A novel numerical algorithm to evaluate transient response for synchronous scanning streak camera using time domain BLT equation

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Abstract

The transient response is of great influence on the electromagnetic compatibility of synchronous scanning streak camera (SSSC). In this paper, a novel numerical method to evaluate the transient response is proposed. To evaluate the transient response of scanning deflection plate (SDP), the simplified circuit model for the SDP used in SSSC was established and the Baum-Liu-Tesche (BLT) equation was derived in frequency domain. From the frequency-domain BLT equation, its transient counterpart is derived. These parameters, together with the transient BLT equation are used to compute the transient load voltage and load current, and then a novel numerical method to fulfill the continuity equation was proposed. Several numerical simulations were conducted to verify this proposed method. The computed results are subsequently compared with transient responses obtained by frequency domain/FFT method, and the agreement is excellent for highly conducting cables. The benefit of deriving the BLT equation in the time domain is that it may be used with slight modifications to calculate the transient response and the error can be controlled by program. The result shows that the transient voltage is up to 1000V and transient current is about 10A, some protection measures should be taken to improve the electromagnetic compatibility.

Key words: synchronous scanning streak camera, scanning deflection plate, transient response, Baum-Liu-Tesche (BLT) equation, numerical method

1. Introduction

Streak camera is an ultra high-speed detector which captures light emission phenomena occurring in extremely short time periods. Not only can the streak camera measure the intensity variation with a superb temporal resolution, but it can also be used for simultaneous measurement of the spatial (or spectral) distribution, therefore, it is widely used as an ultrafast diagnostic tool in physics of Inertial Confinement Fusion (ICF) [1, 2]. Currently high-performance streak camera is being studied in Japan [3], Russia [4], Germany [5], France [6] and the United Kingdom [7]. The SDP is an essential device for streak camera, in Ref [8], [9]; the characteristics of the SDP have been studied. What's more, the large power and high frequency sine wave was applied on SDP. If the transient current and voltage on SDP are exorbitant, it may radiate the high power electromagnetic field, which can deteriorate the performance of streak camera; especially the temporal resolution. In addition; the scanning circuit would be damaged by high transient voltage and large transient current. However, the equivalent circuit and transient response on SDP has not been touched. In order to evaluate the transient response, it is essential to design a numerical algorithm to evaluate the transient response. Then the corresponding measures can be adopted to improve the electromagnetic compatibility of streak camera.

The electromagnetic topology model is widely used in electromagnetic compatibility analysis, and the BLT equation is a useful tool to describe the behavior of voltage and current for this model. The original BLT equation was

formulated in the frequency domain and the transient response can be achieved by Fourier transform methods [10, 11]. Thus, it was implicitly assumed that the entire system should be time invariant and linear in earlier work. However, in some cases, the nonlinear elements may be contained in the system, and it is possible to accumulate the transient response of the nonlinear system with the method of frequency-domain analysis [12]. A time domain BLT equation for the lossless transmission line was derived from its frequency-domain counterpart in an earlier Ref. [13]. The method provided in Ref. [13] was different from other commonly used analysis solution, such as the finite-difference time-domain solution [14], fast Fourier transform [15], distributed circuit model approach[16] and SPICE model methods[17,18]. In Ref. [13], the transient response of voltage and current for linear and nonlinear resistor termination was derived; however, the SDP can be simplified as a resistor in series with a capacitance and the capacitance is the energy storage element.

In this paper, the reflection coefficient of SDP in time domain was derived and then the time domain BLT to evaluate the transient voltage and current was proposed. Moreover, the numerical method to evaluate the transient response was derived and several examples were adopted to verify the proposed method. This solution yield results that are identical to those provided by a frequency-domain/FFT analysis up to a predetermined time. Finally, this proposed numerical method can provide a guide to calculate the transient response. The result shows that the transient voltage in SDP is up to 1000V and the transient current is up to 10A. Therefore, some protection measures should be taken to improve the electromagnetic compatibility.

