquisition system [6] under the irradiation of the two neutron sources and a ^{60}Co gamma source. The weak ^{60}Co source was positioned in the center of the slit. For the calculation of detection efficiency, the neutron flux from the moderated ^{252}Cf source was calibrated using a ^3He tube (8 bar, $\Phi 2.5 \text{ cm} \times 20 \text{ cm}$) with a $1 \text{ cm} \times 1 \text{ cm}$ aperture, while the total gamma-ray rate incident on the slit was calculated from the quoted activity of the ^{60}Co source. The incident neutron rate over the slit is 4.4 cps and the gamma rate is 7.9×10^4 cps.

The waveforms $A(t)$ from a ^3He tube with an anode wire biased at +2000 V were continuously collected at the trigger level of 5 mV. A set of 1×10^4 waveforms was saved in a binary format and analyzed using Python code. In the digital signal analysis, the waveforms were aligned at the maximum-amplitude positions in the time-axis [16]; and a small fraction (about 10%) of events with peak-time shifts larger than 1000 channels was discarded. The average pulse-shape curves were calculated to provide the template (reference) spectra. A constant background was subtracted from the waveform. Five and three sets of waveforms under the irradiation of ^{60}Co gamma-source and a ^{137}Cs source, respectively, were collected to determine the boundary between the neutron and gamma classes and check the reproducibility of the NGD

ratio. The results for the ^{137}Cs source were not shown in this paper due to relatively easier NGD compared to those for the ^{60}Co source.

III. DIGITAL SIGNAL ANALYSIS

Feature extraction

We analyzed a digital waveform by subtracting a constant background, which was the average amplitude of the first few hundred samples of the waveform. Fig. 1(a) shows two examples of background-subtracted waveforms of individual neutron and gamma events. The raw waveform has a large fluctuation in amplitudes. To extract the characteristic feature parameters accurately, the collected digital pulses were first smoothed using the 4th-order Savitzky-Golay filter [21] to eliminate high-frequency noise while keeping the essential time-profile of the pulses. Fig. 1(b) shows the residual (or deviation) spectra between the raw and smoothed pulses, which are random noise with average amplitudes of about 1 mV for both neutron and gamma events.

Fig. 1(a) examples of background-subtracted raw waveforms of neutron and gamma events. The peaks of the two waveforms are aligned at the same time position ($t=0$). The smoothed waveforms were obtained using the 4th-order Savitzky-Golay filter. The inserted plot shows smoothed and raw waveforms of the neutron event near the peak. The vertical dashed line indicates the time (t_{op}) for separating the prompt time-window and the delayed one in charge integration analysis (see Fig. 4). (b) Difference spectra between the raw and smoothed waveforms.

Feature parameters [12,13,16] of each waveform were extracted, among which are the maximum amplitude ($A_{\max}$) and the rise-time (T_r) of the pulse. Here T_r is defined as the time-interval between the two sampling points corresponding to 10 to 50% $A_{\max}$ on the rising part of the waveform. The differences in $A_{\max}$ and T_r between neutron and gamma events are seen in the waveforms in Fig. 1(a), i.e., a typical neutron event has a larger amplitude but a shorter rise-time compared with a typical gamma event. The raw waveform of the gamma event in Fig.1 (a) has a secondary peak in the rising part, probably due to the incomplete charge collection (or Compton scattering) of the gamma ray.

Unlike the previous work [16], we did not apply a Fast Fourier-Transform (FFT) to obtain the maximum power, because it is nontrivial to implement a FFT in a FPGA as will be required for future implementation.

In the charge integration method [16], the optimum time for separating the prompt time-window and the delayed time-window was determined to be $t_{\text{op}}=225$ ns (see Fig. 4) after reaching maximum amplitude (set at $t=0$). The integration areas S_P and S_D of an individual waveform were evaluated within the prompt window (-400 to 225 ns) and the delayed window (225 to 6000 ns), respectively. The prompt-component fraction $F_P=S_P/(S_D+S_P)$ and the total integrated-charge $S_T=S_D+S_P$ were calculated. The F_P and S_T of each waveform were used as two features for additional analysis.

