1 Recoil distance transmission method: Measurement of
2 interaction cross sections of excited states with fast
3 rare-isotope beams

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9 **Abstract**

10 The possible appearance of nuclear halos in ground and excited states close
11 to the particle-decay threshold is of great importance in the investigation of
12 nuclear structure and few-body correlations at the limit of stability. In order to
13 obtain direct evidence of the halo structure manifested in nuclear excited states,
14 we have considered a new method to measure the interaction cross sections of
15 excited states. The combination of the transmission method and recoil distance
16 Doppler-shift method with a plunger device enables us to measure the number
17 of interactions of the excited states in a target. Formulae to determine the
18 interaction cross section are derived, and key issues to realize measurements
19 are discussed. Dominant sources of errors are uncertainties in the excited-state
20 lifetimes and γ -ray yields. We examine prototype experiments and perform
21 simulations to study the impact of each uncertainty on the final result. This
22 method provides a novel opportunity to perform cross section measurements on
23 the excited states of rare isotopes.

24 *Keywords:* Nuclear excited states, Interaction cross section, Recoil distance
25 Doppler-shift method, Rare-isotope beams

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1. Introduction

Nuclear halos at the drip lines are of great interest in the investigation of structure of weakly bound nuclear systems [1–7]. If the energy level of a nuclear state is close to the particle decay threshold, the state tends to have asymptotic cluster structure [8, 9]. The simple cluster structure is a neutron (proton) halo structure comprising valence nucleon(s) and the core, which appears near the neutron (proton) decay threshold. Up to now, the search for halos has mainly focused on ground states of nuclei near the drip lines. Along with the development of heavy-ion accelerators, ground states of drip-line nuclei in the light mass region have been investigated, and several halo nuclei have been identified [1–7]. Amongst the heaviest known halo nuclei are ^{31}Ne [10–12] and ^{37}Mg [13–15], suggesting the appearance of halos across a wider mass region. However, even next-generation facilities currently under construction will not be able to produce all bound nuclei in the nuclear chart [16], indicating that the search for halos will be limited to the medium-mass region. On the other hand, the halo structure can be expected in excited states close to the threshold even for unstable nuclei closer to the stability line. Thus, reaction studies can be important for the identification of halo structure in nuclear excited states.

Ground-state halo structures have been investigated by three types of measurements: (a) interaction cross section [17, 18], (b) momentum distribution [19, 20], and (c) Coulomb breakup reaction [21–23]. The halo structure consists of a dilute neutron or proton cloud of extended radius, which results in a large interaction cross section. Therefore enhancement of the interaction cross section is regarded as direct evidence of halo structure. The measurement of the momentum distribution, following knockout reaction of a valence nucleon, confirms the orbital angular momentum of the halo nucleon. Coulomb breakup reactions have been utilized to investigate the extended wave function characteristic of low- ℓ orbitals.

In this paper, we propose a new experimental method to measure the interaction cross sections (σ_I^{ex}) of nuclei in the excited states. The measurement can

provide direct evidence of the halo structure in the excited states. The present approach combines the transmission method and recoil distance Doppler-shift method. Basic ideas for the method are presented in Sec. 2. Formalism to deduce the interaction cross section and evaluate its error is described in Sec. 3. Section 4 proposes prototype experiments and considers a realistic case along with simulated results. Key considerations in designing future experiments are discussed in Sec. 5, and Sec. 6 is a summary of this paper.

2. Basic ideas of the recoil distance transmission method

This section gives an overview of the new method named “recoil distance transmission method” to extract the interaction cross sections of nuclei in the excited states. This method utilizes the recoil-distance setup [24] to count the excited states before and after a reaction target and applies the transmission method to determine the interaction cross section σ_I^{ex} . In this section, the conventional transmission method to measure the ground-state interaction cross section (σ_I^{gs}) is briefly reviewed, and then the recoil distance transmission method is introduced.

2.1. Brief description of the transmission method

In the transmission method, the interaction cross section is obtained from the following relations (see, e.g., [18]):

$$\sigma_I^{\text{gs}} = -\frac{A}{N_A \rho x_t} \ln \frac{\Gamma}{\Gamma_0}, \quad (1)$$

$$\Gamma = \frac{N_{\text{gs}}(x_t)}{N_{\text{gs}}(0)}, \quad (2)$$

$$\Gamma_0 = \left. \frac{N_{\text{gs}}(x_t)}{N_{\text{gs}}(0)} \right|_{\text{empty target}}, \quad (3)$$

where x_t , A , and ρ represent the thickness, mass number, and density of the reaction target, respectively. N_A is Avogadro’s number. Γ denotes the ratio of the number of non-reacting outgoing nuclei ($N_{\text{gs}}(x_t)$) to that of incoming nuclei ($N_{\text{gs}}(0)$). Γ_0 represents the same ratio measured without a target, which corrects for background contributions such as nuclear reactions in the detectors.

80 In an experiment, $N_{\text{gs}}(x_t)$ and $N_{\text{gs}}(0)$ are observables to determine σ_I^{gs} , but
 81 statistical errors are only important for $N_{\text{gs}}(x_t)$ because $N_{\text{gs}}(0)$ defines the total
 82 number of the incoming beams [25]. However, systematic errors in $N_{\text{gs}}(x_t)$ and
 83 $N_{\text{gs}}(0)$ normally dominate uncertainties in σ_I^{gs} . To study new halo candidates,
 84 a precision better than 14% is typically required for σ_I^{gs} as is the case for earlier
 85 studies [26]. For instance, the differences between the interaction cross sections
 86 of halo nuclei and neighboring non-halo nuclei range between 14% to 30% for the
 87 cases of halo nuclei ^6He , ^{11}Li , ^{11}Be , and ^{14}Be [26]. In the following discussion,
 88 we adopt the criterion that 14% precision of the interaction cross section is
 89 needed to distinguish a halo from a non-halo state.

90 *2.2. Recoil distance transmission method*

91 In the recoil distance transmission method, a “long-lived” excited state
 92 (termed excited product) with a lifetime of the order of 100–1000 ps is pro-
 93 duced by beams (termed beam nuclei) in a production target as illustrated in
 94 Fig. 1(a). The interaction cross section of the excited state on a reaction target
 95 is measured. The number of excited products is determined by measuring γ
 96 decays before the reaction target, while decays after the reaction target give the
 97 number of outgoing excited products. The production and reaction targets are
 98 mounted on a plunger device together with a degrader placed downstream of the
 99 reaction target which helps to properly count the number of outgoing excited
 100 products. Figure 1(b) depicts the number of the excited products ($N_{\text{ex}}(x)$) as
 101 a function of the distance x along the beam axis, where $x = 0$ and $x = x_t$ cor-
 102 respond to the front and back surfaces of the reaction target, respectively. The
 103 reduction rate of $N_{\text{ex}}(x)$ through the reaction target reflects the magnitude of
 104 the interaction cross section of the excited state, excluding other factors which
 105 will be discussed in Sec. 3.2.

