Low-resolution Gamma-ray Spectrometry for an Information Barrier Based on a Multi-Criteria Template-Matching Approach

Malte Götttsche, Janet Schirm, Alexander Glaser

Princeton University

Abstract

Gamma-ray spectrometry has been successfully employed to identify unique items containing special nuclear materials. Template information barriers have been developed in the past to confirm items as warheads by comparing their gamma signature to the signature of true warheads. Their development has, however, not been fully transparent, and they may not be sensitive to some relevant evasion scenarios. We develop a fully open template information barrier concept, based on low-resolution measurements, which, by design, reduces the extent of revealed sensitive information. The concept is based on three signatures of an item to be compared to a recorded template. The similarity of the spectrum is assessed by a modification of the Kolmogorov-Smirnov test to confirm the isotopic composition. The total gamma count rate must agree with the template as a measure of the projected surface of the object. In order to detect the diversion of fissile material from the interior of an item, a polyethylene mask is placed in front of the detector. Neutrons from spontaneous and induced fission events in the item produce 2.223 MeV gamma rays from neutron capture by hydrogen-1 in the mask. This peak is detected and its intensity scales with

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the item’s fissile mass. The analysis based on MCNP Monte Carlo simulations of various plutonium configurations suggests that this concept can distinguish a valid item from a variety of invalid ones. The concept intentionally avoids any assumptions about specific spectral features, such as looking for specific gamma peaks of specific isotopes, thereby facilitating a fully unclassified discussion. By making all aspects public and allowing interested participants to contribute to the development and benchmarking, we enable a more open and inclusive discourse on this matter.

Keywords: gamma-ray spectrometry, information barrier, template approach, nuclear warhead verification, nuclear disarmament, nuclear arms control, prompt-gamma activation analysis

1. Background

Current nuclear arms-control agreements between the United States and Russia limit the number of deployed strategic nuclear weapons each party can have. Future nuclear arms-control agreements, however, are likely to also place limits on the number of nuclear weapons and warheads that a country can maintain in its arsenal. For example, the 2010 U.S. Nuclear Posture Review Report recommended that the United States should engage Russia in negotiations aimed at “achieving substantial further nuclear force reductions and transparency that would cover all nuclear weapons—deployed and non-deployed, strategic and non-strategic” [1]. Such agreements would require the inspection and verification of warheads that are in storage or queued for dismantlement. To accomplish this task, viable verification approaches must be available to confirm the authenticity

46 of nuclear components while also protecting classified information [2, 3, 4]. One
47 such approach would be to analyze the gamma spectrum originating from an
48 item identified for verification, and—through the use of an information barrier—
49 confirm or deny the identity of the item as a valid nuclear component without
50 otherwise revealing any information about the item.

51 Several information barriers have been developed based on the so-called at-
52 tribute method, whereby a given set of attributes, such as mass thresholds
53 and isotopic ratios, are used to characterize (classified) nuclear components.
54 These systems typically require high-resolution detectors, such as high-purity
55 germanium detectors, to perform this task. Prominent attribute systems are
56 the “Trusted Radiation Attribute Demonstration System” (TRADS) [5], the
57 “Attribute Verification System with Information Barrier for Plutonium with
58 Classified Characteristics utilizing Neutron Multiplicity Counting and High-
59 Resolution Gamma-ray Spectrometry” (AVNG) [6] developed as part of the
60 Trilateral Initiative between the United States, Russia, and the International
61 Atomic Energy Agency and, more recently, the information barrier developed
62 as part of the UK/Norway Initiative (UKNI) [7]. A second approach, called
63 template-matching, has been developed in parallel. The template approach
64 does not seek to determine absolute characteristics of the inspected item, such
65 as plutonium mass or isotopics; instead, this complementary method seeks to
66 detect significant differences between two items, for example by means of gamma
67 spectroscopy, without seeking to determine the origin for these differences. Few
68 such systems have been proposed and built; among these, the Trusted Radi-

69 ation Inspection System (TRIS) developed at Sandia National Laboratories is
70 the most prominent and widely tested one [8]. While the general strategy of
71 TRIS has been described in the literature, few relevant details of the developed
72 algorithm have been published. For a more detailed overview of both attribute
73 and template systems, see [3], [9] and [10].

