1 Position-sensitive detection of ultracold neutrons with
2 an imaging camera and its implications to spectroscopy

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18 **Abstract**

19 Position-sensitive detection of ultracold neutrons (UCNs) is demonstrated
20 using an imaging charge-coupled device (CCD) camera. A spatial resolution
21 less than 15 μm has been achieved, which is equivalent to an UCN energy res-
22 olution below 2 pico-electron-volts through the relation $\delta E = m_0 g \delta x$. Here,
23 the symbols δE , δx , m_0 and g are the energy resolution, the spatial resolu-
24 tion, the neutron rest mass and the gravitational acceleration, respectively.

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A multilayer surface convertor described previously is used to capture UCNs and then emits visible light for CCD imaging. Particle identification and noise rejection are discussed through the use of light intensity profile analysis. This method allows different types of UCN spectroscopy and other applications.

25 *Keywords:* Multilayer ^{10}B surface detector, imaging detector, UCN
26 spectroscopy

1. Introduction

A multilayer surface detector for ultra-cold neutrons (UCNs), or neutrons with kinetic energies below 340 nano-electron-volts (neV), has recently been reported [1]. The multilayer detector front end consists of a boron-10 (^{10}B) layer on the top of a ZnS:Ag phosphor layer. The ^{10}B layer, up to several hundred nanometers thick, is exposed to vacuum and directly captures UCNs. A phosphor layer several microns thick is sufficient to stop the charged ions from the $^{10}\text{B}(\text{n},\alpha)^7\text{Li}$ neutron capture reaction, while thin enough so that light due to α and ^7Li escapes for detection by photomultiplier tubes (PMTs).

We extend the previous work by using an imaging charge-coupled device (CCD) camera instead of a photo-multiplier tube (PMT) to detect UCN-induced visible light which peaks around 450 nm. We show that the position resolution of UCN detection is comparable to the size of individual pixels or 15 μm . The multilayer construction of the detector front end remains the same as before. The use of optics allows the separation of the neutron converter in vacuum from the camera in air. Applications of this method include UCN spectroscopy and UCN-based materials research. For spectroscopy, a resolution of 15- μm corresponds to an energy resolution of less than 2 pico-electron-volts (peV) for some designs (see Sec. 4.2 below). Because of the availability of imaging cameras with smaller pixels for scientific and consumer use, further improvements are possible that would yield a spatial resolution of few microns, or sub-peV energy resolutions. It should be mentioned that at a resolution of a few microns, which are comparable to the stopping distances of MeV α 's and ^7Li , the details of ion stopping in ZnS phosphor may no longer be ignored. Examples of fine spatial resolution for

ultra-cold, cold and thermal neutrons can be found in different context [2–8]. Another interesting development is the adoption of a webcam for UCN detection by removing the cover glass on the sensor array [9]. Besides CCD and complementary metal-oxide-semiconductor (CMOS) imaging sensors, hybrid pixelated silicon devices with an application specific integrated circuit such as TimePix were also reported for position-sensitive UCN detection and spectroscopy [10].

Below, we first present the experimental setup, followed by experimental data and analysis, and finally a discussion of UCN spectroscopy and materials applications.

2. Experimental setup

The experiment was carried out in the UCN beamline in the Los Alamos Neutron Science Center (LANSCE) [11]. As shown in Figure 1, the neutron-to-photon converter film was placed against the quartz window at the end of the test port. An ~ 80 nm ^{10}B film faced the incoming flux of UCNs, which were controlled by a gate valve (GV). The products of UCNs captured on the ^{10}B film were primarily ^7Li at 0.84 MeV and α at 1.47 MeV. They induced blue scintillation light in the silver-doped ZnS film. The light peaked at 450 nm with a characteristic decay time of about 200 ns. The ion stopping ranges of ^7Li and α are calculated to be within a few microns in both the ^{10}B and ZnS:Ag films. If fully stopped in the scintillator, each α particle at 1.47 MeV is estimated to yield 7.4×10^4 photons [1]. The scintillation light is first transmitted out of the transparent acrylic plastic substrate through a small gap, *i.e.*, small to human eye but large relative to the wavelengths

76 of scintillation light, passed through the quartz window, and then partially
 77 collected by a 50-mm camera lens before reaching an electron-multiplying
 78 CCD (EMCCD) imaging sensor.

