The effect of deadtime and electronic transients on the predelay bias in neutron coincidence counting

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Abstract

In neutron coincidence counting using the shift register autocorrelation technique, a predelay is inserted before the opening of the (R+A)-gate. Operationally the purpose of the predelay is to ensure that the (R+A)- and A-gates have matched effectiveness, otherwise a bias will result when the difference between the gates is used to calculate the accidentals corrected net real coincidence rate. The necessity for the predelay was established experimentally in the early practical development and deployment of the coincidence counting method. The choice of predelay for a given detection system is usually made experimentally, but even today long standing traditional values (e.g., 4.5 μ s) are often used. This, at least in part, reflects the fact that a deep understanding of why a finite predelay setting is needed and how to control the underlying influences has not been fully worked out. In this paper we attempt to gain some insight into the problem. One aspect we consider is the slowing down, thermalization, and diffusion of neutrons in the detector moderator. The other is the influence of deadtime and electronic transients. These may be classified as non-ideal detector behaviors because they are not included in the conventional model used to interpret measurement data. From improved understanding of the effect of deadtime and electronic transients on the predelay bias in neutron coincidence counting, the performance of both future and current coincidence counters may be improved.

Keywords: predelay; neutron coincidence counting; deadtime

Introduction and background

Passive neutron coincidence counting performed using shift register time correlation analysis logic, is a way of extracting correlated information from a train of detected neutron events. It is routinely applied to the determination of plutonium mass for nuclear safeguards and waste management. The shift register technique is a form of autocorrelation analysis using event triggered coincidence gates. Fast

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neutrons emerging from an item and entering a moderated detector assembly, may become quickly thermalized, and then diffuse over a relatively long period during which they might be detected in thermal neutron detectors embedded in the moderator. Bursts of neutrons will, on the average, remain correlated in time and will be detected over a period commensurate with the average lifetime of neutrons in the detector (usually characterized by an effective die-away time parameter). Mathematically, the action of the shift register can be explained as follows. Following a short predelay, each detected event triggers the opening of a coincidence gate, the (R+A)-gate. A second gate, the A-gate, of equal duration is opened after a long delay (i.e., typically of the order of 5-10 or more die-away times). Every event present in the coincidence gate is counted as a coincidence (one pair) in association with the triggering event, and the number of coincidences recorded on a scaler counter is incremented accordingly. The coincidence rate is also referred to as the doubles rate and we use the two interchangeably in this paper. The predelay is normally much shorter than the characteristic neutron die-away time. Consequently the (R+A)-gate, being close in time to the trigger and hence to the initiating fission burst, gathers genuine or real coincidences associated in time with the trigger along with accidental or chance coincidences. This explains the name (R+A), which stands for (reals plus accidentals). The long delay is many times longer than the die-away time and so the events falling into the A-gate may be treated as being all random chance coincidences, that is with no relation to the trigger (i.e., the neutron events in the A-gate do not originate from the same fission burst of neutrons responsible for the trigger event). The difference between the signal triggered (R+A) and A rates gives the net genuine (or) reals coincidence rate. The width of the coincidence gate is chosen to be commensurate with the die-away time of thermal neutrons in the detector so that there is a near optimum proportion of reals to accidentals events to achieve the best counting precision in a given assay time.^[1] The method of neutron coincidence counting has been known since the 1970s,^[2,3,4] but it was the development of the shift register implementation in hardware that enabled high-rate and precise coincidence measurements to be made on a broad range of items of practical interest.^[3,5] Today the same kind of analysis can be achieved by analyzing list mode (time stamp) data via off-line analysis in a reasonable computational time. In addition, instead of using a single signal triggered accidentals gate, over sampling of the accidentals at a high rate can be used to achieve a useful incremental improvement in precision.^[6,7]

The necessity for a finite predelay was recognized during the early development of neutron coincidence counting to overcome the cause of disparities directly observed in the (R+A)-gate. The most straightforward way to study the effect is by experimentally counting a pseudo-random neutron emitter, for example an (α ,n) radionuclide source comprising AmB or AmLi. In this case, the reals coincidence rate is expected to be low, arising from induced fission of Americium in the α -source and perhaps some due to (n,2n) reactions (although far less than from AmBe which has both a harder spectrum and a significant Be(n,2n) probability).^[8,9] Consequently, we would not expect any significant difference between the (R+A)- and A-rates. From such a measurement a bias factor (B) can be defined, which would be zero in the ideal case. We define this bias factor as follows:

$$B = \frac{(R + A)}{A} - 1, \quad (1)$$

where (R+A) is the reals plus accidentals count rate and A is the accidentals count rate as recorded by the action of a shift register module.