106 The numbers of excited products immediately before the reaction target
 107 ($N_{\text{ex}}(0)$) and after the reaction target ($N_{\text{ex}}(x_t)$) are obtained by counting the
 108 de-excitation γ rays. Figure 1(c) shows the expected γ -ray spectrum obtained
 109 from the present method. The fast, reduced, and slow peaks correspond to γ

110 rays emitted between the production and reaction targets, between the reaction
 111 target and degrader, and after the degrader, respectively. The numbers of γ
 112 decays in the fast region (G_1) and reduced region (G_2) can be related to the
 113 γ -ray yields for the fast peak (g_1) and reduced peak (g_2), respectively:

$$G_1 = g_1/\varepsilon_1, \quad (4)$$

$$G_2 = g_2/\varepsilon_2, \quad (5)$$

114 where ε_1 and ε_2 represent the detection efficiencies of the γ rays. $N_{\text{ex}}(0)$ and
 115 $N_{\text{ex}}(x_t)$ are obtained from G_1 and G_2 as

$$N_{\text{ex}}(0) = \frac{G_1 e^{-t_1/\tau}}{1 - e^{-t_1/\tau}}, \quad (6)$$

$$N_{\text{ex}}(x_t) = \frac{G_2}{1 - e^{-t_2/\tau}}, \quad (7)$$

116 where τ represents the mean lifetime of the excited state, and t_1 (t_2) denotes
 117 the time of flight over the distance d_1 (d_2) (see Fig. 1(b)). As a note for Eq. (7),
 118 if the position of the degrader is far from the reaction target ($d_2 \rightarrow \infty$), the
 119 time t_2 approaches infinity ($t_2 \rightarrow \infty$), so that $N_{\text{ex}}(x_t)$ and G_2 become equal.
 120 The number of the γ rays emitted in the fast region corresponds to the change
 121 of the number of the excited products:

$$G_1 = N_{\text{ex}}(-d_1) - N_{\text{ex}}(0) \quad (8)$$

$$= \frac{N_{\text{ex}}(0)}{e^{-t_1/\tau}} - N_{\text{ex}}(0), \quad (9)$$

122 where $N_{\text{ex}}(-d_1)$ is the number of the excited products at the back surface of
 123 the production target and can be converted as $N_{\text{ex}}(-d_1) = N_{\text{ex}}(0)/e^{-t_1/\tau}$. By
 124 solving Eq. (9) for $N_{\text{ex}}(0)$, one obtains Eq. (6). Similarly, the number of the γ
 125 rays emitted in the reduced region is written as

$$G_2 = N_{\text{ex}}(x_t) - N_{\text{ex}}(x_t + d_2) \quad (10)$$

$$= N_{\text{ex}}(x_t) - N_{\text{ex}}(x_t) e^{-t_2/\tau}, \quad (11)$$

126 where $N_{\text{ex}}(x_t + d_2)$ is the number of the excited products at the front surface of
 127 the degrader and can be converted as $N_{\text{ex}}(x_t + d_2) = N_{\text{ex}}(x_t) e^{-t_2/\tau}$. By solving

Eq. (11) for $N_{\text{ex}}(x_t)$, one obtains Eq. (7). As a result, if one assumes that the number of excited products across the reaction target is affected only by interaction on the reaction target, the interaction cross section σ_I^{ex} is calculated from

$$\begin{aligned}\sigma_I^{\text{ex}} &= -\frac{A}{N_A \rho x_t} \ln \frac{N_{\text{ex}}(x_t)}{N_{\text{ex}}(0)}, \\ &= -\frac{A}{N_A \rho x_t} \ln \left(\frac{G_2}{G_1} \frac{1 - e^{-t_1/\tau}}{e^{-t_1/\tau} (1 - e^{-t_2/\tau})} \right).\end{aligned}\quad (12)$$

If $t_1 = t_2$ is chosen, σ_I^{ex} is expressed as a simple form:

$$\sigma_I^{\text{ex}} = -\frac{A}{N_A \rho x_t} \ln \left(\frac{G_2}{G_1} \frac{1}{e^{-t_1/\tau}} \right). \quad (13)$$

There are two important differences from Eq. (1). First, the G_2/G_1 ratio is associated with γ decays instead of the particle ratio Γ/Γ_0 . Second, the additional term $(1/e^{-t_1/\tau})$ appears for the present method for excited states.

3. Formalism of the recoil distance transmission method

3.1. Derivation of the formula to deduce the interaction cross section

The number of excited products is changed through five processes in the reaction target: (a) interaction of the excited states, (b) γ decay from the excited states, (c) production of the excited states from the unreacted beam nuclei, (d) inelastic scattering from the ground states of reaction products to populate the excited states, and (e) inelastic scattering from the excited states to populate the ground states. Therefore, the differential equation of $N_{\text{ex}}(x)$ with respect to the position x ($0 < x < x_t$) is fully written as

$$\begin{aligned}\frac{dN_{\text{ex}}(x)}{dx} &= -\frac{N_A \rho}{A} \sigma_I^{\text{ex}} N_{\text{ex}}(x) - \frac{1}{\tau v} N_{\text{ex}}(x) + \frac{N_A \rho}{A} \sigma_{\text{prod}}^{\text{be} \rightarrow \text{ex}} N_{\text{be}}(x) \\ &\quad + \frac{N_A \rho}{A} \sigma_{\text{inel}}^{\text{gs} \rightarrow \text{ex}} N_{\text{gs}}(x) - \frac{N_A \rho}{A} \sigma_{\text{inel}}^{\text{ex} \rightarrow \text{gs}} N_{\text{ex}}(x),\end{aligned}\quad (14)$$

where the first term represents the interaction of the excited products of interest, and subsequent terms are related to the γ decay, additional production, and reduction of the excited state in the reaction target. The quantities $N_{\text{be}}(x)$ and

148 $N_{\text{gs}}(x)$ denote the numbers of beam nuclei and nuclei of interest in the ground
 149 state (ground-state products) as a function of the position x , respectively. The
 150 cross section $\sigma_{\text{prod}}^{\text{be} \rightarrow \text{ex}}$ is defined for the production of the excited states from
 151 the beam, while $\sigma_{\text{inel}}^{\text{gs} \rightarrow \text{ex}}$ ($\sigma_{\text{inel}}^{\text{ex} \rightarrow \text{gs}}$) is the inelastic scattering cross section for the
 152 ground-state products leading to the excited states (excited products leading
 153 to the ground states). The velocity v is given for the excited products. In the
 154 present formalism, we assume that reaction product has only two energy levels,
 155 the ground state and single excited state, as a typical physics case for a loosely
 156 bound halo system.

