1 *Characterizations of Double Pulsing in Neutron Multiplicity and Coincidence Counting*
2 *Systems*

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9 **ABSTRACT**

10 *Passive neutron coincidence/multiplicity counters are subject to non-ideal behaviour, such as*
11 *double pulsing and dead time. It has been shown in the past that double-pulsing exhibits a*
12 *distinct signature in a Rossi-alpha distribution, which is not readily noticed using traditional*
13 *Multiplicity Shift Register analysis. However, it has been assumed that the use of a pre-delay in*
14 *shift register analysis removes any effects of double pulsing. In this work, we use high-fidelity*
15 *simulations accompanied by experimental measurements to study the effects of double pulsing on*
16 *multiplicity rates. By exploiting the information from the double pulsing signature peak*
17 *observable in the Rossi-alpha distribution, the double pulsing fraction can be determined.*
18 *Algebraic correction factors for the multiplicity rates in terms of the double pulsing fraction*
19 *have been developed. We discuss the role of these corrections across a range of scenarios.*

20 **KEYWORDS:** *neutron multiplicity counting; coincidence counting; Rossi-alpha distribution;*
21 *double pulsing; dead time*

1. INTRODUCTION

Assay of special nuclear material (SNM) for international nuclear safeguards typically relies on nondestructive assay techniques such as neutron coincidence and multiplicity counting. This is most often done using ^3He -based detector systems and shift register analysis to extract and quantify neutron correlations [1,2]. These techniques suffer from detector and electronics effects such as dead time or can be affected by detector and electronics artifacts like double pulsing. The impact of these effects is often enhanced for high count rates bringing additional challenges to assay of samples with high neutron emission rates. Recent advancements in instrumentation and data acquisition systems (e.g. the feasibility of list mode—or event-by-event—data acquisition) make it possible to further evaluate and potentially mitigate these effects. Because SNM samples of sufficient grade and variability are often difficult or impossible to obtain, high-fidelity simulations frequently become a researcher’s tool of choice for investigating the performance of these assay systems. Further, simulations are uniquely able to isolate and quantify the effects of dead time and double pulsing, since these effects can be individually added, removed or modified, although the validity of any simulated results still needs to be benchmarked against experimental data.

Recent results have shown that contrary to the commonly-held assumption that a pre-delay is effective in eliminating the effect of double pulsing in shift register-based time correlation analysis, double pulsing still affects the measured coincidence rates [3]. This research also showed that double-pulsing yields a distinctive signature in the Rossi-alpha distribution (RAD), giving an advantage to RAD analysis over a traditional multiplicity shift register analysis approach, which as routinely practiced does not yield any immediately obvious evidence of the presence of double-pulsing. A RAD is a histogram of time differences between a trigger pulse and every pulse that comes after it within the RAD gate time period. Each time in a pulse train acts as a trigger pulse. The information contained in this type of RAD is the singles rate, doubles rate, and die-away profile of the counter [4–6]. In traditional multiplicity shift register analysis, every pulse opens a coincidence gate after a short period of time (pre-delay), which is meant to avoid the most severe effects of dead time. The pulses within this gate are counted and recorded as a combination of real coincidences and accidental coincidences, populating a reals+accidentals histogram. After a long delay, a second gate called the accidental gate is opened and these correlations are recorded as purely accidental coincidences, populating the accidentals histogram. The moments of these histograms can then be used to find the higher multiplicity rates in addition to the singles and doubles.

While previous work [3] focused on the effect that double pulsing has on singles and doubles rates and noticed a signature of double pulsing in the RAD, current research characterizes the quantitative effects double pulsing has on both the RAD and the correlated rates, including higher order multiplicities not addressed in [3], so as to be able to recognize and correct for it. Additionally, in this paper the effects of the double pulsing fraction and its temporal distribution are explored, suggestions for correcting the double pulsing effects are made, and the double pulsing relationship with dead time is qualitatively discussed. This is initially done with simulations, where double pulsing can be explored without the contributions from dead time that affect real world measurements, but the general trends are then compared to experimental data.

2. HIGH-FIDELITY NEUTRON COINCIDENCE COUNTER SIMULATIONS

Because in experimental data the effects of non-ideal behaviour like dead time and double pulsing can be inextricably combined, simulations can be used to disentangle their individual

effects and explore a wider range of sources than may be experimentally available. Further, the specific dead time and double pulsing characteristics can be manipulated in a controlled manner. For this reason, simulations were used to probe the effects of double pulsing at a variety of double pulsing fractions, for a range of neutron source strengths, and with different double pulsing characteristics. The double pulsing fraction, r , is defined as the fraction of pulses that do not correspond to an actual neutron detection but are triggered by one. Unlike spurious noise, these pulses are temporally correlated to a pulse that does correspond to a genuine neutron detection.

As the design of ^3He counters can have an effect on the intensity and magnitude of imperfections such as double pulsing, the starting point of these simulations is as accurate and detailed a description of an actual counter as possible. Neutron coincidence and multiplicity counters are typically a composite of many ^3He tubes, with several tubes electronically bundled to the same preamplifier. Note that preamplifiers refer here to a signal processing unit composed of signal shaping and identification and logical pulse generation. Noise and dead time can be introduced in principle anywhere from the initial preamplifiers through to the final electronic OR where the outputs from individual preamplifiers are combined to typically one output for readout. To simulate double pulsing and dead time, the neutron-capture location (i.e. in which ^3He tube the capture occurred) along with the time of a neutron detection is tracked, so it is possible to add these electronic imperfections to the simulations on a preamplifier level.

The Monte Carlo transport code MCNPX [7] was used to model the neutron counters, fission sources, and all neutron interactions and transport. Realistic MCNP models have been made for a representative sample of the standard multiplicity and coincidence counters such as the Active Well Coincidence Counter (AWCC) [8], the Epithermal Neutron Multiplicity Counter (ENMC) with and without the Inventory Sample (INVS) insert [9,10], the High-Level Neutron Coincidence Counter (HLNCC) [11], and the Plutonium Scrap Multiplicity Counter (PSMC) [12]. The AWCC was chosen as a well-characterized representative from this group. It was assumed that when double pulsing manifests on all the preamplifiers equally, the effects on the rates should be similar, if not the same, for all counters. This would not be the case if an individual preamplifier double pulsed at a different rate from the other preamplifiers, however.

The capture-type Particle Track Output Card (PTRAC) output file from the MCNP simulation was used to create a pulse train (i.e. a sequence of neutron capture times) using FastTrain, a C#-based, Los Alamos National Laboratory (LANL) developed code with a graphical user interface [13,14]. The PTRAC file includes information on every neutron capture event of interest (e.g. a neutron captured within a ^3He proportional detector tube), such as final location and time of neutron capture. From the PTRAC file, FastTrain extracts the location of neutron capture in the form of an MCNPX cell number corresponding to an individual ^3He proportional detector tube along with time information for each neutron capture and generates list mode type data, with a separate output file for capture times and cell numbers respectively. The neutron capture times in a PTRAC file are times with respect to a spontaneous fission event within the assayed sample. With a user-defined spontaneous fission frequency, absolute times for neutron captures can be established and recorded with FastTrain.

