Analysis of Coupled-Bunch Instabilities for the NSLS-II Storage Ring with a 500 MHz 7-Cell PETRA-III Cavity

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Abstract

The NSLS-II storage ring is designed to operate with superconducting RF-cavities with the aim to store an average current of 500mA distributed in 1080 bunches, with a gap in the uniform filling for ion clearing. At the early stage of the commissioning (phase 1), characterized by a bare lattice without damping wigglers and without Landau cavities, a normal conducting 7-cell PETRA-III RF-cavity structure has been installed with the goal to store an average current of 25mA. In this paper we discuss our analysis of coupled-bunch instabilities driven by the Higher Order Modes (HOMs) of the 7-cell PETRA-III RF-cavity. As a cure of the instabilities, we apply a well-known scheme based on a proper detuning of the HOMs frequencies based upon cavity temperature change, and the use of the beneficial effect of the slow head-tail damping at positive chromaticity to increase the transverse coupled-bunch instability thresholds. In addition, we discuss measurements of coupled-bunch instabilities observed during the phase 1 commissioning of the NSLS-II storage ring. In our analysis we rely, in the longitudinal case, on the theory of coupled-bunch instability for uniform fillings, while in the transverse case we complement our studies with numerical simulations with OASIS, a novel parallel particle tracking code for self-consistent simulations of collective effects driven by short and long-range wakefields.

Keywords: Coupled-bunch, Instability, HOMs, Head-tail

1. Introduction

To help the transition to the commissioning of the NSLS-II storage ring with superconducting CESR-B 500MHz single RF-cavities, with the final goal to store the nominal average current $I_{av} = 500\text{mA}$, in the phase 1 commissioning a normal conducting 7-cell PETRA-III RF-cavity structure [1] has been used with the aim to accumulate a maximum average current $I_{av} = 25\text{mA}$. Concerns of potential coupled-bunch instabilities driven by the HOMs of the PETRA-III cavity, which was designed without HOM dampers, have motivated the analysis of coupled-bunch instabilities with the goal to prevent unstable multi-bunch operations during the phase 1 commissioning of the NSLS-II storage ring, characterized by a bare lattice without damping wigglers and without Landau cavities [2], [3], with parameters given in Table 1. The analysis presented in this paper is based on a well-known scheme to cure coupled-bunch instabilities [4]-[10]. Besides presenting our analysis, we discuss measurements of coupled-bunch instabilities observed during the phase 1 commissioning of the NSLS-II storage ring [11],[12]. Our analysis relies on

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the coupled-bunch instability for uniform fillings. A justification of this assumption is based on the work of Prabhakar [13] and Kohaupt [14]. A detailed analysis of the coupled-bunch instability driven by non-uniform fillings is discussed in a separate paper [15], where a derivation of the analytical formula for the complex frequency shift induced by arbitrary fillings is given and benchmarked against numerical simulations and measurements in the NSLS-II storage ring. For the instability threshold calculations based on the HOMs of the 7-cell PETRA-III cavity structure we rely on the numerical data computed by R. Wanzenberg [16], complemented by measurements aimed to determine the dependence of the HOM frequencies on the cavity temperature [17]. In the longitudinal case, since in the NSLS-II storage ring the cut-off frequency of the PETRA-III cavity is higher than the case considered by R. Wanzenberg, we computed HOMs up to 1390 MHz [18], as shown in Table 2, with the `GdfidL` code [19]. The beam dynamics simulation results discussed in this paper have been obtained with `OASIS` [20], a parallel code for self-consistent simulations of single and multi-bunch effects. For single-bunch effects, the code uses the same physical model of the `TRANFT` code developed by M. Blaskiewicz [21], [22], while in the case of multi-bunch effects the code implements a novel self-consistent algorithm to take into account long-range effects [23]. The novelty of the algorithm consists of the capability to simulate efficiently the self-consistent coupled-bunch interaction of bunches with finite length distributed in arbitrary filling patterns, via a proper Taylor expansion of the long-range wakefields. The algorithm can be applied to arbitrary wakefields. Codes that allow to study simultaneously short- and long-range wakefield effects, to various degree of self-consistency, are discussed in [21], [24]-[28]. The `MBTRACK` code, for example, developed by Nagaoka et al. [26], has been applied to study both single- and multi-bunch instabilities at the SOLEIL and DIAMOND light sources. Our idea to expand in Taylor series the long-range wakefields, first discussed in our paper [23], has been implemented in an earlier version of `MBTRACK` to model the longitudinal long-range interaction in the study of passive Landau cavities [29], and later abandoned in favor of a different approach, discussed in detail in the recent paper [27].

The paper is organized as follows. In Section 2 we discuss the PETRA-III RF-cavity structure and the HOMs computation. Section 3 and Section 4 are devoted to the analysis of coupled-bunch instabilities. In Section 3 we analyze the longitudinal case and rely on the analytical theory for instability calculations, with the parameters of the longitudinal HOMs as listed in Table 2. We discuss measurements of longitudinal coupled-bunch instabilities observed during the phase 1 commissioning of the NSLS-II storage ring and interpret the results according to our analysis. In Section 4 we analyze transverse coupled-bunch instabilities, with the parameters of the transverse HOMs listed in Table 3, describe the broad-band impedance model used to drive the slow head-tail effect, discuss the measurements of the HOM frequencies as a function of the cavity temperature and give a description of the capability of the `OASIS` code. We show that the beam is potentially unstable at zero chromaticity. We discuss the beneficial effect of the slow head-tail damping at positive chromaticity on the coupled-bunch instability thresholds, and investigate the HOMs frequencies detuning based upon cavity temperature change as a cure of the coupled-bunch instability. We show that the slow head-tail effect for an average current $I_{av} = 25\text{mA}$ evenly distributed in $M = 1320$ bunches, corresponding to a single bunch current $I_0 = I_{av}/M = 0.019\text{mA}$, does not provide enough damping to suppress the coupled-bunch instability for values of the chromaticity in the planned range of operation $[0, 2]$. Since the slow head-tail effect is proportional to the single bunch current I_0 , while the coupled-bunch interaction is proportional to the average current I_{av} , to make the slow head-tail effect more effective we discuss

Figure 1: RF-straight section of Cell 24. Phase 1 commissioning of the NSLS-II Storage Ring.

fillings with a larger single bunch current. Specifically, we consider the case $M = 132$ bunches and single bunch current $I_0 = 0.19\text{mA}$. In Appendix A we discuss and benchmark the analytical theory of the head-tail instability and coupled-bunch instability against numerical simulations with `OASIS`, and give a detailed description of the parallel algorithm for self-consistent simulations of the coupled-bunch instability.

2. PETRA-III RF-Cavity

In Fig.1 we show part of the layout of the high- β RF-straight section (Cell 24) of the NSLS-II storage ring during the phase 1 commissioning with a normal conducting 500MHz 7-cell PETRA-III RF-cavity. The diameter of the pipes attached to the cavity is 90mm. The straight section was set up in a way to minimize the relocation of the vacuum chambers after de-installing the PETRA-III cavity and installing the new superconducting CESR-B RF-cavity as designed for NSLS-II: a circular pipe with diameter of 240mm has been installed downstream with a tapered transition to the cavity pipe with diameter of 90mm, as it is shown in Fig.1. There is no tapered transition from the cavity to the regular vacuum chamber, upstream of the section, where instead a step transition has been applied from the round profile to the octagonal profile at the location of the bellows. Here the circular pipe is connected to the regular vacuum chamber with dimensions $76\text{mm} \times 25\text{mm}$. The resonant modes in the cavity are trapped at frequencies lower than the cutoff frequency of the attached vacuum pipes. The cutoff frequency $f_c = c\nu_{01}/(2\pi R) = 2.6\text{GHz}$ for the E_{01} -Mode in a circular chamber with a radius $R = 45\text{mm}$, where c is the speed of light and $\nu_{01} = 2.405$ is the first zero of the Bessel function of zeroth order. For the elliptical profile, the cutoff frequency is estimated numerically using the `GdfidL` code and it is $f_c = 6.8\text{GHz}$ for the E_{01} -mode.