The best setting for the predelay is as short as possible consistent with achieving a bias factor close to zero and in the region where the bias factor no longer varies with predelay.^[10] A larger than necessary predelay value results in an undesirable loss of the real signal and so the real to accidentals ratio is adversely affected. The penalty of a longer predelay is worse for shorter gates and hence for detectors with shorter decay times. Depending on the counting rate, a bias factor close to zero is required. We can appreciate this from the following argument: If the singles counting rate is S counts per second, then the accidentals gate is opened S times per second and the mean number of events in the coincidence gate is $S \cdot t_G$ where t_G is the duration of the gate. The accidentals rate is therefore $A = S^2 \cdot t_G$. When a bias is present, the bias in the net real rate may therefore be estimated as follows:

$$R_B = B \cdot A = B \cdot S \cdot (S \cdot t_G). \quad (2)$$

Suppose $B = 10^{-5}$, $S = 10^5$ Hz and $t_G = 64 \times 10^{-6}$ s, which are not atypical values; then $R_B = 6.4$ doubles per second. Whether this amount of bias in the net real rate matters or not depends on the magnitude of the real signal coming from the items, and so it must be assessed on a case by case basis. Although far from commonplace, in some analysis software, such as (former) Canberra-Harwell HiNA, provision is made to compensate for the bias by calculating the bias corrected net real rate from:

$$R = \left[(R + A) - \frac{A}{B} \right]. \quad (3)$$

With B set to unity, Eq. 3 returns the traditional real rate, net of chance or accidental coincidences but uncorrected for bias. The bias factor must be determined with all of the instrument settings finalized (amplifier gain matching, high-voltage, gate width, etc.). This expression is approximate because the bias factor is expected to depend on both the counting rate and on the degree of correlation on the pulse train, as we shall now discuss.

The origin of the bias and the necessity of a finite predelay has been described in various ways in the literature. Krick et al.^[11] state that the predelay is needed to allow the amplifiers to recover for detecting subsequent neutrons and that the predelay should be at least as long as the recovery time of the detector and amplifier stages, otherwise the system deadtime affects the (R+A)-gate more than the A-gate. Menlove and Swansen state that the predelay is needed to allow for the amplifier signal pulse to recover to baseline.^[12] They go on to say that for their particular measurement configuration, at high rates with short predelays, the signal pulses can fill in the predelay and be forced into apparent correlation in the (R+A)-gate, giving positive bias. Swansen lists several sources of bias including noise, uncompensated pole-zero and deadtime.^[13] The source of bias considered dominant is amplifier baseline displacement following a pulse. The distribution of pulses in amplitude occurring on this displaced baseline has a different probability of being recorded than pulses occurring much later, when the baseline is at a quiescent level. According to Swansen's explanation, if the displacement extends into the (R+A)-gate, then either a positive or negative bias will result. It is our experience with many systems over the decades that both positive and negative biases are indeed possible, depending on the system implementation. As the predelay is increased from zero, the bias may start off relatively large

with one sign, reduce in magnitude, change sign and even oscillate before approaching zero from either above or below.

The amplitude and shapes of the charge pulses collected from cylindrical ^3He -filled proportional neutron counters are quite variable^[14], with the rise time, width and fall time depending on where in the counter the neutron interacted, as well as on the orientation of the reaction product tracks in the gas relative to the central anode wire. It is imperative, therefore, that if the predelay is to be minimized, careful attention must be paid to the design of the frontend electronics and how best to match the amplifier to the proportional counter and ensure rapid and smooth restoration of the baseline using bipolar shaping and other techniques.

Assuming that the frontend electronics have been carefully designed and engineered so that baseline distortion can be neglected, a shorter predelay can be used. The requirement for a predelay will not however go away completely because at some point the influence of deadtime will come into play. The predelay acts on the combined pulse train from all of the counting chains comprising the system, and the effective system deadtime can therefore be reduced by distributing the detection efficiency between many separate amplifiers.^[15] This adds cost and complexity to the system, so a balance must be struck. For a given application, it is unlikely to be worth trying to eliminate the system deadtime beyond a certain point. That point will depend on the quality data objectives of the measurements and will be governed by the magnitude of the deadtime correction and how accurate the deadtime formalism is believed to be. Presently no guidance exists on how this balance might be decided upon because no formal derivation of the deadtime effect bias has been made for realistic behaviors. It should be noted that with current coincidence counter designs, having a predelay shorter than the system deadtime may not be practical since on average the effective deadtime caused by a single event is inversely proportional to the number of amplifiers used in the coincidence counter system.^[16] However, if future coincidence counter designs use a very large number of amplifiers (tending to one per proportional counter), the average effective system deadtime may be small enough that having a predelay shorter than the system deadtime may be acceptable.