The analysis code FastTap [15], a LANL-developed software expanded from VBTap [16], can extract standard shift register multiplicity rates (singles (S), doubles (D), triples (T), quads (Q), and pents (P)) and create RADs and time interval distributions. FastTap can read both simulated and experimental list mode data. For the simulated data, FastTap has additional features that

allow for improving the fidelity of the synthetic data. FastTap is able to implement both dead time of various levels of sophistication and double pulsing with specific characteristics on the list mode data before analyzing. This feature allows the user to analyze the same pulse train with and without dead time and double pulsing and/or modify the parameters of both effects.

In the most current version of FastTap (FastTapX 1.3) the double pulsing is being added at the preamplifier level with all preamplifiers double pulsing with the same fraction. This considers the generally accepted origin of double pulsing being on the level of the preamplifiers (see Section 3). The option for each preamplifier double pulsing with different fractions may be added in the future. In the context of this research, all simulations were carried out without dead time. Simulations combining double pulsing and dead time is a topic of future research.

For this study, the AWCC was selected as it represents a standard neutron coincidence counter commonly used by safeguard inspectors worldwide. Originally developed at LANL, it is commercially available from ANTECH and Canberra [17,18]. The AWCC has 42 ^3He proportional counters with groups of 7 detectors connected to a single signal processing preamplifier, for a total of 6 preamplifiers. The TTL logic pulse outputs of these 6 preamplifiers is then connected to a master OR, a logic gate that outputs a single pulse train of digital timestamps corresponding to individual neutron detections.

Initially, we use a random neutron source created with a random number generator to observe the effect of double pulsing. Then, we show the effects of double pulsing in the context of nonproliferation and safeguards measurements by simulating the AWCC with a 4-kg sphere of low-grade Pu metal (90% ^{239}Pu and 10% ^{240}Pu). Finally, we explore the effects of double pulsing with simulated assay of several ^{252}Cf sources of various strengths using the AWCC. This is to simplify the problem and show the effects of double pulsing on a correlated neutron source without the additional level of complexity due to the effects of multiplication. The activity of the ^{252}Cf source in simulations was varied using FastTrain.

To study the effects of double pulsing, pulse trains with double pulsing were simulated such that a second pulse was inserted into the pulse train with a user-defined probability in a randomized Gaussian distribution with respect to the triggering pulse. Using this method, the modified pulse trains can be analyzed with either shift-register type analysis or a RAD.

3. ORIGINS AND RECENT STUDIES OF DOUBLE PULSING

Double pulsing is thought to have two primary sources. In the first case, it arises from the electron-ion collection within the ^3He tube. A thermal neutron detection is essentially a detection of the proton and/or triton, which are produced from neutron capture on ^3He . Their paths are roughly rectilinear, so in the case where the paths of the charged particle products of neutron capture are perpendicular to the anode wire, the charge collection from their ionization of the gas can induce a two-peak structure as the two lobes of charge are collected, resulting in a double trigger if the time constant of the shaping amplifier is shorter than the temporal separation of the two peaks [19,20]. Such a phenomenon has been long understood, and can be mitigated with a sufficiently long shaping time constant or implementation of additional blanking circuit to veto the discriminator operation and minimize the double triggering phenomenon. Both of these approaches inevitably increase dead time. In the second case, double pulsing can be due to an electronic feedback in a preamplifier or cross-talk between different channels, when the output TTL pulse from a preamplifier is routed near the sensitive input side. Such features are in general

a result of poor electronic design or implementation, and as such, can be eliminated by improved quality of electronic architecture.

In appropriately designed detection systems, double pulsing does not occur at significant levels because it is mitigated by electronics design and matching ^3He tube and electronics performance. Nevertheless, in some high count rate applications where the neutron flux is coupled to a strong gamma-ray field, the discriminator threshold may be set to a higher level to obtain good gamma-ray discrimination [19] or the operating high voltage may be lowered, which has the same effect as raising the discriminator threshold. Depending on the pulse shape, this could cause double pulsing if the time constant of the shaping amplifier is not sufficiently long. Historically, double pulsing was considered a minor effect that was believed to have no effect on real coincidences because of the pre-delay used in shift register analysis [21]. However, these conclusions were not supported by a rigorous study, and recent research [3] indicates that the pre-delay does not completely remove the influence of double pulsing.

Recent research focusing on temperature stability of neutron counting systems also explored the option of using double pulsing to offset count losses due to temperature-dependent effects in the detector system [22]. By operating at a higher than standard high voltage and from preamplifiers heating up and effectively lowering the threshold, double pulsing was introduced to offset count rate losses from a negative temperature dependence in the neutron capture cross-section. This was shown to keep the singles rate constant, but its effects on multiplicity rates were not well characterized.

To advance the knowledge of double pulsing and fully characterize its effects on correlated counting, current work focuses on high-fidelity simulation and expands the double pulsing investigation to higher order multiplicity rates.

4. EFFECTS OF DOUBLE PULSING ON ROSSI-ALPHA DISTRIBUTIONS

To demonstrate the effect of double pulsing on a RAD, a random neutron source was simulated with a frequency of 750 kHz. Double pulsing was then added to the train using FastTap. The resulting RADs are shown in Figure 1. Figure 1 provides a comparison of a double pulsing free RAD (with $r=0$) and a RAD with a double pulsing fraction of $r=5\%$. In this case, the simulation was of double pulsing with an expectation value (μ) at 2 μs after the triggering pulse and a standard deviation (σ) of 50 ns as can be seen clearly in the peak of the RAD. The peak is created from the time differences between the false double pulses and their trigger. The overall rise in amplitude is created from the time differences between the false double pulses and all the other pulses in the RAD gate time period.

{Figure 1}

Correlated neutron pulses show similar effects in the RAD. Previous research [3,21,23], benefiting from list mode data acquisition, has shown that the presence of double pulsing can be clearly identified in a RAD as well as the time interval distribution (TID) for actual detectors. The RAD and TID with and without double pulsing are illustrated in Figure 2 using simulations performed within the current work. The simulations were of a 4-kg low-grade Pu metal sphere (90% ^{239}Pu and 10% ^{240}Pu) in an AWCC. The double pulsing peak in the RAD is more clearly visible than in the TID, making it a good diagnostic of double pulsing. As such, this research is focused on the RAD rather than the time interval distribution.

