Multi-cell disk-and-ring tapered structure for compact RF linacs

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A tubular disk-and-ring, tapered accelerating structure for small electron linacs and MicroLinacs is considered. It consists of metal and dielectric elements inserted into a metallic tube to eliminate multi-cell, multi-step brazing. The structure enables a wide range of phase velocities (including non-relativistic), a wide bandwidth allowing large number of cells (for standing wave mode) or short filling time (for traveling wave mode), combination of compensated and purely π -mode cells, alternative periodic focusing built-in to the RF structure (the disks), and combining of RF and vacuum windows. RF and accelerating performance of such a long structure having up to four dozens cells is analyzed. Some of beam dynamics, thermal, and vacuum aspects of the structure and MicroLinac performance are considered as well.

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1. Introduction

Linear accelerators of electrons are employed in a wide variety of applications, including radiography and sterilization, radiolysis and imaging, modification of materials and testing of semiconductor electronic components for space applications. Among emerging applications is driving table-top sub-mm wave sources [1] and replacement of radionuclide sources [2] to improve public security and prevent the diversion of radioactive material for radiation dispersion devices. A challenging specific of such a replacement is inexpensive, portable, easy-to-manufacture linac system delivering electrons with energies from a fraction of 1 MeV to several MeVs. The bremsstrahlung X-rays produced by the electron beam on a high-Z converter at the end of the linac may match the penetration and dose rate of a radionuclide source to be substituted.

Linac operation exploits autophasing principle discovered by V.I. Veksler in 1944 [3] and applied to a linear motion. The first traveling wave linac was demonstrated in 1947 [4]. W. Walkinshaw developed in 1948 a classical theoretical design of a tapered corrugated waveguide consisting of disk loaded circular cells [5]. A typical electron RF linac for industrial applications employs a low-gradient, tapered slow wave structure powered by a compact, pulsed microwave source (e.g., S-, C-, or X- band magnetron). The beam injection into the structure is usually provided by a compact thermionic gun operating at voltages limited by a few dozens of kilovolts. Tapering provides beam capture, bunching, and acceleration synchronously with the wave while the beam velocity along the structure elevates starting from low, non-relativistic velocities.

As a rule classical linac system is rather expensive and not portable. MicroLinac technology originally proposed at SLAC [6] employs a compact X-band linear accelerator powered by an inexpensive, low power, magnetron [7,8]. However, to make the MicroLinac

concept suitable for a wider scope of applications, the compact linac technology needs to be advanced to reduce further cost, weight, and dimensions. One such modification may employ of a light-weight, compact modulator, using, e.g., a transformer-free Marx scheme with array of isolated gate bipolar transistors (IGBTs). Such a compact modulator makes development of a light-weight linac and focusing structures even more important.

First practical step towards compact X-band MicroLinacs has been made by utilizing high-impedance, all-copper, multi-cell structures [7,9]. A bi-periodic part of a MicroLinac structure employs side [9] or on-axis [7] coupling cells enabling a somewhat substantial bandwidth and number of cells to achieve about 1 MeV beam energies within a single section at limited power supply (sub-MW in X-band). However, fabrication of a multi-cell, tapered MicroLinac structure remains rather expensive and time consuming. It includes high precision machining, cold-testing and re-machining of the cells, testing of the clamped assembly, multi-step brazing, and individual tuning of the cells and of the brazed assembly.

One unexplored opportunity for eased fabrication of a MicroLinac is using of a hybrid metal-dielectric, so called disk-and-ring (DaR) [10], quasi-periodic slow-wave structure capable of high group velocities $(0.1-0.2)c$ and simplified production for a low-current, low-gradient applications. The accelerating cells in the DaR MicroLinac consist of metal disks (with short drift tubes) that are inserted between the dielectric rings. In this paper we focus solely on resonant, standing wave (SW) mode of operation as being more effective for implementation in miniature RF linacs, though the DaR structure can be employed in a traveling wave mode as well. The high group velocity is provided by employing of so called compensated [10] cells having both frequencies of even and odd TE_{01} π -modes close to the operating frequency.

Note automatic frequency control (AFC) system may easily get unstable being confused by

parasitic and adjacent modes even for (1.5-2)-cell RF guns or few-cell linacs. That limits application of conventional RF SW linacs, RF guns, and MicroLinacs at higher repetition rates or significantly changing ambient temperatures even in S-band. Implementation of stable and reliable frequency control is of vital importance for variety of applications of resonant linacs and microwave injectors of electrons. The key for the problem solving is wide structure bandwidth, which remains rather narrow for state-of-the-art SW RF linacs.