Figure 1: Schematic of the experimental setup for position-sensitive measurement of UCNs. The UCN flux is regulated by a gate valve (GV) before reaching the multilayer UCN detector front end, which sits against the quartz vacuum window. The multilayer UCN detector front end consists of a thin layer of boron-10 on top of a layer of ZnS:Ag scintillator on top of an acrylic plastic sheet, as shown in magnified view. A Nikon Nikkor 50 mm $f/1.2$ lens was used to focus the scintillation light onto the CCD camera.

79 The camera (ProEM-HS made by Princeton Instruments) has a back-
 80 illuminated EMCCD sensor. The full sensor array has 512×512 mono-chrome
 81 pixels with 16-bit well depth. Each pixel is $16 \mu\text{m} \times 16 \mu\text{m}$ in physical size.
 82 A Nikon Nikkor 50 mm lens has a maximum f -number (f) of $f/1.2$ and a
 83 46-mm wide clear aperture. Coupling the lens with a stack of extension tubes
 84 enables close photography of the film (within a few cm). A length calibration
 85 showed that each pixel corresponds to $15 \mu\text{m}$ at the film location, giving a
 86 full field-of-view of $7.7 \times 7.7 \text{ mm}^2$. The camera has a built-in thermoelectric

87 cooling system, which reduces the sensor noise level greatly when the sensor
 88 temperature is below -70°C . During the measurement, the optical window
 89 and camera lens are enclosed in an ambient light shield (two layers of black
 90 Tedlar sheets), while the camera body remains in open air for ventilation.

91 The number of photons that can be ultimately recorded by the camera
 92 is estimated as follows. The indices of refraction of ZnS and acrylic plastic
 93 are about 2.5 and 1.5 respectively. Since the indices of refraction decrease
 94 monotonically from ZnS to acrylic plastic to the vacuum gap, total internal
 95 reflections will occur at each interface. The critical angle that allows the
 96 light to transmit out of the converter film is about 23.6 degrees, or $\theta_c =$
 97 $\text{asin}(1/2.5)$, at the interface of ZnS and acrylic. The associated solid angle
 98 fraction is 4.1%, which is also the fraction of the total scintillation light that
 99 can come out of the converter film for isotropic emission. The light then
 100 passes through the quartz vacuum window, where it experiences no solid
 101 angle loss but a transmission loss of about 10% at 450 nm due to absorption
 102 and reflection. Hence, 3.7% of the scintillation light can exit the window
 103 and spread out in a 2π solid angle. The camera lens is positioned about
 104 31 mm away from the window. The solid angles to the lens from points
 105 in the $7.7 \times 7.7 \text{ mm}^2$ image area vary slightly around 0.4π , corresponding to
 106 the optical setup having about 20% of the light collection efficiency. In the
 107 experiment, the lens aperture was set at $f/2.0$ to balance the depth-of-field
 108 (for ease of focusing) and the field-of-view. The $f/2.0$ aperture only allows
 109 40% of the light collection that would otherwise be obtained at $f/1.2$. The
 110 QE of the CCD sensor is about 85% at 450 nm. Taking all the loss factors
 111 into account, an upper limit of about 190 photons generated by an α particle

112 of 1.47 MeV, or about 0.2% of the 7.4×10^4 photons, can be detected by the
113 image sensor.

114 3. Results and Data Analysis

115 To determine our spatial resolution, we analyze the images of individual
116 scintillation events, modelled as a 2D Gaussian response function.