Just as with the random neutron source, the RAD from a correlated pulse train with false double pulsing shows a double pulsing peak and an overall increase in amplitude. The most immediately obvious signature of double pulsing is a peak on top of the standard RAD located at the time the secondary pulse is measured with respect to the triggering pulse as seen in Figure 2. Figure 2 shows both the double pulsing free RAD (with $r=0$) and a RAD with a double pulsing fraction of $r=5\%$. The double pulsing was simulated with an expectation value (μ) at 2 μ s after the triggering pulse and a standard deviation (σ) of 50 ns. The RAD can be well-approximated by a fit using the following function:

$$Y(t)dt = \left(\frac{D}{f_d} \frac{e^{-t/\tau}}{\tau} + S^2 + A_p \frac{e^{-\frac{1}{2}(\frac{t-\mu}{\sigma})^2}}{\sqrt{2\pi}\sigma} \right) dt, \quad (1)$$

where D and S are the measured doubles and singles rates, f_d is the gate fraction, τ is the decay time of the counter, and t is time after the triggering pulse. The third term is a normalized Gaussian distribution with area A_p , centroid μ , and standard deviation σ . Fitting the double pulsing peak in Figure 2 with this form returns the simulated double pulsing distribution, where $\mu=2$ μ s and $\sigma=50$ ns. The double pulsing was simulated with a Gaussian-shaped probability distribution function; therefore, the characteristics of this function are directly reflected in the fit. Figure 3 illustrates that by simulating double pulsing with no dead time for different double pulsing fractions, the shape and magnitude of the RAD changes. Figure 3 shows ratios of RADs with a range of double pulsing fractions to a RAD with no double pulsing. This trend is not surprising as the origin of this peak is the time between the pulse that “triggers” the double pulsing and the induced second pulse itself.

{Figure 2}

{Figure 3}

The second feature of a RAD with double pulsing is an overall increase in magnitude as shown in Figures 1-3. This is due to the fact that each secondary pulse will become a trigger itself contributing to the RAD and thus increasing its overall magnitude. Normalizing a RAD from a pulse train with double pulsing to a RAD from the original pulse train without double pulsing yields information about the relationship between doubles, singles, and the double pulsing fraction. From Figure 3, it can be seen that the ratio of RADs is constant in time following the triggering pulse. This means that outside of the double pulsing peak in the RAD,

$$\frac{S^2}{S_0^2} = \frac{\frac{D}{f_d} \frac{e^{-t/\tau}}{\tau} + S^2}{\frac{D_0}{f_d} \frac{e^{-t/\tau}}{\tau} + S_0^2}, \quad (2)$$

where the subscript nought denotes inherent rates. We refer to inherent rates as the rates of neutron captures within ^3He tubes, distinct from measured rates which include the effects of dead time and any other electronic artifacts. This can be simplified to:

$$\left(\frac{S}{S_0} \right)^2 = \frac{D}{D_0}. \quad (3)$$

To understand the relationship of doubles and singles to their inherent rates, we can use the definition of the double pulsing fraction. We know that from the definition of the double pulsing fraction,

$$S = S_0(1 + r), \quad (4)$$

where S is the singles rate with double pulsing; S_0 is the inherent singles rate; and r is the double pulsing fraction expressed as a decimal rather than percent. Thus, with Eq. 3,

$$D = D_0(1 + r)^2, \quad (5)$$

where D is the doubles rate with double pulsing and D_0 is the inherent doubles rate. With a sufficiently small r , which is most often the case since the double pulsing fraction is reasonably assumed to be much less than 1 when expressed as a decimal rather than percent, an approximation using Taylor expansion yields a linear relationship, where

$$D \cong D_0(1 + 2r). \quad (6)$$

Figure 3 also provides evidence that, when there is no dead time, the die-away time is not affected by double-pulsing as can be seen in the constant value of the RAD ratio.

Even without assuming the constant nature of the normalized RAD in Figure 3, Eqs. 4 and 5 can be derived from first principles as shown in the Appendix.

In addition to noticing the double pulsing peak signature and an overall increase in magnitude, it is also possible to relate the area of the double pulsing peak in the RAD to the double pulsing fraction. As every secondary pulse follows the triggering pulse at a specific time, each of these secondary pulses has to contribute to the overall area of the double pulsing peak. On the other hand, all of these secondary pulses also lead to an increase in the measured singles rate. Therefore,

$$A_p = S - S_0, \quad (7)$$

where A_p is the area of the peak. From Eqs. 4 and 7, we can extract the double pulsing fraction:

$$r = \frac{A_p}{S - A_p} = \frac{A_p}{S_0}. \quad (8)$$

A comparison of the calculated double pulsing fraction from the simulated RAD to the actual double pulsing fraction defined by the user at the time of simulation shows marked agreement (see Figure 4). The double pulsing peak area, A_p , was extracted from a fit of the entire RAD distribution (500 μ s) using the form of Eq. 1. Uncertainties in the calculated r correspond to the statistical uncertainty of a simulated 100-s measurement of a 2.2 MHz singles rate source of a 4-kg low-grade Pu metal sphere (90% ^{239}Pu and 10% ^{240}Pu) in the AWCC. The doubles rate for this source is 6.9 MHz.

Once the double pulsing fraction is established, Eqs. 4 and 5 can be used to correct for the effects of double pulsing on the measured singles and doubles rates as will be illustrated in Section 5.

{Figure 4}

5. EFFECTS OF DOUBLE PULSING ON MULTIPLICITY RATES

The relationship between S and S_0 has already been defined as has the relationship between D and D_0 (Eqs. 4 and 5). The reliance of these rates on the double pulsing fraction, r , belies the historical assumption that rates were not affected by double pulsing as long as the pre-delay was

long enough to remove the double pulsing peak observed in the RAD from the coincidence gate. To further evaluate the effects of double pulsing on multiplicity rates, pulse trains were created from a simulated assay of a ^{252}Cf source in an AWCC. The double pulsing was added to these trains in a Gaussian distribution with an expectation value of 2 μs and a standard deviation of 50 ns. The shift register analysis performed by FastTap used a pre-delay of 4.5 μs and a gate of 64 μs to calculate multiplicity rates, consistent with the usual parameters for analyzing AWCC experimental data. It should be pointed out that the pre-delay of 4.5 μs provides sufficient time after the double pulsing occurrence at 2 μs to confirm the effects of double pulsing on the multiplicity rates beyond the pre-delay duration. Figure 5 shows the simulated multiplicity rates normalized by the inherent multiplicity rates as a function of the double pulsing fraction.

Using the data in Figure 5, a free fit shows agreement with the previously derived equations for singles (Eq. 4) and doubles (Eq. 5). Additionally, for this case of simulated assay of ^{252}Cf in the AWCC, the relation for the triples can be empirically determined to have the following dependence on r :

$$T \cong T_0(9.4r^2 + 8.6r + 1.0). \quad (9)$$

It is doubtful whether this can be generalized to other sources or counters and should be an area of future research. The conclusion that can be drawn is that the effect of double pulsing on triples rates is significantly more than on doubles or singles rates.

Simulations varying the mean or the width of the distribution of double pulsing times with respect to the triggering pulse showed no effect on the measured multiplicity rates, as shown in Figure 6. This is because regardless of temporal location of the secondary pulse within the pre-delay, it will still create the same number of additional coincidences with other pulses, increasing the multiplicity rates by the same factor for a given double pulsing fraction.