For the present discussion we do not consider ^3He proportional counters to have an intrinsic deadtime *per se*. This is because the physics of analog signal formation is based upon the drifting of charges and charge induction via the Shockley-Ramo theorem, so that analog signals will continue to be formed when charged particles from several $^3\text{He}(n,p)$ reactions are present in the counter gas at the same time. In other words, until charge saturation becomes dominant, the analog signal will simply be the sum of the individual contributions. Instead, the deadtime losses for individual pulse counting can be considered to arise in the discriminator. An individual analog pulse is detected when it crosses some threshold, and another pulse cannot be detected as a distinct event until the signal has fallen below the threshold again. Overlapping analog pulses, therefore, can extend the period of time spent above the electronic threshold and may be recorded as just one rather than two or more events. In this sense, each individual counting chain, typically several proportional counters serviced by a single amplifier/discriminator, behaves as if an extending deadtime element is present. The deadtime per event is not fixed, however, but has a distribution of values reflecting the widths of the pulses and the way they overlap, but we shall assume here that a single effective value, equal for all of the counting chains, is an adequate first order description. It is clear that the deadtime mechanism must have some effect on the early time counting – that is close to the preceding event. Pulse widths of between 1-

3 μ s are typical for the 25.4 mm external diameter tubes most commonly used in thermal well neutron coincidence counters^[17] so that an individual chain may be considered to be effectively dead for a period of a few microseconds after the detection of an initial neutron. The absolute magnitude and scaling of this effect is not known, and in this paper we shall address this. But, our experience suggests that the deadtime mechanism may not be sufficient to explain the short predelay time counting irregularities. In particular, for very short predelay times, the real rate may actually decrease, which would not be possible for a pure deadtime mechanism. Even in the absence of baseline distortion we anticipate an additional mechanism. In this paper we consider in more detail the mechanism for the observed early time counting irregularities, in particular the drop in the measured real rate compared to what would be expected from a pure, single exponential decay of the neutron population. In addition, we propose an additional, neutron transport, mechanism for the early counting effects. What is unknown is the magnitude of both effects and whether either is significant in the context of making practical measurements to achieve acceptable accuracy on a given system, because there are large variations between implementations.

Model

Extracting the correlated doubles rate from the pulse train is part of neutron coincidence counting. The other part is how to relate the rate to properties of the item and detector. Traditionally this has been done using a physical representation known as the point model, which yields equations that can be conveniently inverted algebraically.^[18,19,20,21] A key function in the point model is the probability as a function of time that another neutron will be detected after a triggering neutron event. Because the triggering event is the detection of a neutron in a ^3He proportional counter, and ^3He proportional counters respond primarily to thermalized neutrons in the moderator, in the most basic form the point model equations assume that the signal triggered capture time distribution can be approximated as an exponential decay. The $1/e$ time constant describing the exponential is therefore seen as a measure of the characteristic lifetime of thermal neutrons inside the detector, where neutrons may be removed from the detection system by absorption in ^3He (detection), parasitic absorption (e.g. in hydrogen, the structural materials and cadmium liner if present), or through leakage. For suitably designed detectors, this simple picture is a reasonable first order approximation^[22,23] although real systems exhibit a combination of time components reflecting the heterogeneous combination of materials and geometries used in their construction. Multiple time component models have therefore been developed,^[24] and they are useful in justifying near optimal shift register gate width setting.^[25] In special cases the point model assumptions that the neutronic behavior inside the item is instantaneous, that the item is neutronicallly isolated from the detector, and that the temporal behavior is therefore dominated by the neutron transport processes taking place in the detector are violated. In this case, special account of the item-detector coupling is needed.^[26] In general, therefore, a simple exponential model is a useful approximation but not a complete description.

In this work, we begin with a model which considers that the thermalization and detection of neutrons that enter a neutron detector is really a two-step process. First, the incoming fast neutrons from the item must suffer collisions with hydrogen atoms that reduce the neutron energy to the average thermal energy of the detector (nominally room temperature) to have a high likelihood of being detected. It should be noted that there is some probability of detection of neutrons with higher energy, especially when high pressure ^3He tubes are used (in so-called epithermal detectors), but the probability is small and will be neglected here. After reaching thermal energy, the neutrons must diffuse by a collisional

process to the ^3He detectors where they are absorbed to be detected. There are competing effects for both processes that complicate the analysis. For example, neutrons could pass fully through the detector without suffering a thermalizing collision. Thermalized neutrons can diffuse to the detector, but they can also be absorbed in the hydrogen and other materials, or diffuse out of the detector without being detected. However, for the purposes of this analysis, we ignore these effects. This model differs from the classical point model assumption, which is that only the neutron diffusion is considered and the thermalization process is ignored. Our model, in which the fast neutron population decays, feeding the thermal population which initially builds up before decaying, is more fully described elsewhere.^[26] We note also that, the deadtime effects, which we are presuming here (in the absence of baseline distortion) to be a main reason for needing the predelay, are also not part of the standard point model theory. Because the standard point model does not include baseline, deadtime or other non-ideal behavior, the need for a predelay does not emerge naturally in the theory.

