Title:

A Feature Point Identification Method for Positron Emission Particle Tracking with Multiple Tracers

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Abstract:

A novel detection algorithm for Positron Emission Particle Tracking (PEPT) with multiple tracers based on optical feature point identification (FPI) methods is presented. This new method, the FPI method, is compared to a previous multiple PEPT method via analyses of experimental and simulated data. The FPI method outperforms the older method in cases of large particle numbers and fine time resolution. Simulated data show the FPI method to be capable of identifying 100 particles at 0.5 mm average spatial error. Detection error is seen to vary with the inverse square root of the number of lines of response (LORs) used for detection and increases as particle separation decreases.

Keywords:

Positron emission particle tracking

Feature point identification

Multiple particle tracking

1. INTRODUCTION

Positron emission particle tracking (PEPT) uses positron emission tomography (PET) technology to track the motion of an activated particle in time [1]. Tracer particles are radiolabelled using a positron-emitting isotope such as ^{18}F , ^{11}C , or ^{22}Na . After emission, the positron annihilates with a nearby electron, producing coincident, back-to-back, 511 keV gamma rays. By detecting coincident gamma ray pairs, one can draw a series of lines of response (LORs) between detection pairs. Each LOR will then pass near the location of the tracer. By examining a series of collections of LORs, one can recreate the position of the tracer particle in time.

Since the invention of PEPT, it has been used to study a number of flow and other industrial systems [2-5]. The majority of the experiments detailed in the literature are single particle tracking experiments. Further experiments have been conducted with multiple particle tracking, but the majority of these have required *a priori* knowledge of the number of particles in the system and their initial locations [6-8]. It is desired to have a PEPT method that does not require such *a priori* knowledge and allows particles to enter and leave the field of view (FOV) of the system. Such a measurement technique could be used for Lagrangian flow measurements in which multiple tracer particles can pass through a test section as part of a recirculating flow loop. Wiggins, et al. [8] developed such a method for multiple particle tracking, and this has been used to study both vortical flow in a transparent test section [9] and jet flow induced between baffle plates [10]. In this paper, a new method for multiple particle tracking is introduced and compared to the previous.

2. PARTICLE TRACKING TECHNIQUES

The most commonly used method for single particle PEPT analysis is the Birmingham method developed by Parker et al. [1]. In this method, one collects LORs over a given amount of time and finds the point in space that minimizes the sum of square distances to the lines. This point is taken to be the particle location. An adaptation of this has been created for tracking multiple particles in the case that the particles are of very different activity [6].

A second method developed by Bickell, et al. [7] searches for a region of the FOV of the detector that has the highest density of LORs by segmenting the FOV into a three-dimensional grid, counting the number of LOR crossings at each grid point, finding the maximum grid element, and applying a Gaussian fit to the data at this point. This method is then expanded to a method capable of tracking multiple particles in the case that the number of particles and their initial positions are known. The methods for multiple particle tracking presented herein are derivatives of this “line density” method in that they all begin by dividing the full data into time slices and creating a grid of LOR density in each slice.

2.1. G-means Clustering Approach

Wiggins, et al. [8] developed an algorithm for multiple particle tracking that does not require *a priori* information of the number of particles in the system. In this way, it can be used to track particles entering and leaving the FOV of the detector system as is typical in recirculating flow experiments. In this method, LOR crossings are tallied across a superimposed grid, as in the line density method, and regions of high line density are detected via G-means clustering [11]. This method will henceforth be referred to as the G-means method for PEPT in this work. A brief review of this method follows.

For the LOR dataset, let $N^t(x,y,z)$ be the number of line crossings at grid point (x,y,z) during time frame t , and let N_{max}^t be the maximum value in the LOR grid at this time. Grid elements having LOR crossing values in the upper r^{th} -percentile of their given frame are selected for use in particle location. The value r is adjusted based on the amount of LOR noise in an experiment, as well as the distribution of activities of particles used in an experiment. At each frame, any point (x,y,z) having a number of line crossings greater than $r \times N_{max}^t$ is treated as a number of $N^t(x,y,z) - r \times N_{max}^t$ points in 3D space at location (x,y,z) . G-means, a divisive clustering algorithm based on the well-established k -means algorithm, is then used to group all the points in a given frame. In this method, one begins with a $k=1$ clustering (i.e. universal centroid identification) of the set. This algorithm then subsequently splits and re-clusters any clusters that are not normally distributed along their main principal components. The process terminates when all clusters are Gaussian, as determined by a 1-D Anderson-Darling test [12]. The centroids of the detected

clusters are then taken to be the particle locations, and the standard deviations of the centroids are taken to be the uncertainties.