In the Cross-Correlation (CC) analysis [16, 22, 23], the normalized cross-correlation of a waveform $\mathbf{A}$ and a reference (or template) pulse $\mathbf{R}$ is calculated by $C=\mathbf{R}_0^T\mathbf{A}_0/[\|\mathbf{R}_0\|\cdot\|\mathbf{A}_0\|]$, where the centered reference waveform vector $\mathbf{R}_0=\mathbf{R}-\mathbf{w}^T\mathbf{R}\mathbf{w}/N$, $\mathbf{w}$ is a vector that has the same dimension N as $\mathbf{R}$ and all of its elements are 1; and the centered waveform vector $\mathbf{A}_0=\mathbf{A}-\mathbf{w}^T\mathbf{A}\mathbf{w}/N$. Template waveform $\mathbf{R}$ is useful for quantifying the similarity between an individual waveform $\mathbf{A}$

and the template waveform of a class [16,22,23]. We chose the average waveform from the ^3He tube (with $A_{\text{max}} > 24$ mV) measured at CG1A beam line as the reference pulse $\mathbf{R}$ for neutrons (see Fig. 4), due to the much higher fraction of neutron events among all the events from CG1A compared with those from the moderated ^{252}Cf source (see Fig. 3b). Similarly, the average waveform (with $A_{\text{max}} < 24$ mV) under the irradiation of the ^{60}Co source is chosen as the reference pulse $\mathbf{R}$ for gamma-rays. We calculated the cross-correlation coefficient, C_n (and C_g), between an individual waveform and neutron (and gamma) template waveform, and $R_c = C_n/C_g$ was used as a feature for NGD analysis. C_n (and C_g) describes the similarity between the individual waveform and neutron (and gamma) template waveform. Therefore, their ratio parameter R_c is a unique feature to characterize the pulse shape over the whole time window of the waveform.

The Wiener filter [16, 24], a simplified version of pole-zero model of digital signals in the time domain [25], was applied to the waveforms to obtain two characteristic parameters (a_1 , b_0) of a neutron scintillator responding to a neutron or gamma source. The normalized impulse response, which was denoted as $x(n)$ in Ref. 16 for calculation of a_1 , and b_0 , is assumed to be symmetric; and $x(n)$ is generated from the average of the rising edge of the neutron and gamma average waveforms [16]. From the formula for calculating a_1 and b_0 [16, 24], it is obvious that a_1 is the equivalent of pulse-decay lifetime and b_0 is equivalent to the pulse intensity.

The analytical methods described above result in two classes of features, i.e., A_{max} , S_T , and b_0 are pulse-energy related [16], whereas T_r , F_p , R_c , and a_1 are pulse-shape related. Each feature contains unique discrimination information for neutrons and gammas as indicated in its scattering plot or histogram (see Figs. 3, 5-7) and in the follow-up discussion on the independence of energy-related features in the Principal Component Analysis (PCA). A vector composed of seven features is obtained for each event, i.e., $\mathbf{f} = [A_{\text{max}}, T_r, S_T, F_p, R_c, b_0, a_1]$.

Methods that reduce the dimensionality of $\mathbf{f}$ are needed to simplify the discrimination process, while maintaining as much of the discrimination information of each feature as possible to obtain the highest NGD ratio. Certain subspace projection methods can satisfy our needs, in which a projection vector $\mathbf{W}$ is sought such that $P=\mathbf{fW}$ gives one value which represents the original high-dimensional features. $\mathbf{W}$ is typically an eigenvector of a matrix $\mathbf{S}$ containing the information on the features of all learning events, i.e., $\mathbf{W}$ satisfies eigen-decomposition equation $\mathbf{SW}_i=\lambda_i\mathbf{W}_i$, where λ_i is an eigenvalue and $\mathbf{W}$ is selected from the eigenvectors $\{\mathbf{W}_i\}$.

Fisher Linear Discriminant Analysis (FLDA)

In the FLDA method [20, 26, 27], the $\mathbf{S}$ matrix is defined as $\mathbf{S}=\mathbf{S}_W^{-1}\mathbf{S}_B$. Here the within-class scatter matrix $\mathbf{S}_W = \sum_{i=1}^c \sum_{\mathbf{f}_k \in X_i} (\mathbf{f}_k - \boldsymbol{\mu}_i)^T (\mathbf{f}_k - \boldsymbol{\mu}_i)$, and the between-class scatter matrix $\mathbf{S}_B = \sum_{i=1}^c N_i (\boldsymbol{\mu} - \boldsymbol{\mu}_i)^T (\boldsymbol{\mu} - \boldsymbol{\mu}_i)$; $c=2$ is the total number of classes (neutron and gamma) in our NGD problem; $\boldsymbol{\mu}_i$ and N_i are the vector composed of mean-values of the features for an event and the number of events in the class X_i , respectively; $\boldsymbol{\mu}$ is the mean-value vector of the features of all the events, k is the index for the events. FLDA is designed to enhance the between-class difference and minimized the intra-class difference [20, 26], which improves NGD.