The simple exponential decay describes only the neutron diffusion mechanism to the ^3He detectors.^[27] If we also consider the neutron slowing down and thermalization, then this simple exponential decay is modified. To calculate the neutron detection probability, we consider that each neutron, upon entering the detector, has a probability distribution for thermalization followed by a different probability distribution for diffusion and detection. The diffusion phase begins once the neutron is thermalized, so it is delayed by an amount corresponding to the thermalization probability distribution and is weighted by the same thermalization probability distribution. This is shown graphically in Figure 1. The thermalization process is modeled by a probability distribution. At some point on this curve, the diffusion process starts. The overall probability distribution is the sum (integral) of all possible diffusion curves, each delayed and weighted by the thermalization probability distribution. This process is calculated using the convolution integral. Therefore, the overall neutron detection probability function is the convolution of the neutron thermalization with its subsequent diffusion to the ^3He detector tube.

The typical time constant for thermalization (τ_T) in a hydrogenous moderator such as polyethylene or water is about 1-2 μs .^[28] Once thermalized, the neutron undergoes more collisions in a diffusive process until it is either absorbed by the detector, absorbed by the hydrogen, or escapes from the detector. The typical time constant for diffusion and detection (τ_D) in a high-density polyethylene moderated thermal well neutron coincidence counter is about 50 μs and occurs only subsequent to thermalization. This is the nominal 1/e die-away time for the detector. However, since both the thermalization and the diffusion processes are probabilistic, the combined process involves the convolution of the second process with the first.^[26] The timing is shown in Figure 1. The diffusion and detection process must be considered for each possible thermalization time, weighted by the thermalization probability, and summed.

Figure 1: Plot of two probability functions for neutron detection. The first (black) exponential is the thermalization; the second (gray) is the diffusion. For this plot, $\tau_T = 2 \mu s$ and $\tau_D = 50 \mu s$.

The integral to be evaluated (un-normalized) is then:

$$P(t) = \int_0^\infty \left(e^{-\frac{(t-t_1)}{\tau_D}} \right) \left(e^{-\frac{t_1}{\tau_T}} \right) dt_1, t \geq t_1, \quad (4)$$

where: $P(t)$ is the probability (un-normalized),

t is the overall time for detection,

t_1 is the time for thermalization,

τ_T is the thermalization time constant,

τ_D is the detection time constant.

The integration is readily performed using standard forms, after some manipulation (see Appendix 1), resulting in the following:

$$P(t) = \left(\frac{\tau_D \tau_T}{\tau_D - \tau_T} \right) \left(e^{-\frac{t}{\tau_D}} - e^{-\frac{t}{\tau_T}} \right). \quad (5)$$

The plot of this probability function is shown in Figure 2.

Figure 2: Plot of the convolution probability distribution assuming neutron thermalization (solid line). The thermalization time is 2 μs, and the die-awaytime is 50 μs. The dotted line is the conventional single exponential die-away shown for comparison.

From Figure 2 it is apparent that with a finite thermalization time, the early time neutron detection probability distribution can deviate strongly from the simple exponential model. When measured from the time of neutron creation (for instance supposing one has ^{252}Cf incorporated into a fission chamber which is being used for system characterization), this difference can increase or decrease the real count rate, compared to the case with instantaneous growth because of the way in which the predelay plays out over the region of changing importance. The magnitude of the effect shown in Figure 2 depends on the length of both the die-away time and the thermalization time. From a practical perspective, the uncertainty in the die-away time and thermalization time parameters is not critical. It is indeed why an empirically determined system-specific predelay is used. The predelay delays the start of the (R+A)-gate until after the point where the classical exponential and the thermalization-corrected distribution converge.

From Eq. 5, and seen in Figure 2, we arrive at the probability distribution of the detection of a single neutron. However, in neutron coincidence counting we are primarily concerned with pairs of detected neutrons and an event triggered coincidence gate opening. To obtain a probability distribution of a pair of correlated detected neutrons,^[29] Eq. 6 is used:

$$P(t)_d = 2 \int_0^{t_G} P(t)P(t + t_1) dt_1, \quad (6)$$

where: $P(t)_d$ is the correlated detection probability (un-normalized), and t_G is the gate width.

After carrying out the integration in Eq. 6, $P(t)_d$ becomes:

$$P(t)_d = 2 \left(\frac{\tau_D \tau_T}{\tau_D - \tau_T} \right)^2 \left(e^{-\frac{t}{\tau_D}} - e^{-\frac{t}{\tau_T}} \right) \left[\tau_D \left(e^{-\frac{t}{\tau_D}} - e^{-\frac{t+t_G}{\tau_D}} \right) - \tau_T \left(e^{-\frac{t}{\tau_T}} - e^{-\frac{t+t_G}{\tau_T}} \right) \right]. \quad (7)$$

When plotting Eq. 7, shown in Fig. 3, we can see that its general shape is similar to that of Eq. 5 but more steeply increases and decreases from its maximum at 5.5 μs . This maximum is slightly smaller than that of Eq. 5, which occurs at approximately 6.5 μs .