{Figure 5}

{Figure 6}

It is noted that Eqs. 4, 5, and 9 show no dependence on the inherent singles rate. In fact, the normalized ratio of doubles to singles shows the following relationship from Eqs. 4 and 5:

$$\frac{D/S}{D_0/S_0} = \frac{D/D_0}{S/S_0} = \frac{(1+r)^2}{1+r} = 1 + r. \quad (10)$$

This value is plotted as a function of inherent singles in Figure 7 and the predicted relationship is seen in the simulated results. For example, with $r=5\%$, the ratio should be 1.05, which is shown in the simulation results in Figure 7.

{Figure 7}

6. COMPARISON OF SIMULATION TO EXPERIMENTAL RESULTS WITH DEAD TIME

Experimental measurements utilizing the AWCC with a range of ^{252}Cf sources were performed to verify the impact of double pulsing and associated trends seen in simulations. The measurements were performed in thirty 30-second long cycles to allow for evaluation of the sample-based error. Shift register analysis was used with a pre-delay of 4.5 μs and a gate of 64 μs to calculate multiplicity rates. The measured rates are summarized in Table 1. By operating the counter at a higher than optimum high voltage, it is possible to induce double pulsing by

effectively raising the gain, which means the discriminator is more likely to re-trigger on structures in the pulse shape. The simulation results shown in the previous section do not have any dead time added to the pulse trains, therefore they reflect most directly the effect of double pulsing on measured multiplicity rates, but a qualitatively similar effect comparable in magnitude can be seen with dead-time corrected experimental data as shown below in Figure 8.

{Table 1}

{Figure 8}

By operating the AWCC at a range of elevated high voltages to induce double pulsing and subsequently at the nominal high voltage of 1680 V with a range of ^{252}Cf sources, a double pulsing fraction can be evaluated. The estimated double pulsing fraction was calculated using the dead-time corrected measured singles rates from the high voltage inducing double pulsing relative to the dead-time corrected singles rates at 1680 V and is summarized in Table 2. It is assumed that there is no double pulsing at 1680 V, which can be verified in the experimental RADs in Figure 8, and we assume there is no appreciable increase in detector efficiency at the high voltages used. The dead time was corrected using the process outlined in [1]. The expression $\ln(D_m/S_m)$ is plotted against S_m and fit with a chi-square fit to a linear function. The coefficients of this linear fit give the dead time coefficients (B is the offset and A is $-4/3$ times the linear coefficient). For this study, the dead time coefficient B was set to 0. The dead time coefficient A was determined independently for each high voltage using four ^{252}Cf sources. It is noted that this is an imperfect scenario for looking at double pulsing. The dead time correction is influenced by and partially accounts for the double pulsing present in the data. It is necessary to determine the dead time correction coefficients at each high voltage because this effective rise in gain on the pulses does affect the dead time, but the dead time correction will also naturally be including the effect of double pulsing. It is assumed that this effect is minor within the scope of this paper.

{Table 2}

As shown in Table 2, the experimental data result in slightly different double pulsing fractions than those used in the simulations. However, as shown in Figure 9, the results are indicative of a constant dependence of a normalized dead-time corrected D/S with respect to a dead-time corrected singles rate according to Eq. 10. Specifically, for the experimental results where $r \cong 6\%$ in Table 2, the normalized D/S ratio is about 1.06 as seen in Figure 9. This agrees with the predicted dependence shown in Eq. 10. Likewise, the normalized D/S ratios from the other double pulsing fractions appear to coincide with the relationship predicted by Eq. 10. However, recent research by Nguyen, et al. shows a different relationship in the normalized D/S versus S plot (Figure 9 in [3]). Where the results presented in this paper show a constant value for this relationship, the results of [3] show an increasing exponential relationship with singles rate. Nguyen, et al. saw evidence of what could be double pulsing in one of their neutron coincidence collars (JCC-13). By creating false double pulsing in another working coincidence collar (JCC-31), they replicated the feature they attributed to double pulsing. The false pulses were inserted 1 μs after randomly selected true pulses with a double pulsing fraction varying between 0.1% and 5% and reanalyzed for multiplicity rates and Rossi-alpha distributions. Measurements were done

with PuBe sources with varying amounts of Pu and low D/S ratios (less than 0.3). While the authors do not discuss how dead time interacts with double pulsing or how and when it is corrected in these measurements, the exponentially increasing relationship of Nguyen, et al. could not be replicated in either the simulations or experimental measurements that were performed in this work.

{Figure 9}

To determine r from the RAD with experimental data, it may seem most straightforward to fit the RAD. Due to the effects of dead time, however, the expected Gaussian shape of the double pulsing peak may be severely distorted. Therefore, A_p is calculated by subtracting the underlying RAD, assumed to be linear within the given time interval in which the double pulsing peak is observed, from the sum of all the bins in the RAD within that time interval, as illustrated in Figure 10. To average out statistical fluctuations in the RAD, the time interval for determining A_p is varied with a lower bound between 350 – 550 ns and a higher bound between 1.5 – 1.7 μ s, with time increments of 10 ns, resulting in 400 time intervals. The linear background was determined by the endpoints of these time intervals. The statistical uncertainty in this value A_p was propagated into the calculation of the double pulsing fraction.

{Figure 10}

A comparison of the double pulsing fraction extracted from the measured singles rates with the double pulsing fraction calculated from the RAD of the double pulsed data (using Eq. 8) shows a consistent underestimate of r by 5-16% for the two higher activity sources (see Table 2), but good agreement within statistical uncertainties for the lower activity sources. Because experimental data includes dead time, it is speculated that the underestimate with the higher activity sources may be explained by not correcting A_p for dead time. Using Eq. 8 with dead-time corrected singles rates, but no dead-time correction for A_p results in a double pulsing fraction even further from the truth, so it is clear that a dead-time correction for A_p is critical to get a better estimate of r . The estimated double pulsing fraction is derived from comparing dead-time corrected singles rates at the double pulsing high voltage and at 1680 V. The calculated double pulsing fraction has no explicit dead time corrections included. The combined effects of double pulsing and dead time on the double pulsing peak in the RAD and their correction is beyond the scope of this paper but will be a subject of simulation, experimental, and theoretical investigation in the near future.

Finally, we evaluate the potential to correct the measured multiplicity rates for the double pulsing as outlined in Section 5. Since the effects of double pulsing and dead time are interconnected in the measured data we will evaluate the combined effects of double pulsing and dead time using two methods: (1) the multiplicity rates are first corrected for dead time and then corrected for double pulsing and (2) the multiplicity rates are first corrected for double pulsing and then for dead time. These methods do yield different results because although the double pulsing correction is merely a multiplicative factor, the correction for dead time uses the measured singles in the exponent of the dead time correction. Depending on whether double pulsing is corrected first or second, this changes the measured singles value which in turn affects the dead time correction factor

To test these two methods, the results are summarized in Tables 3 and 4 in the form of ratios of corrected singles and doubles rates relative to the double pulsing free rates extracted from

measurements at the nominal high voltage (1680 V). Using the calculated r from the experimental RADs (see Table 2) and the multiplicity rates for each elevated high voltage measurement (see Table 1), the singles rates are corrected to within 0.7%. This method corrects the doubles rates to within 1.8%. The difference between correcting for dead time first and then double pulsing or vice versa is marginal and nonexistent within statistical uncertainties. For the rates which are not corrected to 0 within statistical uncertainty, they are consistently undercorrected because of the underestimate in the double pulsing fraction. This implies that if the double pulsing fraction can be determined more accurately, the multiplicity rates can be more accurately corrected. It is assumed that the dead time on the measured rates is being adequately corrected and what remains is the effects of uncorrected double pulsing because the measured normalized ratio D/S in Figure 9 is flat.