We calculated the class parameters $\boldsymbol{\mu}_i$ and N_i using the CG1A-induced waveforms with $A_{\max}>40$ mV as the neutron class whereas the ^{60}Co source induced waveforms with $A_{\max}<20$ mV as the gamma class. Here, we increased the amplitude-threshold for the neutrons and reduced the amplitude-threshold for the gamma compared to the 24 mV-threshold used for the primitive NGD methods (Table I). This way, we can differentiate the two classes without any ambiguity. Before calculating $\mathbf{S}$, we also centered the class parameters $\boldsymbol{\mu}_i$ and feature parameters $\mathbf{f}$ [28] and

normalized them by the standard deviations of their components. This standardization process ensures that the dimensionless features are used for FLDA and also for other algorithms in this work regardless of the voltage applied to the ^3He tube. We then evaluated the 7-projection vectors $\mathbf{W}_i$ and their eigenvalues λ_i ($i=1,\dots,7$, sorted from the largest to the smallest). $\mathbf{W}_1$ corresponding to the largest eigenvalue λ_1 gave the best NGD. Therefore, we used the correspondingly projected value P_1 vs. $A_{\max}$ for the NGD ratio calculation. We noticed that the remaining projections along the eigenvectors $\mathbf{W}_i$ ($i=2-6$) can also provide very good NGD, in agreement with the claim that the eigenvectors corresponding to smaller eigenvalues may also be useful [27].

Canonical Correlation Analysis (CCA)

In the CCA method [28], the input feature vector $\mathbf{f}$ was split into two vectors, i.e., $\mathbf{y}_i=[A_{\max}, S_T, a_1]$ and $\mathbf{x}_i=[T_r, F_p, R_c, b_0]$, where $i=1,\dots,N$ and N is the total number of events. Here we designed the target (or output) vector $\mathbf{y}_i$ composed of the event features which provide the maximum neutron-gamma discrimination as possible (Fig. 3, Figs. 5 to 7). Then we define a $N \times 3$ target matrix $\mathbf{Y}^0=\{\mathbf{y}_i\}$ and a $N \times 4$ input matrix $\mathbf{X}^0=\{\mathbf{x}_i\}$. We centered each column of $\mathbf{X}^0$ and $\mathbf{Y}^0$ by shifting the origin to the centroid of each column, and normalized each column by the standard deviation of that column. The centered and normalized event matrices were denoted as $\mathbf{X}$ and $\mathbf{Y}$, and their covariance matrices $\mathbf{C}_{xy}=1/N \cdot \mathbf{X}^T \mathbf{Y}$, $\mathbf{C}_x=1/N \cdot \mathbf{X}^T \mathbf{X}$, and $\mathbf{C}_y=1/N \cdot \mathbf{Y}^T \mathbf{Y}$ were calculated.

The projection vectors $\mathbf{u}$ and $\mathbf{v}$ in the input space and target space, respectively, are determined such that the correlation value $\mathbf{u}^T \mathbf{C}_{xy} \mathbf{v}$ between the target and the input matrices is maximized. The vectors $\mathbf{u}$ and $\mathbf{v}$ form the columns of projection matrices $\mathbf{U}$ and $\mathbf{V}$ in the input

space and target space, respectively. Matrices $\mathbf{U}$ and $\mathbf{V}$ satisfy two orthogonal constraints $\mathbf{U}^T \mathbf{C}_x \mathbf{U} = \mathbf{I}$ and $\mathbf{V}^T \mathbf{C}_y \mathbf{V} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. The vectors $\mathbf{u}$ and $\mathbf{v}$ can be obtained by solving the following eigenvalue problem [28]

$$\begin{pmatrix} \mathbf{C}_x & 0 \\ 0 & \mathbf{C}_y \end{pmatrix}^{-1} \begin{pmatrix} 0 & \mathbf{C}_{xy} \\ \mathbf{C}_{xy}^T & 0 \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix} = \lambda \begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix}. \quad (1)$$

We calculated the product of the first and the second matrices on the left side of eq. (1), which is denoted as matrix $\mathbf{B}$. To get better conditioned solutions and overcome the overfitting issue of a classifier, a Tikhonov's regularization term is applied to the eigen-decomposition equation [27]

$$(\mathbf{B} + \beta \mathbf{I}) \begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix} = \lambda \begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix}, \quad (2)$$

where constant β controls the amount of regularization. The $\mathbf{u}$, $\mathbf{v}$ and λ were obtained using standard linear algebraic tools in Python. We found that $\beta = 1 \times 10^{-7}$ gave consistent results for all sets of neutron and gamma waveform data. Moreover, the projected parameter $P_{1Y} = \mathbf{Y} \mathbf{v}_1$ vs. $A_{\max}$ gave the best NGD among all the projected parameters in both target and input spaces, where the vector $\mathbf{v}_1$ in the target space corresponds to the largest eigenvalue λ_1 .