Figure 3: Plot of the correlated detection probability, $P(t)_d$. The thermalization time is 2 μs , the deadtime is 50 μs , and the gate width is 64 μs .

Evaluation of the impact on the reals rate

In this section we quantify the net loss of reals by thermal population build-up and the deadtime effects as function of predelay.

In the case of signal build-up, the proportion of reals present on the pulse train that are observed within the shift register gate has been derived elsewhere.^[26] The result is:

$$f = 2 \frac{\lambda_0^2 \lambda_1}{(\lambda_0 - \lambda_1)^2} \left[\frac{1}{2\lambda_1} - \frac{1}{\lambda_0 + \lambda_1} \right] f_1 - 2 \frac{\lambda_0 \lambda_1^2}{(\lambda_0 - \lambda_1)^2} \left[\frac{1}{\lambda_0 + \lambda_1} - \frac{1}{2\lambda_0} \right] f_0, \quad (8)$$

where we have introduced as a convenience the short hand notation:

$$f_j = e^{-\lambda_j t_p} (1 - e^{-\lambda_j t_g}), \text{ for } j=0, \text{ and } 1,$$

which we recognize as being the expression for the gate fraction for a pure exponential decay profile with 1/e time constant $\tau_j = 1/\lambda_j$, where subscript 0 refers to the time constant of the in-growth or feed process and subscript 1 refers to the decay process.

In the limit $\lambda_0 \rightarrow \infty$, this expression for f collapses to previously established results^[24] for instantaneous build-up, namely $f = f_1$. Rearranging the expression in terms of the 1/e time constants, we obtain the alternative but equivalent expression:

$$f = \frac{1}{(\tau_1 - \tau_0)^2} \left[f_1 \tau_1^2 \left(1 - \frac{2}{1 + \tau_1/\tau_0} \right) - f_0 \tau_0^2 \left(\frac{2}{1 + \tau_0/\tau_1} - 1 \right) \right]. \quad (9)$$

In the case of the deadtime effect, the probability distribution for the detection of a neutron following a trigger must first be modified by the effect that once a ³He proportional counter – amplifier/discriminator combination has detected a neutron, it cannot detect any subsequent neutrons until the counting chain has recovered. The recovery time may be specified in the amplifier/discriminator unit, which automatically resets after some fixed time, t_R . If there is a number, n , equivalent counting chains in a neutron coincidence counter, then the multiplicative factor effect on the detector efficiency when one chain becomes “dead” is proportional to: $(n-1)/n$. However, the chain is inactive only until the reset time, t_R . Therefore, when considering the signal triggered counts, namely the (R+A) gate, we can model the reduction effect of the system efficiency by the factor:

$$\frac{n-1}{n} u(t - t_R), \quad (10)$$

where $u(t)$ is the usual unit-step function, or the Heaviside step function, which as $u(t-t_R)$ is zero for times less than the recovery period t_R and equal to unity for times which exceed the recovery time. Note we are considering only burst losses here, or losses associated with isolated clusters of correlated neutrons, and not the losses that come about at high rates when time random and fission neutrons are strongly intermingled.

To calculate the deadtime effect on the signal triggered gate in this model, we perform the same gate fraction integral modified by this deadtime factor. For simplicity we shall replace $P(t)_d$ by the simple exponential form to illustrate the approach and isolate the effect of deadtime. The integral is:

$$f = \int_{t_P}^{t_P+t_G} e^{-\frac{t}{\tau_D}} \left[1 - \frac{u(t)}{n} + \frac{u(t - t_R)}{n} \right] dt, \quad (11)$$

where f is the gate fraction, τ_D is the die-away time, t_P is the predelay time, t_G is the gate width, and t_R is the reset time.

The corresponding result for f , calculations shown in Appendix 2, is as follows:

$$f = e^{-\frac{t_P}{\tau_D}} \left(1 - e^{-\frac{t_G}{\tau_D}} \right) - \frac{u(t_R - t_P)}{n} \left(e^{-\frac{t_P}{\tau_D}} - e^{-\frac{t_R}{\tau_D}} \right), \quad (12)$$

where $u(t_R - t_P)$ is the step function in the variable t_R translated by t_P predelay time, which value is one for positive arguments otherwise zero (see also Appendix 2).

Note that the first term of this expression is identical to the standard form of the gate fraction for a detector with a simple exponential die-away profile.^[24] The second term is the first order correction for the deadtime effect. Just as the case with introduction of the thermalization time, this second term only has an effect for short predelay times, less than the tube reset time: $t_P < t_R$. Similarly, for the case of thermalization, we observe that the thermalization curve and the conventional die-away curve asymptotically joined after a few thermalization times have passed: $t_P \geq \sim 3\tau_T$. This behavior is identical for both processes; the effects are only seen for times smaller than normally chosen predelay times. Therefore, the predelay solves both problems and, if present, baseline distortion such as ringing on the analog signal.