74 In this paper, a concept for an open-software template approach is proposed
75 and evaluated based on its capability to detect the most relevant evasion sce-
76 narios. One of them is the diversion of fissile material. While passive gamma
77 spectrometry is generally not well suited for determining the fissile mass in a nu-
78 clear component due to self-shielding effects, we examine conditions under which
79 the technique is able to detect significant differences in the mass, in addition to
80 isotopic composition and the size of the item’s surface, in simple configurations
81 using simulated gamma spectra.

82 2. Validation

83 Below, we will use extensive MCNP6 Monte Carlo simulations [11] to deter-
84 mine the expected performance of the inspection system. In order to validate
85 the models used for these simulations, we draw on the high-quality SINBAD-
86 2013.12 benchmark data for the so-called beryllium-reflected plutonium (BeRP)
87 ball. The dataset is part of a larger collection available from the Radiation
88 Safety Information Computational Center (RSICC) [12]. The BeRP ball is a
89 solid 4484-gram sphere of alpha-phase, weapon-grade plutonium (0.02 wt% Pu-
90 238, 93.27 wt% Pu-239, 5.91 wt% Pu-240, 0.07 wt% Pu-241, 0.03 wt% Pu-242)

91 enclosed in a thin steel casing. The ball was fabricated in 1980 and contained
92 about 0.25 wt% of americium-241 at the time of the measurement (January
93 2009). The SINBAD exercise used a High-Purity Germanium (HPGe) detector
94 for all gamma measurements [13, 14].

95 The simulated spectra are obtained using MCNP’s pulse height tally. The
96 following simulation procedure results in an accurate representation of the mea-
97 sured spectrum: First, a neutron simulation is run with a spontaneous fission
98 source to determine induced fission rates as a function of the distance from the
99 center of the BeRP ball. In a second simulation, induced fission reactions are
100 not simulated during the neutron transport, but induced fission neutrons are
101 directly included in the neutron source alongside spontaneous fission neutrons
102 and the gamma rays from decay. The second step simulates only one source
103 particle per simulation history. The partition into two steps is necessary, be-
104 cause simulating a spontaneous fission event and all subsequent interactions in
105 one history could result in more than one gamma ray being detected per his-
106 tory. The pulse height tally yields the energy deposited per history, i.e., not
107 per gamma ray [11, p. 3-177]. Tally data are broadened following the MCNP
108 simulation using a normal distribution with a width (resolution) determined by
109 the simulated detector type.

110 In order to increase the efficiency of the Monte Carlo simulations, we place
111 a number of identical detectors on a spherical surface. Detectors can be evenly
112 distributed on this surface using a modification of “Vogel’s Method” [15]. Par-
113 ticles that reach one of the detectors but are not absorbed are terminated by

114 the simulation to avoid the possibility of a non-realistic interaction in another
115 detector. In order to further increase the simulation efficiency, we only include
116 gamma energies above 100 keV because lower-energy gammas would be easily
117 shielded and do not provide a robust signature for verification purposes. A typ-
118 ical run corresponding to a 60-minute measurement uses about 1000 hours of
119 single-CPU computer time.

120 Figure 1 shows a comparison between the experimental data acquired during
121 the SINBAD measurement campaign and the results obtained with MCNP com-
122 puter simulations. In the measurement shown, the BeRP ball was surrounded by
123 a series of nesting spherical shells of high-density polyethylene, overall 7.62 cm
124 thick. Neutrons are moderated and subsequently captured by hydrogen in these
125 shells, resulting in the emission of high-energy gammas (2223 keV)—an effect
126 that is further discussed below.