Particle locations in each frame are then linked into trajectories based on a nearest-neighbors approach. A modified Kuhn-Munkres algorithm [13, 14] is used to find the inter-frame linking which minimizes the sum of particle displacements. The primary input parameters to this linking method are v_{max} , a maximum velocity limiting the allowed between frame displacement for particle links, and R , a linking range indicating over how many frames a particle will look for a candidate match, similar to that found in [15]. For further discussion of both the identification and linking phases of this algorithm, see [8].

It is noted that in this G-means method, results depend on the selection of the Anderson-Darling critical value, A_{crit} , used when determining of number of clusters present, especially in the case of multiple particle detection. The standard values of A_{crit} , derived by Stephens [16], are for continuous data and have been found unsuitable for the discrete data created from the LOR crossing grid. It has been found in testing that several values of A_{crit} must often be tried before the resulting number of clusters found by G-means matches the true number of particles. It is desired that a multiple particle detection method be devised that does not depend on a parameter which must be adjusted without full understanding for best results.

2.2. Feature Point Identification Method

When considering the aforementioned grid of LOR crossings, one can view each element of the grid as a 3D counterpart to the pixel called a “voxel”, with the number of line crossings being analogous to a greyscale value. As such, traditional optical particle tracking methods may be adapted for use in PEPT. A new approach to multiple PEPT analysis is presented, based on the feature point identification techniques of Crocker and Grier [17] and Sbalzarini and Koumoutsakos [15]. For this reason, this method will be referred to as the feature point identification (FPI) method. This method is carried out in four stages: smoothing, position estimation, position refinement, and trajectory linking.

As with previous methods, this method begins by collecting LORs for a given amount of time and counting line crossings across a grid superimposed on the system’s FOV. Continuing the previous notation, let $N^t(x, y, z)$ be the number of line crossings at position (x, y, z) during time frame t . The grid is first smoothed via a box-car average over a cubic region of width $2f+1$:

$$N^{t'}(x, y, z) = \frac{1}{(2f+1)^3} \sum_{i=x-f}^{x+f} \sum_{j=y-f}^{y+f} \sum_{k=z-f}^{z+f} N^t(i, j, k) \quad (1)$$

where f is taken to be a smoothing size. In most studies, f is set to 1 (minimal smoothing) or 0 (no smoothing). In the above operation, the outer f elements are neglected in each dimension. Crocker and Grier [17] and Sbalzarini and Koumoutsakos [15] also convolve each frame with a Gaussian surface of rotation of half-width 1 pixel to reduce camera pixelation noise. An analog was attempted for this study; however, no significant benefit was seen, and this operation only added to computational costs. This second smoothing step will be omitted until the noise associated with the grid-based discretization of LORs is further understood.

Next, particle positions are estimated by finding local maxima in the smoothed grid, $N^{t'}$. Local maxima are taken to be grid elements having LOR crossing values in the upper r^{th} -percentile of their given frame and having values greater than any of their neighbors within a cube of width $2w+1$. As in the G-means approach, the value r is adjusted based on the amount of LOR noise in an experiment, as well as the distribution of activities of particles used in an experiment.

The parameter w serves as an apparent particle radius where the virtual particle can be viewed as a cloud of high LOR density. However, w limits the between particle separation that can be resolved in a given frame to w times the side length of the grid elements used. Below this separation, only the particle that appears “brightest” in LOR crossing space, i.e. the one corresponding to the highest LOR crossing peak, will be detected, and any others will only serve to bias this measured position toward the centroid of the particles involved. Thus, in the case of two particles approaching closely, it is likely that the trajectory of the higher activity particle will be continued while there will be a break in the measured history of the other. As such, w can be adjusted in a given experiment according to expected particle density so that it is greater than the expected inter-particle separation. Furthermore, in searching for local maxima, the outer w

elements are neglected in each dimension. This prevents bias along the edges of the scanner but restricts the useable FOV of the scanner based on the selection of w .

Ties in the local maxima search are handled on a “first come, first serve” basis. If two grid elements of identical smoothed LOR crossing value are identified as local maxima and are found in the same cubic region of side length $2w+1$, the first one detected by the algorithm will be accepted as a particle position estimate, and the second will be rejected. It is expected that any bias introduced here is reduced or eliminated during the particle position refinement stage of the algorithm. Furthermore, the frequency of such ties is reduced by the use of the aforementioned smoothing filter.