Ortho-normalized Partial Least-Square Analysis (OPLS)

Analogous to the CCA method, OPLS (also called semi-penalized CCA) [28] is designed to maximize the correlation values $\mathbf{u}^T \mathbf{C}_{xy} \mathbf{C}_{xy}^T \mathbf{u}$ with an orthogonal constraint $\mathbf{U}^T \mathbf{C}_x \mathbf{U} = \mathbf{I}$ and only perform the projection in the input space. It is equivalent to solving the following eigenvalue problem

$$\mathbf{C}_x^{-1} \mathbf{C}_{xy} \mathbf{C}_{xy}^T \mathbf{u} = \lambda \mathbf{u}. \quad (3)$$

Denoting $\mathbf{B}=\mathbf{C}_x^{-1}\mathbf{C}_{xy}\mathbf{C}_{xy}^T$, and adding a regularization term in the eigenvalue equation as in Eq. (2), we solved Eq. (3) using $\beta=1 \times 10^{-7}$ and found that the projected parameter $\mathbf{P}_{1X}=\mathbf{X}\mathbf{u}_1$ vs. $A_{\max}$ gave the best NGD among all the projected parameters in the input space, where the vector $\mathbf{u}_1$ corresponds to the largest eigenvalue λ_1 .

Principal-Component Analysis (PCA)

PCA was performed to check the independence of the elements in energy-related vector $\mathbf{E}=[A_{\max}, S_T, b_0]$ [16] as well as the feasibility of NGD using a number of features smaller than 7. PCA searches a projection vector $\mathbf{u}$ by solving the eigenvalue equation $\mathbf{C}_E\mathbf{u}=\lambda\mathbf{u}$, where $\mathbf{C}_E$ is the covariance matrix of vector $\mathbf{E}$ after the energy-related features are centered and normalized [27, 28]. We calculated the covariance matrices $\mathbf{C}_E$ using the ^{252}Cf -source (or CG1A) generated waveforms with $A_{\max}>40$ mV, and the ^{60}Co -source induced waveforms with $A_{\max}<20$ mV. Three eigenvectors $\mathbf{u}$ and their corresponding eigenvalues λ of $\mathbf{C}_E$ were obtained for the waveforms connected under the irradiation of a ^{252}Cf (or CG1A) neutron source or a ^{60}Co gamma source. The final projected vectors $\mathbf{P}_E=\mathbf{E}\mathbf{u}+\mathbf{E}_c\mathbf{u}$ were calculated for all the events, where the second-term on the right is used to correct the energy bias introduced by the centering process, and $\mathbf{E}_c$ is the standard-deviation-normalized centroid of the vector $\mathbf{E}$. The obtained projected parameter P_{E1} is plotted vs. P_{E2} for the NGD, where P_{E1} and P_{E2} are the projected values corresponding to the largest (λ_1) and second largest (λ_2) eigenvalues, respectively.

IV. RESULTS AND DISCUSSION

Traditional pulse-height analysis (PHA)

Fig. 2(a) shows the pulse-height histograms for the summed and amplified signals from two ends of the ^3He tube detector, which were collected using the moderated ^{252}Cf neutron source and the ^{60}Co gamma source. Under thermal neutron radiation, there is a broad peak centered at channel number $N \approx 680$, corresponding to the total energy of neutron-induced daughter ions. The pulse height spectrum under gamma-ray illumination from the ^{60}Co source (with an average energy of 1.25 MeV) and the background spectrum have much lower count rates. The background spectrum was subtracted from the spectra in the calculation of detection efficiencies. For the spectrum of the moderated ^{252}Cf source, the low-energy tail below the channel $N \approx 400$ may be due to the partial energy deposition of protons and/or tritons near the detector wall (wall effect) [29, 30], so the events in the regions of $N \approx 200-400$ may be counted as neutrons. In Fig. 2(b), the neutron detection efficiency as well as the gamma efficiency vs. threshold channel N_{th} are shown, where N_{th} is defined as start channel for the integration of pulse-height spectra.

Fig. 2 Traditional pulse-height-analysis results for ^3He tube: (a) raw pulse height spectra under the radiation of the moderated ^{252}Cf neutron and ^{60}Co gamma-ray sources, and background ('Bg'); (b) detection efficiencies of neutron (relative to an 8-bar ^3He tube) and gamma-ray vs. the threshold channel N_{th} .