Since it is time consuming to calculate the whole RF-straight section, we performed studies of the HOMs in the PETRA-III cavity structure first and found out that trapped modes at frequencies below $f_c = 2.6\text{GHz}$ already have an effect on the beam stability. However more modes can be trapped at higher frequencies in the attached elliptical pipes, whose cutoff frequency is almost a factor of 3 higher.

We started our studies from time-domain simulations, where the Gaussian bunch is passing through the RF-structure of length $\sim 2\text{m}$. The real parts of the transverse and longitudinal impedances are shown in Fig.2. The beam generates many modes since the PETRA-III cavity is a 7-cell structure and each mode has 7 oscillating sub-modes. The structure has been designed without the possibility of damping the HOMs. The frequency shift

of dangerous modes by the variation of the cavity temperature will be discussed in the next sections as a solution to stabilize the beam and increase the coupled-bunch instability thresholds.

Time-domain simulations (with perfect conducting material) help us to classify and identify the resonant modes and their frequencies. It is a preparation step for further detailed analysis of the shunt impedances and quality factors using frequency-domain simulations including the material properties [30].

First, in frequency-domain, we checked the electrodynamics parameters of some of the modes from Table 2, which was sent to us by Wanzenberg [16] and produced with the 3D MAFIA code [31], then we extended, with 3D GdfidL simulations, the original list of modes from 1270 MHz to 1390 MHz. The shunt impedance is defined as ¹

$$R_{\text{sh}} = \frac{|V|^2}{P_{\text{loss}}}, \quad (1)$$

where V is the complex voltage

$$V = \int_{z_1}^{z_2} E_Z(0, 0, z) e^{i \frac{\omega z}{\beta c}} dz. \quad (2)$$

The total power loss P_{loss} is therefore $P_{\text{loss}} = \omega W / Q$, where Q is the quality factor, ω is the frequency and W is the total stored energy. The well-known ratio R_{sh}/Q depends on the geometrical dimensions only and it is convenient to use as a characteristic parameter for a cavity

$$\frac{R_{\text{sh}}}{Q} = \frac{|V|^2}{\omega W}. \quad (3)$$

Table 2 shows the electrodynamics parameters for two extra longitudinal HOMs (with first sub-mode frequency 1369.88 MHz and 1386.0 MHz respectively) of the PETRA-III 7-cell normal conducting copper (Cu) cavity. In some cases, the frequency separation between the resonant sub-modes is very small, less than 100kHz. This makes the determination of all seven resonances for each mode very difficult, due to the complexity of the structure and the high resonance frequency limit required for the eigenvalues computation ($\sim 2\text{GHz}$). The frequency domain simulations discussed here do not take into account external coupling through the transmission lines used to power the fundamental mode, since GdfidL does not allow this option at the moment. We plan to investigate external coupling in the future.

3. Longitudinal Coupled-Bunch Instability

An analytical estimate of the coherent frequency shift for M equally spaced Gaussian bunches is given, for the longitudinal dipole mode $l = 1$, by Eq.(6.142) of [32], [33]

$$\Omega^{(\mu)} - \omega_s = i \frac{I_0 \eta}{8 \sqrt{\pi} E_0 \sigma_t^3 \omega_s} \frac{\sum_p Z_{||}(\omega') \omega' e^{-\omega'^2 \sigma_t^2}}{\sum_p \omega'^2 e^{-\omega'^2 \sigma_t^2}}, \quad \mu = 0, \dots, M-1, \quad (4)$$

where $\omega' = pM\omega_0 + \mu\omega_0 + \omega_s$, I_0 is the single bunch current, η the slippage factor, E_0 the energy of the reference particle in electron volts, σ_t the bunch length, ω_s the synchrotron angular frequency and $\omega_0 = 2\pi/T_0$, where T_0 is the revolution time, $\nu_s = \omega_s/\omega_0$ the synchrotron tune; $Z_{||}$ is the total longitudinal impedance of the ring. With

¹Notice that the definition of shunt impedance used in accelerator physics is adopted in this paper, which is twice the definition of shunt impedance used in engineering.

Table 1: Parameters for Threshold Calculations

Parameter	Symbol	Value	Unit
Energy	E_0	3	GeV
Revolution period	T_0	2.64	μs
Harmonic number	h	1320	
Average current	I_{av}	25	mA
Momentum compaction	α	3.7×10^{-4}	
Energy loss per turn	U_0	288	keV
RF voltage	V_{rf}	1.86	MV
Synchrotron tune	ν_s	0.0069	
Horizontal tune	ν_x	33.22	
Vertical tune	ν_y	16.26	
Transverse damping time	$\tau_{x,y}$	54	ms
Longitudinal damping time	τ_s	27	ms
Energy spread	σ_δ	5×10^{-4}	
Bunch length	σ_t	11	ps

Figure 2: Real part of the vertical dipole impedance (top frame) and the longitudinal impedance (bottom frame).

Table 2: Simulated Longitudinal HOMs Parameters of the 7-cell PETRA-III Cavity Structure.

Frequency f_r , MHz	Shunt Impedance $R_{sh, }$, MΩ	Quality Factor $Q_{ }$	$R_{sh, }/Q_{ }$, Ω
500	31.6	36900	856.4
501	0.0053	37500	0.14
502	0.0575	38100	1.51
503	0.0006	38800	0.02
505	0.0017	39800	0.04
507	0.0001	40800	0.002
508	0.0001	41400	0.002
712	0.0091	30300	0.3
716	0.0124	31100	0.4
722	1.13	32300	39.98
728	3	33600	89.29
733	0.7245	35000	20.7
738	0.479	35500	13.49
739	0.457	36000	12.69
1130	0.0002	34300	0.006
1140	0.0008	35500	0.02
1150	0.0006	37100	0.02
1170	0.025	38900	0.64
1250	0.067	55100	1.22
1260	0.155	55200	2.81
1270	0.01	49500	0.2
1369.88	0.6	39600	15.2
1369.94	0.5	39600	12.6
1370.95	1.14	39750	28.7
1386.3	1.1	39900	27.6
1386.9	0.3	39950	7.5
1387.5	1.8	39960	45
1387.3	1.3	40000	32.5
1388.1	0.2	40000	5
1389.7	0.1	40000	2.5

the condition $M\omega_0\sigma_t \ll 1$, satisfied for short bunches, the sum at the denominator of Eq.(4) can be approximated by an integral

$$\sum_{p=-\infty}^{+\infty} \omega'^2 e^{-\omega'^2 \sigma_t^2} \approx \frac{1}{M\omega_0\sigma_t^3} \int d\omega \omega^2 e^{-\omega^2} = \frac{\sqrt{\pi}}{2M\omega_0\sigma_t^3}. \quad (5)$$

According to the parameters listed in Table 1, $M\omega_0\sigma_t = 0.003$, thus to good approximation Eq.(4) reads

$$\Omega^{(\mu)} - \omega_s = i \frac{I_0 M \eta}{4\pi E_0 \nu_s} \sum_{p=-\infty}^{+\infty} \omega' e^{-\omega'^2 \sigma_\tau^2} Z_{||}(\omega'). \quad (6)$$

105 The growth rate $1/\tau^{(\mu)} \equiv \text{Im}(\Omega_\mu - \omega_s)$ of bunch mode μ therefore reads

$$\frac{1}{\tau^{(\mu)}} = \frac{I_0 M \eta}{4\pi E_0 \nu_s} \sum_{p=-\infty}^{+\infty} (pM\omega_0 + \mu\omega_0 + \omega_s) e^{-(pM\omega_0 + \mu\omega_0 + \omega_s)^2 \sigma_t^2} \text{Re}Z_{||}(pM\omega_0 + \mu\omega_0 + \omega_s). \quad (7)$$

