Simulations of Electron Avalanches in an Ultra-Low-Background Proportional Counter

John W. Robinson^a, Craig Aalseth^a, Michael P. Dion^{a,1,*}, Cory Overman^a,
Allen Seifert^a, Brent VanDevender^a

^a*Pacific Northwest National Laboratory Richland, WA 99354 USA*

Abstract

New classes have been added to the simulation package Garfield++ to import the potential and electric field solutions generated by ANSYS® Maxwell™ v.16. Using these tools we report results on the simulation of electron avalanches and induced signal waveforms in comparison to experimental data of the ultra-low-background gas proportional counters being developed at Pacific Northwest National Laboratory. Furthermore, an improved mesh search algorithm based on Delaunay triangulation was implemented and provided at least a three order of magnitude time savings when compared to the built-in point-location search class of Garfield++.

Keywords: gas proportional counter, ultra-low-background, detector modeling and simulation, Garfield, gaseous detectors

1. Introduction

There is ongoing research at Pacific Northwest National Laboratory (PNNL) to develop a new capability with ultra-low-background gas proportional counters (ULBPC). The detectors are constructed from ultra-pure electroformed copper that is grown in the 30 meter-water-equivalent underground laboratory at PNNL [1]. The electroforming process reduces the majority of uranium and thorium contamination in the copper to produce extremely low-background starting material. These counters are being developed for ground water dating with ³⁹Ar and age dating with ¹⁴C and ³H [2].

The applications described above demand an understanding of every pulse registered in the active gas volume to reduce the minimum detectable activity of the system. This includes electron avalanche response from a particle of interest interacting in the gas volume, background induced signals, micro-discharge pulses and other gas breakdown phenomena. In parallel to the experimental

*Corresponding author

Email address: michael.dion@pnnl.gov (Michael P. Dion)

¹Address: P.O. Box 999 MSIN: J4-65 Tel.: +1-509-372-6983

15 effort, simulations can also provide insight to geometrical construction and ma-
16 terials used to increase the fiducial volume while understanding spurious pulses
17 (e.g., gas induced breakdown).

18 The electric field of a cylindrical, single-wire proportional counter can be
19 solved analytically for much of its length (i.e., $E(r) \approx \frac{1}{r}$) but regions near
20 the ends of the wire can present unforeseen behavior and dielectric material
21 imposed on the geometry increase the difficulty in analytical methods. There
22 are computational methods that can accurately solve (e.g., see Refs. [3, 4]) this
23 (non-linear) field configuration with dielectric materials. Another more popular
24 technique for a numerical solution of the electric potential and hence the electric
25 field for a detector model is the finite element method (FEM). The FEM divides
26 an object of interest into smaller elements, in this case tetrahedra, and then
27 calculates the potential and electric field vectors at the nodes which can then
28 be used to interpolate the electric field within each element. Increasing the
29 number of tetrahedra increases the accuracy of the field solution, allowing areas
30 of interest such as the anode wire of a gas proportional counter to be modeled
31 in greater detail than other regions of a model. ANSYS® Maxwell™ v.16 [5] is
32 available at PNNL for a variety of projects to perform electromagnetic modeling.
33 An advantage of using Maxwell is the ability to import the full CAD model of
34 the detector and subsequently solve the potential field. However, implementing
35 FEM to solve the (non-linear) potential of a proportional counter with a linear
36 interpolation can lead to singularities. A common method to “force” an accurate
37 solution near the wire surface is the introduction of “dummy” cylinders into the
38 model, as described in Section 3.1.

39 Multi-wire proportional chambers, time-projection chambers and single-wire
40 proportional counters are used for measurements in many domains. Garfield [6]
41 was developed to allow interfaces between finite element analysis solutions and a
42 program providing Monte-Carlo techniques for electron-ion drift and subsequent
43 signal development of a gas detector. Recently, Garfield has been transitioned
44 to a C++ based program known as Garfield++ (termed here G++) which is
45 used for simulations in this work. G++ has updated transport code (compared
46 to the Fortran version) and a ROOT [7] interface. Magboltz [8] is called by
47 G++ and calculates the parameters of a user defined gas mixture (e.g., the
48 electron drift velocity and Townsend coefficients as a function of electric field)
49 and HEED [9] calculates the ionization created from a particle interaction in the
50 detector medium (primary electron/ion pairs). The Monte-Carlo simulation of
51 electron avalanches is computational intensive and time consuming, especially
52 when the FEM mesh becomes finely spaced and contains many elements (orders
53 of 10^6). This is not an unknown or new problem and other works have imple-
54 mented techniques to reduce computational time by orders of magnitude. For
55 example, Bonzi et al. found several orders of magnitude (time) improvement by
56 assuming an exponential distribution of electron path lengths between ionizing
57 collisions [10]; eliminating the calculation between successive steps. Also, Alkaa
58 et al. described a technique to reduce computation time by an order of mag-
59 nitude by eliminating numerical integration of Newtons equations due to the
60 cylindrical symmetry of proportional counters (at pressures above 0.1 atm) [11].