The initial concept of an X-band MicroLinac is shown in Fig. 1. It employs a commercially available electron gun and X-band magnetron that can be powered by a light-weight, custom-built, Marx modulator.

Fig. 1. Conceptual scheme proposed for an X-band MicroLinac system for radiography source replacement.

In the next section we present and analyze design of two-section, X-band, DaR structure accompanied by beam dynamics results including alternating periodic focusing (APF) option with magnets integrated with the disks. In Section 3 we analyze RF design and performance of a long, single-section, X-band SW structure including various dielectric materials, beam dynamics simulations, and vacuum quality simulated along the structure. The electromagnetic and beam dynamics simulations presented below have been performed with the CST Studio Suite™ [11] and ASTRA [12] codes respectively. In all designs presented here we use ~9.4 GHz as the

approximated operation frequency, $\varnothing 0.75''$ standard copper tubing for the tubular structure housing, and about 3 mm beam aperture. Note the resonant operation frequency for multi-cell assemblies may noticeably deviate from the target frequency (initially obtained as the optimization goal for each cell at relative permittivity $\epsilon=9.9$) dependently on concrete properties of the material used, temperature, and RF coupling.

2. Two-section disk-and-ring linac structure

To achieve and exceed 1 MeV beam energy at limited power of RF source we have initially designed two multi-cell accelerating structures shown in Fig. 2. In designing the 1st section we have combined both compensated and non-compensated DaR cells, whereas the 2nd section is composed entirely from compensated DaR cells.

a)

b)

Fig. 2: DaR MicroLinac sections: 13 keV-0.4 MeV (a) and (0.4-1.2)MeV with normalized phase velocities (b).

Combination of different types of inter-cell coupling can provide better balance between shunt impedance and bandwidth to enable satisfactory (for capture and bunching) field profile along the long section and wide enough frequency separation between adjacent modes. That balance is especially critical for the first section. In particular design presented in Fig. 2a, the 1st section starts from four purely π -mode (i.e. non-compensated) cells operating at $\beta_{ph}=0.34$ normalized phase velocity followed by two compensated cells ($\beta_{ph}=0.41$), two compensated cells at $\beta_{ph}=0.4$, and three pairs of purely π -mode cells at $\beta_{ph}=0.48$, $\beta_{ph}=0.51$, and $\beta_{ph}=0.55$ respectively. The rest of the 1st section is composed by compensated cells with phase velocities ranging from $\beta_{ph}=0.62$ to $\beta_{ph}=0.83$. Phase velocities along the 2nd section are shown in Fig. 2b.

The S11 parameter simulated in Fig. 3 for high grade Alumina ($\epsilon=9.9$) indicates a significant mode separation between the operating (geometrically-pure π -mode) and adjacent modes. This separation is more than an order of magnitude higher compared to a conventional π -mode X-band system. Such a wide bandwidth may ensure reliable operation of the AFC system even with elevated temperature sensitivity of the resonant frequency in presence of dielectrics.

Fig. 3: S11 parameter simulated for the operating modes (central peaks) of the 1st (a) and 2nd (b) DaR sections of Fig. 5.

The electric field E_z distributions along the sections of Fig. 2 is given in Fig. 4 for the operating mode.

Fig. 4: Longitudinal electric field profiles [V/m] simulated along DaR MicroLinac [mm] for operating modes at ~9.4 GHz for the 1st (a) and 2nd (b) sections of Fig. 2.

One interesting feature of the multi-cell tapered DaR structure is capability to operate at significant deviations of the material properties from nominal. In Fig. 5 and Fig. 6 we demonstrate S-parameters (coupling is not optimized) and magnitude of the accelerating field along the 1st section for low grade Alumina and anisotropic (monocrystalline) Sapphire respectively.

Fig. 5. The 1st section RF performance with rings made of material with relative permittivity $\epsilon=9.6$ (low grade Alumina): S11 curve (a) and field magnitude along the section (b) for $f=9.465$ GHz.

Fig. 6. The 1st section RF performance with rings made of Sapphire with diametrical C-axis orientation $\epsilon_{\perp}=11.6$, $\epsilon_{\parallel}=9.39$ ($\tan\sigma_{\perp}=8.6\cdot 10^{-5}$, $\tan\sigma_{\parallel}=3\cdot 10^{-5}$): S11 curve (a) and field magnitude along the section for $f=9.402$ GHz.