After particle positions are estimated, refinements are made according to a centroid calculation over a cubic region of side length $2w+1$ centered at each estimate. Given an estimate position of $\mathbf{x}=(x, y, z)^T$, a final position, $\mathbf{x}'=(x', y', z')^T$ is calculated by:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \frac{1}{K} \sum_{i=x-w}^{x+w} \sum_{j=y-w}^{y+w} \sum_{k=z-w}^{z+w} \begin{pmatrix} i \\ j \\ k \end{pmatrix} N^{t'}(i, j, k) \quad (2)$$

with normalization:

$$K = \sum_{i=x-w}^{x+w} \sum_{j=y-w}^{y+w} \sum_{k=z-w}^{z+w} N^{t'}(i, j, k). \quad (3)$$

Individual particle positions are linked into trajectories via the aforementioned nearest-neighbors method found in [8]. A four-frame projection method like that of Hassan and Canan [18] has since been implemented, but for consistency in comparison of detection methods, results presented herein use the original, two-frame method. Furthermore, the cases examined in this study are all low particle density cases in which the between-particle separations in each frame are less than the inter-frame displacements for each particle. In such cases, a simple linking method is sufficient as well as computationally efficient.

3. VALIDATION EXERCISES

3.1. MicroPET P4

A series of experiments were designed and conducted at the University of Tennessee Medical Center to test the validity of the original multiple PEPT algorithm of [8]. Selected experiments are here analyzed via the FPI method and compared to the results obtained via G-means clustering. The experiments are conducted in a MicroPET P4 preclinical PET scanner. The P4 contains 168 detector modules, each of these made up of an 8x8 lutetium oxyorthosilicate (LSO) crystal array, arranged in a cylinder of diameter 26 cm and length 7.8 cm. The P4 allows timing resolution up to 1 ms and has an average intrinsic spatial resolution for PET reconstruction of 1.75 mm [19]. All experiments are performed using an energy acceptance window of 425-625 keV, chosen to reduce the random coincidence count rate caused by the decay of the ^{176}Lu present in the LSO crystals. Both tracking algorithms are implemented in C++ and are run using an Apple LLVM 6.1 compiler on an Apple Macbook Pro computer with 2.66 GHz Intel Core 2 Duo processor.

In a first experiment, a single particle of diameter roughly 1 mm is activated with 430 μCi of ^{18}F and placed in the center of the FOV of the scanner and left stationary. The particle’s position was recorded for 1 s using 1 ms time steps and a grid of element size $2\text{x}2\text{x}2 \text{ mm}^3$. Grid sizes and time steps are chosen to match those of [8]. Given the particle’s activity and the scanner’s sensitivity, this resulted in an average of 140 LORs per time step. The particle’s average position and standard deviation were calculated from these 1000 histories. Using the FPI method with $f=1$, $r=0.3$, and $w=3$ the particle’s location was found to be $(1.22, -5.49, 39.4) \pm (0.51, 0.48, 0.24) \text{ mm}$ in the x , y , and z -directions, respectively. In the coordinates used herein, z denotes the axial, x the horizontal transaxial, and y the vertical transaxial direction, with the origin located in the radial center of the bore, aligned to its front axial edge. When analyzed with G-means, the particle’s location was calculated to be $(1.13, 5.19, 39.5) \pm (0.32, 0.36, 0.32) \text{ mm}$. The two show similar

1 results in position measurement with the G-means result showing lower variances in the transaxial
2 direction, while that of FPI is lower in the axial direction. The runtime for FPI was 7.48 s, compared with
3 9.65 s for G-means.

4 Next, a study is conducted to test the ability of both methods to track multiple particles simultaneously.
5 17 particles of diameter 1 mm and activated with between 20 and 50 μCi of ^{18}F are placed in a polyethylene
6 holder with average separation 20 mm and moved through the field of view of the scanner using the
7 scanner's bed motion controls at a rate of approximately 3 mm/s. First, a $1\times 1\times 1\text{ mm}^3$ grid and 100 ms time
8 steps are used, with on-average 230 LORs per particle per time step. For both studies, a value of 0.3 was
9 used for r . For G-means, an A_{crit} value of 25 was used, and FPI used values of $f=1$ and $w=3$. An identical
10 trajectory linking routine is used for all cases. The resulting trajectories can be seen in Fig. 1. Here, each
11 color represents a different particle. It can be seen that FPI is capable of replicating the multiple particle
12 detection ability of G-means. Furthermore, FPI appears to be more capable of detecting particles near the
13 axial edge of the scanner. This is shown by the slightly longer trajectories detected by FPI.