In Section 2 we discussed the accuracy of the simulated HOMs frequencies listed in Table 2. Here, as a worse case scenario, we estimate the growth rates in Eq.(7) assuming the real part of the impedance is sampled at its peak. According to Table 2, the HOM at frequency $f_r = 728\text{MHz}$ is the HOM with the highest shunt impedance $R_{sh,||}$, and with the parameters listed in Table 1 the argument $\omega' = pM\omega_0 + \mu\omega_0 + \omega_s$ of $\text{Re}Z_{||}$ is $\omega' = 2\pi \times 727.9486\text{MHz}$ for $p = 1$ ($M=1320$ and $\mu = 603$), which differs $\approx 50\text{kHz}$ from $f_r = 728\text{MHz}$, thus within the numerical accuracy of the simulated HOM frequencies. The fastest growth time driven by the HOM at frequency $f_r = 727.9486\text{MHz}$ is therefore

$$\tau = \frac{2E_0 \nu_s}{I_{av} \eta f_r R_{sh,||}} = 2.1\text{ms}, \quad (8)$$

much smaller than the longitudinal radiation damping time $\tau_s = 27\text{ms}$, thus capable of driving a coupled-bunch instability. Notice that we do not specify the sign of τ and τ_s . In this paper we assume implicitly that growth times are positive and damping times are negative. Notice that this convention does not apply to growth rates and damping rates.

3.1. Commissioning Studies

During the phase 1 commissioning of the NSLS-II storage ring [11],[12], longitudinal coupled-bunch instabilities have been observed in multi-bunch operation (≈ 1000 bunches) at an average current $I_{av} \approx 10\text{mA}$ and chromaticity $\xi_x = +2$ and $\xi_y = +2$ [37], before properly setting the RF-cavity temperature to a value that allowed the accumulation of $I_{av} = 25\text{mA}$. Longitudinal HOMs with frequencies $f_r \approx 1374.5\text{MHz}$ and $f_r \approx 728\text{MHz}$ have been identified as driving the instability at the cavity temperature $T_c = 38.4^\circ\text{C}$ and $T_c = 39.2^\circ\text{C}$ respectively.

As discussed in Section 2, the discrepancy between the simulated HOM with frequency $f_r = 1370.95\text{MHz}$ shown in Table 2 (not present in the original list of simulated values provided by R. Weizenberg [16]) and the measured value $f_r = 1374.5\text{MHz}$, can be justified by the accuracy of the simulations and by the difference between the simulated and real geometry of the PETRA-III RF-cavity.

Although the instability has been seen with a non-uniform bunch filling pattern, in our analysis we assume that the average current is distributed evenly in 1320 bunches. A justification of this assumption is discussed in [15]. In our interpretation of the measurements, we assume that the onset of the instability is observed when the growth time induced by the HOMs is of the same order of the radiation damping time $\tau_s = 27\text{ms}$.

Figure 3: Left frames: Real part of the longitudinal impedance at $T_c = 38.4^\circ\text{C}$ (red circle) and $T_c = 39.2^\circ\text{C}$ (blue square), sampled at $\omega = pM\omega_0 + \mu\omega_0 + \omega_s$ for: a) HOM at frequency $f_r \approx 1374.5$ MHz ($p = 2$ and $\mu = 991$), and. c) HOM at frequency $f_r \approx 728$ MHz ($p = 1$ and $\mu = 603$). Right frames: Growth rates as a function of bunch mode at $T_c = 38.4^\circ\text{C}$ (red circle) and $T_c = 39.2^\circ\text{C}$ (blue square) driven by: b) HOM at frequency $f_r \approx 1374.5$ MHz and, d) HOM at frequency $f_r \approx 728$ MHz.

At the cavity temperature $T_c = 38.4^\circ\text{C}$ and $I_{\text{AV}} = 10\text{mA}$, we observed an unstable bunch mode with $\mu = 991$ driven by the HOM at frequency $f_r \approx nf_{rf} + 991f_0$ (where f_0 is the revolution frequency and $n \in \mathbb{N}$). With $n = 2$, we identified the HOM at frequency $f_r \approx 1374.5$ MHz as driving the instability. With $R_{sh,||} = 1.14\text{M}\Omega$ and $Q = 39750$ (see Table 2), tuning the HOM frequency to the value $f_r = 1374.4739$ MHz, the bunch mode $\mu = 991$ acquires a growth time equal to the radiation damping time, as shown in Fig.3a in red. The growth rates induced by the HOM with frequency $f_r \approx 1374.5$ MHz at $T_c = 38.4^\circ\text{C}$ are plotted in Fig.3b in red as a function of bunch mode, where the fastest growing mode is $\mu = 991$.

In an attempt to suppress the instability, we increased the cavity temperature T_c from 38.4°C to 39.2°C , with an expected HOM frequency shift $\Delta f_r = -8\text{kHz}$ (see the discussion on the HOMs measurements in Section IV). We observed a new unstable bunch mode with $\mu = 603$ at $I_{\text{AV}} = 11.64\text{mA}$, driven by a HOM with frequency $f_r \approx nf_{rf} + 603f_0$. The measurement of the time evolution of modes, shown in Fig.4, gives for the bunch mode $\mu = 603$ a growth rate $1/\tau_m^{(\mu)} = 6.72\text{ s}^{-1}$, thus a growth time $\tau_m^{(\mu)} = 147\text{ms}$. The measured oscillation frequency is $f_m^{(\mu)} = 276.78\text{kHz}$. Assuming the measured growth rate $\alpha_m = 1/\tau_m^{(\mu)}$ equal to the sum of the growth rate due to radiation damping $\alpha_s = 1/\tau_s$ and the growth rate due to the HOM $\alpha = 1/\tau^{(\mu)}$, we obtain $\alpha = \alpha_m - \alpha_s = 43.7\text{s}^{-1}$,

Figure 4: Time evolution of bunch modes during a transient grow/damp measurement, where the feedback loop is opened under software control for a predetermined period of time and then closed. In the open-loop conditions, unstable modes grow exponentially due to noise, and a feedback system records the motion of the bunches during the transient [34]. The exponential growth of bunch mode $\mu = 603$ gives a measured growth time $\tau_m^{(\mu)} = 147\text{ms}$, corresponding to a growth rate $\alpha_m = 1/\tau_m^{(\mu)} = 6.72\text{s}^{-1}$. The observed multi-bunch instability, measured at the cavity temperature $T_c = 39.2^\circ$ and at an average current $I_{av} = 11.64\text{mA}$, is excited by the HOM at frequency $f_r \approx 728\text{MHz}$. The measured oscillation frequency $f_m^{(\mu)} = 2.7678\text{kHz}$. Assuming the measured growth rate α_m is equal to the sum of the growth rate due to radiation damping $\alpha_s = 1/\tau_s$ and the growth rate due to the HOM $\alpha = 1/\tau_m^{(\mu)}$, we obtain $\alpha = \alpha_m - \alpha_s = 43.7\text{s}^{-1}$, thus a growth time $\tau^{(\mu)} = 23\text{ms}$ for the bunch mode $\mu = 603$ induced by the HOM at frequency $f_r \approx 728\text{MHz}$.

thus a growth time $\tau^{(\mu)} = 23\text{ms}$ for the bunch mode $\mu = 603$. With $n = 1$, we identified the HOM at frequency $f_r \approx 728\text{MHz}$ as driving the instability. With $R_{sh,||} = 3M\Omega$ and $Q = 33600$ (see Table 2), tuning the HOM frequency to the value $f_r = 727.9689\text{MHz}$, the growth time of bunch mode $\mu = 603$ is equal to the measured value of 23ms, as shown in Fig.3d in blue.