117 An example raw image and its processed versions are shown in Figure 2.
118 The image was taken with an exposure time of 1200 ms. The raw image
119 shows a poor signal-to-noise ratio due to background noise. An initial noise
120 reduction is achieved by averaging over the 7×7 neighboring pixels. As shown
121 in Figure 2b, the sharpness of the image is sacrificed to improve the signal-
122 to-noise ratio. The image is evenly divided into 8×8 sub-regions where the
123 mean background noise level is relatively flat. Only pixels above a fixed
124 threshold are kept, and the mean background of the subregion is subtracted
125 (Figure 2c). The threshold is set to a few times the standard deviation above
126 the mean pixel value of the image. Thus we generally separate signal pixels
127 from those of the background. In order to avoid double counting of signals on
128 the boundary of the sub-regions, the pixel extraction is repeated on the final
129 image. Limitations of this approach include (a.) false identification of some
130 uneven backgrounds as signals, and (b.) failure in separating overlapping
131 signals if two neutron captures are too close to each other (pulse pile-up).

132 Scintillator light is emitted along the paths of MeV α and ${}^7\text{Li}$ ions in
133 the scintillator, which have ion-stopping ranges of a few microns. The 15-
134 μm pixel resolution in this work is at least a few times the ion ranges, and
135 therefore can not resolve the details of ion stopping. Furthermore, only

Figure 2: An example of a raw image and processing. (a.) The raw image. (b.) The processed image after initial noise reduction by averaging over the neighboring 7×7 pixels. (c.) The processed image after signal extraction and background removal.

136 certain neutron events are at sharp focus due to the shallow depth of focus.
 137 To account for the different image blurring mechanisms, we use a 2D elliptical
 138 Gaussian function to fit each spot. The signal S is centered around (x_0, y_0) ,
 139 the position of neutron capture,

$$S = A \exp \left[-a(x - x_0)^2 + 2b(x - x_0)(y - y_0) - c(y - y_0)^2 \right] + B, \quad (1)$$

140 where

$$\begin{aligned} a &= \frac{\cos^2 \theta}{2\sigma_1^2} + \frac{\sin^2 \theta}{2\sigma_2^2}, \\ b &= -\frac{\sin 2\theta}{4\sigma_1^2} + \frac{\sin 2\theta}{4\sigma_2^2}, \\ c &= \frac{\sin^2 \theta}{2\sigma_1^2} + \frac{\cos^2 \theta}{2\sigma_2^2}. \end{aligned} \quad (2)$$

141 Here A is the height of the Gaussian blob, B is the value of the base plane,

σ_1 and σ_2 are the widths along the major and minor semi-axes, and θ is
the angle between the major semi-axis and the x-axis in the Cartesian coordinate system. A typical fit is compared with the original signal as shown
in Figure 3, where $(x_0, y_0) = (308.13, 39.14)$ pixels, $A = 37.6$, $\sigma_1 = 3.15$
pixels, and $\sigma_2 = 2.51$ pixels. The signal intensity is taken from the integral
over the processed signal, which is assumed to be proportional to the number
of photons detected. For instance, the integrated intensity from the data in
Figure 3 is 1811.6.

Figure 3: An example of the two-dimensional (2D) Gaussian fitting of the scintillation signal from an UCN capture. The pre-processed original signal, as described in Figure 2 is in the left column (top and bottom for the 3D and 2D views) and the 2D Gaussian fit is on the right.

We have taken data at different exposure times ranging from 30 ms up
to a few seconds. Since our algorithm for signal identification works best

152 for a low signal density, data at exposure times longer than 1200 ms are
 153 not included in the following analysis. There are a total of 4.5×10^4 events
 154 identified by the analysis. The quality of the 2D Gaussian fit is used to
 155 exclude events due to uneven background, dark currents and piled-up signals.
 156 The latter two are rare events for exposure times under 1200 ms. About 76%
 157 of these preliminary identified events are well fitted by the 2D Gaussian.
 158 Pulse height spectra of peak intensities for both Gaussian and non-Gaussian
 159 events are shown in Figure 4. The quality of the fit removes a large fraction
 160 of background events (low-intensity) and a small fraction of real UCN events
 161 (from pile-up rejection). Signals with poor fit are excluded from the analysis
 162 that follows.

Figure 4: Pulse height spectra for Gaussian and non-Gaussian signals. Only Gaussian signals are counted as real neutron-induced signals.

163 To investigate the nature of background noise, we compare data taken for

164 a fixed 800-ms exposure time with and without a UCN flux. We note that,
 165 although the UCN flux was turned off to the detector, the spallation source
 166 was still running. A total of 112 out of the 300 frames were taken without a
 167 UCN flux.