Both the thermalization effect and the deadtime effect govern the gate fraction. However, the magnitudes may be significantly different. The effective system deadtime correction is proportional to $1/n$, the inverse number of counting chains comprising the neutron coincidence counter. The thermalization effect, as can be appreciated from inspection of Figure 3 and from the expression for f , depends on the growth period, die-away time and the details of the gating structure (predelay and gate). The optimum choice of gate width also depends on the growth period, although a value of about 1.2 times the die-away time will generally remain reasonable^[25] because the growth period is generally much smaller than the die-away time.

Extension and numerical examples

There is another difference that the thermalization and deadtime model brings to the picture of neutron coincidence counting that can be tested. The calculation above for the deadtime correction was relatively simple in that it only considered the reduction of the reals in the signal triggered (R+A)- gate, and made a first order computation which does not consider the pile-up of dead periods. However, the signal-triggered gate is also triggered by uncorrelated neutrons. Therefore, the (R+A) count should be reduced even for a purely uncorrelated neutron stream; i.e., for a radionuclide neutron source such as $\text{AmLi}(\alpha, n)$. Moreover, the model above does not include the possibility that the deadtime effect might change with neutron count rate; the higher the neutron count rate, the more pronounced the deadtime effect. For the calculation of the reals rate exclusively, the model above may be sufficient in some cases because the reals rate is relatively small compared to the (R+A) rate. Therefore, the count rate changes might not be manifest on the reals count for modest count rates. However, the larger (R+A) effect would be count rate dependent, and it would affect the measured reals rate because the accidentals component of the signal triggered gate (R+A) would be suppressed, but the accidentals measured in the random triggered gate (A) would not be. These effects can be estimated directly.

We begin our study of these possibilities by comparing the effective reduction in the real rate for nominal (i.e., modest) count rates in a thermal well neutron counter. The predelay can be reduced progressively while the gate width is held constant. We can then calculate the relative reduction in the real rate compared with the ideal case and compare to the measured results. Consider then the case for a typical gate time, $t_G = 64 \mu s$. We will consider the ratio of no predelay to the normal predelay, and how the real rate will be affected. There are three possible cases: (1) the ideal case, (2) the effect is due to deadtime, or (3) the effect is due to the thermalization described above.

1) Ideal case

In the ideal case there is no bias effect. From this the ratio would be:

$$\frac{f}{f_1} = \frac{\int_0^{t_G} e^{-t/\tau_D} dt}{\int_{t_P}^{t_P+t_G} e^{-t/\tau_D} dt} \quad (13)$$

In this case, the ratio is just the effect of extending the integration over the entire exponential decline. Note that the ratio, numerically, is equal to e^{t_P/τ_D} . This is greater than unity because we are including the neutrons normally blocked by the predelay.

2) Due to deadtime

The assumption here is that once a neutron detector tube has detected a neutron it is effectively unusable for a prolonged period of time. Therefore, the predelay is used to accommodate the detector deadtime. For the deadtime case, the ratio would be:

$$\frac{f}{f_1} = \frac{e^{-\frac{t_P}{\tau_D}} \left(1 - e^{-\frac{t_G}{\tau_D}} \right) - \frac{u(t_R - t_P)}{n} \left(e^{-\frac{t_P}{\tau_D}} - e^{-\frac{t_R}{\tau_D}} \right)}{e^{-\frac{t_P}{\tau_D}} \left(1 - e^{-\frac{t_G}{\tau_D}} \right)} \quad (14)$$

In this case, it is the same as for the ideal case, except that we have accounted for the deadtime of the single counting chain that detected the first neutron. When the predelay is less than the reset time, this ratio is less than unity because events are lost to the deadtime. When the predelay is equal to or greater than the reset time, the ratio is unity.

3) Due to the thermalization

The effect of initial thermalization of neutrons gives rise to the following ratio:

$$\frac{f}{f_1} = \frac{1}{(\tau_1 - \tau_0)^2} \left[\tau_1^2 \left(1 - \frac{2}{1 + \tau_1/\tau_0} \right) - \frac{f_0}{f_1} \tau_0^2 \left(\frac{2}{1 + \tau_0/\tau_1} - 1 \right) \right] \quad (15)$$

418

419 The value of this ratio depends on the exact combination of parameters. In general, a large die-away
 420 time will result in a small ratio, and a small gate time will also result in a small ratio. The tube reset time
 421 has no effect on the ratio, which is expected since it is not part of the equation.