The observed instabilities are interpreted by noticing that the HOMs at $f_r \approx 1374.5\text{MHz}$ and $f_r \approx 728\text{MHz}$ give the fastest growth time (of the order of the radiation damping time) at the cavity temperatures $T_c = 38.4^\circ\text{C}$ and $T_c = 39.2^\circ\text{C}$ respectively. Indeed, with the cavity temperature change $\Delta T = 0.8^\circ\text{C}$ and an induced frequency shift $\Delta f_r = -8\text{kHz}$, and recalling that the bunch modes $\mu = 991$ and $\mu = 603$ are excited by the HOMs at frequencies $f_r \approx 1374.5\text{MHz}$ and $f_r \approx 728\text{MHz}$ respectively, the growth times are $\tau^{(991)} = 27\text{ms}$ and $\tau^{(603)} = 35\text{ms}$ at $T_c = 38.4^\circ\text{C}$, and $\tau^{(991)} = 29\text{ms}$ and $\tau^{(603)} = 23\text{ms}$ at $T_c = 39.2^\circ\text{C}$, as shown in Fig.3, where the results are shown in red at $T_c = 38.4^\circ\text{C}$ and in blue at $T_c = 39.2^\circ\text{C}$.

4. Transverse Coupled-Bunch Instability

We now discuss the scheme for stable operations of storage rings that use normal conducting multi-cell RF-cavity structures, and apply it to the NSLS-II storage ring with a 7-cell PETRA-III RF-cavity. The scheme helps in preventing coupled-bunch instabilities driven by the transverse HOMs of the RF-cavity, and is based on the slow head-tail effect at positive chromaticity and a proper detuning of the HOMs frequencies as a function of RF-cavity temperature change. For a proper application, the scheme requires self-consistent beam slow head-tail dynamics simulations of short range (slow head-tail effect) and long range (coupled-bunch interaction) wakefield

Table 3: Simulated Transverse HOMs Parameters of the 7-cell PETRA-III Cavity Structure

	Frequency f_r , MHz	Shunt Impedance $R_{sh,\perp}$, MΩ/m	Quality Factor $Q_\perp$	$R_{sh,\perp}/Q_\perp$, Ω/m
HOM1	860.17	14.7	55700	263.91
HOM2	867.04	17.5	56800	308.1
HOM3	869.47	56.1	58200	963.92
HOM4	870.88	19.7	59400	331.65
HOM5	1043.41	83.6	40400	2069.31
HOM6	1047.32	26.2	40900	640.59
HOM7	1089.00	17.0	49400	344.13
HOM8	1464.92	15.5	54600	283.88
HOM9	1545.11	26.8	44300	604.97

effects, and the precise knowledge of the frequency shift as a function of RF-cavity temperature change for every HOM. During the phase 1 commissioning of the NSLS-II storage ring we did not apply the scheme to the transverse case, we solely detuned the frequencies of the HOMs to stabilize the beam longitudinally, as discussed in Section 3. However, as pointed out in the introduction, the goal of the phase 1 commissioning with the PETRA-III cavity was to reach the average current $I_{av} = 25\text{mA}$ with no demanding conditions of reproducibility and no stringent requirements for stable operations with different multi-bunch configurations. We remind here that the accumulation of $I_{av} = 25\text{mA}$ was achieved at chromaticity $\xi_x = +2$ and $\xi_y = +2$ with approximately 1000 bunches. In addition, the accumulation was achieved with the use of a transverse feedback system.

4.1. Measurements of HOMs

To apply the scheme to the NSLS-II storage ring we use the parameters listed in Table 1. For the parameters of the transverse HOMs, listed in Table 3, we integrated the simulated values with the measurements of the HOMs done in [17]. To facilitate the discussion we label the nine HOMs as shown in Table 3, i.e. as HOM1, ..., HOM9. The fundamental mode is denoted as HOM0. In Fig.5 we discuss the measurements of the HOM frequency dependence on cavity temperature T_C for HOM2 (Fig.5a), HOM6 (Fig.5b) and HOM8 (Fig.5c) at $T_C = 29.5^\circ\text{C}$. The measurements show a linear dependence. The slopes, found by linear fitting, are plotted as a function of the HOM frequencies in Fig.5d, showing the linear dependence $\Delta f_r / \Delta T_C = \alpha(f_r - f_{r0})$, where $\alpha = -2.4 \times 10^{-5} \text{ 1}/^\circ\text{C}$. In the measurements, the fundamental mode HOM0 was always tuned at $f_{r0} = 499.68\text{MHz}$. Since only three HOMs have been measured, for the purpose to illustrate the detuning scheme, we assume the HOMs listed in Table 3 are measured values at $T_C = 29.5^\circ\text{C}$. We next proceed to analyze the growth rates induced by the transverse HOMs at $T_C = 38.4^\circ\text{C}$ and $T_C = 39.2^\circ\text{C}$, at which temperatures longitudinal coupled-bunch instabilities have been observed, as discussed in Section 3, with the HOMs frequencies determined by the measurements shown in Fig.5. For the analysis of short-range wakefield effects, we use a broadband resonator model, with parameters listed in Table 4, together with a description of the impedance model [35]. The long-range wakefields are modeled with the HOMs of the PETRA-III cavity and the resistive wall impedance of the aluminum vacuum chamber.

Table 4: Impedance Model

PETRA-III cavity HOMs (see Table 2 and Table 3), $\beta_x^{\text{cav}} = 20\text{m}$, $\beta_y^{\text{cav}} = 7.5\text{m}$
Transverse broadband impedance: $f_r = 30\text{GHz}$, $R_{sh,\perp} = 1\text{M}\Omega/\text{m}$, $Q_{\perp}=1$
Resistive wall impedance: 792m of aluminum with vertical half-gap of 12.5mm

Figure 5: Measurements of the transverse HOMs frequency dependence on cavity temperature T_C [17]. HOM frequency as a function of cavity temperature for the HOMs at frequencies, at $T_C = 29.5^\circ\text{C}$, $f_r = 867.04\text{MHz}$ (HOM2) a), $f_r = 1047.32\text{MHz}$ (HOM6) b), and $f_r = 1464.92\text{MHz}$ (HOM8) c), showing a linear dependence. d) Slopes of the measurements in a), b) and c) as a function the HOM frequency, showing the linear dependence $\Delta f_r / \Delta T_C = \alpha(f_r - f_{r0})$, where $\alpha = -2.4 \times 10^{-5} \text{ 1/}^\circ\text{C}$ and f_{r0} is the fundamental mode. In the measurements, f_{r0} is always tuned at 499.68MHz.

4.2. OASIS: Self-Consistent Simulations of Single and Coupled-Bunch Instabilities

The self-consistent modeling of collective effects in numeric codes can be computationally very demanding, even with the use of the most powerful computer clusters presently available. This is evident when the code has to model the self-consistent dynamics of a large number of bunches (of the order of one thousand in modern third generation light sources) interacting with each other and itself simultaneously via long and short-range wakefields. In particular, the calculation for arbitrary long-range wakefields² in time domain can not only be very time consuming, but very memory demanding as well, since the algorithm requires to store the “history” of the bunches for several revolutions in order to calculate the wake forces at the present time. Moreover, in many applications one can not rely on point bunch approximations and has to follow the dynamics of the internal structure of the bunches.