168 In Figure 5, we plot the pulse height spectra of integrated signal inten-
 169 sity in the GV-open and GV-closed cases, respectively. The spectrum looks
 170 qualitatively similar to those obtained previously by PMT-based detectors,
 171 as expected for a camera sensor array with a uniform response to light. The
 172 GV-closed dataset is dominated by background, and is used to set a lower
 173 intensity threshold.

Figure 5: Pulse height spectra of the integrated peak intensities for the GV-open periods (light-shaded) and for the GV-closed periods (dark-shaded). The same data set as shown in Figure 6 below is used. It is clear that the noise dominates the pulses below 180.

174 The UCN density per frame is plotted in Figure 6. The two GV-closed

175 periods are clearly identifiable with essentially no UCN counts.

Figure 6: UCN flux density as a function of frame (equivalent to time) using the movie mode of the camera. The exposure time is fixed at 800 ms for each image frame. The upper plot includes all signals well-fit by a 2D Gaussian. The lower plot imposes an lower intensity threshold.

176 In Figure 7, two examples of images taken during GV-closed periods are
 177 presented: Figure 7a shows a signal with a 2D Gaussian profile, while Fig-
 178 ure 7b shows a non-Gaussian profile. Both signals are non-UCN induced
 179 events on the image sensor since the GV was closed. In the GV-closed pe-
 180 riods, the mean background rate is about 2.09 ± 0.17 events $\text{cm}^{-2} \text{s}^{-1}$. In
 181 the GV-open period, 7% of the total signals are below the threshold, with
 182 a mean rate of 4.07 ± 0.24 events $\text{cm}^{-2} \text{s}^{-1}$. Up to 3.4% of the total UCN
 183 signals may be lost using a threshold of 180 to exclude low intensity data.
 184 The mean UCN rate above the threshold is 53.89 ± 0.89 UCNs $\text{cm}^{-2} \text{s}^{-1}$

185 in the GV-open periods, and 0.11 ± 0.05 UCNs $\text{cm}^{-2} \text{s}^{-1}$ in the GV-closed
 186 periods. A PMT-based detector nearby that used a similar converter film
 187 read about 60 UCNs $\text{cm}^{-2} \text{s}^{-1}$.

Figure 7: Two examples of background noise (small amplitude less than 180) that are taken during the GV closed periods. (a.) The first row, a background noise with a Gaussian profile. (b.) The second row, a background noise with a non-Gaussian profile.

188 4. Discussion

189 4.1. Position resolution

190 For a 2D Gaussian light intensity distribution, the error in position deter-
 191 mination or the position resolution along one semi-axis is given by $\sigma_i/\sqrt{N_\nu}$,
 192 where σ_i is the width of the Gaussian along the semi-axis and N_ν is the

193 number of photons in the signal. The fitting values of σ_1 and σ_2 for all
 194 the down-selected data are shown in Figure 8. The values of σ_1 are in the
 195 range of 1.5 to 3.8 pixels and peak around 2.77 pixels. The correlation of
 196 the peak intensities with the corresponding σ_1 value is shown in Figure 9,
 197 which indicates that σ_1 of 2.77 pixels is associated with signals of intensities
 198 around 1000. Each photon is estimated to result in an amplitude of ~ 9 in
 199 raw camera counts, so that an amplitude of 1000 corresponds to ~ 111 pho-
 200 tons. Therefore, a typical error in position determination for a strong signal
 201 can be 0.3 pixels, or about $4 \mu\text{m}$, along the major semi-axis. For weaker
 202 signals and larger Gaussian widths (*e.g.*, 20 photons and σ_1 of 4 pixels), the
 203 error of position could be as large as 0.9 pixels, which is equivalent to $13 \mu\text{m}$.
 204 Depending on the orientation angle θ , the error projecting on the Cartesian
 205 coordinate should be a propagation of errors from both semi-axes, but the
 206 maximal error does not exceed that on the major semi-axis.