157 Dividing Eq. (14) by $N_{\text{ex}}(x)$ and integrating from the front surface of the
 158 reaction target ($x = 0$) to the back surface ($x = x_t$) give the following equation:

$$\ln \frac{N_{\text{ex}}(x_t)}{N_{\text{ex}}(0)} = -\frac{N_A \rho x_t}{A} \langle \sigma_I^{\text{ex}} \rangle - \frac{x_t}{\tau} \left\langle \frac{1}{v} \right\rangle + \frac{N_A \rho x_t}{A} \left\langle \sigma_{\text{prod}}^{\text{be} \rightarrow \text{ex}} \frac{N_{\text{be}}(x)}{N_{\text{ex}}(x)} \right\rangle \\ + \frac{N_A \rho x_t}{A} \left\langle \sigma_{\text{inel}}^{\text{gs} \rightarrow \text{ex}} \frac{N_{\text{gs}}(x)}{N_{\text{ex}}(x)} \right\rangle - \frac{N_A \rho x_t}{A} \langle \sigma_{\text{inel}}^{\text{ex} \rightarrow \text{gs}} \rangle, \quad (15)$$

159 where $\langle \sigma_I^{\text{ex}} \rangle$, $\langle \frac{1}{v} \rangle$, $\left\langle \sigma_{\text{prod}}^{\text{be} \rightarrow \text{ex}} \frac{N_{\text{be}}(x)}{N_{\text{ex}}(x)} \right\rangle$, $\left\langle \sigma_{\text{inel}}^{\text{gs} \rightarrow \text{ex}} \frac{N_{\text{gs}}(x)}{N_{\text{ex}}(x)} \right\rangle$, and $\langle \sigma_{\text{inel}}^{\text{ex} \rightarrow \text{gs}} \rangle$ are averages
 160 with respect to x :

$$\langle \sigma_I^{\text{ex}} \rangle = \frac{1}{x_t} \int_0^{x_t} \sigma_I^{\text{ex}} dx, \quad (16)$$

$$\left\langle \frac{1}{v} \right\rangle = \frac{1}{x_t} \int_0^{x_t} \frac{1}{v} dx, \quad (17)$$

$$\left\langle \sigma_{\text{prod}}^{\text{be} \rightarrow \text{ex}} \frac{N_{\text{be}}(x)}{N_{\text{ex}}(x)} \right\rangle = \frac{1}{x_t} \int_0^{x_t} \sigma_{\text{prod}}^{\text{be} \rightarrow \text{ex}} \frac{N_{\text{be}}(x)}{N_{\text{ex}}(x)} dx, \quad (18)$$

$$\left\langle \sigma_{\text{inel}}^{\text{gs} \rightarrow \text{ex}} \frac{N_{\text{gs}}(x)}{N_{\text{ex}}(x)} \right\rangle = \frac{1}{x_t} \int_0^{x_t} \sigma_{\text{inel}}^{\text{gs} \rightarrow \text{ex}} \frac{N_{\text{gs}}(x)}{N_{\text{ex}}(x)} dx, \quad (19)$$

$$\langle \sigma_{\text{inel}}^{\text{ex} \rightarrow \text{gs}} \rangle = \frac{1}{x_t} \int_0^{x_t} \sigma_{\text{inel}}^{\text{ex} \rightarrow \text{gs}} dx. \quad (20)$$

161 It is noted that $\langle \sigma_I^{\text{ex}} \rangle$, $\left\langle \sigma_{\text{prod}}^{\text{be} \rightarrow \text{ex}} \frac{N_{\text{be}}(x)}{N_{\text{ex}}(x)} \right\rangle$, $\left\langle \sigma_{\text{inel}}^{\text{gs} \rightarrow \text{ex}} \frac{N_{\text{gs}}(x)}{N_{\text{ex}}(x)} \right\rangle$, and $\langle \sigma_{\text{inel}}^{\text{ex} \rightarrow \text{gs}} \rangle$ are intro-
 162 duced to leave out the kinetic energy dependence of σ_I^{ex} , $\sigma_{\text{prod}}^{\text{be} \rightarrow \text{ex}}$, $\sigma_{\text{inel}}^{\text{gs} \rightarrow \text{ex}}$, and
 163 $\sigma_{\text{inel}}^{\text{ex} \rightarrow \text{gs}}$, respectively, through the reaction target. By solving Eq. (15) for $\langle \sigma_I^{\text{ex}} \rangle$
 164 and substituting $N_{\text{ex}}(x_t)/N_{\text{ex}}(0)$ with Eqs. (6) and (7), one finally obtains the

165 formula

$$\begin{aligned} \langle \sigma_I^{\text{ex}} \rangle = & -\frac{A}{N_A \rho x_t} \ln \left(\frac{G_2}{G_1} \frac{1 - e^{-t_1/\tau}}{e^{-t_1/\tau} (1 - e^{-t_2/\tau})} \right) - \frac{A}{N_A \rho \tau} \left\langle \frac{1}{v} \right\rangle \\ & + \left\langle \sigma_{\text{prod}}^{\text{be} \rightarrow \text{ex}} \frac{N_{\text{be}}(x)}{N_{\text{ex}}(x)} \right\rangle + \left\langle \sigma_{\text{inel}}^{\text{gs} \rightarrow \text{ex}} \frac{N_{\text{gs}}(x)}{N_{\text{ex}}(x)} \right\rangle - \langle \sigma_{\text{inel}}^{\text{ex} \rightarrow \text{gs}} \rangle. \quad (21) \end{aligned}$$

166 The first term of right-hand side of Eq. (21) is based on the ratio of the
 167 numbers of incoming and outgoing excited products as determined from the
 168 γ -ray yields G_2/G_1 . The second term corresponds to the γ -decay rate of the
 169 excited state in the reaction target. The third term is for the production of
 170 the excited states from the beam nuclei. The fourth and fifth terms represent
 171 the production and reduction of the excited states by inelastic scattering from
 172 the ground-state and excited products, respectively. As discussed later, the first
 173 three terms of Eq. (21) are dominant, whereas the last two terms $\left\langle \sigma_{\text{inel}}^{\text{gs} \rightarrow \text{ex}} \frac{N_{\text{gs}}(x)}{N_{\text{ex}}(x)} \right\rangle$
 174 and $\langle \sigma_{\text{inel}}^{\text{ex} \rightarrow \text{gs}} \rangle$ have the smallest contributions. Actual experiments should be
 175 designed so that the first term of Eq. (21) dominates the measured value of
 176 $\langle \sigma_I^{\text{ex}} \rangle$. Therefore other terms need to be controlled or reduced by experimental
 177 techniques.