To meet the requirements to accurately model collective effects, we have developed the self-consistent parallel tracking code **OASIS** (Optimal Algorithm for Self-consistent Instability Simulations). The code is written in Fortran and the parallel algorithms use MPI libraries. **OASIS** can efficiently study short and long-range wakefield effects. The general structure of the code is the following. M bunches, each with $\mathcal{N}$ simulation particles, are distributed to M processors. The short-range (single bunch) wakefield interaction is calculated in serial (locally) by each processor, while the long-range wakefield calculation is done in parallel (globally) via master-to-slave processor communications, by storing the “history” of moments of the bunches. **OASIS** has also the capability to efficiently study the microwave instability, where a large number of simulation particles $\mathcal{N}$ is required to study the evolution of microstructures in the bunch. In this case, since only one bunch is tracked, the calculation of the single-bunch interaction is done in parallel by distributing $\mathcal{N}/M$ simulation particles between the M processors. Efficient algorithms for density estimation from particles are implemented in **OASIS** to accurately calculate the short-range wakefields and monitor the evolution of distribution densities [36]. For the study of the microwave instability threshold for the NSLS-II storage ring with the **OASIS** code see [22]. More details on the parallel algorithm for microwave instability simulations will be discussed elsewhere [20]. In Appendix A we discuss in some detail the self-consistent calculation of the transverse long-range wakefield interaction and its benchmark against the analytical theory at zero chromaticity.

4.3. Instability Analysis

An analytical estimate of the coherent frequency shift for M equally spaced Gaussian bunches is given, for an impedance distributed along the circumference C , by the following Sacherer-Zotter formula [38]

$$\Omega^{(\mu)} - \omega_\beta = -i \frac{I_0 c}{4\sqrt{\pi} \omega_\beta E_0 \sigma_t} \frac{\sum_p Z_\perp(\omega') e^{-(\omega' - \omega_\xi)^2 \sigma_t^2}}{\sum_p e^{-(\omega' - \omega_\xi)^2 \sigma_t^2}}, \quad (9)$$

where $\omega' = pM\omega_0 + \mu\omega_0 + \omega_\beta$, ω_β is the betatron frequency, I_0 the single bunch current, η the slippage factor, E_0 the energy of the reference particle in electron volts, σ_t the bunch length, $\omega_\xi = \omega_0 \xi / \eta$ with ξ the chromaticity, η the slippage factor and ω_0 the revolution angular frequency, and $Z_\perp$ is the total transverse impedance of the ring

²By arbitrary here we mean the wakefield is given either analytically, or numerically as input file.

(here $\perp$ is either x or y). With the condition $M\omega_0\sigma_t \ll 1$, satisfied for short bunches, the sum at the denominator of Eq.(9) can be approximated by an integral

$$\sum_{p=-\infty}^{+\infty} e^{-\omega'^2\sigma_t^2} \approx \frac{1}{M\omega_0\sigma_t} \int d\omega e^{-\omega^2} = \frac{\sqrt{\pi}}{M\omega_0\sigma_t}. \quad (10)$$

According to the parameters listed in Table 1, $M\omega_0\sigma_t = 0.003$, thus to good approximation Eq.(9) reads

$$\Omega^{(\mu)} - \omega_\beta = -i \frac{I_0 M c}{4\pi E_0 \nu_\perp} \sum_{p=-\infty}^{+\infty} Z_\perp(\omega') e^{-(\omega' - \frac{\xi}{\eta}\omega_0)^2 \sigma_t^2}, \quad (11)$$

where we used $\nu_\perp = \omega_\beta/\omega_0$. For a localized impedance with local beta function $\beta_\perp$, $\nu_\perp$ in Eq.(11) is replaced by $C/(2\pi\beta_\perp)$. The transverse growth rate $1/\tau^{(\mu)}$ of bunch mode μ driven by the HOMs of the PETRA-III cavity is therefore given by

$$\frac{1}{\tau^{(\mu)}} = -\frac{I_0 M c \beta_\perp^{\text{cav}}}{2E_0 C} \sum_{p=-\infty}^{+\infty} e^{-(pM\omega_0 + \mu\omega_0 + \omega_\beta - \frac{\xi}{\eta}\omega_0)^2 \sigma_t^2} \text{Re}Z_\perp(pM\omega_0 + \mu\omega_0 + \omega_\beta), \quad (12)$$

where we used the notation $\beta_\perp^{\text{cav}}$ for the local beta function of the cavity. Clearly for point bunches Eq.(12) reads

$$\frac{1}{\tau^{(\mu)}} = -\frac{I_0 M c \beta_\perp^{\text{cav}}}{2E_0 C} \sum_{p=-\infty}^{+\infty} \text{Re}Z_\perp(pM\omega_0 + \mu\omega_0 + \omega_\beta). \quad (13)$$

For the NSLS-II parameters with zero-current bunch length $\sigma_t = 10\text{ps}$ and for low to moderate values of the chromaticity ξ , the growth rates in Eq.(12) depend weakly on σ_t and ξ , thus we will use the point bunch formula in our analysis.

4.3.1. Zero Chromaticity

Since the uniform filling configuration with $M = 1320$ bunches does not provide enough slow head-tail damping at an average current $I_{\text{av}} = 25\text{mA}$, as discussed in the introduction, we consider the storage ring filled with $M = 132$ equally spaced bunches (every tenth RF-bucket filled), thus a single bunch current $I_0 = 0.19\text{mA}$. In Fig.6 we study, at zero chromaticity, the horizontal and vertical coupled-bunch instabilities driven by the HOMs of the PETRA-III cavity at $T_c = 38.4^\circ\text{C}$ (left frames) and $T_c = 39.2^\circ\text{C}$ (center frames), the same cavity temperatures at which longitudinal coupled-bunch instabilities have been observed, and at $T_c = 40^\circ\text{C}$ (right frames). The growth rates as a function of mode number μ are plotted in Fig.6a, Fig.6b and Fig.6c. The fastest horizontal growing mode $\mu = 114$ has a growth time $\tau^{(\mu)} = 0.9\text{ms}$ at $T_c = 38.4^\circ\text{C}$, $\tau^{(\mu)} = 1.9\text{ms}$ at $T_c = 39.2^\circ\text{C}$ and $\tau^{(\mu)} = 3.4\text{ms}$ at $T_c = 40^\circ\text{C}$, much smaller than the transverse radiation damping time $\tau_x = 54\text{ms}$. Therefore the beam would be unstable at zero chromaticity. In Fig.6d, Fig.6e and Fig.6f, we plot the real part of the total impedance, $\text{Re}Z_\perp$, evaluated at $\omega(p) = (pM + \mu + \nu_\perp)\omega_0$ as a function of p and μ at $T_c = 38.4^\circ\text{C}$, $T_c = 39.2^\circ\text{C}$ and $T_c = 40^\circ\text{C}$ respectively. The two fastest growing modes are $\mu = 114$ and $\mu = 52$ in the horizontal plane, and $\mu = 131$ and $\mu = 69$ in the vertical plane, whose growth rates are driven by the HOMs with angular frequency $\omega_r \approx 6556.7\text{MHz}$ and $\omega_r \approx 5448.3\text{MHz}$ respectively, as shown in Fig.6g and Fig.6l at $T_c = 38.4^\circ\text{C}$, Fig.6h and Fig.6m at $T_c = 39.2^\circ\text{C}$, and Fig.6i and Fig.6n at $T_c = 40^\circ\text{C}$. The shift of the HOM frequencies has the effect of lowering the growth rates of the fastest growing modes, with the growth time of the fastest horizontal growing mode $\mu = 114$ increasing from $\tau = 0.9\text{ms}$ at $T_c = 38.4^\circ\text{C}$ to $\tau = 3.4\text{ms}$ at $T_c = 40^\circ\text{C}$ [39].