2. Equivalent circuit model of SDP

The synchronous scanning streak tube is comprised of photoelectric cathode, grid, focusing electrode, SDP, microchannel plate (MCP), focusing electrode and fluorescent screen et al, as shown in Fig.1.

Fig.1. Architecture of SSSC

Fig.2. Simplified circuit

The SSSC converts the photon pulse varying with time into an electron pulse and applies a time dependent transverse deflection. This results in a transverse image size proportional to the photon pulse length. The main challenge of the streak camera is to preserve the photon pulse varying with time and properly focus the electron beam at the image plane. In order to deal with this challenge, the SDP was adopted. It can superpose the weak signal to fluorescent screen precisely by repetitive high frequency synchronous scanning in order to improve the detection sensitivity and dynamic range. However, it should be supplied with sine wave with high power about 100W and high frequency ranging from 40MHz to 400MHz, which is imported by a coaxial line. The transient voltage and transient current in SDP may be larger due to the high power and high frequency sine wave. It could deteriorate the temporal resolution and damage the scanning circuit. Therefore, the equivalent circuit model of SDP should be established firstly to evaluate the transient response. The SDP can be simplified as a resistor in series with a capacitance, which is derived from the experiments measurement and the simplified circuit model is shown in Fig.2. We note that the resistor and capacitance vary with the frequency. The impedance of SDP is measured by vector network analyzer, and the resistor and capacitance are listed in Fig.3.

Fig.3. Equivalent capacitance and resistor

3. Proposed method

The high frequency and large power sine signal is imported into the SDP by a single coaxial line as shown in Fig.4(a). The excitation is the sine signal and the SDP is equivalent to the load. In order to analyze the transient voltage and transient current, the electromagnetic topology model [11] can be expressed as shown in Fig.4 (b). It includes of an excitation source and two nodes representing the loads. The transmission line can be simplified as tube, which has the characteristic parameters such as propagation function $g(x, t)$, characteristic impedance $Z_c(t)$ and characteristic admittance $y_c(t)$.

Fig.4 (a) Circuit architecture of SDP

Fig.4 (b) Simple transmission-line representation with lumped source excitation

It comprises of a lossy transmission line of length L , with the linear or nonlinear termination $Z_{L1}(t)$ and $Z_{L2}(t)$, the lumped transient voltage and current source at $x=x_s$. The transient load voltage and current at each end ($v_1(t)$, $i_1(t)$) and ($v_2(t)$, $i_2(t)$) are to be determined. To derive the time domain BLT equation, we will first review the BLT equation in frequency domain [11].

$$\bar{V} = [\bar{U} + \bar{R}][\bar{U} - \bar{\Gamma}(L)\bar{R}]^{-1}\bar{S} \quad (1)$$

$$\bar{I} = Y_c(s)[\bar{U} - \bar{R}][\bar{U} - \bar{\Gamma}(L)\bar{R}]^{-1}\bar{S} \quad (2)$$

The equation (1), (2) is the BLT equation and it can describe the behavior of voltage and current at all of the junctions of the transmission line in frequency domain. The coaxial line can be simplified by a circuit model [19] as shown in Fig.5. The inner conductor has a radius a , and the outer conductor has an inner radius b and thickness Δ . Each conductor is assumed to be lossy with the same electrical conductivity σ , and the cable interior is filled with a lossless dielectric having a relative permittivity ϵ_{rel} . The magnetic permeability is assumed to be that of free space μ_0 .