14
15 (Figure 1)

16
17 Next, the same dataset is considered using 25 ms time steps and the same detection parameters listed
18 previously, resulting in only about 60 LORs per particle per time step. Results are seen in Fig. 2. It can be
19 seen that G-means was only capable of tracking 13 of the particles, and these were only tracked over a
20 fraction of their transit. While the trajectories found by FPI show significantly more waggle than their 100
21 ms counterparts (here demonstrated by a thicker line), it is seen that this method was capable of finding all
22 17 particles throughout the length of the scanner.

23
24 (Figure 2)

25
26 Lastly, two of the particles from the previous experiment were used to test the resolving power of this
27 method. In this study, one particle was held stationary while the other was moved toward it until the two
28 made contact. The resolving power of G-means with 25 msec time steps and a $2\times 2\times 2\text{ mm}^3$ mesh was
29 previously found to be roughly 5 mm [8]. Using FPI with $w=3$ and meshes of $1\times 1\times 1\text{ mm}^3$ and $2\times 2\times 2\text{ mm}^3$,
30 the particles were resolved down to a separation of 2.7 mm and 4.5 mm, respectively. Below this and until
31 contact, tracking of the moving particle continued while the trajectory corresponding to the stationary
32 particle ended. These least resolved separations seen are slightly lower than the respective values of 3 mm
33 and 6 mm that are expected based on the selection of w and mesh size. It is believed that the nearness of the
34 two particles resulted in a bias during the position refinement stage, i.e. initial particle guesses were outside
35 the expected resolving limits, but final particle positions were shifted toward the center of the particles
36 during the refinement step.

37 38 3.2. Simulation Comparison

39
40 Since the acquisition of the above data, a Siemens Inveon preclinical PET scanner has been acquired by
41 the University of Tennessee, and PEPT experiments have been conducted using this machine [10]. Similar
42 to the P4, the Inveon is made up of 64 detector modules arranged in a cylinder of diameter 16.1 cm and
43 length 12.7 cm. Each of these modules is comprised of a 20×20 array of LSO crystals. The Inveon allows
44 timing resolution up to 200 μs and has spatial resolution below 1.8 mm for PET reconstruction [20]. A
45 GEANT4 Application for Tomographic Emission (GATE) [21] model of this machine has been developed
46 at the University of Tennessee [22]. The geometry of this model, as well as an example of LORs from an
47 emission scan, can be seen in Figure 3.

48
49 (Figure 3)

50
51 In order to further test the FPI and G-means methods in higher particle number environments, this
52 model was used to simulate emission scans using varying numbers of particles. Simulations were
53 conducted using 25, 50, 75, and 100 particles randomly distributed in a cylinder of diameter 80 mm and
54 length 80 mm centered in the bore of the scanner and aligned axially with the scanner bore. The simulated
55 particles were ^{22}Na point sources of activity 5 μCi each in cubic volumes of side length 0.5 mm. Energy
56 blurring is induced via random assignment of crystal resolutions between 15% and 35% based on a

reference energy of 511 keV. In each case, the particles were left stationary and scanned for 10 s using an acquisition energy window of 425-625 keV and the default coincidence timing window of 3.432 ns.

It should be noted that such GATE models have been seen to overpredict the number of LORs generated by a source during a scan [23, 24]. In [23] the magnitude of this overprediction was determined, and a number of the simulated LORs were rejected based on a Bernoulli random process. In the case of our simulation, machine dead time effects began to affect the outcomes of the experiments with higher particle numbers, making rejection of events in post-processing unphysical. For this reason, all LORs generated by GATE are kept, but results will be cited in terms of the number of LORs used for particle location, not the actual activity of the particles.

For each of the four aforementioned cases, particle locations were determined using both G-means and FPI with 100 ms time steps. Thus for the 25, 50, 75, and 100 particle cases, an average of 1012, 770, 575, and 417 LORs per particle, respectively, were used for particle location at each time step. In both methods, a $0.5 \times 0.5 \times 0.5$ mm³ grid was used for line-density tallying, along with a filtering value of $r=0.3$. For all cases, G-means used an A_{crit} value of 10, and FPI used values of $f=1$ and $w=6$. These values were determined based on experience and testing and were held constant to avoid influences of parametric changes between cases. Identical linking parameters are used for all cases.

Results are evaluated based on the number of particles found and the continuity and accuracy of the detection. In these simulations, a particle is deemed “found” if it is located at least 20 times in the 100-frame set, and these locations are linked by the common linking algorithm. In this way any detected short, spurious trajectories are treated as false positives and rejected. Furthermore, instances where a given particle is tracked in time, lost for a number of time steps, and then found again are treated as “gaps” in trajectories and considered below. Lastly, accuracy, is determined in the x (radial, horizontal) direction by comparing the measured position of each particle, $x_{meas,i}$, to that of the original particle in the simulation, $x_{sim,i}$, by

$$\Delta x = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_{meas,i} - x_{sim,i})^2} \quad (4)$$

and similarly for the y (radial, vertical) and z (axial, horizontal) directions, where N is the number of detections of a given particle. A summary of the results is seen in Table 1.