178 3.2. Error evaluation of the interaction cross section

179 Based on Eq. (21), the error in $\langle \sigma_I^{\text{ex}} \rangle$, denoted as $\Delta \langle \sigma_I^{\text{ex}} \rangle$ hereafter, can be
 180 described as

$$\begin{aligned} (\Delta \langle \sigma_I^{\text{ex}} \rangle)^2 = & \delta_\rho^2 + \delta_{x_t}^2 + \delta_{G_2/G_1}^2 + \delta_{1/v}^2 \\ & + \delta_\tau^2 + \delta_{t_1}^2 + \delta_{t_2}^2 + \delta_{\sigma_{\text{prod}}^{\text{be} \rightarrow \text{ex}}}^2 + \delta_{\sigma_{\text{inel}}^{\text{gs} \rightarrow \text{ex}}}^2 + \delta_{\sigma_{\text{inel}}^{\text{ex} \rightarrow \text{gs}}}^2, \quad (22) \end{aligned}$$

181 where the individual contributions δ_ρ , δ_{x_t} , δ_{G_2/G_1} , $\delta_{1/v}$, δ_τ , δ_{t_1} , δ_{t_2} , $\delta_{\sigma_{\text{prod}}^{\text{be} \rightarrow \text{ex}}}$,
 182 $\delta_{\sigma_{\text{inel}}^{\text{gs} \rightarrow \text{ex}}}$, and $\delta_{\sigma_{\text{inel}}^{\text{ex} \rightarrow \text{gs}}}$ originate from uncertainties in measurable quantities ρ ,
 183 x_t , G_2/G_1 , $\langle 1/v \rangle$, τ , t_1 , t_2 , $\left\langle \sigma_{\text{prod}}^{\text{be} \rightarrow \text{ex}} \frac{N_{\text{be}}(x)}{N_{\text{ex}}(x)} \right\rangle$, $\left\langle \sigma_{\text{inel}}^{\text{gs} \rightarrow \text{ex}} \frac{N_{\text{gs}}(x)}{N_{\text{ex}}(x)} \right\rangle$, and $\langle \sigma_{\text{inel}}^{\text{ex} \rightarrow \text{gs}} \rangle$,
 184 respectively. The relations between the individual contributions to $\Delta \langle \sigma_I^{\text{ex}} \rangle$ and

the original uncertainties in the observables are given as

$$\delta_\rho = \left[\frac{A}{N_A \rho x_t} \ln \left(\frac{G_2}{G_1} \frac{1 - e^{-t_1/\tau}}{1 - e^{-t_2/\tau}} \right) + \frac{A}{N_A \rho \tau} \left\langle \frac{1}{v} \right\rangle \right] \frac{\Delta \rho}{\rho}, \quad (23)$$

$$\delta_{x_t} = \frac{A}{N_A \rho x_t} \ln \left(\frac{G_2}{G_1} \frac{1 - e^{-t_1/\tau}}{1 - e^{-t_2/\tau}} \right) \frac{\Delta x_t}{x_t}, \quad (24)$$

$$\delta_{G_2/G_1} = -\frac{A}{N_A \rho x_t} \frac{\Delta (G_2/G_1)}{G_2/G_1}, \quad (25)$$

$$\delta_{1/v} = -\frac{A}{N_A \rho \tau} \left\langle \frac{1}{v} \right\rangle \frac{\Delta \langle 1/v \rangle}{\langle 1/v \rangle} \quad (26)$$

$$\delta_\tau = \frac{A}{N_A \rho} \left[\frac{1}{\tau} \left\langle \frac{1}{v} \right\rangle + \frac{1}{x_t} \frac{t_1/\tau}{1 - e^{-t_1/\tau}} - \frac{e^{-t_2/\tau}}{x_t} \frac{t_2/\tau}{1 - e^{-t_2/\tau}} \right] \frac{\Delta \tau}{\tau}, \quad (27)$$

$$\delta_{t_1} = -\frac{A}{N_A \rho x_t} \frac{t_1/\tau}{1 - e^{-t_1/\tau}} \frac{\Delta t_1}{t_1}, \quad (28)$$

$$\delta_{t_2} = \frac{A}{N_A \rho x_t} e^{-t_2/\tau} \frac{t_2/\tau}{1 - e^{-t_2/\tau}} \frac{\Delta t_2}{t_2}, \quad (29)$$

$$\delta_{\sigma_{\text{prod}}^{\text{be} \rightarrow \text{ex}}} = \Delta \left\langle \sigma_{\text{prod}}^{\text{be} \rightarrow \text{ex}} \frac{N_{\text{be}}(x)}{N_{\text{ex}}(x)} \right\rangle, \quad (30)$$

$$\delta_{\sigma_{\text{inel}}^{\text{gs} \rightarrow \text{ex}}} = \Delta \left\langle \sigma_{\text{inel}}^{\text{gs} \rightarrow \text{ex}} \frac{N_{\text{gs}}(x)}{N_{\text{ex}}(x)} \right\rangle, \quad (31)$$

$$\delta_{\sigma_{\text{inel}}^{\text{ex} \rightarrow \text{gs}}} = \Delta \langle \sigma_{\text{inel}}^{\text{ex} \rightarrow \text{gs}} \rangle. \quad (32)$$

Figure 2 shows the relative contribution from each component of Eq. (22) to the total uncertainty $\Delta \langle \sigma_I^{\text{ex}} \rangle$ plotted as a function of x_t , where typical experimental conditions are assumed. The reaction target is beryllium with $A = 9$ and $\rho = 1.85 \text{ g/cm}^3$. The lifetime $\tau = 500 \text{ ps}$, flight times $t_1 = t_2 = 250 \text{ ps}$, and beam velocity $\langle 1/v \rangle = 1/(10 \text{ cm/ns})$ are considered. For a constant $\langle \sigma_I^{\text{ex}} \rangle$ of 1 b, the G_2/G_1 ratio varies between $G_2/G_1 = 0.589$ for $x_t = 1 \text{ mm}$ to $G_2/G_1 = 0.035$ for $x_t = 100 \text{ mm}$. For simplicity, the uncertainty in each quantity is fixed at