207 4.2. UCN spectroscopy

208 Accurate real-time determination of UCN positions allows precise deter-
 209 mination of UCN energies and other kinetic information. Several methods
 210 have already been reported based on CR-39 plastic or a CCD camera [7, 12,
 211 13]. Using imaging cameras has some advantages, including real-time data
 212 collection and analysis. There is no difficulty in replacing CR-39 plastic with
 213 imaging cameras to perform experiments similar to these previous works. The
 214 spatial resolution can be converted to energy resolution through the factor
 215 $102 \text{ neV}/\mu\text{m}$. Therefore, a $15\text{-}\mu\text{m}$ spatial resolution is equivalent to an energy
 216 resolution of 1.5 peV, which is small enough to probe quantum bounce states
 217 of UCNs near surfaces and other fundamental physics [14, 15]. One concern

Figure 8: 2D histogram of the populations of UCN signals with respect to σ_1 and σ_2 , the Gaussian widths in the major and minor semi-axes. The gray scale represents the frequency of events. Most signals populate around σ_1 of 2.76 pixels and σ_2 of 2.44 pixels.

Figure 9: Signal intensity correlation with the Gaussian widths σ_1 along the major semi-axis for all the signals as shown in Figure 8. Each circle represents a scintillation signal. The most probable σ_1 of 2.77 pixels corresponds to more than 100 photons using estimates described in the text.

218 with this type of measurement is that a relatively small UCN phase space
 219 is sampled and most of the UCNs are not used. We shall discuss two types
 220 of UCN spectroscopy, one based on UCN position measurements alone, the
 221 other on both position and time-of-flight information. In both cases, larger
 222 UCN phase spaces can be sampled.

223 Measuring the energy spectrum of a UCN source is equivalent to deter-
 224 mining the distribution function $f(E, \alpha, \phi)$, where E is the UCN energy, α is
 225 the angle of launch as shown in Figure 10 and ϕ stands for the azimuthal an-
 226 gle in a cylindrical geometry. Here we assume a steady UCN source. Below,
 227 we also ignore the azimuthal variations of the distribution function. With-
 228 out loss of generality, we only need to consider the 2D motion in the plane
 229 defined by the cross product of $\mathbf{v} \times \mathbf{g}$, where $\mathbf{v}$ is the initial velocity vector
 230 and $\mathbf{g}$ the vector of gravitational acceleration. The equations of motion are
 231 elementary and given by

$$R = R_0 + vt \cos \alpha, \quad (3)$$

$$h = h_0 + vt \sin \alpha - \frac{1}{2}gt^2, \quad (4)$$

232 where $R_0, h_0 = 0$ are the initial UCN coordinates or the position of the UCN
 233 entrance slit and t is the time of flight. Only the magnitude of velocity v is
 234 needed. We also ignore magnetic field and consider gravity as the only force
 235 during the UCN free flight.

236 If the time-of-flight (ToF) information t is known, the initial positions R_0
 237 and h_0 , which are defined by design, and the final positions R, h , which are
 238 measured through a camera as shown in Fig. 10, are sufficient to determine
 239 v and the launch angle α uniquely through Eqs. (3) and (4). The measured

Figure 10: The cylindrical cross section of a ToF+gravity hybrid UCN spectrometer. UCNs enter the detection volume via a pinhole at the ceiling of the storage volume. The detector is placed on the ceiling of the cylindrical detection volume. α is the angle of launch of an UCN from an entrance aperture/pinhole. Zero in height ($h = 0$) is defined at the level of the pinhole.

240 UCN kinetic energy is given by

$$E = \frac{1}{2}m_0v^2 = \frac{m_0(R - R_0)^2 + m_0(h - h_0 + 0.5gt^2)^2}{2t^2}, \quad (5)$$

241 where m_0 is for the UCN rest mass. The energy resolution, $\delta E = m_0v\delta v$,
 242 can be derived as

$$\frac{\delta E}{E} = e_1 \frac{\delta \tilde{R}}{\tilde{R}} + e_2 \frac{\delta \tilde{h}}{\tilde{h}} + e_3 \frac{\delta t}{t}, \quad (6)$$