(Table 1)

It can be seen that the FPI method was more capable of finding individual particles than G-means, with G-means showing extreme deterioration in the 100 particle case. Furthermore, in all but the 100 particle case, FPI also outperforms G-means in terms of gaps in tracking and average measurement accuracy, with the comparison between the 100 particle cases being skewed by the disparity of particles detected by G-means. It is of note that the requirement that all trajectories must be at least 20 frames long eliminated all false positives for both methods. For visual comparison, Figure 4 shows the particles detected via G-means (a), those detected with FPI (b), and the true particle positions (c) for the 100 particle case. It is easily observed that the measurements of FPI very closely recreate the true particle distribution seen in panel (c), while G-means only captures a fraction of the true distribution.

(Figure 4)

4. APPLICATION EXERCISE

An experiment is performed to test the ability of the FPI method to track particles moving in a circular motion, similar to that performed in [1,2]. In this experiment, two ²²Na sources of activity 7 μCi each are attached to the ends of a rod, and the rod is rotated about its near-midpoint via a drill. An acquisition scan is performed in a Siemens Inveon scanner with the drill mounted to the scanner’s bed and the sources inside the scanner bore. The axis of rotation is aligned with the axis of symmetry of the scanner bore. The radii of rotation of the two particles are measured with a caliper and found to be 26 ± 2 mm and 24 ± 2 mm. The

particle with the larger radius will be henceforth referred to as “particle 1” and the other as “particle 2”. The uncertainty here is due to uncertainty of the exact location of the sources within their mounts. The frequency of rotation is measured with a laser triggering system and found to be 6.9 Hz. As such, the expected velocities of the two particles are 1.1 ± 0.1 m/s and 1.0 ± 0.1 m/s, and the expected accelerations are 48 ± 4 m/s² and 43 ± 4 m/s², respectively.

PEPT data are analyzed with FPI in time steps of 3 ms, resulting in the use of 30 LORs per particle per detection, on average. LOR crossings are tallied across a $1 \times 1 \times 1$ mm³ grid, and FPI parameters of $r=0.4$, $f=1$, and $w=10$ are used. 3D representations of the measured trajectories and a sample of the measured radial coordinates (x and y) of the particles are seen in Figure 5.

(Figure 5)

Here it can be seen that the FPI method is capable of tracking the motion of the particles. The x and y coordinates of each particle are fitted to functions of the form

$$x = x_0 + R \cos(\omega t + \phi) \quad (5a)$$

$$y = y_0 + R \sin(\omega t + \phi). \quad (5b)$$

Best fit curves are seen as black lines in Figure 5. The radii of the trajectories measured via this best fit are 23.9 mm for particle 1 and 23 mm for particle 2. Such an underprediction of the radius of the particle trajectories is predicted by Parker, et al. as the motion of a particle in an arc during a finite time step will cause the centroid of its LOR distribution to fall slightly inside the circle [1]. Furthermore, it is known that in PET measurements there is a biasing of measurements toward the radial center of the scanner that increases as one moves away from the radial center due to “radial elongation”. This effect is caused by a gamma ray that impinges obliquely onto a crystal face and penetrates into a neighboring crystal before interaction, resulting in a false assignment of the detection site [25]. It is expected that this same effect is further biasing this measurement.

To make use of this technique for Lagrangian measurements, it is desired that velocity and acceleration information can be extracted from measured trajectories. Voth, et al. [26] emphasize that one must low-pass filter position data before using it for calculation of velocity and acceleration information. Both the filtering and differentiating of the position signal can be achieved via convolution with Gaussian kernels, as is done in [27]. By convolving with differentiated discrete Gaussian kernels of the form

$$k(\tau) = A \exp\left(-\frac{\tau^2}{\sigma^2}\right) \quad (6a)$$

$$k'(\tau) = A' \tau \exp\left(-\frac{\tau^2}{\sigma^2}\right) \quad (6b)$$

$$k''(\tau) = A'' \left(\frac{2\tau^2}{\sigma^2} - 1\right) \exp\left(-\frac{\tau^2}{\sigma^2}\right) + B \quad (6c)$$

filtered position, velocity, and acceleration data are obtained, respectively. Here normalization constants are selected so that $k * 1 = 1$, $k' * \tau = 1$, $k'' * 1 = 0$, and $k'' * \tau^2 = 2$, i.e. proper filtering and first and second derivatives of simple functions are achieved, where “*” is the convolution product. The filter width, σ , can be adjusted for best results. This is not explored in this work, but an example of such is seen in [26].