One can see from Fig. 5 and Fig. 6 that the operating mode is relatively stable though the frequency changes for different materials.

2.1 Beam dynamics and focusing means

Two focusing variants have been considered to confine the accelerated beam within the small 3 mm aperture. One variant utilizes the same solenoidal type of focusing implemented with permanent magnet blocks in prior MicroLinac designs [9] (see also Fig. 20). ASTRA simulation results for transverse and longitudinal beam dynamics are shown in Fig. 7 and Fig. 8 respectively for the total RF power ~ 200 kW and 10^{-4} loss tangent for high grade Alumina.

Fig. 7. Beam rms transverse dimensions [mm] simulated along the 1st and 2nd sections of Fig. 2 using solenoidal focusing with 0.15 T on-axis magnetic field magnitude.

Fig. 8: Beam kinetic energy [MeV] simulated along the 1st (a) and 2nd (b) sections [m].

Another focusing variant utilizes alternating periodic focusing built-in to the RF accelerating structure. The disks can be made from a magnetic material such as high purity iron and magnetized. Then the disks can serve the same way as external focusing system. By interchanging the direction of magnetization from disk to disk (or group of disks) as shown in Fig. 9 and Fig. 10, it is possible to achieve APF operation with stable beam confinement by proper choice of magnetization strength and focusing lattice (not optimized in our example). To ensure sufficient Q-factor of the RF structure integrated with APF the disks can be coated with copper using, e.g., electro-plating. The Cu-plated iron disks in the model are assumed being magnetized to ~ 0.7 T (which is still well below the ~ 1.1 T saturation field). Note magnetization of iron is much easier than that for rare earth magnets whereas the same equipment and procedures can be used for the disks magnetization.

Fig. 9. Magnetostatic field lines for the central part of the 1st section structure integrated with APF. No external magnetic shielding is applied.

Longitudinal magnetostatic field profile along the DaR-APF structure is shown in Fig. 10. The focusing is sufficient at 0.7 T magnetization of the disks for beam confinement and capture (>12%) into the acceleration process (see Fig. 11 and Fig. 12).

Fig. 10. Profile of longitudinal magnetostatic field along the DaR-APF structure at 0.1 T and 0.7 T magnetizations of the disks.

Beam injection was simulated assuming 18 kV voltage beam waist having 1.5 mm diameter at the entrance. Bunch forming process in the RF-magnetostatic integrated structure is demonstrated in Fig. 11.

Fig. 11. Beam capture, confinement, bunching, propagation, and acceleration along the DaR-APF integrated structure magnetized with magnitude 1T. RF longitudinal electric field amplitude averaged along the section length is ~ 12 MV/m (18 MV/m peak on-axis field).

Beam capture as a function of magnetization strength is shown in Fig. 12. One can see sufficient capture and acceleration is feasible in the integrated DaR-APF system. To make the DaR-APF system competitive with more conventional focusing above, the disks should be magnetized above ~ 0.65 T.

Fig. 12. Relative number of captured particles as a function of disk magnetization at ~ 12 MV/m RF longitudinal electric field amplitude averaged along the section.

Thus the APF effect can be achieved in the dielectric micro-linac structure by magnetizing the metal disks in the interlaced way: odd disks have positive magnetization, even disks have negative magnetization. Such a focusing system may reduce weight of a compact MicroLinac or at least only the radiating “head” containing the RF system with the magnetron, linac structure, and the target (converter). The simulations above demonstrate feasibility and effectiveness of this type of “zero-weight”, integrated focusing. The integrated DaR-APF structure can be shielded by wrapping with μ -metal tape to mitigate magnetic distortions and demagnetization.

2.2 Thermal performance

Power dissipation density across the structure is illustrated in Fig. 13. In spite of the lowest field amplitudes occurring in the low-beta cells, the power deposition density is the highest in those initial cells. Therefore thermal simulations for these cells are performed separately (see Fig. 14). Simulations indicate that though water cooling is not necessary, enforced air cooling to be applied. The rest of the structure as well as the second section can use natural convection cooling.

Fig. 13. Distribution of power deposition density along the 1st section.

Fig. 14. Thermal simulation for the low-beta cells at 0.1% duty factor and 50 W/(m²K) heat transfer coefficient for the tubular housing that corresponds to enforced air flow cooling.