243 where we have used $\tilde{R} = R - R_0$ and $\tilde{h} = h - h_0$. Here δt , $\delta \tilde{R} = \sqrt{\delta R_0^2 + \delta R^2}$
 244 and $\delta \tilde{h} = \sqrt{\delta h^2 + \delta h_0^2}$ are the measurement errors in time, the two radii and
 245 the two heights. The coefficients e_i 's are given by

$$\begin{aligned} e_1 &= 2 \cos^2 \alpha, \\ e_2 &= \frac{\tilde{h}}{\tilde{R}} \sin 2\alpha, \\ e_3 &= 2(1 - e_1 - e_2). \end{aligned} \quad (7)$$

246 Here we express e_i 's in terms of α for simplicity. It is straight-forward
 247 to reformulate α and Eqs. (6) in terms of the measured quantities R_0 , R , h ,
 248 h_0 and t . The special case of $\tilde{R} = 0$ and $\alpha = \pi/2$ needs to be considered
 249 separately and will not be elaborated here. Based on these formulas, a spec-
 250 trometer that combines the time-of-flight with the initial and end positions
 251 can be set up. The performance of an example spectrometer of this type is
 252 shown in Figure 11. The cylindrical spectrometer is 10 cm in height and 2
 253 cm in radius with a pinhole (UCN entrance aperture) of 1.6 mm in radius.
 254 The UCN scintillation signals can be optically detected on the ceiling plate.
 255 As the height h is predetermined, we assume that $\delta \tilde{h}$ comes from a typical

256 machining accuracy of $25\ \mu\text{m}$. Here $\delta\tilde{R}$ is dominated by the pinhole radius
 257 up to 1.6 mm. The spatial resolution at the camera location (less than 15
 258 μm) is negligible in comparison. Furthermore, if we assume the use of a fast
 259 mechanical shutter with an opening time around 1 ms at the entrance and
 260 the temporal resolution of the camera of 1 ms, the total time error δt is 0.7
 261 ms. The error in time δt therefore dominates in the energy resolution in such
 262 a hybrid spectrometer.

Figure 11: Estimated UCN energy (solid lines) and associated error (dashed lines) as functions of ToF at different radius positions for a cylindrical spectrometer. The overall error is dominated by the ToF uncertainty, even for a relatively large entrance aperture of 1.6 mm in radius.

263 Figure 11 includes UCNs passing the height h in both ascending and
 264 descending motions as if the ceiling does not exist in the detector volume.
 265 When considering the case of imaging UCN signals on the ceiling, we only

266 need to consider the ascending section. Hence, each cycle of measurement
 267 takes about 0.15 sec. It is apparent that the UCNs of high energy arrive on
 268 the detector plane faster than the low energy ones. Within the 2 cm radius of
 269 the ceiling plate, the measured UCN energy precision is significantly affected
 270 by ToF but not the R positions. The error in energy decreases from 18 neV
 271 for 200 neV UCNs to 0.32 neV for 20 neV UCNs; below 20 neV, it keeps
 272 decreasing but becomes more radius dependent; e.g., for 10.3 neV UCNs, the
 273 error in energy can achieve 16 peV at $R = 2$ cm and 2.6 peV at $R = 0$ cm.

274 As an alternative to overcome energy resolution limited by ToF, one could
 275 choose to experimentally select the angle of launch in order to resolve the
 276 energy spectrum. Without going into details, we derive the energy and its
 277 resolution based on known angle of launch. The kinetic energy of the neutron
 278 is given by

$$E = \frac{m_0 g \tilde{R}^2}{4(\tilde{R} \sin \alpha \cos \alpha - \tilde{h} \cos^2 \alpha)}. \quad (8)$$

279 The corresponding energy resolution satisfies

$$\delta E = f_1 \delta \tilde{R} + f_2 \delta \tilde{h} + f_3 \delta \alpha, \quad (9)$$

280 where

$$\begin{aligned}
 f_1 &= \frac{2E}{\tilde{R}} - \frac{2E^2 \sin 2\alpha}{m_0 g \tilde{R}^2}, \\
 f_2 &= \frac{4E^2 \cos^2 \alpha}{m_0 g \tilde{R}^2}, \\
 f_3 &= \frac{2E \sin \alpha}{\cos \alpha} - \frac{4E^2}{m_0 g \tilde{R}}.
 \end{aligned} \quad (10)$$