61 We report the successful results of an effort that produced a simulation
62 path from the computer-aided design (CAD), to accurate finite element method
63 (FEM) solutions of the electric potential of the detector that were then imported
64 into G++. We have reduced the electron avalanche simulation time of G++ by
65 three orders of magnitude by implementing a new point-search algorithm based
66 on a stochastic walk [12] and the Delaunay triangulation. Single-electron and
67 avalanches induced by photon absorption were simulated in a mixture of 3 atm
68 90% Ar 10% CH₄ (P-10). The gas gain extracted from the simulations is com-
69 pared to experimental data using a Diethorn plot [13]. Furthermore, simulations
70 provided induced signals in the detector which were then further processed to be
71 directly compared to experimental waveforms acquired by the data acquisition
72 system.

73 2. The ULBPC Apparatus

74 This section gives a brief description of the ULBPC, more details can be
75 found in Ref. [2]. The detectors are manufactured from ultra-pure electroformed
76 copper which is grown at PNNL [1]. The “crimp end” of the detector is shown
77 in Figure 1. This end contains a dielectric spring structure (light blue structure
78 in Figure 1) that centers a small copper cylinder which is crimped around the
79 anode wire inside the cathode wall. The ULBPC gas gain was experimentally
80 measured using $\approx 100 \mu\text{Ci}$ of ^{241}Am in sealed source form. The ULBPC was
81 filled with ultra-high purity (99.997%) pre-mixed P-10 (uncertainty of $\pm 5\%$ on
82 the CH₄ component) to 2286.38 torr (3.01 atm) [14]. At this pressure, the wire
83 was biased from +2150–2500 V in 50 V increments (counter wall held at ground)
84 for gas gain measurements. Prior to final gas filling, the detector was evacuated
85 to ~ 0.1 torr and backfilled to 3 atm with P-10 three times. The ^{241}Am allows
86 measurement of the 59.5 keV gamma-ray and also K-shell fluorescence x-rays
87 (8.05 keV [100%], 8.03 keV [$\approx 51\%$] and 8.9 keV [$\approx 17\%$]) [15] from the copper
88 cathode wall which then interact in the gas volume. The ^{241}Am sealed source
89 was placed directly against the cathode wall and midway along the wire length
90 to reduce any end effects. A CANBERRA 2006 charge sensitive preamplifier was
91 connected to an SHV-5 bulkhead connector which connects to the wire of the de-
92 tector through a compression fitting [2]. The preamplifier has adjustable charge
93 sensitivity of either 47 or 235 mV/Mion-pair (47 for this work), diode protected
94 field-effect transistor, pole-zero adjustment and low noise/fast rise time over a
95 wide range of input capacitance making this a good choice for a proportional
96 counter. Specific details of the preamplifier can be found in the user manual [16].
97 The output analog waveforms of the preamplifier were digitized with a 75 MHz
98 (1023 samples) Pixie-4 acquisition card [17]. The digital trapezoidal filter had
99 an energy filter flat top of 4.68 μs , rise time of 20.85 μs and a trigger filter
100 flat top and rise time of 0.1064 μs . Waveforms of 13.6 μs with a pre-trigger
101 of 1.5 μs were digitized, analyzed and stored for every event above the trigger
102 threshold. The Pixie-4 ADC channel-to-voltage calibration was accomplished in
103 parallel to taking gas gain spectra and consisted of injecting a fixed charge into
104 the preamplifier and measuring the Pixie-4 ADC channel number. This allowed

105 each photopeak in the energy MCA to be converted from digital channel num-
 106 ber to voltage. The experimental energy spectra of each applied voltage was
 107 analyzed using ROOT [7] by manually applying Gaussian fits to each peak of
 108 interest (8 and 59.5 keV photopeak and test pulse). Subsequently, the number
 109 of secondary electrons created in the electron avalanche was calculated using
 the charge sensitivity of the preamplifier from which the gas gain was derived.