2.3 RF coupler and vacuum port

Two variants of RF coupler configuration are used in simulations: with stub (see Fig. 2) and without stub (see Fig. 11 and Fig. 13). Surface current distributions for both variants are given in Fig. 15. RF coupling for DaR structure requires presence of dielectric material within the hole in the tubular housing, whereas in the stub the RF fields are strongly evanescent (see Fig. 15). Therefore presence of the stub does not change the RF performance dramatically and it can be placed anywhere between the disks. The stubs can be used not only as vacuum ports, but also for RF tuning (including fine tuning of the coupling).

The RF coupler can in principle be combined with RF window. In that case vacuum sealing can be provided by local brazing applied to the dielectric ring (made from, e.g., Sapphire or Alumina) and a springing metal ring attached to correspondingly enlarged hole in the tubular housing. That approach allows a vacuum-sealed variant of the linac structure using internal non-evaporable getter (NEG) pumps. That variant requires a number of evanescent holes in the

tubular housing surrounded by an additional, external tube with the NEG placed between the internal and external tubes.

Fig. 15. Distribution of surface currents near RF coupler without stub (a) and with evanescent stub (b) that can be used as a vacuum port.

Providing and sustaining about the same resonant frequency (within $\sim 1/Q_{\text{loaded}}$ relative bandwidth) for both sections is not a simple task. That may require development of additional tuning means (e.g., differential thermo-stabilization and control). An alternative solution is using a much longer section.

3. Long structure of a single-section DaR linac

The modal spectra of Fig. 3, Fig. 5a, and Fig. 6a are not as dense as that for conventional π -mode multi-cell linacs. That suggests a capability to operate a longer section. We consider here a 44-cell long, single-section DaR structure shown in Fig. 16 to obtain beam energy exceeding 1 MeV using less than 220 kW pulse power in X-band. Configuration of Fig. 16 has been chosen from several variants with different positions of the port. The selection is based on the field distribution along the structure (and beam dynamics respectively).

Fig. 16. A cut-view of the 44-cell DaR structure with a single RF port.

221 RF performace for the configuration of Fig. 16 is presented in Fig. 17-Fig. 19 for Alumina and
 222 anisotropic Sapphire having two different orientations of C-axis.

223 **Fig. 17: S11 curve (a) and profile of the longitudinal electric field magnitude (b)**
 224 **simulated for operating mode of DaR MicroLinac (see Fig. 16) with rings made**
 225 **from a high grade Alumina ($\epsilon=9.9$, $\tan\sigma=10^{-4}$).**

Fig. 18. S11 curve (a) and profile of the longitudinal electric field magnitude (b) simulated for operating mode of DaR MicroLinac (see Fig. 16) with rings made from a longitudinally oriented Sapphire ($\epsilon_{\perp}=9.39$, $\epsilon_{\parallel}=11.6$; $\tan\sigma_{\perp}=3\cdot 10^{-5}$, $\tan\sigma_{\parallel}=8.6\cdot 10^{-5}$).

Fig. 19. S11 curve (a) and profile of the longitudinal electric field magnitude (b) simulated for operating mode of DaR MicroLinac (see Fig. 16) with rings made from a diametrically oriented Sapphire ($\epsilon_{\perp}=11.6$, $\epsilon_{\parallel}=9.39$; $\tan\sigma_{\perp}=8.6\cdot 10^{-5}$, $\tan\sigma_{\parallel}=3\cdot 10^{-5}$).

3.1 Beam dynamics

Beam dynamics has been simulated for the DaR structure of Fig. 16 with rings made from a crystalline Sapphire having diametrically oriented C-axis (i.e. transversely to the beam, see also Fig. 19). Solenoidal focusing field used in simulations is characterized in Fig. 20. Beam dimensions, emittance, and energy gain along the structure are shown in Fig. 21. The results are obtained for 150 kW RF power and indicate substantial ~19% capture.

Fig. 20. On-axis profile of longitudinal magnetic field along the accelerating section.

Fig. 21. Beam rms size [mm] and normalized emittance [μm] (a) and energy gain (b) along the long DaR structure (see Fig. 16 and Fig. 19).

As it can be seen from Fig. 21 the beam normalized rms emittance is relatively low at the MicroLinac exit ($\sim 3 \text{ mm} \times \text{mrad}$). This is a typical result for MicroLinacs due to filtering effect in a long and narrow accelerating channel which is rather lossy for the higher emittance fraction. Along inherent energy chirp it makes the microlinacs of potential interest as drivers for THz sources [1]. Note, a thermionic, 2-3 MeV, 1.5 cell RF gun produces $\sim 12\text{-}15 \text{ mm} \times \text{mrad}$ normalized rms emittance. An alpha-magnet with beam scraper followed by an accelerating section are usually applied [13] to achieve the same emittance with RF thermionic gun.