Fig.5. Cross section of the coaxial transmission line and its per-unit-length equivalent circuit

For the lossless coaxial line, the Per-Unit-Length of resistor R' , inductance L' and capacitance C' have been provided by Paul in Ref. [18].

$$L' = \frac{\mu_0}{2\pi} \ln\left(\frac{r_a}{r_b}\right) \quad C' = \frac{2\pi\epsilon_0\epsilon_{rel}}{\ln(r_a/r_b)} \quad R' = \frac{1}{\pi\sigma} \left(\frac{1}{a} + \frac{1}{2b\Delta} \right) \quad (3)$$

With these parameters defined, the total per-unit-length impedance and admittance of the coaxial line can be written as

$$Z' = sL' + R' + \xi\sqrt{s} \quad Y' = sC'$$

Where, the Z' and Y' are in Laplace domain. For a coaxial line with conductor radii a and b , the parameter ξ is given by

$$\xi = \frac{1}{2\pi} \sqrt{\frac{\mu_0}{\sigma}} \left(\frac{1}{a} + \frac{1}{b} \right) \quad (4)$$

The parameters Z'_a and Z'_b in Fig.5 can be approximated as in Ref.19.

$$Z'_a = \frac{1}{2\pi a} \sqrt{\frac{s\mu_0}{\sigma}} \quad Z'_b = \frac{1}{2\pi b} \sqrt{\frac{s\mu_0}{\sigma}}$$

For $\left| \sqrt{s\mu_0/\sigma} \right| a \gg 1$ and $\left| \sqrt{s\mu_0/\sigma} \right| b \gg 1$. For the radii b is 2.5mm and the conductor made of copper, the approximations are valid for frequencies above 100 kHz.

1) The transmission line propagation constant[19] can be written as

$$\gamma(s) = \sqrt{(sL' + R' + \xi\sqrt{s})(sC')} \approx s\sqrt{L'C'} + \frac{R'}{2} \sqrt{\frac{C'}{L'}} + \frac{\xi\sqrt{s}}{2} \sqrt{\frac{C'}{L'}} \quad (5)$$

For $\left| \frac{R'}{L's} \right| \ll 1$ and $\left| \frac{\xi}{L'\sqrt{s}} \right| \ll 1$

$$\gamma(s) \equiv \frac{s}{v} + a_1\sqrt{s} + a_2$$

Where the following definitions are used

$$v = \frac{1}{\sqrt{L'C'}}, \quad a_1 = \frac{\xi}{2} \sqrt{\frac{C'}{L'}}, \quad a_2 = \frac{R'}{2} \sqrt{\frac{C'}{L'}}$$

The parameter v is the wave propagation speed on the line. The spectral wave propagation term of the line can be expressed as

$$G(s, x) = e^{-\gamma(s)x} = e^{-sx/v} e^{-\sqrt{s}a_1x} e^{-a_2x} \quad (6)$$

And the inverse transformation provides the impulse function $g(t, x)$. It provides the distortion of an applied δ function wave at $x=0$, as it propagates on the lossy line. The first exponential term $e^{-sx/v}$ corresponds to a time shift of $T=x/v$. The time domain propagation function can be derived by using Laplace transformation.

$$g(t, x) = e^{-a_2x} \frac{a_1x}{2\sqrt{\pi(t-x/v)^3}} e^{-(a_1x)^2/4(t-x/v)} \quad (7)$$

It shows that $g(t, x)$ is similar to a delta function whose amplitude decreases with the distance as does the exponential term e^{-a_2x} .

1) Surge impedance and admittance: For the low-loss line, the frequency domain characteristic impedance can be approximated as

$$Z_c(s) = \frac{\sqrt{R' + sL' + \xi\sqrt{s}}}{\sqrt{sC'}} \approx z_c \left(1 + \frac{R'}{2sL'} + \frac{\xi}{2\sqrt{sL'}} \right) \quad (8)$$

Where $z_c = \sqrt{L'C'}$, the time domain surge impedance $z_c(t)$ is the invert Laplace transformation of $Z_c(s)$, and then it can be expressed as

$$z_c(t) = z_c \delta(t) + z_c \frac{R'}{2L'} \Phi(t) + z_c \frac{\xi}{2L'} \frac{1}{\sqrt{\pi t}} \quad (9)$$