When the threshold channel N_{th} is set at 157, the neutron-gamma-discrimination ratio is about 4×10^5 , and the neutron detector efficiency is about 96% that of the 8-bar ^3He tube

detector normalized to equivalent detection areas. Increasing N_{th} to 200 or 400 reduces the neutron detection efficiency into 71% and 44% respectively and increases the NGD ratio to 7×10^5 and 9×10^7 respectively. The enhanced NGD ratio unavoidably results in significantly reduced neutron detection efficiency, which is not optimal for a typical neutron-flux limited scattering measurement. Hence, new methods that discriminate gamma-rays while maintaining reasonable neutron efficiency need to be developed.

Primitive DSP methods

As in the case of a GS20 neutron scintillator [16], pulse-shape features can be directly used for NGD without additional processing. Fig. 3(a) shows the scattering plot of rise-time T_r vs. the maximum amplitude A_{max} for individual events, and Fig. 3(b) shows the histograms of T_r and A_{max} . Though the T_r has limited capability to separate neutrons from gamma-rays, the A_{max} alone can provide reasonable gamma discrimination as well as excellent neutron detection efficiency. Here we set up a threshold $A_{max}^{th}=24$ mV to eliminate the ^{60}Co gamma-rays from thermal neutrons. The waveforms with $A_{max}>A_{max}^{th}$ are assigned to neutron events. A slant straight-line [12-14] may also be used to separate neutrons from gammas in the range of $A_{max}=12\text{-}24$ mV, it provides a higher neutron detection efficiency but a lower NGD ratio compared with those from the vertical threshold line in Fig. 3 (a).

The peak-normalized average pulse-decay curves from thermal neutrons ($A_{max}>24$ mV) and ^{60}Co gamma-rays ($A_{max}<24$ mV) are shown in Fig. 4. Differences between neutron and gamma events are obvious, i.e., gamma events have increased rise- and decay-times. There is a large difference in the average decay curve near $t_{op}=225$ ns (after the peaking time) between neutron and gamma-rays, which was implemented in the charge integration method for NGD.

1

4 Fig. 3 Rise-time and maximum amplitude analyses for ^3He tube: (a) rise-time T_r vs. maximum amplitude
5 $A_{\max}$; (b) histograms of $A_{\max}$ and T_r . The dashed lines indicate the thresholds to separate thermal neutrons
6 from ^{60}Co gamma-rays.

14 Fig. 4 Peak-normalized average pulse-decay curves for both neutron ($A_{\max} > 24$ mV) and ^{60}Co gamma
15 ($A_{\max} < 24$ mV) events. The vertical dashed line indicates the location corresponding to the maximum
16 difference between neutron and gamma [16].

17 The prompt-component fraction F_P vs. the total integrated-charge S_T is shown in Fig. 5

18 (a). As shown in the histogram of S_T in Fig. 5(a), the total charge parameter S_T alone can provide
19 reasonable gamma discrimination. We tend to use the F_P vs. S_T for NGD, since it provides
20 excellent discrimination capability against ^{60}Co gamma-rays while maintaining relatively high
21 detection efficiency for neutrons. For this purpose, a slant straight-line is set as the boundary

1 between neutrons and gamma-rays. The events on the right side of the straight line are regarded
 2 as neutron events. Fig. 5(b) shows the histogram of the projected values along the y_1 -axis, which
 3 is perpendicular to the boundary line (x_1 -axis). The projected value y_1 provides a limited neutron-
 4 gamma separation.

5 Fig. 5 Charge-integration for NGD: (a) prompt-component fraction F_p vs. integrated charge S_T (arbitrary
 6 unit) in charge-integration analysis. The dashed line indicates the boundary between gamma and neutron
 7 events. Inserted are the histograms of S_T and F_p . (b) Histograms of projected values y_1 . The vertical
 8 dashed line in (a) is drawn to separate neutrons from gammas.
 9

10 The cross-correlation ratio R_c vs. $A_{\max}$ is shown in Fig. 6. A joint application of R_c and
 11 $A_{\max}$ provides an excellent NGD ratio while maintaining a high neutron detection efficiency. A
 12 boundary composed of two straight lines and a parabola curve is used to separate neutron and
 13 ^{60}Co gamma-rays. Events on the right side of the boundary are assigned as neutrons.

Fig. 6 Cross-correlation ratio R_c vs. maximum
 amplitude $A_{\max}$. The curve is drawn to separate
 thermal neutrons from ^{60}Co gamma-rays.