422

423 Now let us consider some numerical examples that will illustrate these three cases. We shall assume
 424 the following parameters which are typical for a thermal well neutron counter:

425

- 426 • Die-away time: $\tau_D = 50 \mu s$
- 427 • Thermalization time: $\tau_T = 2 \mu s$
- 428 • Recovery/reset/deadtime: $t_R = 1 \mu s$ per event for each chain
- 429 • Number of counting chains in the system: $n = 6$
- 430 • Predelay time: $t_p = 0$ to $4.5 \mu s$
- 431 • Gate time: $t_G = 64 \mu s$

432

433 The numerical results are summarized in Table 1. From these numerical examples we can see that in
 434 Case 1 the fractional gains from reducing the predelay can be fairly substantial. For Case 2, we can see
 435 the effect of deadtime is rather small compared to that of Case 1, even as the predelay approaches the
 436 limiting value of $0 \mu s$. For Case 3, the effect of thermalization is even smaller than that of deadtime.
 437 The thermalization effect does have the interesting property of being less than unity for predelay times
 438 less than approximately $0.68 \mu s$ and larger than unity for predelay values bigger than approximately
 439 $0.68 \mu s$, for the typical thermal well neutron counter values used.

440

Predelay $t_p (\mu s)$	Ratio (f/f_1)			Gate fraction (f)		
	Case 1 Ideal case	Case 2 Due to deadtime	Case 3 Due to the thermalization	Case 1 Ideal case	Case 2 Due to deadtime	Case 3 Due to the thermalization
0.00	1.00000	0.99543	0.99938	0.72196	0.71866	0.72152
0.25	1.00501	0.99656	0.99963	0.72196	0.71589	0.71810
0.50	1.01005	0.99770	0.99986	0.72196	0.71314	0.71468
0.75	1.01511	0.99885	1.00005	0.72196	0.71040	0.71125
1.00	1.02020	1.00000	1.00023	0.72196	0.70767	0.70783
1.25	1.02532	1.00000	1.00038	0.72196	0.70414	0.70441
1.50	1.03045	1.00000	1.00052	0.72196	0.70063	0.70099
1.75	1.03562	1.00000	1.00064	0.72196	0.69713	0.69758
2.00	1.04081	1.00000	1.00075	0.72196	0.69365	0.69418

2.25	1.04603	1.00000	1.00085	0.72196	0.69019	0.69078
2.50	1.05127	1.00000	1.00093	0.72196	0.68675	0.68739
2.75	1.05654	1.00000	1.00101	0.72196	0.68333	0.68402
3.00	1.06184	1.00000	1.00108	0.72196	0.67992	0.68065
3.25	1.06716	1.00000	1.00114	0.72196	0.67653	0.67730
3.50	1.07251	1.00000	1.00119	0.72196	0.67315	0.67395
3.75	1.07788	1.00000	1.00124	0.72196	0.66980	0.67062
4.00	1.08329	1.00000	1.00128	0.72196	0.66646	0.66731
4.25	1.08872	1.00000	1.00131	0.72196	0.66313	0.66400
4.50	1.09417	1.00000	1.00135	0.72196	0.65982	0.66071

Table 1. Illustrative numerical values for the three cases for the parameter set given in the text.

Conclusions

This work represents an initial discussion and preliminary theoretical investigation based on a rudimentary physical picture into the underlying causes and quantification of predelay effects. The practical application of neutron coincidence counting has proceeded empirically dealing with biases in an approximate *ad hoc* way, by bundling the effects into calibration functions, or by including them by default into empirical rate corrections. We have discussed the important role that the design of the frontend electronics coupled to the traditional ^3He filled proportional counter plays in the minimization of predelay bias. This is an aspect we feel deserves additional attention in future system development because it is possibly the dominant reason for non-ideal detector behavior. A theoretical model of the predelay effects was evaluated with three specific numerical examples. These examples helped reveal that the predelay effects require a far more complete picture of the detailed signal formation stages. For example, the required predelay is not fixed but varies according to an implementation specific distribution. In other words, the pre-delay should be evaluated and varied on an application-specific basis. Considerably more work is needed in understanding the formation of the signal from neutron coincidence counters. This theoretical model would also benefit from more examples and testing, specifically from measured and simulated data streams. By improved understanding of the effect of deadtime and electronic transients on the predelay bias in neutron coincidence counting, the performance of both future and current coincidence counters maybe improved. In particular, as we move towards using neutron coincidence and neutron multiplicity methods in an absolute way for highly accurate reference source characterization, it is imperative that these long neglected fine details be better understood and quantified.

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Appendix 1. Evaluation of the P(t) integral

The integral to be evaluated (un-normalized) is:

$$P(t) = \int_0^{\infty} \left(e^{-\frac{(t-t_1)}{\tau_D}} \right) \left(e^{-\frac{t_1}{\tau_T}} \right) dt_1, t \geq t_1,$$

591

592 where: $P(t)$ is the probability(un-normalized),

593 t is the overall time for detection,

594 t_1 is the time for thermalization,

595 τ_T is the thermalization time constant,

596 τ_D is the detection time constant.