Where, $\Phi(t)$ is unit step function. A similar procedure can be used to derive the characteristic admittance of the line and the result is

$$y_c(t) = y_c \delta(t) - y_c \frac{R'}{2L'} \Phi(t) - y_c \frac{\xi}{2L'} \frac{1}{\sqrt{\pi t}} \quad (10)$$

Where $y_c = z_c^{-1}$

2) Transient Load Reflection Coefficients: the transient reflection coefficients for each end of the line are different with the previous parameters. It depends both on line properties and the termination impedance. The voltage reflection coefficient in the spectral domain for load can be expressed as

$$\rho_R = \frac{R_L - Z_c(s)}{R_L + Z_c(s)} \quad (11)$$

A. The reflection coefficient in time domain

The reflection coefficient $\rho(s)$ in the frequency domain can be described as

$$\rho(s) = \frac{R_L - Z_c(s)}{R_L + Z_c(s)}$$

Where R_L denote the load attached to the end of the line. For the SDP the equivalent circuit as shown in Fig.2, the impedance can be described as $R_L = R + \frac{1}{sC}$ for a single frequency. The characteristic impedance of coaxial line is

$$Z_c(s) = 1 + \frac{R_0}{2sL} + \frac{\varepsilon}{2\sqrt{sL}} \quad (12)$$

Then the reflection coefficient can be formulated as

$$\rho_L(s) = \frac{R_L - \frac{a}{s} - \frac{b}{\sqrt{s}}}{R_L + \frac{a}{s} + \frac{b}{\sqrt{s}}} + \frac{-2}{R_L + \frac{a}{s} + \frac{b}{\sqrt{s}}} + \frac{\frac{2}{sC}}{R_L + \frac{a}{s} + \frac{b}{\sqrt{s}}} \quad (13)$$

Define $a = \frac{Z_c R_0}{2L} + \frac{1}{C}$, $b = \frac{Z_c \varepsilon}{2L}$, $R_L = (R + Z_c)$ and define $\rho_L(s) = f_1(s) + f_2(s) + f_3(s)$, then

$$f_1(s) = \frac{R_L - \frac{a}{s} - \frac{b}{\sqrt{s}}}{R_L + \frac{a}{s} + \frac{b}{\sqrt{s}}} \quad f_2(s) = \frac{-2 + \frac{2}{sC}}{R_L + \frac{a}{s} + \frac{b}{\sqrt{s}}} \quad f_3(s) = \frac{\frac{2}{sC}}{R_L + \frac{a}{s} + \frac{b}{\sqrt{s}}} \quad (14)$$

The invert Laplace transformation was made for $f_1(s)$

$$f_1(s) = 1 - \frac{\frac{2b}{R_L} \sqrt{s} + \frac{2a}{R_L}}{s + \frac{b}{R_L} \sqrt{s} + \frac{a}{R_L}} \quad (15)$$

Factoring the denominator as

$$s + \frac{b}{R_L} \sqrt{s} + \frac{a}{R_L} = (\sqrt{s} - Q_1)(\sqrt{s} - Q_2)$$

Where the roots are calculated as

$$Q_{1,2} = \frac{1}{2R_L} \left[-b \pm \sqrt{b^2 - 4aR_L} \right]$$

Then the Eq. (15) can be written in the factored form as

$$f_1(s) = 1 + \frac{m_1}{\sqrt{s} - Q_1} - \frac{m_2}{\sqrt{s} - Q_2}$$

Here two additional parameters are defined as

$$m_1 = \frac{\frac{2a}{R_L}}{Q_2 - Q_1} \left(Q_1 + \frac{b}{a} \right) \quad m_2 = \frac{\frac{2a}{R_L}}{Q_2 - Q_1} \left(Q_2 + \frac{b}{a} \right)$$

The leading term in Eq. (15) converts to a delta function in time and two remaining terms can be transformed into the time domain.