Two Wiener-filter parameters (a_1 , b_0) extracted from the ^3He waveforms are shown in Fig. 7(a). A boundary composed of straight line and a parabola-shaped curve is proposed for NGD. Events confined within this boundary are assigned to the neutron class. The majority of neutron events concentrate in a well-defined region ($a_1=0.995-1.0$ and $b_0=0-0.2$). The histograms $f(a_1)$ and $f(b_0)$ are shown in Fig. 7(a). The lifetime-related parameter a_1 alone can provide better NGD than the energy-related parameter b_0 . Moreover, the histogram of b_0 has a similar shape as that of other two energy-related parameters $A_{\max}$ in Fig. 3(b) and S_T in Fig. 5(a). Fig. 7(b) shows the histogram of the projected values along the x_1 -axis, which is perpendicular to the boundary (y_1 -axis). The projected value x_1 provides a neutron-gamma separation slightly better than the projected value y_1 in the charge-integration method (Fig. 5b).

Fig. 7 Wiener-filter for NGD: (a) parameters a_1 vs. b_0 . The dashed line and curve are used to separate neutrons from gammas. Histograms of a_1 and b_0 are inserted. (b) Histograms of projected values along the x_1 -axis. The vertical dashed line in (b) indicates the threshold to separate neutrons from gammas.

FLDA, CCA, OPLS, and PCA

The projected parameter P_1 vs. the maximum amplitude $A_{\max}$ is shown in Fig. 8. To separate neutron and gamma events unambiguously, we set up boundaries between neutron and gamma events using 5 straight lines, whose positions are determined to achieve high NGD as

- 1 possible while maintaining the neutron detection efficiency as close as possible to that from
 2 PHA. The events enclosed in this boundary shown in Fig. 8 are assigned to the neutron class.

Fig. 8 Projected feature P_1 from Fisher Linear Discriminant Analysis (FLDA) as a function of the maximum amplitude $A_{\max}$. Two data sets collected under the irradiation of the ^{60}Co source are included to illustrate the reproducibility of the data.

- 17 The projected feature P_{1Y} from CCA as a function of $A_{\max}$ is shown in Fig. 9(a). Four
 18 straight lines were set up to separate the neutron and gamma based on the scattering profiles of
 19 all data sets collected in this work. The events enclosed in this boundary shown in Fig. 9(a) are
 20 assigned to the neutron class. The feature P_{1X} obtained from OPLS vs. $A_{\max}$ is shown in Fig. 9(b).
 21 Compared with the profile of the rise time T_r vs. $A_{\max}$ (Fig. 3a), the P_{1X} of ^{60}Co gamma-ray
 22 events are shifted upwards to result in a better NGD. The empirical boundaries between neutron
 23 and gamma classes are established as shown in Fig. 9(b). Events inside the closed boundary are
 24 assigned as neutrons.

Fig. 9 Projected features vs. the maximum amplitude $A_{\max}$: (a) P_{1Y} from Canonical Correlation Analysis (CCA); (b) P_{1X} from Orthonormalized Partial Least-Square (OPLS). Dashed lines in (a) and (b) define the neutron domains.

In PCA, the projected features P_{E1} vs. P_{E2} obtained from individual events are shown in Fig. 10(a). Obviously there is a clear separation between neutron and gamma events. Based on the 2D P_{E1} vs. P_{E2} profiles of 5 sets of waveforms collected under the irradiation of the ^{60}Co source, a closed boundary is determined to have a best NGD ratio while maintaining neutron detection efficiency as high as possible. Events inside the boundary are assigned to the neutron class.

Fig.10 Principal-component analysis (PCA): (a) projected energy-parameter P_{E2} vs. P_{E1} ; (b) histogram of P_{E2} . The dashed lines in (a) define the neutron domain, while the vertical line in (b) indicates the neutron-gamma boundary for the 1D feature P_{E2} .

NGD ratios from digital analysis of waveforms

We estimated the average NGD ratios from the digital analysis methods discussed above using the previous procedure [16], one waveform set collected using the ^{252}Cf neutron source, and 5 waveform sets collected using the ^{60}Co gamma source. The results are listed in Table I. In the case of CCA, neutron and ^{60}Co gamma events are completely separated (Fig. 9a) within the limit of statistics used in this work. However, only one event in three data sets collected under

the ^{137}Cs gamma-ray irradiation (not shown in Fig. 9a) is located in the neutron domain. This misidentified event was used to evaluate the lower limit of the NGD ratio for CCA as listed in Table I. There we also list the neutron detection efficiencies ($\eta_{\text{eff}}^{\text{n}}$, relative to a ^3He 8-bar tube detector). In the traditional PHA method (Fig. 2b), the threshold channel N_{th} is set such that the $\eta_{\text{eff}}^{\text{n}}$ from the neutron PHA is equal to that from a digital method, and then the NGD ratio is directly calculated from the ratio of detection efficiencies of neutrons and gamma-rays. The enhanced NGD factor, the ratio of the NGD values of a digital method and PHA, is also listed in Table I.