The previously shown rotating particle data are filtered and differentiated in this way. The filtered position, velocity, and acceleration time histories of particle 1, with sinusoidal trend lines added, are seen in Figure 6. Here, convolution has been performed with kernels with filter width $\sigma=2$ time steps.

(Figure 6)

Velocity and acceleration values are obtained from sinusoidal fits to data and found to be 1.0 m/s and 45.4 m/s² for particle 1 and 0.99 m/s and 43.7 m/s² for particle 2, respectively. These values undershoot those expected from the measured particle radii and drill frequency. This is anticipated, given the inward biasing seen earlier. However, these values closely match the velocity and acceleration calculated using the measured frequency and the radii calculated from fitting position data to equations 5, indicating that the majority of the error here has been introduced in the measurement of particle positions, not in the differentiating process. In the future, this can be reduced by conducting measurements closer to the radial center of the scanner and using particles whose mean motion is in the axial direction, as would be the case in a through-passing, recirculating flow experiment.

Lastly, it can be seen in Figure 6 that a large amount of erraticism is seen in the differentiated data, especially acceleration. It is for this reason that finer time resolution is desired in order to resolve if such behavior is noise or a physical attribute of the system (shaking of the drill mount and irregularity of motion in this measurement; turbulent, chaotic behavior in flow measurements). In order to achieve this, higher activity tracer particles are used in physical measurements. In this experiment, the sources were of activity only 7 μ Ci, whereas in physical flow experiments, tracers used are of activity of order 1 mCi [9, 10].

5. ERROR ASSESSMENT

For past PEPT methods, it has been shown or postulated that accuracy varies as $\frac{1}{\sqrt{N}}$ where N is the average number of LORs used for each detection [1, 7]. To assess this for the FPI method, the four previous simulations are further examined using varying time steps. All other detection parameters are held the same as the previous test, but the time steps used are varied from 10 ms to 100 ms in 10 ms steps. The accuracy of measurement is then assessed via an overall detection error:

$$\Delta r = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} \quad (7)$$

for each particle. Figure 7 shows the average error for each run plotted against the inverse of the square root of the average number of LORs used for detection, with trendlines added. For the 100 particle simulation with 10 ms and 20 ms time steps and for 75 particle simulation with 10 ms time steps, results were exceptionally poor and were deemed unuseable. For this reason, these three cases are omitted from Figure 7 and the following discussion.

(Figure 7)

Here, one can see that in each case, average error varies linearly with $\frac{1}{\sqrt{N}}$, as expected, with the linear fits to the 25, 50, 75, and 100 particle cases having R^2 values of 0.967, 0.985, 0.988, and 0.973, respectively. Also, it is seen that this dependence is stronger for higher particle cases; however, the constant of proportionality is still unknown and is expected to be a function of many factors (number of particles, velocity, scanner geometry, the parameters f and w , etc.). Further work is underway to understand the effects of these various parameters in PEPT uncertainty. A user will be required to select time steps that allow for a time resolution suitable for a given application (as seen in the previous section), while maintaining an ample number of LORs per time step for location accuracy. An increase in particle specific activity will help to relieve this burden.

Lastly, it is expected that measurement error would increase in cases where two particles are very near each other as the distributions of LORs from the two particles begin to overlap. Figure 8 shows the measurement error of each particle as a function of distance to its nearest neighbor for the 100 particle case, as examined by FPI using 100 ms time steps and the same detection parameters as before. Here one sees an increase in measurement error as particles are found more closely together. However, it is also observed that beyond a separation of about 7 mm, the error appears to stay at a steady constant of about 0.5 mm. As such, use of environments with low particle number densities (i.e. greater average particle separation) will allow for the acquisition of cleaner data at the cost of poorer statistics.

(Figure 8)

6. CONCLUSIONS AND FUTURE WORK

A method for multiple-particle PEPT analysis based on the principles of optical feature point identification (FPI) is presented herein. This method is shown to be of similar quality to the previously developed G-means method in the case of a single stationary particle and two particles slowly approaching one another. In the case of multiple, slow-moving particles, FPI is found to recreate particle trajectories with finer spatial resolution than previously possible using G-means. Simulations also indicate the ability of FPI to outperform G-means in the case of locating large numbers of particles. The FPI method is also seen to be able to recreate the trajectories of two particles of activity $7 \mu\text{Ci}$ moving at speeds of roughly 1 m/s in a circular path around the scanner major axis. There is a slight biasing toward the radial center of the scanner, and velocity and acceleration data are extracted from the trajectories.