$$f_1(t) = \delta(t) + m_1 \left[\frac{1}{\sqrt{\pi t}} - Q_1 e^{Q_1^2 t} \operatorname{erfc}(Q_1 \sqrt{t}) \right] - m_2 \left[\frac{1}{\sqrt{\pi t}} - Q_2 e^{Q_2^2 t} \operatorname{erfc}(Q_2 \sqrt{t}) \right] \quad (16)$$

The invert Laplace transformation was made for $f_2(s)$

$$f_2(s) = -2 \left\{ \frac{1}{R_L} \left[\frac{R_L s - a - b\sqrt{s}}{R_L s + a + b\sqrt{s}} \right] + \frac{1}{R_L} \left[\frac{a + b\sqrt{s}}{R_L s + a + b\sqrt{s}} \right] \right\} \quad (17)$$

$\frac{R_L s - a - b\sqrt{s}}{R_L s + a + b\sqrt{s}}$ is similar to $f_1(s)$, for the second item

$$\frac{\frac{a}{R_L} + \frac{b}{R_L} \sqrt{s}}{s + \frac{a}{R_L} + \frac{b}{R_L} \sqrt{s}} = \frac{m_1}{\sqrt{s} - Q_1} - \frac{m_2}{\sqrt{s} - Q_2} \quad (18)$$

The invert Laplace transformation made for Eq. (18) and the time domain formulation can be described as

$$f_2(t) = \frac{-2}{R_L} \left[m_1 \left(\frac{1}{\sqrt{\pi t}} - Q_1 e^{Q_1^2 t} \operatorname{erfc}(Q_1 \sqrt{t}) \right) + m_2 \left(\frac{1}{\sqrt{\pi t}} - Q_2 e^{Q_2^2 t} \operatorname{erfc}(Q_2 \sqrt{t}) \right) \right] + f_2'(t)$$

And

$$f_2'(t) = \frac{-2}{R_L} \left[\delta(t) + m_1 \left[\frac{1}{\sqrt{\pi t}} - Q_1 e^{Q_1^2 t} \operatorname{erfc}(Q_1 \sqrt{t}) \right] - m_2 \left[\frac{1}{\sqrt{\pi t}} - Q_2 e^{Q_2^2 t} \operatorname{erfc}(Q_2 \sqrt{t}) \right] \right] \quad (19)$$

The invert Laplace transformation was made for $f_3(s)$

$$f_3(s) = \frac{2}{R_L C} \frac{1}{s + \frac{a}{R_L} + \frac{b}{R_L} \sqrt{s}}$$

Factoring the denominator as

$$f_3(s) = \frac{2}{R_L C} \left(\frac{A}{\sqrt{s} - Q_1} + \frac{B}{\sqrt{s} - Q_2} \right) \quad (20)$$

Where two additional parameters are defined as

$$A = -B = \frac{1}{Q_2 - Q_1}$$

The Laplace transformation made for $f_3(s)$ is similar to $f_1(s)$ and $f_2(s)$, and the $f_3(t)$ can be described as

$$f_3(t) = \frac{2}{R_L C} \left[\frac{1}{Q_2 - Q_1} \left(\frac{1}{\sqrt{\pi t}} - Q_1 e^{Q_1^2 t} \operatorname{erfc}(Q_1 \sqrt{t}) \right) + \frac{1}{Q_2 - Q_1} \left(\frac{1}{\sqrt{\pi t}} - Q_2 e^{Q_2^2 t} \operatorname{erfc}(Q_2 \sqrt{t}) \right) \right] \quad (21)$$

The reflection coefficient in time domain can be represented as $\rho_L(t) = f_1(t) + f_2(t) + f_3(t)$.