As shown in Table I, five primitive digital methods (histogram of A_{max} , T_{r} vs. A_{max} , F_{r} vs. S_{T} , a_1 vs. b_0 , and R_{c} vs. A_{max}) provide a NGD ratio about 7-10 times better than using traditional PHA. Such high NGD ratios from the primitive pulse-shape-based digital methods may be explained by the following reason. The shape of the pulses induced in the anode wire in the ^3He tube is influenced by the energy-loss processes of the secondary particles (electrons and ions), the charge drift in the gas phase under electric fields, and the transient charges collected by the anode wire and the detection system [30]. The differences in the shape and height of pulses between neutrons and gammas are demonstrated in Fig. 1(a), Fig. 3(b) and Fig. 4. These differences indicate that secondary electrons generated upon gamma interactions result in small signals and a wider range of rise-time, whereas the daughter ions induced by the thermal neutrons have higher specific energy loss and generate shorter tracks and relatively narrower rise-time [12]. In this work, the differences in both pulse-shape and pulse-height between neutrons and gamma are exploited [31], which should be more effective in NGD than solely using the height of integrated pulses as in PHA.

Table I NGD ratio and neutron detection efficiency for traditional pulse-height analysis and digital waveform analysis. Neutron detection efficiency is relative to an 8-bar ^3He tube with PHA for NGD.

Methods	NGD	$\eta_{\text{eff}}^{\text{n}}$	NGD for PHA ^(b)	Enhanced NGD factor ^(c)
Primitive DSP methods ^(a)	$(3-4) \times 10^6$	95%	4×10^5	7-10
FLDA	5×10^8	99%	4×10^5	1×10^3
CCA	$> 1 \times 10^9$	95%	4×10^5	$> 2 \times 10^3$
OPLS	8×10^7	104%	3×10^5	2×10^2
PCA	4×10^7	99%	3×10^5	1×10^2

^(a) Using 1-D histogram or 2-D scattering plots of A_{max} , T_r vs. A_{max} , F_r vs. S_T , a_1 vs. b_0 , or R_c vs. A_{max} .

^(b) From the pulse-height analysis (PHA) with a variable threshold channel N_{th} to get the same neutron detection efficiency as a digital method.

^(c) The ratio of NGD values between a digital method and PHA.

So far, CCA provides the best NGD ratio among all the digital signal methods. Compared with the primitive DSP methods, we intentionally include the most discriminative features in the target matrix $\mathbf{Y}$ in CCA, which can initially label an individual event with a class type [28] as precisely as a primitive DSP method can. Furthermore, CCA maximizes the correlation between the projected input and output (target) data, and it could simultaneously decide the directions in the input or output spaces with high variances [28]. Therefore, CCA could provide better NGD ratios than other DSP methods presented in this work.

By reducing the number of features used for NGD, PCA provides a NGD enhancement factor nearly as well as OPLS. However, one may question if there is a redundancy among three energy-related features A_{max} , S_T , and b_0 in the vector $\mathbf{E}$. In fact, the rank of the covariance matrix $\mathbf{C}_E$ is 3 due to the large variations of these features, which indicates that none of the three parameters is redundant. Moreover, we solved the eigenvalue problems of $\mathbf{C}_E$ and obtained three eigenvalues λ_i ($i=1$ to 3 and $\lambda_1 > \lambda_2 > \lambda_3$). We found that $\lambda_3/\lambda_1 \approx 0.004-0.01$ and $\lambda_2/\lambda_1 \approx 0.07$ for the neutron events. In the framework of spectral analysis of eigenvalues [27], the eigenvalue λ_1 is the

variance of the event data of a class projected on the eigenvector $\mathbf{u}_i$. Our result suggests that the projection using the third eigenvector may be regarded as noise. We then focus on the relation of P_{E1} and P_{E2} in this work. In fact, the histogram of P_{E1} (not shown) is very similar to that of $A_{\max}$, indicating that P_{E1} correctly catches the general variations of the three energy-related parameters. More interesting is the histogram $f(P_{E2})$ of P_{E2} shown in Fig. 10(b). Obviously, $f(P_{E2})$ has sharper peaks for neutron events, as compared with traditional PHA (Fig. 2a) and the histograms of three energy-related parameters in Fig. 3(b), Fig. 5(a) and Fig. 7(a). P_{E2} has much less variation than the P_{E1} , which can be explained by the facts that $\lambda_2 < \lambda_1$ for both neutron and gamma classes. Though P_{E2} has apparent negative values for neutron events, a mirror operation (or using absolute values) of P_{E2} is allowed, because P_{E2} originated from the transformation between the input and output spaces and the directions of the eigenvectors $\mathbf{u}_i$ ($i=1$ to 3) in the output space can be reversed. Moreover, previous works showed that the improved energy resolution is helpful for improving the spatial resolution of ^3He detectors [29, 32]. It would be interesting to investigate whether a projected energy-related parameter, e.g., P_{E2} or a similar one obtained from other dimension-reduction methods, can improve the spatial resolution of ^3He detectors.