Preliminary testing of these methods indicates that the FPI method is superior to our previous reconstruction algorithm. Presently, the G-means method shows a slightly higher precision in single-particle detection experiments. However, FPI seems more capable of detecting particles using fewer LORs, ideal for achieving fine time resolution. Furthermore, the user-input parameters needed for FPI are easily understood and adjusted and do not suffer from the need for major user refinement as does the A_{crit} input of G-means. As shown herein via simulation, the FPI method also outperforms G-means in detecting many particles simultaneously, which is an important capability in flow characterization where an increased number of measured particle trajectories leads to improved measurement of turbulence statistics. Further, the FPI method requires less computational time, supporting movement toward near real time assessment of flow data.

Experimental testing has begun using the Siemens Inveon PET scanner [10]. Experiments will be conducted to test the performance of our current methods for flow characterization.

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REFERENCES

1. D. J. PARKER, et al., "Positron Emission Particle Tracking-a Technique for Studying Flow Within Engineering Equipment," *Nucl. Instr. Meth. Phys. Res. A*, **326**, 592 (1993).
2. M. BARIGOU, "Particle Tracking In Opaque Mixing Systems: An Overview of the Capabilities of PET and PEPT", *Chem. Eng. Res. Des.*, **82**, 1258 (2004).
3. Y. CHANG, et al., "PEPT: An Invaluable Tool for 3-D Particle Tracking and CFD Simulation Verification in Hydrocyclone Studies," *EPJ Web of Conf.*, **05001**, 1 (2013).
4. W. GRIFFITHS, et al., "The Use of Positron Emission Particle Tracking (PEPT) to Study the Movement of Inclusions in Low-Melting-Point Alloy Castings", *Metall. Mater. Trans. B*, **43B**, 370 (2011).
5. D. J. PARKER, et al., "Positron Imaging Techniques for Process Engineering: Recent Developments at Birmingham", *Meas. Sci. Technol.*, **19**, 1 (2008).
6. Z. YANG, et al., "Multiple-particle Tracking – an Improvement for positron particle tracking." *Nucl. Instr. Meth. Phys. Res. Sec. A*, **564**, 332 (2006).
7. M. BICKELL, et al., "A New Line Density Tracking Algorithm for PEPT and its Application to Multiple Tracers," *Nucl. Instr. Meth. Phys. Res. Sec. A*, **682**, 37 (2012).
8. C. WIGGINS, et al., "A novel clustering to positron emission particle tracking," *Nucl. Instr. Meth. Phys. Res. Sec. A*, **811**, 18 (2016).
9. S. LANGFORD, et al., "Positron Emission Particle Tracking (PEPT) for Fluid Flow Measurements," *Nucl. Engr. Des.*, **302**, 81 (2016).
10. S. LANGFORD, et al., "Positron Emission Particle Tracking (PEPT) Validation for Jet Flow,"