B. Time domain BLT equation

In equation (1), the term $[\bar{U} - \bar{\Gamma}(L)\bar{R}]^{-1}$ is hard to calculate. On the other hand, it may create the transmission line resonances at an infinite number of points in the complex frequency plane. This inverse matrix function can be expanded in a power series as

$$[\bar{U} - \bar{\Gamma}(L)\bar{R}]^{-1} = [\bar{U} + (\bar{\Gamma}(L)\bar{R}) + (\bar{\Gamma}(L)\bar{R})^2 + \dots] \quad (22)$$

Performing this expansion for the voltage BLT equation (21), we obtain

$$\bar{V} = [\bar{U} + \bar{R}] [\bar{U} + (\bar{\Gamma}(L)\bar{R}) + (\bar{\Gamma}(L)\bar{R})^2 + \dots] \bar{S} \quad (23)$$

Then Eq.(24) can be expressed in the time domain as

$$\bar{v}(t) = [\bar{u}(t) + \bar{r}(t)] * [\bar{u}(t) + (\bar{r}(t, L) * \bar{r}(t)) + (\bar{r}(t, L) * \bar{r}(t))^2 + \dots] * \bar{s}(t) \quad (24)$$

Where $\bar{u}(t)$, $\bar{r}(t)$, $\bar{r}(t, L)$ and $\bar{s}(t)$ are the invert Laplace transformation of $\bar{U}$, $\bar{R}$, $\bar{\Gamma}(L)$ and $\bar{S}$ correspondingly. The matrix $\bar{r}(t, L)$ is given by

$$\bar{r}(t, L) = \begin{bmatrix} 0 & g(t, L) \\ g(t, L) & 0 \end{bmatrix} \quad (25)$$

The $\bar{r}(t, L) * \bar{r}(t)$ can be expressed as

$$\begin{aligned} \bar{r}(t, L) * \bar{r}(t) &= \begin{bmatrix} 0 & g(t, L) \\ g(t, L) & 0 \end{bmatrix} * \begin{bmatrix} r_1(t) & 0 \\ 0 & r_2(t) \end{bmatrix} \\ &= \begin{bmatrix} 0 & g(t, L) * r_2(t) \\ g(t, L) * r_1(t) & 0 \end{bmatrix} \end{aligned} \quad (26)$$

Then the $(\bar{r}(t, L) * \bar{r}(t))^2$ can be described as

$$\begin{aligned} (\bar{r}(t, L) * \bar{r}(t))^2 &= \begin{bmatrix} 0 & g(t, L) * r_2(t) \\ g(t, L) * r_1(t) & 0 \end{bmatrix} * \begin{bmatrix} 0 & g(t, L) * r_2(t) \\ g(t, L) * r_1(t) & 0 \end{bmatrix} \\ &= \begin{bmatrix} r_{z_1}(t) * r_{z_2}(t) & 0 \\ 0 & r_{z_1}(t) * r_{z_2}(t) \end{bmatrix} \end{aligned} \quad (27)$$

Thus, we can easily obtain that

$$(\bar{r}(t, L) * \bar{r}(t))^3 = \begin{bmatrix} 0 & r_{z_1}(t) * r_{z_2}^2(t) \\ r_{z_1}^2(t) * r_{z_2}(t) & 0 \end{bmatrix} \quad (28)$$

Where the notation, $r_{z_1}(t) = g(t, L) * r_1(t)$, $r_{z_1}^n(t) = r_{z_1}(t) * r_{z_1}(t) * r_{z_1}(t) * \dots$ (n terms)

Define the matrix

$$H = \bar{U} + (\bar{\Gamma}(L)\bar{R}) + (\bar{\Gamma}(L)\bar{R})^2 + \dots$$

Then

$$H(t) = \begin{bmatrix} h(t) & r_{z_2}(t) * h(t) \\ r_{z_1}(t) * h(t) & h(t) \end{bmatrix} \quad (29)$$

Where $h(t) = \delta(t) + r_{z_1}(t) * r_{z_2}(t) * \{(\delta(t) + r_{z_1}(t) * r_{z_2}(t) * [\delta(t) + r_{z_1}(t) * r_{z_2}(t)](\dots))\}$