127 **3. Concept**

128 In order to limit the amount of sensitive information acquired during a
129 measurement, we examine whether robust results can be obtained with low-
130 resolution detectors. While the information barrier concept is introduced to
131 prevent inspectors from accessing the spectral data, acquiring low-resolution
132 spectra can add an additional layer of security.¹ Similar to Sandia’s TRIS, the

¹Even with low-resolution data, a knowledgeable inspector (for example, from another weapon state) could attempt a “dictionary attack,” in which possible configurations based on “educated guesses” are forward-modeled until a matching spectrum is found.

Figure 1: Measured (black) and simulated (red) high-resolution benchmark gamma spectra for the BeRP ball in a 7.62-cm polyethylene reflector. While the simulated background is overall slightly lower, the spectral features are well reproduced. Simulated (tally) data are broadened following the MCNP simulation to reproduce the resolution of the detector type used in the experiment (see insert of the 350–450 keV region). Note the peak at 2223 keV, which is due to neutron capture in polyethylene.

133 proposed setup uses a sodium-iodide (NaI) detector with a 5.08-cm by 5.08-cm
 134 crystal (Figure 2). The detector resolution has been determined by measure-
 135 ments of various radioactive sources with the *Canberra* Model 802 NaI detector
 136 [16], and the respective parameters are used for subsequent MCNP calculations
 137 to broaden gamma lines. A 15-cm-thick polyethylene mask is placed between
 138 the source and the detector; this mask enables sensitivity to the neutron signa-
 139 ture of the item via gamma detection from neutron capture in hydrogen. As the
 140 polyethylene also acts as gamma shielding, the mask has a central hole to allow

141 gammas to pass through without attenuation. A 5-cm-thick steel plate (with a
 142 corresponding hole) serves to attenuate gammas that would otherwise scatter in
 143 the polyethylene and arrive in the detector, *inter alia* contributing to Compton
 144 background. The mask has an outer radius of 20 cm and a polyethylene mass of
 145 17.6 kg. The detector is positioned 10 cm behind the plate. The mask's thick-
 146 ness and its distance to the detector are chosen based on MCNP simulations
 147 performed to identify a geometry to yield a sufficient 2223-keV peak intensity.

Figure 2: Simulated experimental setup for the multi-criteria template-matching approach.

148 Two principles guide the development of the system proposed here. First,
 149 we seek to avoid making assumptions about the expected radiation signatures;
 150 in particular, we are not trying to tailor the algorithms to particular spectral
 151 features of expected elements and nuclides such as uranium or plutonium. Can-
 152 didate algorithms should be simple, versatile, and robust. Such an approach
 153 may also be easier to use and optimize in an unclassified setting because it
 154 should perform equally well with standard calibration sources. Second, we ex-
 155 tend the standard template approach, which relies on a comparison of gamma

spectra, by including two additional criteria in order to be more sensitive to important diversion scenarios that may otherwise be missed. Combined, the current criteria are:

Spectral features. The level of similarity between the acquired gamma spectrum and the template is determined by an “empirical” Kolmogorov-Smirnov (KS) test. In statistics, the standard two-sample KS test is a nonparametric hypothesis test that measures the similarity of two distributions. To do this, the test uses the largest absolute difference (D_{KS}) between the two cumulative distribution functions of the datasets as a measure of disagreement (Figure 3). It accepts the hypothesis that the distributions are equal if D_{KS} is less than a threshold value D_T , which is determined by choosing a significance level [17, 18]. The method has been previously and successfully applied to compare gamma spectra [19]. The function determining D_T for a given significance level assumes that both cumulative distribution functions are samples of the exact same parent distribution. In gamma spectrometry measurements, however, the measurement configuration, the detector voltage, and other experimental conditions inevitably vary to a small extent; the cumulative distribution functions of the reference item (template) and an identical inspected item are not samples of the exact same parent distribution.² The hypothesis that two measured spectra