- 1 *ICONE 24*, Charlotte, North Carolina, June 26-30, 2016, American Society of Mechanical Engineers
2 (2016).
- 3 11. G. HAMERLY and C. ELKAN, "Learning the k in k -means," *Advances in Neural Information*
4 *Processing Systems 16* (2003).
- 5 12. T. W. ANDERSON and D. A. DARLING, "Asymptotic Theory of Certain 'Goodness of Fit' Criteria
6 Based on Stochastic Processes," *Ann. Math. Stat.*, **23**, 193, (1952).
- 7 13. H. KUHN, "The Hungarian Method for the Assignment Problem," *Nav. Res. Logist., Q*, **2**, 83 (1955).
- 8 14. J. MUNKRES, "Algorithms for the Assignment and Transportation Problem," *J. Soc. Ind. Appl.*
9 *Math.*, **5**, 32 (1957).
- 10 15. I. F. SBALZARINI and P. KOUMOUTSAKOS, "Feature Point Tracking and Trajectory Analysis for
11 Video Imaging in Cell Biology," *J. Struct. Biol.*, **151**, 182 (2005).
- 12 16. M. A. STEPHENS, "EDF Statistics for Goodness of Fit and Some Comparisons," *J. Am. Stat. Assoc.*,
13 **69**, 730 (1974).
- 14 17. J. C. CROCKER and D. G. GRIER, "Methods of Digital Video Microscopy for Colloidal Studies," *J.*
15 *Colloid Interf. Sci.*, **179**, 298 (1996).
- 16 18. Y. A. HASSAN and R. E. CANAAN, "Full-Field Bubbly Flow Velocity Measurements Using a
17 Multiframe Particle Tracking Technique," *Exp. Fluids*, **12**, 49 (1991).
- 18 19. Y. C. TAI, et al., "Performance Evaluation of the MicroPET P4: a PET System Dedicated to Animal
19 Imaging," *Phys. Med. Biol.*, **46**, 1845 (2001).
- 20 20. Q. BAO, et al., "Performance Evaluation of the Inveon Dedicated PET Preclinical Tomograph Based
21 on the NEMA NU-4 Standards," *Journ. Nucl. Med.*, **50**, 401 (2009).
- 22 21. S. JAN, et al., "GATE a Simulation Toolkit for PET and SPECT," *Phys. Med. Biol.*, **49**, 4543 (2004).
- 23 22. S. LEE, et al., "Development and Validation of a Compete GATE Model of the Siemens Inveon
24 Trimodal Imaging Platform," *J. Mol. Imaging*, **12**, 1 (2013).
- 25 23. K. S. LEE, et al., "Single-Cell Tracking With PET Using a Novel Trajectory Reconstruction
26 Algorithm," *IEEE Trans. Med. Imaging*, **34**, 994, (2015).
- 27 24. L. LU, et al., "Performance Evaluation of the Inveon PET Scanner Using GATE Based on the NEMA
28 NU-4 Standards," *2013 NSS/MIC*, Seoul, Korea, October 27-November 2, 2013, Institute of Electrical and
29 Electronics Engineers, (2013).
- 30 25. W. W. MOSES, "Fundamental limits of spatial resolution in PET," *Nucl. Instr. Meth. Phys. Res. A*,
31 **648**, 5236 (2011).
- 32 26. G. A. VOTH, et al., "Measurement of particle accelerations in fully developed turbulence," *J. Fluid*
33 *Mech.*, **469**, 121 (2002).
- 34 27. N. MORDANT, et al., "Experimental Lagrangian acceleration probability density function
35 measurement," *Physica D*, **193**, 245 (2004).
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Figure

Fig. 1. Particle trajectories calculated using 100 ms time steps via G-means (a) and FPI (b).

Figure

Fig. 2. Particle trajectories calculated using 25 ms time steps via G-means (a) and FPI (b).

Figure

Fig. 3. Geometry of GATE model of Siemens Inveon PET scanner. Shown are LORs as generated by a source particle in center FOV of the scanner.

Figure

Fig. 4. Particle positions from 100 particle GATE simulation as measured by G-means method (a) and FPI method (b) using 100 ms time steps and $0.5 \times 0.5 \times 0.5 \text{ mm}^3$ grid. Each different color indicates a different particle, as measured. Panel (c) shows the true particle positions input into GATE.

Figure

Fig. 5. Trajectories of measured in rotating particle experiment for particle 1 (red +) and particle 2 (blue ×). Shown are 3D representation (a) and radial coordinates, x (b) and y (c), over the first half-second of measurement with sinudoidal trend line seen black.

Figure

Fig. 6. Measured acceleration (a), velocity (b), and filtered position (c) of particle 1 in rotating particle experiment with sinusoidal trend line shown in black. Data are obtained via convolution with Gaussian kernels of filter-width $\sigma=2$.

Figure

Figure 5. Location accuracy variation with inverse square root of average number of LORs used in each time step for multiple particle measurements with FPI with $0.5 \times 0.5 \times 0.5 \text{ mm}^3$ grid.

Figure

Figure 6. Location accuracy variation with nearest neighbor proximity for 100 particle simulation using $0.5 \times 0.5 \times 0.5 \text{ mm}^3$ grid and 100 ms time steps.

Table

Number of Particles	G-means					FPI				
	Number Found	Number of Gaps	Δx (mm)	Δy (mm)	Δz (mm)	Number Found	Number of Gaps	Δx (mm)	Δy (mm)	Δz (mm)
25	25	13	0.28	0.25	0.26	25	0	0.11	0.12	0.14
50	46	6	0.21	0.29	0.22	50	0	0.14	0.17	0.19
75	60	8	0.32	0.38	0.30	75	0	0.16	0.19	0.23
100	13	2	0.28	0.22	0.22	100	8	0.26	0.27	0.33

Table 1. A comparison of results obtained for simulated multiple particle tracking using G-means and FPI methods. All results obtained using LOR time slices of 100 ms tallied over $0.5\times0.5\times0.5\text{ mm}^3$ grid.