For BLT equation in the time domain, there are several transformations that must be calculated from their frequency domain counterparts. Thus, the transient BLT equation (23) in component form becomes

$$\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} \delta(t) + r_1(t) & 0 \\ 0 & \delta(t) + r_2(t) \end{bmatrix} * \begin{bmatrix} h(t) & r_{z_2}(t) * h(t) \\ r_{z_1}(t) * h(t) & h(t) \end{bmatrix} * \begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix} \quad (30)$$

With the transient source vector being expressed as

$$\bar{s}(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}[v_s(t) - z_c(t) * i_s(t)] * g(t, x_s) \\ \frac{1}{2}[v_s(t) + z_c(t) * i_s(t)] * g(t, L - x_s) \end{bmatrix} \quad (31)$$

In equation (30), we note that once the reflection coefficient is computed, the voltage and current can be evaluated.

C. Proposed numerical method

We assume that the initial system energy is zero, so it implicitly assumes that $v(t_0) = 0$, the transient voltage of SDP can be written as

$$v(t) = i(t)R + \frac{1}{C} \int_{-\infty}^t i(t) dt$$

If t_0 is set as the start point and in this time interval the transient current and voltage is zero, define $t_1 = t_0 + \Delta t$, then

$$\begin{aligned} v(t_1) &= i(t_1)R + \frac{1}{C} \int_{-\infty}^{t_0} i(t) dt + \frac{1}{C} \int_{t_0}^{t_0 + \Delta t} i(t) dt \\ &= i(t_1)R + \frac{1}{C} \int_{t_0}^{t_0 + \Delta t} i(t) dt \end{aligned}$$

If Δt is small enough, the equation can be written as

$$v(t_1) = i(t_1)R + \frac{1}{C} \int_{-\infty}^{t_0} i(t) dt + \frac{1}{C} \int_{t_0}^{t_0 + \Delta t} i(t) dt$$

The following equation can be achieved from above equation when $t_2 = t_0 + 2\Delta t$

$$\begin{aligned} v(t_2) &= i(t_2)R + \frac{1}{C} \int_{-\infty}^{t_0 + \Delta t} i(t) dt + \frac{1}{C} \int_{t_0 + \Delta t}^{t_0 + 2\Delta t} i(t) dt \\ &= i(t_2)R + v_c(t_1) + \frac{1}{C} \int_{t_0 + \Delta t}^{t_0 + 2\Delta t} i(t) dt \end{aligned}$$

Then the iteration equation can be achieved when $t_n = t_0 + n\Delta t$,

$$v(t_n) = i(t_n)R + v_c(t_{n-1}) + \frac{1}{C} \int_{t_0 + (n-1)\Delta t}^{t_0 + n\Delta t} i(t) dt \quad (32)$$

In order to achieve the numerical method which can be easily realized by program, the Eq. (32) can be evaluated by numerical iteration as

$$\begin{aligned} v(t_0) &= 0; v(t_1) \approx i(t_0 + \Delta t)R + \frac{\Delta t}{6C} \left[i(t_0) + 4i(t_0 + \frac{\Delta t}{2}) + i(t_0 + \Delta t) \right] + R(t) \\ v(t_2) &\approx i(t_0 + 2\Delta t)R + v_c(t_1) + \frac{\Delta t}{6C} \left[i(t_0 + \Delta t) + 4i(t_0 + \frac{3\Delta t}{2}) + i(t_0 + 2\Delta t) \right] + R(t) \dots \\ v(t_n) &\approx i(t_0 + n\Delta t)R + v_c(t_{n-1}) + \frac{\Delta t}{6C} \left[i(t_0 + (n-1)\Delta t) + 4i(t_0 + \frac{(2n-1)\Delta t}{2}) + i(t_0 + n\Delta t) \right] + R(t) \end{aligned} \quad (33)$$

It shows that the transient voltage of SDP can be evaluated by a numerical iteration equation. It has some benefits to calculate the transient current with this numerical method. First, it is easy to be computed by program. Second, the error $R(t)$ can be effectively controlled by Δt , which could easily be realized by program.