²In practice, we envision that the system is allowed to thermally stabilize before it is calibrated or before other measurements are made. Ideally, the template is recorded using a reference item immediately after an energy calibration has been performed. For optimum performance, the system can be recalibrated prior to each inspection to minimize the impact

originate from the same source should, however, be accepted for insignificant variations. We therefore use an empirical KS test, which uses a large number of repeated measurements on a valid item to obtain the distribution of D_{KS} and guarantee a specified false-positive rate (for example, fewer than 1% of valid items are incorrectly flagged as invalid in a single measurement).³

Total gamma count rate. The features of a gamma spectrum are largely determined by the isotopics of the source and only weakly depend on the size of the object. In order to be directly sensitive to size, we therefore also compare the total gamma count rates observed for the template and the inspected item. To first order, these count rates scale with the projected surface area of the items (as seen by the detector) and provide a useful indicator for the similarity of their shapes.

Total neutron count rate. The gamma signature of massive objects, i.e., objects with thicknesses that exceed several millimeters, is dominated by the isotopics near the surface. The signature is much less sensitive to the presence of radioactive material away from the surface. For this reason, simple gamma measurements will not be robust for estimating the mass of an inspected item; at best, they can provide a minimum-mass estimate [3, p. 38–39]. Most diversion scenarios involve removing fissile material from an item, and an indicator that scales with mass would therefore be highly valuable. Here, we use neutron

of detector drift. We have experimentally confirmed that the proposed procedure is robust even for extended measurement times.

³See [20] for a discussion of optimum repetition strategies for inspections.

Figure 3: Quantifying the differences between two radiation spectra. Simulated gamma spectrum of the original BeRP ball compared against the spectrum of a similar ball made of reactor-grade plutonium (top, shown in black and red). The cumulative distribution function is equivalent to the fraction of the counts observed as a function of energy, and the maximum distance found in this distribution can be used to characterize the similarity of the spectra (bottom). For reference purposes, results from a measurement on the BeRP ball are also shown, though the experimental setup was slightly different from the modeled one (top, shown in gray).

195 emissions from the item for this purpose.⁴ In order to keep the inspection

⁴This works particularly well for plutonium components, but could be adapted for uranium components when combined with a small “driver source.” Also, in a real configuration, neu-

196 system as simple as possible, one measurement using a single type of detector
 197 is preferable. To do this, we use Prompt Gamma Neutron Activation Analysis
 198 (PGNAA) [21] in reverse and place the polyethylene mask shown in Figure 2
 199 between the source and the detector. This mask thermalizes neutrons emanating
 200 from the inspected item and converts them into high-energy gammas (2223 keV)
 201 following neutron capture in hydrogen [22]. The intensity of this gamma ray will
 202 be used as an indicator scaling with the mass of the object (Figure 4). If neutron
 203 multiplication in the inspected item is significant, then the dependency will be
 204 nonlinear. Since, for the template approach, we only compare the equivalence
 205 of radiation signatures and are not interested in absolute mass measurements,
 206 understanding the functional relationship is not necessary.

207 **4. Results**

208 To evaluate the viability of the proposed method, we examine a series of
 209 diversion scenarios from the (bare) BeRP ball. These include scenarios in which
 210 the isotopics and the apparent size of the object don't match the values of the
 211 reference item. More importantly, we also examine the sensitivity of the inspec-
 212 tion system to material diversions while the outer dimensions of the component
 213 remain unchanged (Figure 5). To achieve this, in the simulations, material is
 214 gradually removed from the interior of the BeRP ball, which is difficult to detect
 215 with standard gamma spectrometry.

216 Table 1 summarizes the results of the empirical KS test for a series of com-

trons from a plutonium component could drive fission events in a nearby uranium component.

Figure 4: Gamma peak at 2223 keV following neutron absorption in hydrogen. Shown are the simulated detector counts for the solid BeRP ball and for a hollow ball from which 15% of the material has been removed (3830 grams vs 4484 grams). The relative peak area drops by about 40% due to reduced mass and lower neutron multiplication in the hollow item. Shown absolute counts are for (simulated) 30-minute measurements.