We have presented excellent NGD performances of FLDA and MVA algorithms (CCA, OPLS, and PCA) for neutrons that hit a specific position along the ^3He tube. These algorithms can be applied for the NGD regardless of neutron positions along the tube, if the effect of neutron hit position on the selected features is known. To do so, we performed a quick test by placing the 3-mm slit at two positions along the tube and the slit was irradiated by the moderated ^{252}Cf source. When neutrons hit the middle of the tube, the obtained histogram of centered-and-normalized S_T , an energy-related feature, has the same shape as that at the neutron position 16.2 cm away from the end B of Tube 2. Moreover, we found that the histogram of F_P , a pulse-shape-

related feature, is independent of the neutron hit position. Therefore, to use FLDA and MVA algorithms for any neutron position x , we need to find the position-dependent centroids (μ_F) and standard deviations (σ_F) of the energy-related features ($A_{\max}$, S_T , b_0) before the centering and normalizing these features. This can be realized by installing the 3-mm slit at several neutron positions x along the tube, measuring the waveforms under the irradiations of the ^{252}Cf and ^{60}Co sources, and fitting the obtained $\mu_F(x)$ and $\sigma_F(x)$ for each energy-related feature. This process may be tedious, but it is necessary in order to get excellent NGD ratios from FLDA and MVA algorithms. For the pulse-shape-related features (T_r , F_p , R_c , and a_1), their μ_F and σ_F obtained at one neutron position can be used for the centering and normalizing these features obtained at any neutron position. The sole use of pulse-shape-related features (T_r , F_p , R_c , and a_1) in FLDA and MVA algorithms may be more convenient for the NGD; however, in our initial attempts it provides a lower NGD ratio compared with the application of all seven features reported in this work.

Now that we have excellent DSP methods to discriminate the gamma-rays of energies of about 1-MeV, it becomes important for the ^3He tube detectors to discriminate thermal neutrons from fast neutrons (1-20 MeV) or other high-energy neutrons (100s MeV to multiple GeV) from spallation neutron sources [33]. A passive method for eliminating these high-energy neutrons is to utilize the shielding materials properly designed for this purpose [33]. However, it is possible to modify a subspace learning method to discriminate both high-energy neutrons and the gamma-ray background, e.g., by setting proper thresholds for the projected energy-related features. This possibility will be tested in future work.

V. CONCLUSION

Neutron-gamma discrimination capability of ^3He gas based neutron detectors was studied using traditional pulse-height analysis (PHA) and digital analysis of waveforms. Traditional pulse-height analysis provides a NGD ratio of about 4×10^5 . In comparison, primitive DSP methods directly based on the seven features obtained from the maximum pulse-amplitude, rise time, charge-integration, Wiener filter, and cross-correlation methods can give an improved NGD ratio [$\sim(3 - 4) \times 10^6$]. Compared to PHA, additional correlation analyses of the seven feature parameters can boost the NGD ratio by 2 to 3 orders of magnitude using a Fisher Linear-Discriminant Analysis (FLDA) and three multivariate-analysis (MVA) based algorithms, i.e., CCA, OPLS, and PCA.

In this work, four proposed algorithms are based on the global structure of the features extracted from the waveforms. There are other dimensionality-reduction methods based on the local geometric structure of the features, e.g., kernel-MVAs (KMVAs) [28], locally linear embedding (LLE) [34], and their variants. These methods can potentially produce new prospects to improve NGD and to other particle identification problems compared with the four algorithms in this work. Analogously, the local structure obtained from intermediate-range-order neutron/X-ray diffractometers [35,36] provide new insights for materials science compared with the global structure extracted from conventional diffractometers. However, the computational costs of KMVAs and LLE are expected to be much higher than the methods presented in this paper. In addition, we did not present the results using the variants of FLDA methods [27] in this paper. Nevertheless, the present work illustrates that the developments of digital-signal analysis and pattern-recognition methods are beneficial to the NGD of ^3He -gas based neutron detectors and other neutron detector systems used in neutron scattering, homeland security, and high-energy physics research.

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