{Table 3}

{Table 4}

7. CONCLUSION

Double pulsing is a phenomenon often present during operation of multiplicity and coincidence counters used for characterization or verification of SNM. Because detection systems that exhibit double pulsing are affected by uncharacterized interplay between the double pulsing and the dead time in the system, high fidelity simulations are used to explore the effects of double pulsing without the added complication of dead time. From these simulations, it is seen that all the multiplicity rates (including doubles and triples in this study) increase as a function of the double pulsing fraction. This demonstrates that contrary to traditional expectations that using a pre-delay is sufficient to remove the double pulsing effects from the true multiplicity rates. In contrast to the results reported by [3], simulations and experimental results did not show a dependence of $(D/S)/(D_0/S_0)$ on the singles rate, but rather a simple $(1+r)$ dependence.

Detailed investigation of double pulsing effects on the RAD revealed that double pulsing has a characteristic signature, which can best be seen in list-mode data. In the RAD, it increases the overall magnitude of the distribution as well as adding a peak whose characteristics echo the distribution of the secondary pulses following the triggering true pulse. A new method was developed that illustrates the potential to utilize the area of this peak to extract the double pulsing fraction which can be used to recover the inherent multiplicity rates from the measured multiplicity rates.

The experimental data support the observations and conclusions based on the simulations. While it is more complicated and less accurate to extract a double pulsing fraction from the RAD with dead time, it can still be determined to within uncertainties for low source activity items (e.g. Source #1 and #2) and to within 16% for higher source activity items (e.g. Source #3 and #4). Double and Singles were corrected to within 2%.

Extensions of this research could be expected to explore the effects of double pulsing in low coincidence sources and high-alpha SNM. Further, extending this research could investigate the effects of just one preamplifier double pulsing or double pulsing at a different rate from other preamplifiers.

As the future research will focus on understanding the interplay of double pulsing and dead time, using simulation, experiment, and theory it is expected to further improve the ability to correct or even eliminate the effects of double pulsing.

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APPENDIX

What follows is an argument for why S and D can be described by the previously proposed relations to the inherent rates S_0 and D_0 (Eqs. 4 and 5). We begin by making our argument to show that $S = S_0(1 + r)$. Then, we argue that $D = D_0(1 + r)^2$.

Case 1 (see Table 5): Let us begin with a random neutron source and the simplest possible pulse train consisting of a time of a single detected neutron. Let us add double pulsing with a double pulsing fraction r . A measurement of singles is performed by counting the number of triggers or gates opened. Therefore, the probability of measuring two neutrons, one of them being artificial due to double pulsing is r , while the probability of measuring only a single neutron is $(1 - r)$. Note, that the probability is normalized, i.e. $(1 - r) + r = 1$.

The probability weighted average singles rate S is then:

$$S = \overline{S_m} = 1 \cdot (1 - r) + 2 \cdot (r) = 1 + r. \quad (11)$$

In this case, the inherent singles rate is $S_0 = 1$, so $S = S_0(1 + r)$.

Case 2 (see Table 6): Let us consider the second simplest pulse train possible, which consists of detection times of two true neutrons. After injecting double pulsing with double pulsing fraction r , four possible scenarios may occur: (1) no artificial pulse is present, and only two true neutrons are detected with a probability of $(1 - r)^2$; (2) the first true neutron induces a secondary artificial pulse, but the second neutron does not, with a probability of occurrence being $r(1 - r)$; (3) the first true neutron does not induce a secondary pulse, but the second one does, with probability $(1 - r)r$; and (4) both true neutrons induce a secondary pulse with a probability of occurrence of r^2 . The singles rates measured in these scenarios would then be 2, 3, 3, and 4 respectively.

The probability weighted average singles rate S is then:

$$S = \overline{S_m} = 2 \cdot (1 - r)^2 + 3 \cdot (r(1 - r)) + 3 \cdot ((1 - r)r) + 4 \cdot r^2 = 2(1 + r). \quad (12)$$

In this case, the inherent singles rate is $S_0 = 2$, so $S = S_0(1 + r)$.

Case 3 (see Table 7): Let us consider the third simplest pulse train possible, which consists of detection times of three individual true neutrons. After injecting double pulsing after each true pulse with double pulsing fraction r , eight possible scenarios may occur resulting in a measured singles rate between three and six, as depicted in detail in Table 7. Considering each probability of occurrence for the individual scenarios, the probability weighted average singles rate S is then:

$$\begin{aligned}
S = \overline{S_m} &= 3 \cdot (1 - r)^3 + 4 \cdot (r(1 - r)^2) + 4 \cdot ((1 - r)r(1 - r)) + 4 \cdot ((1 - r)^2 r) \\
&\quad + 5 \cdot (r^2(1 - r)) + 5 \cdot (r(1 - r)r) + 5 \cdot ((1 - r)r^2) + 6 \cdot r^3 \\
&= 3(1 + r).
\end{aligned} \tag{13}$$

462 In this case, the inherent singles rate is $S_0 = 3$, so $S = S_0(1 + r)$.

463 Following this reasoning, for $S_0 = N$, in the presence of double pulsing with fraction r , the
464 measured singles rate S is $S = S_0(1 + r)$.

465 Let us now consider doubles in Case 1. Here a correlation is observed if a trigger is followed by
466 another trigger within the gate. This is called a doubles event. The gate is shifted with respect to
467 its trigger by a predelay. Therefore in neither of the two possibilities in Case 1 is a doubles event
468 counted. In the first one, it is due to lack of the second pulse, and in the second scenario it is
469 because the secondary artificial pulse arrives during the predelay period and can't be thus
470 counted. Hence for both scenarios, the measured doubles rate is 0.

471 In Case 2, let us consider the scenario where a second true pulse arrives within the gate of the
472 first pulse, so the inherent doubles rate D_0 is nonzero. Then four scenarios may occur as depicted
473 in Table 6. In the first scenario, there are no double pulses, so this scenario has a probability
474 $(1 - r)^2$ of occurring. The first trigger has one correlation within the gate and the second trigger
475 has 0, so the measured doubles for this scenario is 1. In the second scenario, the first true neutron
476 induces a double pulse, but the second true neutron does not. This scenario has a probability of
477 $r(1 - r)$. The first two triggers each have one correlation within their respective gates and the
478 third trigger has 0, so the measured doubles for this scenario is 2. In the third scenario, the
479 second true neutron induces a double pulse, but the first true neutron does not. This scenario has
480 a probability of $(1 - r)r$. The first trigger has two correlations within its gate and the last two
481 triggers have 0, so the measured doubles for this scenario is 2. In the last scenario, both of the
482 true neutrons induce double pulses, so this scenario has a probability r^2 of occurring. In this
483 scenario, the first two triggers each have two correlations within their respective gates and the
484 last two triggers have 0, so the measured double for this scenario is 4. The doubles rates
485 measured in each of these scenarios would then be 1, 2, 2, and 4 respectively.