193 1%:

$$\begin{aligned} \frac{\Delta\rho}{\rho} = \frac{\Delta x_t}{x_t} = \frac{\Delta(G_2/G_1)}{G_2/G_1} = \frac{\Delta\langle 1/v \rangle}{\langle 1/v \rangle} \\ = \frac{\Delta\tau}{\tau} = \frac{\Delta t_1}{t_1} = \frac{\Delta t_2}{t_2} = 1\%. \end{aligned} \quad (33)$$

194 All of $\Delta\left\langle\sigma_{\text{prod}}^{\text{be}\rightarrow\text{ex}}\frac{N_{\text{be}}(x)}{N_{\text{ex}}(x)}\right\rangle$, $\Delta\left\langle\sigma_{\text{inel}}^{\text{gs}\rightarrow\text{ex}}\frac{N_{\text{gs}}(x)}{N_{\text{ex}}(x)}\right\rangle$, and $\Delta\langle\sigma_{\text{inel}}^{\text{ex}\rightarrow\text{gs}}\rangle$ are chosen as 0.01 b.
 195 Here, possible errors due to the unknown energy dependence of all these reaction
 196 cross sections are ignored.

197 The components δ_{G_2/G_1} , δ_τ , δ_{t_1} , and δ_{t_2} in Fig. 2 have a dependence inversely
 198 proportional to the reaction target thickness x_t , since the number of interactions
 199 are calculated from G_2/G_1 , τ , t_1 , and t_2 , and the ratio of the uncertainty
 200 in G_2/G_1 , τ , t_1 , and t_2 to the number of interactions determines the total
 201 uncertainty $\Delta\langle\sigma_I^{\text{ex}}\rangle$. For instance, if a large x_t is chosen, the number of the
 202 interactions becomes large. In such a case, the uncertainties in G_2/G_1 , τ , t_1 ,
 203 and t_2 do not make much contribution with respect to the large number of
 204 the interactions. The other components are independent of the reaction target
 205 thickness x_t and thus provide constant contributions as shown in Fig. 2.

206 In actual experiments with fast rare-isotope beams, the reaction target thick-
 207 ness is expected to be ≈ 20 mm or less. For that thickness, the dominant contri-
 208 butions in $\Delta\langle\sigma_I^{\text{ex}}\rangle$ come from G_2/G_1 , τ , t_1 , and t_2 as shown in Fig. 2. For ρ , x_t ,
 209 and $\langle 1/v \rangle$, a 1% uncertainty is achievable, giving only marginal contributions to
 210 the total uncertainty. The $\delta_{\sigma_{\text{inel}}^{\text{gs}\rightarrow\text{ex}}}$ and $\delta_{\sigma_{\text{inel}}^{\text{ex}\rightarrow\text{gs}}}$ terms are expected to be small.
 211 In general, the $\sigma_{\text{inel}}^{\text{gs}\rightarrow\text{ex}}$ is on the order of 0.01 b with uncertainty on the order of
 212 10% [15, 27], and $N_{\text{gs}}(x)/N_{\text{ex}}(x)$ is around 1–10 [28]. The $\sigma_{\text{inel}}^{\text{ex}\rightarrow\text{gs}}$ is on the or-
 213 der of $\sigma_{\text{inel}}^{\text{gs}\rightarrow\text{ex}}$. Therefore $\Delta\left\langle\sigma_{\text{inel}}^{\text{gs}\rightarrow\text{ex}}\frac{N_{\text{gs}}(x)}{N_{\text{ex}}(x)}\right\rangle$ and $\Delta\langle\sigma_{\text{inel}}^{\text{ex}\rightarrow\text{gs}}\rangle$ can remain as small
 214 as 0.01 b. The third term $\Delta\left\langle\sigma_{\text{prod}}^{\text{be}\rightarrow\text{ex}}\frac{N_{\text{be}}(x)}{N_{\text{ex}}(x)}\right\rangle$ can be excluded by an energy-
 215 loss measurement of beam nuclei as explained in Sec. 4.2. A 1% uncertainty
 216 in t_1 and t_2 is usually achievable because these values are calculated from the
 217 distances, d_1 and d_2 , and velocity of the excited products. The distances can
 218 be constrained to within 1% precision using a plunger device, and the velocity
 219 can be measured precisely by a magnetic spectrograph. An error in τ of 2%

is considered for lifetimes measured with the recoil distance method [29]. For that error in τ , δ_τ amounts to 0.07 b at $x_t = 20$ mm and dominates the total uncertainty $\Delta\langle\sigma_I^{\text{ex}}\rangle$. It is important to note that only the averaged cross section $\langle\sigma_I^{\text{ex}}\rangle$ is measured in the present method and the energy dependence of σ_I^{ex} is not taken into account.

To realize the present measurement, the main experimental challenge is controlling the statistical and systematic errors in G_2/G_1 . To achieve 1% statistical error in G_2/G_1 , the number of the detected γ rays, g_1 and g_2 , should be at least 2×10^4 events. For the systematic error in G_2/G_1 , the method to obtain G_2/G_1 should be considered. In the analysis of real experimental data, the measured γ -ray spectrum is compared with simulated spectra for various G_2/G_1 values, and the ratio G_2/G_1 should be obtained with less than 1% systematic error.

4. Experimental considerations

This section describes experimental instruments suited for the recoil distance transmission method and considers requirements to realize actual experiments based on simulated results. For quantitative discussion, we have exploited experimental conditions at the Coupled Cyclotron Facility (CCF) [30] of the National Superconducting Cyclotron Laboratory (NSCL), Michigan State University (MSU), but our conclusions are generally applicable to present and future rare-isotope beam facilities.

4.1. Experimental setup

The experimental setup for the recoil distance transmission method measurements is essentially based on the recoil-distance setup to measure excited-state lifetimes using fast rare-isotope beams. In this subsection, the actual equipment including the TRIPLEX plunger device [31, 32] and the γ -ray tracking array GRETINA [33] is briefly introduced. More detailed experimental conditions considered in this section are summarized in Table 1.

Table 1: Summary of the experimental setup for $^AZ^*$ and $^{17}\text{F}(1/2^+)$ case.