Figure 6: Growth rates vs mode number μ at zero chromaticity for a cavity temperature $T_c = 38.4^\circ\text{C}$ a), $T_c = 39.2^\circ\text{C}$ b) and $T_c = 40^\circ\text{C}$ c). The fastest horizontal growing mode $\mu = 114$ has a growth time $\tau^{(\mu)} = 0.9\text{ms}$ at $T_c = 38.4^\circ\text{C}$, $\tau^{(\mu)} = 1.9\text{ms}$ at $T_c = 39.2^\circ\text{C}$ and $\tau^{(\mu)} = 3.4\text{ms}$ at $T_c = 40^\circ\text{C}$. Real part of $Z_\perp$ evaluated at $(pM + \mu + \nu_{x/y})\omega_0$ as a function of p and μ at $T_c = 38.4^\circ\text{C}$ d), $T_c = 39.2^\circ\text{C}$ e) and $T_c = 40^\circ\text{C}$ f). The two fastest growing modes are $\mu = 114$ and $\mu = 52$ (horizontal), and $\mu = 131$ and $\mu = 69$ (vertical), with growth rates driven by the HOMs $\omega_r \approx 6556.7\text{MHz}$ and $\omega_r \approx 5448.3\text{MHz}$, as shown in g) and l) at $T_c = 38.4^\circ\text{C}$, in h) and m) at $T_c = 39.2^\circ\text{C}$, and i) and n) at $T_c = 40^\circ\text{C}$. The shift of the HOMs frequencies has the effect of lowering the growth times of the fastest growing modes, with the growth time of the fastest horizontal growing mode $\mu = 114$ increasing from $\tau = 0.9\text{ms}$ at $T_c = 38.4^\circ\text{C}$ to $\tau = 3.4\text{ms}$ at $T_c = 40^\circ\text{C}$.

Figure 7: Self-consistent simulations ($\langle x \rangle / \sigma_{x0}$) of the horizontal head-tail effect and coupled-bunch instability at $\xi = 2$ for $M = 132$ bunches ($I_0 = 0.19\text{mA}$) up to 30000 turns. Unstable evolution at the cavity temperature $T_c = 38.4^\circ\text{C}$ a) and $T_c = 39.2^\circ\text{C}$ b). c) Stable evolution at the cavity temperature $T_c = 40^\circ\text{C}$. The horizontal damping time provided by the slow head-tail effect at $\xi = 2$ is $\tau = 2.5\text{ms}$, while the growth times induced by the HOMs of the PETRA-III cavity are $\tau^{(\mu)} = 0.9\text{ms}$ at $T_c = 38.4^\circ$, $\tau^{(\mu)} = 1.9\text{ms}$ at $T_c = 39.2^\circ$ and $\tau^{(\mu)} = 3.4\text{ms}$ at $T_c = 40^\circ$.

4.3.2. Positive Chromaticity

At positive chromaticity short-range wakefields, in our case modeled with a broadband resonator wake, produce damping of single bunch dipole oscillations, a mechanism driven by the slow head-tail effect. The damping rate provided by the slow head-tail effect is proportional to the single bunch current I_0 , to the strength of the wakefield, and to the chromaticity ξ (for low to moderate values of ξ). Therefore, to take advantage of the slow head-tail damping, it is convenient to accumulate a high single bunch current, avoiding any unwanted single bunch instability. For a discussion of the slow head-tail effect and its benchmark against our simulations see Appendix A, where we show that the slow head-tail effect gives rise to a quadrupole instability for $\xi > 3$ [40].

We will now discuss the coupled-bunch instability at chromaticity $\xi = +2$. In the vertical case, since for the cavity temperatures in the range of study ($38.4^\circ\text{C} < T_c < 40^\circ\text{C}$) the growth time induced by the HOMs of the PETRA-III cavity is bigger than 5ms, and the slow-head damping is $\tau = 1.25\text{ms}$ at $\xi = 2$ (see Fig.A.8d), we expect the filling with $M = 132$ bunches and broadband impedance with parameters as given in Table 4 to be stable. In the horizontal case and at the cavity temperature $T_c = 38.4^\circ\text{C}$, since the fastest growth time induced by the HOMs of the PETRA-III cavity is $\tau^{(\mu)} = 0.9\text{ms}$, we expect the filling with $M = 132$ bunches to be unstable at chromaticity $\xi = +2$, in which case the slow head-tail effect provides a damping time $\tau = 2.5\text{ms}$ (twice the vertical one, according to Eq.(A.1)). At the cavity temperature $T_c = 39.2^\circ\text{C}$, the fastest growth time induced by the HOMs is $\tau^{(\mu)} = 1.9\text{ms}$, still faster than the slow head-tail damping time, giving an unstable configuration as well. On the other hand, at the cavity temperature $T_c = 40^\circ\text{C}$, the fastest growth time induced by the HOMs is $\tau^{(\mu)} = 3.4\text{ms}$, slower than the damping time provided by the slow head-tail effect. Therefore we expect, at the cavity temperature $T_c = 40^\circ\text{C}$, the uniform filling with $M = 132$ bunches to be stable at chromaticity $\xi = +2$. Self-consistent simulations with the OASIS code corroborate our analysis. The numerical results show that at $\xi = +2$ the horizontal head-tail effect is not able to suppress the coupled-bunch instability at the cavity temperature $T_c = 38.4^\circ\text{C}$ (Fig.7a) and $T_c = 39.2^\circ\text{C}$ (Fig.7b), while it is effective in the suppression at the cavity temperature $T_c = 40^\circ\text{C}$ (Fig.7c). The choice of the broadband resonator parameters listed in Table 4 has been based on estimates of the transverse impedance performed during the NSLS-II commissioning (see [11] for partial results), where the horizontal and vertical broadband resonators with frequency $f_r = 30\text{GHz}$ and quality factor $Q = 1$, and shunt

impedance $R_{sh,x} = 0.5\text{M}\Omega/\text{m}$ and $R_{sh,y} = 1\text{M}\Omega/\text{m}$ respectively, have been able to reproduce reasonably well the observed tune shifts vs single bunch current at zero chromaticity, and slow head-tail damping times vs single bunch current at positive chromaticity. Although $R_{sh,y} \approx 2R_{sh,x}$, since the coupled-bunch instability is faster in the horizontal plane than in the vertical plane, we discussed slow head-tail damping simulations assuming the shunt impedances of equal strength in both planes. Moreover, given the uncertainty in the estimates of the shunt impedances, and since our discussion of the transverse case has been primarily motivated by our intention to illustrate the scheme for stable multi-bunch operations, we chose for the broadband shunt impedance the value $R_{sh,y} = R_{sh,x} = 1\text{M}\Omega/\text{m}$. We conclude our analysis with a discussion of the case with $R_{sh,y} = R_{sh,x} = 0.5\text{M}\Omega/\text{m}$. By the linear dependence of the slow head-tail damping rate on the shunt impedance [11], the slow head-tail damping time is $\tau = 5\text{ms}$, longer than the HOM fastest growth time $\tau^{(\mu)} = 3.4\text{ms}$ at $T_c = 40^\circ\text{C}$. By reducing the number of bunches of the uniform filling from $M = 132$ to $M = 66$, and by doubling the single bunch current, we can restore the slow head-tail damping time to the value $\tau = 2.5\text{ms}$, thus obtain a stable multi-bunch configuration. The results of self-consistent simulations for the case with $M = 66$ and $R_{sh,y} = R_{sh,x} = 0.5\text{M}\Omega/\text{m}$, that we do not show here, are very similar to the results for $M = 132$ and $R_{sh,y} = R_{sh,x} = 1\text{M}\Omega/\text{m}$ shown in Fig.7. Adjusting the cavity temperature properly would provide more options to enhance stability.

5. Conclusions

We showed that harmful coupled-bunch instabilities can be driven by the HOMs of a 7-cell PETRA-III cavity structure, and discussed a general scheme to cure the instabilities. The scheme is based on a proper detuning of the HOM frequencies by changing the temperature of the RF-cavity, and, in the transverse case, by a proper choice of positive chromaticity to take advantage of the damping of dipole oscillations provided by the head-tail instability. The scheme requires the measurements of the HOMs as a function of RF-cavity temperature, and numerical simulations for the self-consistent modeling of the head-tail instability, driven by short-range wakefields, and the coupled-bunch instability, driven by long-range wakefields. We accomplished the latter with the OASIS code.