486 The probability weighted average singles rate D is then:

$$D = \overline{D_m} = 1 \cdot (1 - r)^2 + 2 \cdot (r(1 - r)) + 2 \cdot ((1 - r)r) + 4 \cdot r^2 = (1 + r)^2. \tag{14}$$

487 In this case, the inherent doubles rate is $D_0 = 1$, so $D = D_0(1 + r)^2$.

488 In Case 3, let us consider the third simplest pulse train possible, which consists of detection times
489 of three individual true neutrons, such that the inherent doubles rate is 3. After injecting double
490 pulsing after each true pulse with double pulsing fraction r , eight possible scenarios may occur
491 resulting in a measured doubles rate between three and twelve, as depicted in detail in Table 7.
492 Considering each probability of occurrence for the individual scenarios, the probability weighted
493 average doubles rate D is then:

$$\begin{aligned}
D = \overline{D_m} &= 3 \cdot (1 - r)^3 + 5 \cdot (r(1 - r)^2) + 5 \cdot ((1 - r)r(1 - r)) + 5 \cdot ((1 - r)^2 r) \\
&\quad + 8 \cdot (r^2(1 - r)) + 8 \cdot (r(1 - r)r) + 8 \cdot ((1 - r)r^2) + 12 \cdot r^3 \\
&= 3(1 + r)^2.
\end{aligned} \tag{15}$$

In this case, the inherent doubles rate is $D_0 = 3$, so $D = D_0(1 + r)^2$.

Following this reasoning, for $D_0 = N$, in the presence of double pulsing with fraction r , the measured doubles rate D is $D = D_0(1 + r)$.

While this is not a rigorous proof, it is highly suggestive that the relationships defined in Eqs. 4 and 5 have some theoretical basis, in addition to simulation and experimental evidence.

{Table 5}

{Table 6}

{Table 7}

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Figure Captions

Figure 1. The Rossi-alpha distribution from a simulated assay of a random neutron source (750 kHz) by an AWCC with and without artificially added double pulsing. The double pulsing is added in a Gaussian distribution at $\mu=2\ \mu\text{s}$ with a standard deviation (σ) of 50 ns with a frequency with respect to the original train of 5%.

Figure 2. The Rossi-alpha distribution (left) and the time interval distribution (right) from a simulated pulse train of 4-kg of Pu metal exhibiting double pulsing characteristics. Insets show a zoomed-in view of the double pulsing peak, both in linear-linear scale for direct comparison.

Figure 3. RADs from simulated assay of 4-kg of Pu metal with various fractions of artificially added double pulsing are normalized by the RAD from a double pulsing free pulse train.

Figure 4. The double pulsing fraction calculated from the area of the double pulsing peak in the RAD shows strong agreement with the actual user-defined double pulsing fraction. The grey line is included as a visual aid. Error bars correspond to the statistical uncertainty of a simulated 100-second measurement of 4-kg of Pu with inherent singles rate of 2.2 MHz in an AWCC.

Figure 5. The normalized multiplicity rates shown as a function of double pulsing fraction r . The pulse trains were created from simulated assay of a ^{252}Cf source by an AWCC with a measured inherent singles rate of 100 kHz. The double pulsing is added at 2 μs after the triggering pulse with a Gaussian distribution and standard deviation of 50 ns. The errors calculated as the standard error of the mean are smaller than the markers used. The importance of this figure is to show that a free fit for doubles and singles shows the relationship previously hypothesized (Eqs. 4-5). The triples have been fit with a quadratic function merely to show the effect of double pulsing on triples is significantly more than on doubles.

Figure 6. The normalized multiplicity rates are shown as a function of double pulsing fraction in percentage for a simulated assay of ^{252}Cf source by an AWCC with an inherent singles rate of 100 kHz. In the left graph, the double pulsing is simulated at 2 μs after the triggering pulse with a Gaussian distribution and standard deviation of 50 ns and 200 ns. In the right graph, the double pulsing is simulated at 1 μs , 2 μs , and 3 μs all with a Gaussian distribution and standard deviation of 50 ns.

Figure 7. The ratio of D/S with double pulsing to the inherent D_0/S_0 is shown as a function of the inherent singles rate for simulated ^{252}Cf data. The double pulsing is simulated at 2 μs after the triggering pulse with a Gaussian distribution and standard deviation of 50 ns. Error bars correspond to adding the standard error of the mean of S_0 and D_0 in quadrature. The errors of S and D are not independent from S_0 and D_0 and are therefore not included.

Figure 8. RADs from 5-minute measurements of ^{252}Cf sources at varying voltages. The sources are: Source #1 (inherent 0.4 kHz singles and 0.4 μCi activity), Source #2 (inherent 1.9 kHz singles and 1.6 μCi activity), Source #3 (inherent 26 kHz measurement and 20.2 μCi activity), and Source #4 (inherent 42 kHz measurement and 31.1 μCi activity).

Figure 9. The ratio of dead-time-corrected D/S with double pulsing to inherent D_0/S_0 is shown as a function of S_0 , the inherent singles rate for experimental ^{252}Cf data. Rates free of double pulsing were estimated from counts at 1680 V. Error bars represent the standard error of the mean from S_0 , D_0 , S, and D added in quadrature.

599 **Figure 10.** *The determination of the double pulsing fraction comes from the area under the*
600 *double pulsing peak (A_p). This area is found by subtracting the area of the underlying RAD*
601 *(A_{uRAD}), assumed to be linear within the short time interval. The time interval's lower bound*
602 *varies between 350 ns and 550 ns and its upper bound varies between 1.5 μ s and 1.7 μ s.*

Table Captions

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Table 5. Given the simplest case of a pulse train with a single neutron, a second artificial neutron is added with a probability of r . The average measured singles rate, given the probability of each scenario, is $(1 + r)$. Given $S_0 = 1$, the average measured singles rate can be written as $S_0(1 + r)$. For the doubles, a predelay indicated by the dashed lines must be used and in both scenarios (with and without double pulsing), no pulse lands within the gate, so the measured doubles are 0.

Table 6. Given the case of a pulse train with two neutrons, a second artificial neutron can be added to each true neutron with a probability of r . The average measured singles rate, given the probability of each scenario, is $2(1 + r)$. Given $S_0 = 2$, the average measured singles rate can be written as $S_0(1 + r)$. For the doubles, a predelay indicated by dashed lines is used and the number of correlated neutrons within the gate are counted for each scenario. The average measured doubles rate, given the probability of each scenario is $(1 + r)^2$. Given $D_0 = 1$, the average measured doubles rate can be written as $D_0(1 + r)^2$.