Figure 1: Cross section of the model used in simulations with a zoom on the dummy cylinders used to control the meshing process. The proportional counter wire was $25 \mu\text{m}$ in diameter and the cathode had an inner diameter of 2.54 cm. The center black cylinder is the anode wire and the green, blue, magenta, and orange cylinders represent imaginary surfaces of vacuum where Maxwell must form a 2D surface mesh.

110

111 3. Prototyping method and Simulations

112 3.1. Finite Element Modeling in Maxwell

113 To simulate the gas gain of the ULBPC, a geometrical model of the physical
 114 device was created in SolidWorks®. The SolidWorks model was exported in
 115 ACIS-SAT format then imported into Maxwell where materials and boundary
 116 conditions are assigned. Maxwell iteratively generates the finite element mesh
 117 using second-order tetrahedra until the solution reaches a set minimum error

energy. To reduce simulation time and focus on areas of interest, the simulations presented in this work modeled about one-tenth of the ULBPC design as shown in Figure 1.

Preliminary models of the ULBPC solved with Maxwell were accurate in low-field regions; however, high-field regions near the anode wire were distorted and unsuitable for simulations. Field distortions were caused by tetrahedral elements with high aspect ratios, meaning the ratio of their longest and shortest dimensions is much greater than one. To reduce aspect ratios and improve accuracy, “dummy” structures were introduced to manually force the creation of a 2D mesh at these surfaces. As shown in Figure 1, the dummy structures in the detector model were concentric cylinders running along the wire. Strategic selection of the radius of each dummy cylinder and the number of line segments used to approximate curved objects in the model kept the aspect ratio near one for most elements in the final mesh. The radii of the cylinders were set to 1.25, 1.5, 2 and 4 times the radius of the wire ($12.7 \mu\text{m}$). The cylinders were approximated with line segments covering 15 degrees of the surface. This led to the tetrahedra of the mesh adjacent to the wire surface to have one edge of approximately one quarter of the wire radius. Thus setting the dummy cylinder with 1.25 times the wire radius led to an aspect ratio close to one. Radii of the outer dummy cylinders were set in similar way. The gas mixture was 3 atm P-10 for all simulations. Another parameter available in G++ is the Penning ratio. The Penning effect is a mechanism where a metastable excited state of a gas atom (i.e., Ar^*) can ionize the quench gas (i.e., methane) and therefore increase the number of secondary electron/ion pairs during the multiplication process. Without the presence of a quench gas this energy would be lost and therefore less electron/ion pairs would be generated. The Penning ratio was set at 0.24, an approximation based on the work of Şahin et al. [18]. Simulations were performed at typical experimental operating parameters given in Section 2.

3.2. Data conversion and structure

The Maxwell solution files containing the potential, electric field and boundary conditions must be imported into G++ for subsequent electron and ion drift simulations. The Fortran version of Garfield stopped updating the model import code at Maxwell v.11 and G++ also lacked a class for the v.16 import. Two paths were identified to interface Maxwell v.16 and G++: either a new class could be written to directly import the v.16 data into G++ or the data could be converted to an intermediary format prior to import. The latter approach was chosen to eliminate redundancies in the original data, reducing load time in subsequent simulations. The general ASCII format of Gmsh [19], a 3D open-source finite element mesh program, was chosen as the intermediate format as it is well documented and allows viewing of the mesh outside of Maxwell and G++ for verification. The Maxwell mesh converted into Gmsh format for the counter geometry is shown in Figure 2. Software engineers from ANSYS provided assistance by developing a Matlab script which sorted the mesh solution. The code was converted into Python to be run from the interpreter built into Maxwell. Enhancements were also made to the script to include the midpoints of

Figure 2: Gmsh rendering of the Maxwell mesh after importing into Gmsh. The dummy cylinders are visible in the zoomed portion of the figure and mesh boundary conditions are seen.