	Experimental cases	
	$^AZ^*$	$^{17}\text{F}(1/2^+)$
Secondary beam	$A+1(Z+1)$	^{19}Ne
Energy of the secondary beam	140 MeV/nucleon	140 MeV/nucleon
Lifetime (τ) of the excited state	500 ps	406.8 ± 8.7 ps
Energy of the de-excitation γ ray	500 keV	495.3(1) keV
TRIPLEX setup		
Production target	2-mm Be + 0.3-mm Si	2-mm Be + 0.3-mm Si
Reaction target	14-mm Be	10-mm Be
Degradar	1-mm Ta	0.7-mm Ta
d_1	30 mm	20 mm
d_2	45 mm	30 mm
β_{fast}	0.482	0.479
β_{reduced}	0.387	0.387
β_{slow}	0.309	0.318

4.1.1. TRIPLEX setup

The secondary beam is delivered to a plunger device, the TRIPLE PLunger for EXotic beams (TRIPLEX), which is mounted at the target position of the spectrograph [34]. The experimental setup we consider for the present measurement is based on the typical lifetime measurement setup with the TRIPLEX plunger device [32] (see Fig. 4 of Ref. [32] for a photo of the setup). The TRIPLEX can hold three metal foils as presented in Fig. 1(a). In the present experiment, the foil configuration comprises a production target, a reaction target, and a degrader to produce excited products and subsequently measure interaction cross sections of interest. The foil materials and distance settings considered are listed in Table 1.

4.1.2. GRETINA setup

For the γ -ray detection, the Gamma-Ray Energy Tracking In-beam Nuclear Array (GRETINA) is considered. The configuration of the array is assumed to be the same as a previous experiment [35], where seven GRETINA detector modules were used. The four forwardmost detectors cover the laboratory angles θ_{lab} of 20° – 50° with respect to the beam axis and are used to measure γ rays emitted before and after the reaction target.

4.2. Simulated results

We first examine the case of an ideal nucleus with AZ , where $Z = 12$ and $A = 40$ are arbitrarily taken. The energy and lifetime of the excited state are 500 keV and 500 ps, respectively. A secondary beam of ${}^{A+1}(Z+1)$ is used to populate the excited state (${}^AZ^*$) in AZ . The parameters used in the simulation are listed in the first column of Table 1. For inelastic scattering in the reaction target, $\langle \sigma_{\text{inel}}^{\text{gs} \rightarrow \text{ex}} \frac{N_{\text{gs}}(x)}{N_{\text{ex}}(x)} \rangle$ of 0.1 b and $\langle \sigma_{\text{inel}}^{\text{ex} \rightarrow \text{gs}} \rangle$ of 0.1 b are assumed.

The simulated γ -ray spectra are shown in Fig. 3. The spectra are obtained by a simulation package [36], which is based on the GEANT4 framework [37] with modification to incorporate the experimental setup, similar to Refs. [38–40]. In the analysis of the simulated results, the four detectors of GRETINA at forward angles are used exclusively, with a further angular cutoff applied ($\theta_{\text{lab}} < 40^\circ$) to obtain good separation between the fast, reduced, and slow peaks. In the simulation, we consider cross sections $\langle \sigma_I^{\text{ex}} \rangle$ of 1.0, 1.5, and 2.0 b, which result in G_2/G_1 values of 0.752, 0.691, and 0.633, respectively. The sensitivity to the different cross sections is evident in the variation of the height of the reduced peak.

The errors in the cross section are estimated as shown in the first column of Table 2, where $\langle \sigma_I^{\text{ex}} \rangle$ is assumed to be 1.5 b, and the uncertainties in ρ , x_t , G_2/G_1 , and $\langle 1/v \rangle$ are each assumed to be 1%:

$$\frac{\Delta \rho}{\rho} = \frac{\Delta x_t}{x_t} = \frac{\Delta (G_2/G_1)}{G_2/G_1} = \frac{\Delta \langle 1/v \rangle}{\langle 1/v \rangle} = 1\%. \quad (34)$$

For τ , an uncertainty of 2% is adopted, which is as good as the precision achieved

286 in a recent recoil distance measurement [29]. An uncertainty of 0.5% is consid-
 287 ered for t_1 and t_2 which can be constrained using the S800 spectrograph [34]
 288 and plunger device [31, 32]. The velocity uncertainty is estimated to be 0.5%,
 289 which includes the ambiguities in the momentum analysis in the S800 as well
 290 as energy straggling in the foils mounted in the plunger. The uncertainty in the
 291 absolute distances d_1 and d_2 of the plunger is $\approx 30 \mu\text{m}$ [32], which corresponds
 292 to 0.1%. As a result, a total uncertainty of 0.5% is chosen for t_1 and t_2 , in-
 293 stead of 1% as used in Fig. 2. To reduce contributions from $\sigma_{\text{prod}}^{\text{be} \rightarrow \text{ex}}$, a silicon
 294 detector is attached to the back surface of the production target [41] as listed
 295 in Table 1. Because a charge-changing (one-proton removal) reaction is used to
 296 populate the excited state, the Z information obtained from the detector can
 297 select the production of the nuclei of interest in the production target. Therefore
 298 for this simulation, $\sigma_{\text{prod}}^{\text{be} \rightarrow \text{ex}}$ and $\delta_{\sigma_{\text{prod}}^{\text{be} \rightarrow \text{ex}}}$ are assumed to be negligible. Finally,
 299 both $\delta_{\sigma_{\text{inel}}^{\text{gs} \rightarrow \text{ex}}}$ and $\delta_{\sigma_{\text{inel}}^{\text{ex} \rightarrow \text{gs}}}$ are fixed to be 0.01 b.

300 The error $\Delta\langle\sigma_I^{\text{ex}}\rangle$ amounts to 0.12 b corresponding to 7.9 % uncertainty out
 301 of the total $\langle\sigma_I^{\text{ex}}\rangle$ of 1.5 b. In general, the difference of $\langle\sigma_I^{\text{ex}}\rangle$ between halo and
 302 non-halo states is 14% or larger [26], and thus we can distinguish the halo state
 303 from the non-halo state for this case. The error $\Delta\langle\sigma_I^{\text{ex}}\rangle$ is dominated by the
 304 components δ_{G_2/G_1} of 3.8% and δ_τ of 6.0%, which indicates that the precise
 305 determination of G_2/G_1 and τ is essential in obtaining a reasonable precision
 306 in $\langle\sigma_I^{\text{ex}}\rangle$.