Our analysis for the NSLS-II Storage Ring has been helpful for the preparation of the phase 1 commissioning with the 7-cell PETRA-III cavity structure, with the goal to accumulate an average current of 25mA. In the process to reach 25mA, we observed longitudinal coupled-bunch instabilities driven by the HOMs of the PETRA-III cavity at an average current of $\approx 10\text{mA}$, and have been able to suppress them by a proper detuning of the HOM frequencies with a RF-cavity temperature change. An interpretation of the measurements of the coupled-bunch instabilities has been given in Section 3.

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Figure A.8: a) Constant wake (red solid line) used in benchmarking the head-tail effect compared with the broadband resonator wake used in the paper. The bunch length is 10ps. b) Benchmark of the head-tail effect at $\xi = +2$: exponential fit (blue dotted line) of the simulated dipole oscillations ($l = 0$) of the beam (red line) driven by the constant wake shown in a). The dipole oscillations are very well fit by the damping time $\tau = 2.8\text{ms}$ predicted by Eq.(A.1). c) Transverse bunch size vs number of turns at $\xi = +2$ for the constant wake shown in a). The exponential fit (blue line) with the growth time $\tau = 8.4\text{ms}$ according to Eq.(A.1) for the quadrupole mode $l = 1$, fits very well the simulated bunch size. d) Damping time $\tau = 1.25\text{ms}$ at $\xi = +2$ provided by the vertical head-tail effect driven by the broadband impedance with parameters as in Table 4. e) Simulated normalized average bunch displacement (bottom red line) and bunch size (top blue line) vs number of turns at $\xi = +3$. f) Same as e) at $\xi = +4$. Here the slow head tail effect gives rise to a quadrupole mode instability.

Appendix A. Discussion and Benchmark of Simulations

Appendix A.1. Head-tail Instability

The head-tail instability plays a crucial role in suppressing coupled-bunch instabilities. At positive chromaticity the damping of dipole oscillations provided by the head-tail effect increases with the single bunch current and the intensity of the transverse impedance. In this appendix we discuss an explicit formula for the head-tail damping in the case of a constant wake and a longitudinal air-bag distribution, and benchmark it against numerical simulations with OASIS.

The head-tail effect is purely a collective effect and its analysis is usually decoupled from decoherence, a single particle dynamics effect [32], [41]. This is justified since decoherence due to linear chromaticity does not provide damping of dipole oscillations. Decoherence effects due to chromaticity in the NSLS-II storage ring have been studied in [42]. The analysis of the interplay between decoherence and slow head-tail effect is the subject of a paper in preparation [43]. The formula for the head-tail instability growth rate we use to benchmark our simulations is given by Eq. (6.216) of [32] (note that in [32] the chromaticity is defined as $\xi = (\Delta\omega_\beta/\omega_\beta)/(\Delta E/E)$, and CGS units are used [33]). Assuming an air bag model for the longitudinal density and a constant wake, the head-tail growth rate for mode l reads

$$\frac{1}{\tau^{(l)}} = \frac{Nr_0 c W_0}{\gamma T_0 \omega_\beta} \chi \frac{2}{\pi^2 (4l^2 - 1)}, \quad (\text{A.1})$$

where $\chi = \xi \hat{z} \omega_0 / \eta$ is the head-tail phase, and $\hat{z} = \sqrt{2} \sigma_{\text{rms}}$ for an air-bag distribution.

Eq.(A.1) is valid for finite but small χ ($0 < |\chi| \ll 1$). Mode index $l = 0$ corresponds to the dipole mode, $l = 1$ to the quadrupole mode, and so forth. It follows that at chromaticity $\xi > 0$, the dipole mode is stable and the higher order modes unstable above transition ($\eta > 0$). It must be pointed out that Eq.(A.1) is model dependent [32]. For the NSLS-II parameters, the head-tail phase is $\chi = 0.18$ at $\xi = +2$ and the resonator wake function is not constant within the bunch. To benchmark our simulations against Eq.(A.1) we assumed a constant wake as shown is Fig.A.8a (red solid line). The results for a Gaussian bunch of length σ_t at $\xi = +2$ are shown in Fig.A.8b and Fig.A.8c. A single bunch is tracked subjected to the constant wake. A dipole moment equal to the equilibrium transverse bunch size σ_{x0} is induced on the beam after 4000 turns. The analytical damping time $\tau^{(0)} = 2.8\text{ms}$ and growth time $\tau^{(1)} = 8.4\text{ms}$ fit very well the simulated envelope of the dipole oscillations and transverse bunch size respectively (Fig.A.8b and Fig.A.8c).

In Fig.A.8d we calculate the damping time of vertical dipole oscillations driven by the resonator wake with parameters given in Table 4 for $\xi = +2$. The damping time is $\tau = 1.25\text{ms}$. Notice that from Eq.(A.1) it follows that the ratio of the horizontal and vertical damping times is ν_x / ν_y , where ν_x is the horizontal tune and ν_y the vertical tune. For the NSLS-II storage ring (see Table 1) this implies that the horizontal damping time is approximately twice the vertical one. In Fig.A.8e and Fig.A.8f we show the evolution of the average normalized bunch displacement and bunch size at $\xi = +3$ and $\xi = +4$. We see that the slow head tail effect gives rise to a quadrupole mode instability at $\xi = +4$.

Appendix A.2. Coupled-Bunch Instability

We first discuss the self-consistent algorithm for couple bunch instabilities in the case of point bunches. We limit the discussion to the transverse case. We consider the equations of motion [44] of M point bunches separated by the distance C/M , where C is the accelerator circumference

$$\ddot{x}_n(t) + \omega_\beta^2 x_n(t) = -NA \sum_{k=0}^{+\infty} \sum_{m=0}^{M-1} W_1 \left(-kC - \frac{m-n}{M} C \right) x_m \left(t - kT_0 - \frac{m-n}{M} T_0 \right), \quad (\text{A.2})$$

where $n = 0, \dots, M-1$, $\dot{} = d/dt$, N is the number of particles per bunch, $T_0 = C/c$, $A = ec/(T_0 E_0)$, c the speed of light, E_0 the energy of the reference particle in electron volts, and $W_1(z)$ is the transverse wake. We solve Eq.(A.2) numerically assuming that the wake W_1 is generated by a RF-cavity localized at position s in the storage ring using a one-turn map approximation. The point bunches are injected into the ring with initial conditions x_{n0} , p_{xn0} and evolve according to the betatron motion for $\hat{n}_t$ turns, after which the coupled-bunch interaction is turned on. The sum in k over the “history” of the bunches is truncated to a predefined value $\hat{k}$ determined by the long range behavior of W_1 . To compare the numerical solution of Eq.(A.2) with Eq.(11) (replacing $\nu_\perp$ with $C/(2\pi\beta_\perp)$ for a localized impedance) we study the evolution of the multi-bunch mode $x_n^{(\mu)}$ defined by

$$x_n^{(\mu)}(t) = ae^{\frac{(t-t_1)}{\tau^{(\mu)}}} \left[\cos \frac{2\pi\mu n}{M} \cos(\omega_\beta + \omega_r^{(\mu)})(t - t_1) + \sin \frac{2\pi\mu n}{M} \sin(\omega_\beta + \omega_r^{(\mu)})(t - t_1) \right], \quad (\text{A.3})$$

where $1/\tau^{(\mu)} := \text{Im}(\Omega^{(\mu)} - \omega_\beta)$ and $\omega_r^{(\mu)} := \text{Re}(\Omega^{(\mu)} - \omega_\beta)$. In this appendix we consider the case with $M = 1320$ (all RF-buckets filled). The growth rates according to Eq.(A.2) are shown in Fig.A.9a. The fastest growing mode