163 the second-order tetrahedra that could be used for more accurate interpolations
 164 in G++. As noted by Veenhof [20], FEM programs have drawbacks associated
 165 with solving non-linear fields. In particular, the use of tetrahedral elements will
 166 most likely lead to a discontinuous electric field on tetrahedra boundaries. In-
 167 corporating the midpoints of the field solution suggests the interpolated field in
 168 G++ is continuous across normal tetrahedral boundaries. A comparison of the
 169 magnitude of the electric field solved with Maxwell to the interpolation after
 170 import into G++ is shown in Figure 3. The G++ solution is shown as the solid
 171 (black) curve and the Maxwell solution is the dash (red) curve. Overall, there
 172 is good agreement between the two solutions except at the dummy cylinder
 173 boundaries (singularities).

174 3.3. Mesh structures in Garfield++

175 The basic finite element field class in G++, `ComponentFieldMap`, uses a
 176 simple unstructured mesh: an array for the mesh elements and an array for the
 177 element nodes. Existing FEM import classes with second-order tetrahedra avail-
 178 able in G++, especially the `ComponentElmer` class contributed by Renner [21]

Figure 3: Plot of the magnitude of the electric field, $E(r)$, as a function of r ; i.e., from the cathode wall to the wire surface. The G++ interpolated solution is the solid (black) curve and the Maxwell solution is the dash (red) curve. The line plot was made at a coordinate absent from end effects. The dummy cylinder edges are indicated by the vertical dash lines.

179 and the `ComponentAnsys123` class added by Schindler [22], provided the tem-
 180 plate for the new component class developed for the Gmsh files created from
 181 Maxwell models, i.e., `ComponentGmsh`.

182 Computation time for larger meshes is dominated by searching for the tetra-
 183 hedron that contains a particular point, referred to as a point-location search.
 184 Models used in this work contained in excess of a million elements and it was
 185 apparent in early trials that the existing point-location algorithm needed modi-
 186 fication to enable use of larger meshes. Since Maxwell uses straight-edged tetra-
 187 hedra, `ComponentGmsh` was able to use the less computationally intensive search
 188 algorithm used by first-order tetrahedra. Additionally, the mesh was modified
 189 to store the minimum and maximum X, Y, and Z values of each tetrahedron
 190 reducing redundant calculations which further decreased computation time.

191 The afore mentioned modification decreased simulation time but the un-
 192 derlying search algorithm was still impractical for larger meshes. Thus a new
 193 class was written on the premise that Maxwell’s meshes meet the requirements
 194 for Delaunay triangulation, specifically that the meshes are convex, i.e., the
 195 model cannot contain any internal holes or indentations in the outer boundary.
 196 Two new classes, `DelaunayGmsh` and its parent `DelaunayFieldMap`, were writ-
 197 ten using a structured mesh in which each element contained information about
 198 neighboring tetrahedra. The improved search algorithm works by stepping to

neighbors that are progressively closer to the point of interest. The specific algorithm can be referred to as a “stochastic walk” as it randomizes orientation tests to prevent infinite search loops in non-Delaunay meshes [12]. To the authors knowledge this technique has not been applied to gaseous detectors before but Delaunay triangulation has been applied to solid-state detectors [23]. For a more detailed description of the Delaunay method see Ref. [24].

Maxwell does not use a true Delaunay triangulation though we found the results behaved similarly so a “visibility walk” similar to that described in Devillers et al. [12] worked for most cases. This work initially relied on the visibility walk and would fall back on a global search when infinite loops were encountered. Though the stochastic walk can have search times that scale exponentially with number of elements [12], Maxwell does not create the specialized configurations that cause exponential search length. The results of the global search algorithms and the stochastic walk are compared in Table 1. The authors expect other finite element packages will produce meshes that are similar as long as high aspect ratio elements are avoided.

Table 1: Speed improvements of drifting 1000 electrons using **AvalancheMC** in the proportional counter model starting 1.2 cm from the anode wire.

Algorithm	Time for 1000 electrons (s)
ComponentGmsh, min/max calculated	43600
ComponentGmsh, min/max stored	6200
DelaunayGmsh	21.0

4. Results and Comparison to Measurements

4.1. Avalanche Simulations and Gas Gain

An electron avalanche is the underlying mechanism for signal generation in gas proportional counters. The absorption of a photon or the ionization from a charged particle creates primary electron/ion pairs in the gaseous medium. In G++ the number of primary charge is calculated by HEED [9]. The charge drift under the influence of an applied electric field to the corresponding detector electrodes. The electrons are accelerated towards the wire and gain enough energy to ionize additional gas atoms (usually a noble gas based mixture) and begin an avalanche. The drift motion of the electrons and ions generated from the avalanche induce a current on the wire that is detected by electronics. The majority ($\approx 99\%$) of the signal induced in a traditional coaxial proportional counter is due to the relatively slow drift velocity of the ions and long drift path.