217 parisons for configurations similar to those shown in Figure 5. In principle,
 218 many removals from the interior of a spherical shell are detectable in simulated
 219 measurements, but the respective D_{KS} values can be very small even when sta-
 220 tistical noise is insignificant, i.e., for high count rates and/or long measurement
 221 times. In supplementary measurements using standard calibration sources, we
 222 have determined that, under real-world experimental conditions, a threshold

Figure 5: Diversion scenarios for the BeRP ball.

value of $D_T \approx 0.002$ is necessary to ensure a false-positive rate of less than 1% (i.e., in a single inspection, at most 1 out of 100 matching items is incorrectly flagged as invalid) for measurements with about 1,000,000 detector counts. In this case, some of the diversion scenarios listed in Table 1 can no longer be identified. In other words: using the D_{KS} criterion alone, depending on the thickness of the shell, only diversions on the order of 20–30% of the total mass are effectively detectable, which may not be considered adequate for treaty verification purposes.

	Reference item (Template)					
	4480 g	4400 g	3830 g	3200 g	2270 g	960 g
4480 g	(match)	—	—	—	—	—
4400 g	0.0004	(match)	—	—	—	—
3830 g	0.0015	0.0012	(match)	—	—	—
3200 g	0.0021	0.0018	0.0006	(match)	—	—
2270 g	0.0029	0.0026	0.0015	0.0009	(match)	—
960 g	0.0069	0.0066	0.0055	0.0050	0.0044	(match)

Table 1: Simulated D_{KS} values for comparisons between spherical shells with identical outer dimensions but different masses, i.e., with material removed from the interior (see Figure 5 for geometries). As expected, D_{KS} values increase with the amount of material removed, but small differences ($D_{KS} < 0.0010$) are likely to be masked by experimental conditions and statistical noise for shorter measurement times (see text).

Despite its limited usefulness of detecting diversions from the interior of an item that has the same outer geometry as a reference item, spectral features

233 remain a powerful indicator for the comparison of the radiation spectra acquired
 234 for an inspected item and the template. Figure 6 shows the results of the
 235 empirical KS-test for a number of simulated five-minute measurements when
 236 the isotopics of the items differ. In this example, the Pu-240 content in the
 237 inspected item is increased by 5% compared to the template (with a respective
 238 decrease in the Pu-239 content). Even for this rather small isotopic change,
 239 the KS test returns significantly higher D_{KS} values with a mean of about 0.017;
 240 these values are easily distinguished from the valid case.⁵

Figure 6: Empirical KS test for spectral features. Shown are results for 50 simulated five-minute measurements with about 3,000,000 detector events each.

241 Total gamma counts are used as the second criterion, and Figure 7 summa-
 242 rizes the main results. The total counts are largely determined by two factors,

⁵For more drastic changes in the isotopics, as shown for example in Figure 3, the observed D_{KS} values will be significantly larger. In the case of weapon-grade versus reactor-grade plutonium, $D_{\text{KS}} \approx 0.450$.

the projected surface area (seen by the detector) and the isotopic composition of the item near the surface. Figure 7 shows results for an item that has the same mass as the original BeRP ball but a larger outer diameter and results for a 5% change in Pu-240 content. Even small differences in these features are easily detected. Accordingly, the same total counts could be obtained by simultaneously changing both surface and isotopics. Changed isotopics would, however, be detected by the empirical KS test. In contrast, total gamma counts are not particularly sensitive to the interior of the item. As shown in Figure 7, the total count decreases marginally even for scenarios where on the order of 50% of the BeRP ball's mass have been removed. Only when the thickness of the shell drops below 5–7 millimeters, the decrease becomes more significant. To be sensitive to the scenarios shown in Figure 5 and Table 1, an extra indicator is needed.