Table 7. Given the case of a pulse train with three neutrons, an additional artificial neutron can be added to each initial true neutron with a probability of r . The average measured singles rate, given the probability of each scenario, is $3(1 + r)$. Given $S_0 = 3$, the average measured singles rate can be written as $S_0(1 + r)$. For the doubles, a predelay indicated by a dashed line is used and the number of correlated neutrons within the gate are counted for each scenario. The average measured doubles rate, given the probability of each scenario, is $3(1 + r)^2$. Given $D_0 = 3$, the average measured doubles rate can be written as $D_0(1 + r)^2$.

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712 mean from S_0 , D_0 , S , and D added in quadrature.

713

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High Voltage [V]	1680	1760	1780	1820	1860
Dead Time Parameter A (s)	8.7e-7 $\pm 1.2\text{e-}7$	6.3e-7 $\pm 3.5\text{e-}7$	1.16e-6 $\pm 1.2\text{e-}7$	1.13e-6 $\pm 1.0\text{e-}7$	1.18e-6 $\pm 1.2\text{e-}7$
Source	#1 (0.4 kHz / 0.4 μ Ci)				
S_m [Hz]	444 $\pm$ 1	447 $\pm$ 1	456 $\pm$ 1	468 $\pm$ 1	472 $\pm$ 1
D_m [Hz]	105 $\pm$ 1	107 $\pm$ 1	111 $\pm$ 1	116 $\pm$ 1	118 $\pm$ 1
Dead-time corrected S [Hz]	444 $\pm$ 1	447 $\pm$ 1	456 $\pm$ 1	468 $\pm$ 1	472 $\pm$ 1
Dead-time corrected D [Hz]	105 $\pm$ 1	107 $\pm$ 1	111 $\pm$ 1	116 $\pm$ 1	118 $\pm$ 1
Source	#2 (1.9 kHz / 1.6 μ Ci)				
S_m [Hz]	1910 $\pm$ 2	1932 $\pm$ 2	1956 $\pm$ 2	2002 $\pm$ 2	2017 $\pm$ 2
D_m [Hz]	452 $\pm$ 1	465 $\pm$ 1	476 $\pm$ 1	498 $\pm$ 1	506 $\pm$ 1
Dead-time corrected S [Hz]	1911 $\pm$ 2	1933 $\pm$ 2	1957 $\pm$ 2	2004 $\pm$ 2	2018 $\pm$ 2
Dead-time corrected D [Hz]	453 $\pm$ 1	466 $\pm$ 1	477 $\pm$ 1	499 $\pm$ 1	507 $\pm$ 1
Source	#3 (26 kHz / 20.2 μ Ci)				
S_m [Hz]	25596 $\pm$ 6	25857 $\pm$ 5	26150 $\pm$ 7	26760 $\pm$ 7	26981 $\pm$ 8
D_m [Hz]	5950 $\pm$ 15	6068 $\pm$ 15	6210 $\pm$ 13	6497 $\pm$ 13	6612 $\pm$ 14
Dead-time corrected S [Hz]	25739 $\pm$ 20	25964 $\pm$ 60	26350 $\pm$ 22	26963 $\pm$ 19	27197 $\pm$ 24
Dead-time corrected D [Hz]	6083 $\pm$ 24	6169 $\pm$ 59	6401 $\pm$ 24	6697 $\pm$ 22	6826 $\pm$ 27
Source	#4 (42 kHz / 31.1 μ Ci)				
S_m [Hz]	42047 $\pm$ 9	42434 $\pm$ 9	42936 $\pm$ 11	43937 $\pm$ 11	44293 $\pm$ 10
D_m [Hz]	9698 $\pm$ 18	9888 $\pm$ 26	10073 $\pm$ 26	10505 $\pm$ 22	10679 $\pm$ 27
Dead-time corrected S [Hz]	42432 $\pm$ 52	42720 $\pm$ 161	43475 $\pm$ 57	44487 $\pm$ 49	44876 $\pm$ 62
Dead-time corrected D [Hz]	10058 $\pm$ 52	10158 $\pm$ 155	10587 $\pm$ 61	11041 $\pm$ 52	11252 $\pm$ 68

Table 2. The double pulsing fraction calculated using Eq. 8 from an experimental assay of a ^{252}Cf source with an AWCC and the resultant RAD. The estimated r extracted using measured singles is shown for each operating high voltage. Measurements correspond to 30 runs of a 30-second duration. The error represents the standard error of the mean of the 30 measurements.

High Voltage [V]	1760	1780	1820	1860
Source	#1 (0.4 kHz / 0.4 μCi)			
Calculated r from RAD (%)	1.1 $\pm$ 0.2	2.3 $\pm$ 0.4	5.0 $\pm$ 0.5	6.2 $\pm$ 0.1
Estimated r (%)	0.7 $\pm$ 0.3	2.6 $\pm$ 0.3	5.3 $\pm$ 0.3	6.2 $\pm$ 0.2
Source	#2 (1.9 kHz / 1.6 μCi)			
Calculated r from RAD (%)	0.9 $\pm$ 0.1	2.1 $\pm$ 0.3	4.6 $\pm$ 0.4	5.6 $\pm$ 0.1
Estimated r (%)	1.1 $\pm$ 0.1	2.4 $\pm$ 0.1	4.9 $\pm$ 0.1	5.6 $\pm$ 0.1
Source	#3 (26 kHz / 20.2 μCi)			
Calculated r from RAD (%)	0.7 $\pm$ 0.1	1.7 $\pm$ 0.3	4.0 $\pm$ 0.4	5.0 $\pm$ 0.1
Estimated r (%)	0.9 $\pm$ 0.2	2.4 $\pm$ 0.1	4.8 $\pm$ 0.1	5.7 $\pm$ 0.1
Source	#4 (42 kHz / 31.1 μCi)			
Calculated r from RAD (%)	0.8 $\pm$ 0.1	1.9 $\pm$ 0.1	4.2 $\pm$ 0.1	5.0 $\pm$ 0.04
Estimated r (%)	0.7 $\pm$ 0.4	2.5 $\pm$ 0.2	4.8 $\pm$ 0.2	5.8 $\pm$ 0.2

Table 3. The corrected coincidence rates are recovered from the double pulsed experimental data using the r calculated from the RAD, which has no dead-time correction. The corrected rates are corrected first for dead time and then for double pulsing. They are compared to the dead-time-corrected coincidence rates measured when operating at 1680 V.