307 4.3. The case of $^{17}\text{F}(1/2^+)$

308 This subsection discusses experimental feasibility and possible improvement
 309 with a current realistic case. In the light mass region, there are some candi-
 310 dates of excited halo state with small separation energies and lifetimes on the
 311 order of 100–1000 ps [40, 42–44], e.g., the first $1/2^+$ excited state of ^{17}F , first
 312 $1/2^+$ excited state of ^{17}C , and first $3/2^+$ excited state of ^{19}C . In the following
 313 discussion, we focus on the ^{17}F case. The aim of the experiment is to obtain
 314 direct evidence of halo structure in the state denoted by $^{17}\text{F}(1/2^+)$.

315 The nuclear physics interest in the measurement of $\langle\sigma_I^{\text{ex}}\rangle$ of $^{17}\text{F}(1/2^+)$, with

Table 2: Individual error components (%) with respect to $\langle\sigma_I^{\text{ex}}\rangle$ for the experimental cases of $^A Z^*$ and $^{17}\text{F}(1/2^+)$.

Error components with respect to $\langle\sigma_I^{\text{ex}}\rangle$ (%)	Experimental cases	
	$^A Z^*$	$^{17}\text{F}(1/2^+)$
δ_ρ	0.9	0.9
δ_{x_t}	1.8	2.2
δ_{G_2/G_1}	3.8	6.7
$\delta_{1/v}$	0.8	1.3
δ_τ	6.0	8.8
δ_{t_1}	2.4	4.0
δ_{t_2}	1.3	2.4
$\delta_{\sigma_{\text{prod}}^{\text{be}\rightarrow\text{ex}}}$	< 0.1	< 0.1
$\delta_{\sigma_{\text{inel}}^{\text{gs}\rightarrow\text{ex}}}$	0.7	0.8
$\delta_{\sigma_{\text{inel}}^{\text{ex}\rightarrow\text{gs}}}$	0.7	0.8
$\Delta\langle\sigma_I^{\text{ex}}\rangle$	7.9	12

316 a proton separation energy of 104.9(3) keV [45, 46], is to demonstrate the pres-
 317 ence of the halo structure in the excited states. The state has been suggested
 318 to have an s -orbital proton halo structure [46], although there is no direct ev-
 319 idence by reaction studies. Therefore, measurement of $\langle\sigma_I^{\text{ex}}\rangle$ of $^{17}\text{F}(1/2^+)$ is
 320 desired to confirm the excited-state proton halo. The lifetime of the state is
 321 406.8 ± 8.7 ps [44], which is expected to be long enough to apply the present
 322 method. This study can be considered as an extension of the proton radii study
 323 by means of charge-changing reaction measurements performed by Estradé *et*
 324 *al.* [47].

325 The experimental setup is summarized in the second column of Table 1,
 326 and the simulated γ -ray spectrum (black histogram) is shown in Fig. 4. In the
 327 simulation, a cross section of 1.2 b is considered. As before, $\left\langle\sigma_{\text{inel}}^{\text{gs}\rightarrow\text{ex}} \frac{N_{\text{gs}}(x)}{N_{\text{ex}}(x)}\right\rangle$ of
 328 0.1 b and $\langle\sigma_{\text{inel}}^{\text{ex}\rightarrow\text{gs}}\rangle$ of 0.1 b are assumed. We adopt the ^{19}Ne secondary-beam
 329 intensity of 10^4 pps because of the counting limitation of detectors, while a

330 maximum intensity on the order of 10^7 pps is available. To collect a few 10^4
 331 γ -ray events necessary for $\approx 1\%$ statistical error, a beam intensity of around
 332 10^4 pps is needed for a typical one-week experiment.

333 To obtain the G_2/G_1 value from the expected γ -ray spectrum (black his-
 334 togram) in Fig. 4, a procedure to fit the spectrum with simulated γ -ray spectra
 335 is considered. In the procedure, simulated γ -ray spectrum with high statistics
 336 is divided into three components: fast peak (dot-dashed magenta line), reduced
 337 peak (dashed green line), and background (dotted blue line), which come from
 338 decay components behind the production target, behind the reaction target,
 339 and inside of the production and reaction targets, respectively. The solid red
 340 line is the sum of these components and should be fitted to the black histogram.
 341 From the fit result, the G_1 and G_2 values are obtained.

342 It should be noted that the line shapes of the fast and reduced peaks (dot-
 343 dashed magenta and dashed green lines) can be simulated by GEANT4 even if
 344 all cross-section information related to this measurement is unknown, because
 345 the γ rays making the fast and reduced peaks are emitted from spaces between
 346 the production and reaction target and between the reaction target and de-
 347 grader. On the other hand, the shape of the background (dotted blue line)
 348 depends slightly on the interaction cross section of interest and inelastic scat-
 349 tering cross sections. For instance, a 0.2-b uncertainty in the interaction cross
 350 section leads to a 0.3% uncertainty in G_2/G_1 . That uncertainty, however, has
 351 no significant effect in determining $\langle\sigma_I^{\text{ex}}\rangle$. Finally, if one takes into account the
 352 uncertainty in $\varepsilon_2/\varepsilon_1$ which will be discussed in Sec. 5, around 1% uncertainty
 353 in G_2/G_1 is obtained.

354 The errors in the cross section are estimated as shown in the second column
 355 of Table 2, where $\langle\sigma_I^{\text{ex}}\rangle$ of 1.2 b is taken. The uncertainties in ρ , x_t , G_2/G_1 , and
 356 $\langle 1/v \rangle$ are assumed to be 1%, and the uncertainty of 0.5% is assumed for t_1 and
 357 t_2 . For τ , the experimental uncertainty of 2% is used. As a result, the overall
 358 error $\Delta\langle\sigma_I^{\text{ex}}\rangle$ amounts to 0.15 b, corresponding to 12% uncertainty in $\langle\sigma_I^{\text{ex}}\rangle$ of
 359 1.2 b.

360 According to the Glauber model calculation [48], the interaction cross sec-

tion of $^{17}\text{F}(1/2^+)$ is expected to be 1.17 b for a pure s -orbital halo, while the interaction cross section of neighboring core nucleus ^{16}O is calculated to be 0.99 b. The difference is 0.18 b, which is comparable to the uncertainty of the present measurement of 0.15 b. Although the halo state might be identified, more precise measurement should be considered to make sure of existence of a halo, for instance, by improving the lifetime information. The error is dominated by the component δ_τ of 8.8%, as well as the component δ_{G_2/G_1} of 6.7% as shown in Table 2.

5. Discussion for designing experiments

The discussion for the error in $\langle\sigma_I^{\text{ex}}\rangle$ in Sec. 4 gives some insights for designing actual experiments. In this section, requirements for designing such experiments are explored in more detail.