Neutrons emitted from the item are converted to high-energy (2223 keV) gammas via neutron capture in hydrogen. As illustrated in Figure 4, we use a 200-keV region-of-interest centered around this peak and use the total number of counts in that region as our third indicator. Figure 8 summarizes the results. As expected, items that have the same mass but different sizes may be difficult to distinguish based on their neutron signature. Figure 8 shows that this indicator is sensitive both to mass differences between two plutonium components and changes in the Pu-240 isotopics, as spontaneous fission neutrons, which induce further fission events in the item, primarily come from Pu-240. Accordingly, the same peak counts could be obtained by simultaneously changing both fissile

Figure 7: Total gamma counts per second as function of the mass present. Using the total count rate, it is difficult to distinguish the solid BeRP ball from a shell as long as the thickness of the shell exceeds 10 mm (40% diversion with a remaining mass of about 2700 grams). The total count rate is most sensitive to projected surface area (geometry changes) and isotopics. Shown error bars are for simulated 30-minute measurements.

mass and isotopics. As changed isotopics would, however, be detected by the KS test, the proposed method is able to reject items with material withdrawn from the interior.⁶

5. Discussion and Outlook

In this article, we propose a measurement approach to support treaty verification applications based on low-resolution gamma-spectrometry for use in a

⁶An adversary could colocate a concealed neutron source with the inspected item in an attempt to reproduce the total neutron count rate observed for the reference item; this strategy would, however, run the risk of generating additional detectable signatures; for example, an extra neutron source could affect features of the gamma spectrum.

Figure 8: Total neutron counts as a function of the mass present. The nonlinearity is due to the increasing neutron multiplication as the mass of the item increases, i.e., as the size of the cavity in the item decreases. 4484 grams corresponds to the solid BeRB ball. Also shown are two additional diversion scenarios. Shown absolute counts are for simulated 30-minute measurements.

272 template-matching information barrier. The goal is to increase the robustness
273 against evasion attempts compared to previously built systems. In particular,
274 we propose the use of multi-criteria templates to simultaneously detect changes
275 in isotopics, mass, and size or shape. As summarized in Table 2, each indicator
276 can be optimized for one particular attribute, but it will often also be sensitive
277 to other relevant characteristics of the inspected item, making cheating partic-
278 ularly difficult. As one specific implementation of the proposed concept, our
279 analysis shows that the combination of an empirical Kolmogorov Smirnov test,
280 the total gamma count rate test, and the total neutron count rate test (com-
281 bined with a polyethylene mask) constitute an effective concept to distinguish
282 valid from invalid plutonium items.

	MODIFIED KS TEST	TOTAL GAMMA	TOTAL NEUTRONS
ISOTOPICS	Excellent indicator	Good indicator (in combination with others)	Good indicator (in combination with others)
SIZE (PROJECTED SURFACE)	Good indicator (in combination with others)	Excellent indicator (in combination with others)	Inadequate
MASS	Weak indicator	Weak indicator	Excellent indicator (in combination with others)

Table 2: Summary of the capabilities of our proposed system to detect differences in isotopic composition, projected surface area (size), and plutonium mass.

As a next step, extensive measurement campaigns (ideally with special nuclear materials, with both high and low neutron emission rates) will help provide a more nuanced understanding of systematic uncertainties from factors that typically cannot be taken into account in the simulations, as for example noise from the experimental equipment or small variations in the placement of the inspected item and the detector.

Our information barrier concept intentionally avoids the necessity of assumptions about specific spectral features, such as looking for gamma peaks of specific isotopes. This approach is followed in order to facilitate a fully unclassified discussion, which we believe is necessary for the successful implementation of a multinational verification regime. Furthermore, openness and transparency are essential principles to optimize the robustness of information barriers, as it enables cooperative research. Interested participants will then have the possibility to peer review and improve the algorithms⁷ or use their own algorithms

⁷The C++ and Python code used for the first prototype of the device (Information Barrier

for benchmarking purposes, contributing to an inclusive scientific discourse on this matter.

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eXperimental, IBX) used to generate some of the results presented in this article is available at github.com/nuclearfutureslab/ibx.

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