High Voltage [V]	1760	1780	1820	1860
Source	#1 (0.4 kHz / 0.4 μ Ci)			
Relative Difference for Corrected Singles (%)	-0.4 $\pm$ 0.4	0.3 $\pm$ 0.5	0.3 $\pm$ 0.6	0.0 $\pm$ 0.3
Relative Difference for Corrected Doubles (%)	-0.5 $\pm$ 0.9	1.0 $\pm$ 1.0	-0.2 $\pm$ 1.2	-0.4 $\pm$ 0.8
Source	#2 (1.9 kHz / 1.6 μ Ci)			
Relative Difference for Corrected Singles (%)	0.2 $\pm$ 0.2	0.3 $\pm$ 0.3	0.3 $\pm$ 0.4	0.0 $\pm$ 0.2
Relative Difference for Corrected Doubles (%)	0.9 $\pm$ 0.5	1.1 $\pm$ 0.7	0.8 $\pm$ 0.9	0.3 $\pm$ 0.4
Source	#3 (26 kHz / 20.2 μ Ci)			
Relative Difference for Corrected Singles (%)	0.2 $\pm$ 0.3	0.7 $\pm$ 0.3	0.7 $\pm$ 0.4	0.6 $\pm$ 0.2
Relative Difference for Corrected Doubles (%)	0.0 $\pm$ 1.1	1.8 $\pm$ 0.8	1.8 $\pm$ 0.9	1.7 $\pm$ 0.6
Source	#4 (42 kHz / 31.1 μ Ci)			
Relative Difference for Corrected Singles (%)	-0.1 $\pm$ 0.4	0.6 $\pm$ 0.2	0.6 $\pm$ 0.2	0.7 $\pm$ 0.2
Relative Difference for Corrected Doubles (%)	-0.6 $\pm$ 1.6	1.4 $\pm$ 0.8	1.1 $\pm$ 0.7	1.5 $\pm$ 0.8

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High Voltage [V]	1760	1780	1820	1860
Source	#1 (0.4 kHz / 0.4 μ Ci)			
Relative Difference for Corrected Singles (%)	-0.4 $\pm$ 0.4	0.3 $\pm$ 0.5	0.3 $\pm$ 0.6	0.0 $\pm$ 0.3
Relative Difference for Corrected Doubles (%)	-0.5 $\pm$ 0.9	0.6 $\pm$ 1.1	-0.2 $\pm$ 1.2	-0.4 $\pm$ 0.8
Source	#2 (1.9 kHz / 1.6 μ Ci)			
Relative Difference for Corrected Singles (%)	0.2 $\pm$ 0.2	0.3 $\pm$ 0.3	0.3 $\pm$ 0.4	0.0 $\pm$ 0.2
Relative Difference for Corrected Doubles (%)	0.9 $\pm$ 0.5	1.1 $\pm$ 0.7	0.8 $\pm$ 0.9	0.3 $\pm$ 0.4
Source	#3 (26 kHz / 20.2 μ Ci)			
Relative Difference for Corrected Singles (%)	0.2 $\pm$ 0.3	0.7 $\pm$ 0.3	0.7 $\pm$ 0.4	0.6 $\pm$ 0.2
Relative Difference for Corrected Doubles (%)	-0.1 $\pm$ 1.1	1.7 $\pm$ 0.8	1.7 $\pm$ 0.9	1.6 $\pm$ 0.6
Source	#4 (42 kHz / 31.1 μ Ci)			
Relative Difference for Corrected Singles (%)	-0.1 $\pm$ 0.4	0.6 $\pm$ 0.2	0.6 $\pm$ 0.2	0.7 $\pm$ 0.2
Relative Difference for Corrected Doubles (%)	-0.6 $\pm$ 1.6	1.3 $\pm$ 0.8	0.9 $\pm$ 0.7	1.2 $\pm$ 0.8

Table 5. Given the simplest case of a pulse train with a single neutron, a second artificial neutron is added with a probability of r . The average measured singles rate, given the probability of each scenario, is $(1 + r)$. Given $S_0 = 1$, the average measured singles rate can be written as $S_0(1 + r)$. For the doubles, a predelay indicated by the dashed lines must be used and in both scenarios (with and without double pulsing), no pulse lands within the gate, so the measured doubles are 0.

Case 1: $S_0 = 1$ $D_0 = 0$	Probability	S_m	D_m
	$(1-r)$	1	$0 = 0$
	r	2	$0 + 0 = 0$
		Probability-weighted average S_m	Probability-weighted average D_m
		$S = \overline{S_m}$ $= (1 + r)$ $= S_0(1 + r)$	$D = \overline{D_m} = 0$ $[= D_0(1 + r)^2]$

Table 6. Given the case of a pulse train with two neutrons, a second artificial neutron can be added to each true neutron with a probability of r . The average measured singles rate, given the probability of each scenario, is $2(1 + r)$. Given $S_0 = 2$, the average measured singles rate can be written as $S_0(1 + r)$. For the doubles, a predelay indicated by dashed lines is used and the number of correlated neutrons within the gate are counted for each scenario. The average measured doubles rate, given the probability of each scenario is $(1 + r)^2$. Given $D_0 = 1$, the average measured doubles rate can be written as $D_0(1 + r)^2$.

Case 2: $S_0 = 2$ $D_0 = 1$	Probability	S_m	D_m
	$(1-r)^2$	2	$1 + 0 = 1$
	$r(1-r)$	3	$1 + 1 + 0 = 2$
	$(1-r)r$	3	$2 + 0 + 0 = 2$
	r^2	4	$2 + 2 + 0 + 0 = 4$
		Probability-weighted average S_m	Probability-weighted average D_m
		$S = \overline{S_m}$ $= 2(1 + r)$ $= S_0(1 + r)$	$D = \overline{D_m}$ $= (1 + r)^2$ $= D_0(1 + r)^2$

Table 7. Given the case of a pulse train with three neutrons, an additional artificial neutron can be added to each initial true neutron with a probability of r . The average measured singles rate, given the probability of each scenario, is $3(1+r)$. Given $S_0 = 3$, the average measured singles rate can be written as $S_0(1+r)$. For the doubles, a predelay indicated by a dashed line is used and the number of correlated neutrons within the gate are counted for each scenario. The average measured doubles rate, given the probability of each scenario, is $3(1+r)^2$. Given $D_0 = 3$, the average measured doubles rate can be written as $D_0(1+r)^2$.

Case 3: $S_0 = 3$ $D_0 = 3$	Prob.	S_m	D_m
	$(1-r)^3$	3	$2 + 1 + 0 = 3$
	$r(1-r)^2$	4	$2 + 2 + 1 + \dots + 0 = 5$
	$(1-r)r(1-r)$	4	$3 + 1 + 1 + \dots + 0 = 5$
	$(1-r)^2r$	4	$3 + 2 + 0 + \dots + 0 = 5$
	$r^2(1-r)$	5	$3 + 3 + 1 + \dots + 1 + 0 = 8$

	$r(1-r)r$	5	$3 + 3 + 2 + \dots$ $\dots + 0 + 0 = 8$
	$(1-r)r^2$	5	$4 + 2 + 2 + \dots$ $\dots + 0 + 0 = 8$
	r^3	6	$4 + 4 + 2 + \dots$ $\dots + 2 + 0 + 0$ $= 12$
		Probability-weighted average S_m	Probability-weighted average D_m
		$S = \overline{S_m}$ $= 3(1+r)$ $= S_0(1+r)$	$D = \overline{D_m}$ $= 3(1+r)^2$ $= D_0(1+r)^2$

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