4. Simulation verification

To validate the formulation and numerical implement of time domain BLT equation, the coaxial cable was adopted as transmission line in the experiment. In simulation, we assume the parameters for the coaxial cable as following: the inner radius r_a is 2.5mm, the outer radius r_b is 9.345mm, ϵ is 2.5, the loss tangent is 0.01 and the high frequency characteristic impedance of the line is 50Ω. The power of input signal is 100W. For the transmission line, we assume that the total length of the line is $L = 1.1$ m and the excitation is located at $x_s = 0$. The direct time domain solution for the SDP has been obtained using the transient time domain BLT equation for the load voltage. For the

case of a highly conducting coaxial line (copper with $\delta=5.76 \times 107\text{s/m}$), the transient voltage and transient current for the excitation frequency 75MHz and 98MHz were simulated by Matlab software based on the time domain BLT equation proposed. The frequency-domain/FFT method was taken to calculate the transient response in order to verify the simulated result for same coaxial line and excitation frequency, and this solution employed the rigorous formulas in Ref.19 for the per-unit-length parameters for the conductor loss. In simulation, the Δt was set as 0.0015ns in time domain BLT equation and the 16784 samples were adopted in frequency-domain/FFT method. The voltage and current were derived as shown in Fig.6 and Fig.7 when the excitation frequency is 75MHz and 98MHz respectively.

Fig.6 (a). Transient voltage when the excitation is 75MHz

Fig.6 (b). Transient current when the excitation is 75MHz

Fig.7 (a). Transient voltage when the excitation is 98MHz

Fig.7 (b). Transient current when the excitation is 98MHz

The black short dotted lines in Fig6, Fig.7 are the result of an analysis using the frequency domain/FFT method to convert the response to time domain and the solid lines in Fig.6, Fig.7 are the result of an analysis using the method proposed in this paper. We note that there is excellent agreement between the time domain BLT equation and frequency-domain/FFT response up to about 99ns. At this time, the transient calculation begins to deviate from the frequency-domain/FFT method due to the truncation of the infinite convolution function (24). If more terms in this expression are retained; the agreement between the two responses would last longer. In addition, the residual $R(t)$ is about 0.1V for voltage and 0.01A for current. However, it is interesting to point out that if the Δt is the smaller, the residual $R(t)$ could be lower. The benefit of deriving the BLT equation in the time domain is that it can be used with slight modifications to evaluate the transient response in engineer. The transient response can be evaluated by this numerical iterative equation, and the calculation error can be controlled by computer. In addition, the transient voltage calculated by this numerical method is up to 1000V and the transient current is up to 10A. So some protection measures should be taken to protect the scanning circuit such as inserting a matching impedance circuit between the scanning circuits and SDP to reduce the reflection wave, or adopting some transient voltage suppressors.

5. Conclusions

The equivalent circuit model of SDP was set up to evaluate the transient response and the electromagnetic

topology for calculating transient response was established. Moreover, the time domain reflection coefficient, time domain surge impedance, time domain admittance and time domain propagation function were derived by taking inverse Laplace transformation. What's more, the numerical method to evaluate the transient response was given and several numerical examples were conducted to verify the proposed method. The result showed that the transient response evaluated by proposed method agree with the frequency-domain/FFT method excellently. The benefit of deriving the BLT equation in the time domain is that it could be used to calculate the transient response with slight modifications and the error can be reduced by sampling the more data and increasing the convolution operations. In addition, the SSSC usually works in environment of nuclear electromagnetic pulse. The temporal resolution could be deteriorated by lager transient response due to coaxial coupling and this numerical method is also suitable for evaluating the transient response in this case. In next step, we will extend the transient-BLT solution to the case of nonlinear or time varying loads.

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