We have also simulated transverse shunt impedances. For example, for the DaR cell having 0.94 phase velocity the transverse shunt impedance is $0.19 \text{ M}\Omega/\text{m}$ for the most dangerous hybrid dipole mode occurring at 13 GHz. This is more an order less than that we calculated for corresponding disk loaded cell of MicroLinac [9] at 14 GHz and 18 GHz. That large difference is due to lower Q-factor of the DaR structure and significantly shifted dipole mode energy density peak away from the axis closer to the structure periphery. To evaluate, e.g., beam break-up (BBU) effect we applied analytical BBU model [14] assuming 0.5 MeV beam energy, magnetic field profile given in Fig. 20, and $\sim 0.3 \text{ m}$ effective interaction length. We conservatively found threshold current of $\sim 7 \text{ A}$ neglecting effects of tapering (geometry and momentum change).

More generally, the BBU effect for X-band MicroLinacs is not meaningful for the following reasons: a) structure tapering destroys the synchronism with parasitic modes; b) focusing applied along the structure; c) the beam currents in microlinacs are much lower than in regular linacs and is always below the threshold; d) during the bunching at low energies the beam current Fourier component at the dipole mode frequencies is too small to seed BBU; e) “tapering” related to the substantial momentum and velocity growth also increases BBU threshold; and f) beam emittance distortion caused by dipole modes is not important for MicroLinac applications.

3.2 Vacuum simulations

The structure of Fig. 16 may have limited vacuum conductivity. That implies using of more than one vacuum port. We have simulated vacuum performance of the structure with three vacuum ports. Two discrete vacuum ports can be seen in Fig. 22. The third vacuum port is not shown. It is built-in to the flange interfacing the electron gun and the structure.

For vacuum simulations we applied thermal model using the same approach applied earlier for C-band structure [15]. The model parameters of temperature, heat, heat conductance, and heat density on the boundaries correspond to pressure, gas flow rate, vacuum conductance, pumping speed, and outgassing rate respectively. The results are presented in Fig. 22 and Fig. 23. The simulations are performed for three vacuum ports with 60 l/s pumping speed and $5.5 \cdot 10^{-10}$ mBar·cm²·l/s outgassing rate for all surfaces (worst case scenario).

Fig. 22. DaR structure model (a) for vacuum simulations with two discrete side ports. The third vacuum port is built-in to the flange between the electron gun and the structure (b).

Fig. 23. Pressure profile along the long DaR structure with three vacuum ports shown in Fig. 16 and Fig. 22.

In case of using only two vacuum ports (i.e. without the port near the HV injector) the maximum pressure occurs near the gun and reaches 0.6 μTorr . That may present a problem for the thermionic HV gun operation.

4. Discussion

The multi-cell SW DaR structure considered here demonstrates a certain potential for beam energies well above 1 MeV by simply increasing number of compensated DaR cells enabling a wide bandwidth.

Anisotropic Sapphire with diametrical orientation is a very attractive material for the rings of the DaR structure. That includes also reduced loss tangent in longitudinal direction, which remains dominant for the RF electric field of the operating mode.

Disk charging remains an unexplored problem, though it can probably be mitigated by sub-skin-depth (local) metallization of the dielectric rings. Multipactor in DaR structure also needs further study. We do not exclude that it can even be utilized to discharge the disks during a small fraction of the pulse duration (e.g., on the trailing front edge) or by using some of the

pulses for that discharge (at the expense of insignificant drop of duty factor).

Presence of dielectrics requires a special attention to be paid to the temperature control and AFC. More careful design of the tapered part of the DaR structure is required to minimize influence of the temperature change (regulated by AFC) on the bunching process driven by field profile that cannot be controlled directly.

Design of vacuum system features a high aspect ratio of the long and narrow DaR structure that may ultimately imply a distributed array of vacuum ports and additional, vacuum housing, which, in turn, to be sufficiently conductive to the heat.

Although the structure can in principle be assembled as a stack of non-brazed cells in a tubular housing, the technology needs yet to be developed and adapted to such enormously long assemblies. Nevertheless the DaR structure can be considered as a candidate for MicroLinacs capable of, e.g., replacing radioactive sources. It also offers a certain potential for racetrack microtrons, traveling wave tubes, drivers for mm-sub-mm wave pulsed sources, and RF deflectors employed for phase space manipulation, beam diagnostics, and material spectroscopy with ultrafast electron diffraction.

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