281 We consider the special case when $\tilde{h} = h - h_0 = 0$, *i.e.*, the detectors are
 282 at the same level as the neutron entrance aperture. Similar discussions can be
 283 found, for example, in the book by Golub, Richardson and Lamoreaux [16].
 284 The neutron energy $E = m_0 g \tilde{R} / 2 \sin 2\alpha$. The resolution is

$$\frac{\delta E}{E} = \frac{\delta \tilde{R}}{\tilde{R}} - \frac{2 \cos 2\alpha}{\sin 2\alpha} \delta \alpha. \quad (11)$$

285 The best energy resolution is achieved at $\alpha = \pi/4$. The energy resolution,
 286 independent of the initial neutron kinetic energy, is about 1.5 peV for $\delta \tilde{R} = 30$
 287 μm . From $\delta \tilde{R} = \sqrt{\delta R_0^2 + \delta R^2}$ and $\delta R = 15 \mu\text{m}$ at the detector location, one
 288 may use a neutron entrance as wide as $\delta R_0 = 26 \mu\text{m}$. For some applications
 289 when the energy resolution does not need to be this high, a wider neutron
 290 entrance aperture may be used, *i.e.*,

$$\delta R_0 \sim \delta \tilde{R} = \frac{2\delta E}{m_0 g}. \quad (12)$$

291 For $\delta E = 1 \text{ neV}$, entrance aperture can be close to 2 cm wide, which is quite
 292 attractive for the UCN energy spectra measurement since a wider aperture
 293 means less time required to measure the spectra.

294 4.3. Material applications

295 Position-sensitive measurements discussed here could also enable new
 296 studies of UCN interactions with materials. One limitation would be the very
 297 low UCN fluxes available in current sources, which preclude the use of colli-
 298 mation and velocity choppers to develop a monochromatic and narrow UCN
 299 “beam”. However, rough characterizations are possible with coarse control
 300 of the distribution of neutrons and a position-sensitive detector. UCNs could

301 be used to characterize areal variations of thin films. In Figure 12a, UCNs
302 are collimated by dropping down an absorbing guide, and a ToF chopper or
303 a thin film with low potential could be used to select desired energies.

304 The transmission as a function of position can then be measured using the
305 detector described here. An example of the sensitivity of such a setup was
306 explored using a Monte-Carlo simulation. Assuming a transmitted flux of 1
307 UCN/(cm² · s) through the sample and maximum incident angle of 30°, and
308 neglecting reflection (UCN energy $E >$ material Fermi potential), a 1 mm
309 diameter spot with 20% lower transmission (such as due to surface contami-
310 nation or variations in thickness) on a 1 mm thick sample could be resolved
311 after about 1 day of measurement, Figure 12b. With control of the range
312 of UCN energies from the loss due to reflection from the material potential,
313 variations in density or isotopic concentrations could also be studied.

314 5. Conclusion

315 We have successfully demonstrated position-sensitive measurements of
316 UCNs by optically coupling a multilayer surface detector to an imaging CCD
317 camera with 16- μ m pixels. The UCN position accuracy is as small as 4 μ m
318 for a typical strong scintillation signal and about 13 μ m for weak ones. Sev-
319 eral feasible UCN spectroscopy concepts are discussed with the best energy
320 resolution below 2 peV. Therefore, this method can be employed as a real-
321 time detector in studying UCN quantum states in the gravitational field.
322 Other possible applications include UCN microscopy and reflectometry for
323 material science.

Figure 12: Application of the position-sensitive camera to perform studies of areal variations in transmission. (a) Example experiment geometry for transmission measurements. UCNs drop from the guide, are collimated by absorbing walls, and pass through a thin film before being counted by the detector. (b) Monte-Carlo study of transmission through a thin film with a 1 mm diameter, 20% absorbing spot with 10^5 accumulated UCN counts.

324 **Acknowledgments** We gratefully acknowledge the support of the U.S.
325 Department of Energy through the LANL/LDRD Program. L.J.B. is partly
326 supported by the G. T. Seaborg Institute and Los Alamos Science Campaign
327 C1 for this work.

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