Avalanche simulations can be performed with either the **AvalancheMC** or **AvalancheMicroscopic** classes in G++. In this work the **AvalancheMC** class was used for the drifting of ions and The **AvalancheMicroscopic** class was used for electron avalanche simulations since it is generally more accurate given its more rigorous treatment of collisions and ionization.

One method of avalanche simulations was performed by drifting electrons from a discrete point in the active gas volume of the ULBPC model to the wire surface for subsequent avalanche formation. The number of electrons created in each avalanche was populated in a histogram that produces a single-electron spectrum (SES). The Polya distribution from the work of Jahoda et al. [25] was used to fit to the SES distribution:

$$P(q, \mu, h) = \frac{1}{\mu} \frac{h^h}{\Gamma(h)} \cdot \left(\frac{q}{\mu}\right)^{h-1} \exp\left(\frac{-hq}{\mu}\right), \quad (1)$$

where q is the number of electrons produced in a single avalanche, μ is the mean number of electrons produced (the gas gain M), h is a width or shape parameter, and Γ is the gamma function. In Eq. (1), h can also be related to the relative variance on the gain, f , as [26]:

$$h = 1/f. \quad (2)$$

The SES was based on 2.5×10^5 electron avalanches originating at a 1.2 cm radius from the anode wire (the cathode wall is a radius of 1.25 cm) and 1.6 cm from the wire crimp along the wire axis to reduce end effects. It is assumed that this location is a good representation of the majority of the counter volume. The resulting SES distribution is shown in Figure 4, ignoring avalanches with zero gain. Net zero gain avalanches result from electron attachment, drift outside of the model sensitive volume or hitting the cathode wall. A Polya curve defined in Eq. (1) was fit to the SES distribution and the average gain of the detector was 1209 ± 7 . In addition to the SES, the gas gain was also modeled by photon absorption using the HEED interface. To represent the different photon energies, HEED simulations were run with 1000 photons at 8.05 keV, 500 at 8.03 keV, and 170 at 8.9 keV. The method using HEED will generate a Gaussian distribution similar to what is measured experimentally. Figure 4 also contains a histogram of the simulated gas gain of full photon absorption and primary electron/ion pair calculations using HEED. The gas gain using the HEED calculation was determined by dividing the total number of secondary electrons produced in the avalanche simulation by the number of primary electrons produced by the photon absorption in the gas. The latter quantity is equal to the ratio of the photon energy (8.05 keV) by the mean energy needed to create an electron/ion pair in P-10, W-value of 26 eV/(e-/ion pair) [27]. A Gaussian curve was fit to the histogram yielding a mean gas gain of 1172 ± 2 . The lower energy peak at a gas gain of ~ 680 is the argon escape peak equal to full photon absorption minus the Ar k-edge, i.e., 4.7 keV. The gain calculated using HEED is 3% lower compared to the value returned using SES. However, only using 310 primary electrons (i.e., the expected number of primaries for an 8.05 keV photon) and not accounting for the higher-energy fluorescent photons should shift the gas gain higher than that calculated using SES. Therefore, a simple explanation for the lower calculated gas gain of HEED comes from the number of primary electrons determined by HEED compared to the number expected by simple division of photon energy by

Figure 4: A) G++ SES used to extract the gain of the ULBPC model detector filled with 3 atm P-10 at an anode voltage of +2150 V. B) A distribution of the calculated gain for each avalanche originating from HEED photon events. The mean and standard deviation are found by a Gaussian fit to the data.

273 the asymptotic W-value. HEED often produces $\approx 3\text{--}4\%$ fewer primary electrons,
 274 299 for 8.05 keV photons, which correlates to the 3% deficiency in difference
 275 amongst the two methods. Though the number of primary electrons produced
 276 by HEED could be used for simulated gain calculations, the value of 310 primary
 277 electrons was used to remain consistent with the experimental gain calculation
 278 of ULBPC.