• Uncertainty in G_2/G_1 (δ_{G_2/G_1})

Reducing δ_{G_2/G_1} is important, because this component is large in any case as discussed in Sec. 4. The uncertainty is dominated by the systematic error in the determination of G_2/G_1 . In the analysis of experimental data, the measured γ -ray spectrum is compared with simulated spectra for various G_2/G_1 values, and then G_2/G_1 is determined. In the fit procedure, good separation between the reduced and slow peaks in the γ -ray spectrum as shown in Fig. 4 is critical in determining continuous background contributions in measured spectra. Therefore, the thickness of the degrader should be optimized so that the separation between the reduced and slow peaks is large enough to resolve the background contributions.

In addition, uncertainty in the ratio of the γ -ray detection efficiencies $\varepsilon_2/\varepsilon_1$ should be considered because it is involved in the uncertainty of $G_2/G_1(=(g_2/g_1)/(\varepsilon_2/\varepsilon_1))$. The typical energy difference of 50 keV between the fast and reduced peaks for a 500-keV transition corresponds to 4.9% difference between ε_2 and ε_1 ($\varepsilon_2/\varepsilon_1 = 1.049$) for GREY [49, 50].

To control systematic errors of $\approx 1\%$ for G_2/G_1 , this energy dependence of the efficiencies needs to be well understood within 20% accuracy. It is noted that measurements with different settings of d_1 and d_2 provide results for different G_2/G_1 values, which can be used to evaluate the systematic uncertainty in G_2/G_1 .

- *Thickness of the reaction target*

A thicker reaction target leads to a smaller error in $\langle\sigma_I^{\text{ex}}\rangle$ by reducing the components δ_{G_2/G_1} , δ_τ , δ_{t_1} , and δ_{t_2} as shown in Fig. 2. Therefore, the thickest possible reaction target should be chosen, as long as the excited products are not stopped within the reaction target or degrader.

- *Energy of the secondary beam*

To adopt a thicker reaction target, the energy of the secondary beam should be as high as the recoil distance method allows. On the other hand, a beam with a higher energy produces a larger background component in the γ -ray spectrum in general, and thus such a component should be considered quantitatively for each specific experimental case.

- *Plunger device*

The uncertainty in the absolute distance d_1 or d_2 is $\approx 30\ \mu\text{m}$ [32], which corresponds to $\approx 0.3\%$ with respect to $d_1 = d_2 = 1\ \text{cm}$. The uncertainty in d_1 or d_2 is propagated in the error in the flight time t_1 or t_2 . Therefore, d_1 and d_2 should be at least 1 cm.

- *Lifetime of excited states*

The ranges in d_1 and d_2 available with the plunger device restrict the lifetimes for which we can apply our method. The lifetime should be at least as long as the flight time t_1 or t_2 , but short enough to detect sufficient γ rays. For the experimental setup combining the TRIPLEX and GRETINA, required lifetimes are in the range ≈ 100 – $1000\ \text{ps}$. As discussed in Sec. 4, the uncertainty in τ has a large contribution to the final error in $\langle\sigma_I^{\text{ex}}\rangle$. Therefore, the uncertainty of $\approx 2\%$ or less is required.

Consequently, we conclude that the experimental setup using the TRIPLEX and GRETINA is able to obtain the interaction cross sections for the identification of excited halo states if lifetime of the states ranges from ≈ 100 ps to ≈ 1000 ps and its value is known with precision of better than $\approx 2\%$. The use of a thicker reaction target is also advantageous to increase the interaction events relative to the γ -decay contributions.

6. Summary

In summary, a new method has been proposed to measure the interaction cross section of nuclei in the excited states. Combination of the transmission method and recoil distance Doppler-shift method allows for the counting of excited-state interactions in a reaction target. Formulae were derived to calculate the interaction cross section and its uncertainty. The uncertainty was found to depend strongly on the uncertainties in the lifetime τ of the excited state and γ -ray yields G_2/G_1 . We illustrated prototype experiments to study the feasibility and the impact of uncertainty in each observables. We concluded that the precise data of the lifetime or the use of a thicker reaction target can realize the actual measurement. This method provides a first opportunity to perform cross section measurements on nuclear excited states, and these measurements are especially important in obtaining evidence of halo structures in such states.

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Figure 1: Illustrations of (a) the experimental setup, (b) the number of excited states $N_{\text{ex}}(x)$ as a function of the position x along the beam axis, and (c) expected Doppler-shift corrected γ -ray spectra are shown. Different Doppler-shift components in panels (a) and (c) are termed fast, reduced, and slow. In panel (b), the time axis t is shown in top as an additional reference.

Figure 2: Contributions from the individual components to the total error $\Delta\langle\sigma_I^{\text{ex}}\rangle$ (see Eq. (22)) are shown as a function of the reaction target thickness x_t for an interaction cross section of 1 b. A 1% uncertainty is assumed for the individual quantities ρ , x_t , G_2/G_1 , $\langle 1/v \rangle$, τ , t_1 , and t_2 , and 0.01-b uncertainty is considered for $\langle\sigma_{\text{prod}}^{\text{be}\rightarrow\text{ex}} \frac{N_{\text{gs}}(x)}{N_{\text{ex}}(x)}\rangle$, $\langle\sigma_{\text{inel}}^{\text{gs}\rightarrow\text{ex}} \frac{N_{\text{bs}}(x)}{N_{\text{ex}}(x)}\rangle$, and $\langle\sigma_{\text{inel}}^{\text{ex}\rightarrow\text{gs}}\rangle$.

Figure 3: The simulated Doppler-shift corrected γ -ray spectra of $^AZ^*$ are shown for $\langle \sigma_I^{\text{ex}} \rangle = 1.0$ b (solid black line), 1.5 b (red band), and 2.0 b (dotted blue line), respectively. The red band represents the error of 7.9% in $\langle \sigma_I^{\text{ex}} \rangle$ of 1.5 b. The angular cutoff of $\theta_{\text{lab}} < 40^\circ$ is applied to obtain good separation between the different Doppler-shift components (f: fast, r: reduced, and s: slow).

Figure 4: The simulated Doppler-shift corrected γ -ray spectra of $^{17}\text{F}(1/2^+)$ for $\langle\sigma_I^{\text{ex}}\rangle = 1.2$ b is shown as black histogram. The angular cutoff of $\theta_{\text{lab}} < 40^\circ$ is applied. The solid red line shows the fit to the histogram based on simulated three components: fast peak (dot-dashed magenta line), reduced peak (dashed green line), and background (dotted blue line), which come from decay components behind the production target, behind the reaction target, and inside of the production and reaction targets, respectively.