$\mu = 338$ has a growth rate of $1347s^{-1}$ ($\tau^{(\mu)} = 0.74\text{ms}$). In Fig.A.9b we fit the envelope $x_0^{(\mu)}$ with $a \exp(-T_0(n_t - \hat{n}_t)/\tau^{(338)})$, where $a = 2\mu\text{m}$, T_0 is the revolution period, $\hat{n}_t = 50$ turns and $\tau^{(338)} = 0.74\text{ms}$. The fit is very good. In Fig.A.10a we plot the initial conditions for the fastest growing mode $\mu = 338$ in the range $n \in [0, 50]$ while in Fig.A.10b we plot the comparison of $x_n^{(\mu)}(t)$ for the fastest growing mode $\mu = 338$ after $n_t = 400$ turns ($t = n_t T_0$) calculated numerically (red points) and analytically (blue line) according Eq.(A.3) with growth time $\tau^{(\mu)} = 0.74\text{ms}$ and frequency shift $\omega_r^{(\mu)} = -737.44s^{-1}$, and $t_1 = 50T_0$. The agreement is very good. To study the equations of motion with radiation damping and quantum fluctuations, we consider the evolution of bunch mode $\mu = 0$ with initial conditions $x_{n0} = p_{xn0} = 0$. In Fig.A.10c we plot the exponential fit (blue line) with the analytical growth time $\tau^{(\mu)} = 0.74\text{ms}$ of $x_0(t)/\sigma_{x0}$ calculated numerically (red line). The fit with $a \exp(-T_0(n_t - \bar{n}_t)/\tau^{(338)})$ uses $\bar{n}_t = 2500$ turns and $a = 20\sigma_{x0}$. The fit is very good, as well as the comparison of $x_n(t)/\sigma_{x0}$ for the fastest growing mode $\mu = 338$ after 3500 turns, as shown in Fig.A.10d.

In the case of extended bunches, we calculate the coupled bunch kick as follows. The coupled bunch force on bunch 0, for example, after n_t turns is

$$V_1^0(\tau; n_t) = \alpha \sum_{m=0}^{M-1} \sum_{k=\hat{k}}^{n_t} c_{mk} \int d\tau' W_1(\tau - \tau') D_x^m(\tau'; k), \quad (\text{A.4})$$

where $\alpha = \text{const.}$, $c_{mk} = 1 - \delta_{0m}\delta_{0k}$, δ_{ij} the Kronecker delta ($\delta_{ij} = 1$ if $i = j$, 0 otherwise), τ is arrival time and $D_x^m(\tau; k) = \int dx f_x^m(\tau, x; k)$ is the instantaneous transverse dipole density of bunch m at turn k , and f_x^m is the two-dimensional spatial density of bunch m . Notice that $D_x^m(\tau; k)$ is ≈ 0 for $\tau \notin A_s = [\tau_c - 5\sigma_t, \tau_c + 5\sigma_t]$, where $\tau_c(m, k) = (n_t - k)T_0 + mT_0/M$ and σ_t is the bunch length, i.e. we assume the particle distribution densities zero outside an interval of the order of $5\sigma_t$, much smaller than the time difference T_0/M between RF-buckets. If we assume that the long-range wakefield $W_1(\tau)$ is slowly varying (roughly constant) for $\tau \in A_s$ we can conveniently calculate Eq.(A.4) by expanding W_1 in Taylor series around $\tau_c(m, k)$. With the change of integration variable $\tau'' = \tau' - \tau_c(m, k)$, Eq.(A.4) reads

$$\begin{aligned}
 V_1^0(\tau; n_t) &= \alpha \sum_{m=0}^{M-1} \sum_{k=\hat{k}}^{n_t} c_{mk} \int d\tau'' W_1(-\tau_c(m, k) + \tau - \tau'') D_x^m(\tau_c(m, k) + \tau''; k) \\
 &= \alpha \sum_{m=0}^{M-1} \sum_{k=\hat{k}}^{n_t} c_{mk} \int d\tau' D_x^m(\tau_c(m, k) + \tau'; k) \left[W_1(-\tau_c(m, k)) + W_1'(-\tau_c(m, k))(\tau - \tau') \right. \\
 &\quad \left. + W_1''(-\tau_c(m, k)) \frac{(\tau - \tau')^2}{2} + \dots \right] \\
 &= \alpha \sum_{m=0}^{M-1} \sum_{k=\hat{k}}^{n_t} c_{mk} \left[W_1(-\tau_c(m, k)) \langle x \rangle_{m,k} + W_1'(-\tau_c(m, k)) (\tau \langle x \rangle_{m,k} - \langle \tau x \rangle_{m,k}) + \right. \\
 &\quad \left. + W_1''(-\tau_c(m, k)) \frac{(\tau^2 \langle x \rangle_{m,k} - 2\tau \langle \tau x \rangle_{m,k} + \langle \tau^2 x \rangle_{m,k})}{2} + \dots \right], \quad (\text{A.5})
 \end{aligned}$$

where $\langle \tau^n x \rangle_{m,k} = \int d\tau dx \tau^n x f_x^m(\tau, x; k)$. The calculation of the coupled bunch force $V_1^0(\tau; n_t)$ can therefore be done by storing the moments $\langle \tau^n x \rangle_{m,k}$ of bunch m over previous turns k . Eq. (A.5) is the formula we use in our simulations. The arbitrary wakefield W_1 , given by the sum of wakefields with analytical expressions (HOM resonators, resistive wall wakefields, etc.) and calculated numerically with electromagnetic codes (such as **Gdfid1**), is evaluated upfront, before the particle tracking, on the longitudinal grid together with its derivatives, which

Figure A.9: a) Growth rates as a function of mode number μ for the case with $M = 1320$ bunches. The fastest growing mode $\mu = 338$ has a growth rate $1/\tau^{(\mu)} = 1347\text{s}^{-1}$ ($\tau^{(\mu)} = 0.74\text{ms}$). b) Exponential fit (blue line) of $x_0^{(338)}(t)$ ($t = nT_0$, n is number of turns) calculated numerically (red line) with the analytical growth time $\tau^{(\mu)} = 0.74\text{ms}$.

can be accurately calculated with high-order finite-difference schemes. For point bunch simulations, only the first term in the rhs of Eq.(A.5) is calculated. A similar expansion in the longitudinal case allows to take into account finite bunch length effects, as discussed in [20], [23], [45] and [46], important for example in the study of passive Landau cavity effects [23], [45]. As already mentioned, the method is general and applicable to arbitrary long-range wakefields. In the case of a resonator wake, the integration over history can be almost totally eliminated by using invariance properties under translation of the resonator wake function [47], [48], [46]. The idea to express a general wakefield as a sum of resonators has been implemented recently by Migliorati and Palumbo in the tracking code MuSiC to simulate longitudinal multi-bunch effects [24], and in the most recent version of MBTRACK [27]. We plan to compare the computational efficiency of this method with our method based on a Taylor expansion in our future work on arbitrary wakefield effects. The importance of benchmarking simulation codes based on different methods is pointed out in [27]. In Fig.A.10f we plot the exponential fit (blue line) with the analytical growth time $\tau^{(\mu)} = 0.74\text{ms}$ of $\langle x_0(t) \rangle / \sigma_{x0}$ calculated numerically (red line) with initial conditions $\langle x_{n0} \rangle = \langle p_{xn0} \rangle = 0$ for bunches with bunch length σ_t . The agreement is very good as in the case of point bunch simulations, as well as the comparison of $\langle x_n(t) \rangle / \sigma_{x0}$ for the fastest growing mode $\mu = 338$ after 3700 turns calculated numerically (red points) and calculated analytically (blue line), as shown in Fig.A.10e. In Fig.A.11 we plot the transverse density distribution of bunch $m \in [10, 19]$ at $t = 3700T_0$. The average normalized bunch displacement $\langle x_n \rangle$ is distributed according to the fastest growing mode $\mu = 338$ (blue dotted line).

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Figure A.11: Transverse density distribution of bunch $n \in [10, 19]$ at $t=3700T_0$. The average bunch displacement $\langle x_n \rangle$ is distributed according to the fastest growing mode $\mu=338$ (blue dotted line).

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