597

598 Separating terms:

$$P(t) = \int_0^{\infty} \left(e^{-\frac{t}{\tau_D}} \right) \left(e^{\frac{t_1}{\tau_D}} \right) \left(e^{-\frac{t_1}{\tau_T}} \right) dt_1, t \geq t_1$$

599 Extract terms which are not a function of dt_1 out of the integral and simplify to obtain:

$$P(t) = e^{-\frac{t}{\tau_D}} \int_0^{\infty} e^{-\left(\frac{t_1}{\tau_T} - \frac{t_1}{\tau_D}\right)} dt_1, t \geq t_1$$

600 To simplify the notation, define:

$$\gamma \equiv \frac{1}{\tau_T} - \frac{1}{\tau_D}.$$

601 Substitute and change the limit of integration to reflect: $t \geq t_1$:

$$P(t) = e^{-\frac{t}{\tau_D}} \int_0^t e^{-\gamma t_1} dt_1$$

602 Perform the integration:

$$P(t) = e^{-\frac{t}{\tau_D}} \left(\frac{-1}{\gamma} \right) e^{-\gamma t_1} \Big|_0^t$$

603 Evaluate and simplify:

$$P(t) = e^{-\frac{t}{\tau_D}} \left(\frac{1}{\gamma} \right) (1 - e^{-\gamma t})$$

604 Multiply through and collect exponential terms:

$$P(t) = \left(\frac{1}{\gamma} \right) \left(e^{-\frac{t}{\tau_D}} - e^{-t\left(\frac{1}{\tau_D} + \gamma\right)} \right)$$

605 Simplify the second exponent:

$$\frac{1}{\tau_D} + \gamma = \frac{1}{\tau_D} + \frac{1}{\tau_T} - \frac{1}{\tau_D} = \frac{1}{\tau_T}.$$

606 Back substitute

$$P(t) = \left(\frac{1}{\gamma}\right) \left(e^{-\frac{t}{\tau_D}} - e^{-\frac{t}{\tau_T}}\right)$$

607

608 Appendix 2. Evaluation of the predelay effect integral

609

$$f = \int_{t_P}^{t_P+t_G} e^{-\frac{t}{\tau_D}} \left[1 - \frac{u(t)}{n} + \frac{u(t-t_R)}{n}\right] dt,$$

610

611 where: f is the gate fraction,

612 τ_D is the die-away time,

613 t_P is the predelay time,

614 t_G is the gate width,

615 t_R is the tube reset time.

616 and $u(t)$ is the unit step function whose value is zero for $t < 0$, and 1 for $t \geq 0$.

617 Solving the integral, the follow results are obtained:

$$f = \left(1 - \frac{1}{n}\right) \int_{t_P}^{t_P+t_G} e^{-\frac{t}{\tau_D}} dt + \begin{cases} \left(\frac{1}{n}\right) \int_{t_P}^{t_P+t_G} e^{-\frac{t}{\tau_D}} dt & \text{if } t_P \geq t_R \\ \left(\frac{1}{n}\right) \int_{t_R}^{t_P+t_G} e^{-\frac{t}{\tau_D}} dt & \text{if } t_P < t_R \end{cases}$$

$$f = \begin{cases} \int_{t_P}^{t_P+t_G} e^{-\frac{t}{\tau_D}} dt & \text{if } t_P \geq t_R \\ \left(1 - \frac{1}{n}\right) \int_{t_P}^{t_P+t_G} e^{-\frac{t}{\tau_D}} dt + \left(\frac{1}{n}\right) \int_{t_R}^{t_P+t_G} e^{-\frac{t}{\tau_D}} dt & \text{if } t_P < t_R \end{cases}$$

618 Perform the integration, normalize (i.e. multiply by τ_D) and put into standard-point model form:

$$f = \begin{cases} e^{-\frac{t_P}{\tau_D}} \left(1 - e^{-\frac{t_G}{\tau_D}}\right) & \text{if } t_P \geq t_R \\ e^{-\frac{t_P}{\tau_D}} \left(1 - e^{-\frac{t_G}{\tau_D}}\right) - \left(\frac{1}{n}\right) \left(e^{-\frac{t_P}{\tau_D}} - e^{-\frac{t_R}{\tau_D}}\right) & \text{if } t_P < t_R \end{cases}$$

619 Finally, by means of the properties of the step function $u(t_R - t_P)$, whose value is one for positive argument

620 otherwise zero, we can re-write f in compact form as follows:

$$f = e^{-\frac{t_P}{\tau_D}} \left(1 - e^{-\frac{t_G}{\tau_D}}\right) - \frac{u(t_R - t_P)}{n} \left(e^{-\frac{t_P}{\tau_D}} - e^{-\frac{t_R}{\tau_D}}\right)$$