279 The gas gain curves from simulations and experimental measurements from
 an ULBPC are shown as a Diethorn plot in Figure 5. The Diethorn formula is

Figure 5: Diethorn plot for SES-based simulations, HEED-based simulations, and a proportional counter calibrated with copper K_{α} lines (8.05 keV and 8.9 keV) or 59.5 keV gamma-rays from ^{241}Am .

280 defined as:
 281

$$\ln(G) = \frac{V}{\ln(b/a)} \cdot \frac{\ln(2)}{\Delta V} \left(\ln \frac{V}{p a \ln(b/a)} - \ln(K) \right), \quad (3)$$

where G is the measured gas gain, a is the radius of the anode, b is the radius of the cathode, p is the gas pressure and V is the applied voltage. The Diethorn parameters K and ΔV are the minimum electric field needed to start an electron avalanche and the potential difference an electron experiences between successive collisions, respectively [28]. There is good agreement between the Diethorn parameters of the experimental measurement and the simulation results as shown in Table 2 following linear fits to the data. If the gas gain at +2150 V is compared between the mathematical average of gas gain from simulation and measurement, the simulated gain is $\approx 15\%$ deficient. The difference in the gas gain for specific voltages between the simulations and measurements can most likely be attributed to the choice of (a constant) Penning transfer ratio in the simulation, uncertainty of the gas fill composition and/or pressure and other fine mechanical details of the actual detector that are not captured in the simulation (e.g. surface of the fine anode wire). Previous simulation studies have used multiple Penning ratios to calculate the value that best matches experiment [29] and the same method will likely be applied in future simulations of ULBPCs.

The 59.5 keV data points at high gas gain diverge slightly from a linear relationship and this could be space charge effects. It has been shown that the relative channel shift due to space charge effects can be calculated for cylindrical proportional counters [30] and there is a dependence on pulse rate and photon energy (i.e., the number of primary electron-ion pairs created). The count rate during experimental testing was low (≈ 1 Hz) and therefore pile-up is not an issue. Detailed calculations for the ULBPC are planned in the future to determine if the photon energy is contributing to the observed disagreement.

Table 2: Diethorn fit parameters comparing the simulated and experimental measurements of the ULBPC in Figure 5 and the gas gain for all methods at +2150 V. The uncertainty on the Diethorn parameters only includes statistical standard deviation from the linear fit to the data. Experimental uncertainty is estimated at $\approx 15\%$.

Data set	K [kV atm ⁻¹ cm ⁻¹]	ΔV [V]	Gain +2150 V
SES simulation	31.4(1.1)	29.3(0.8)	1212.3
HEED simulation	31.5(1.2)	29.3(0.8)	1172.0
Copper K_α	30.4(0.4)	29.2(0.3)	1408.8
²⁴¹ Am 59.5 keV	30.8(0.3)	29.3(0.2)	1327.3

4.2. Induced current and signal development

The induced signal from the drifting of the liberated charge was simulated in two parts: the electron drift during avalanche and subsequent drifting of ions. G++ uses the Shockley-Ramo theorem to calculate the induced current produced by the movement of charged particles through the detector [31, 32]. The required weighting field is calculated in Maxwell by setting the potential of the anode wire to unity and ground potential elsewhere. The simulated induced current signal was further processed by a circuit simulator to mimic the front-end electronics of the detector (i.e., CANBERRA 2006 preamplifier). Therefore, the

316 induced current on the anode wire was simulated using an avalanche of multiple
 317 primary electrons produced by photoionization in HEED. A single photon with
 318 energy 8.05 keV was released into the detector, producing primary electrons that
 319 were subsequently avalanched microscopically; ions produced by the avalanche
 320 were then drifted toward the cathode wall using the Monte-Carlo method with
 321 time steps of 10 ns for a total of 14 μ s. This provided an induced signal of
 322 sufficient time to compare to experimental waveforms acquired by the ULBPC
 323 data acquisition cards (13.6 μ s). Though ions continue drifting for 4–5 ms, most
 324 of the induced signal develops in less than 100 μ s making it acceptable to ignore
 325 most of the drift time. As a first approximation, the ion mobility for P-10 was
 326 set at $1.9 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$, which corresponds to the low-field limit given in Ref. [33].

327 The induced current signals that corresponded to full photon absorption (i.e.,
 328 escape peak events were not considered) were used for further processing. The
 329 simulated signal is a sum of the electron and ion induced currents. The functional
 330 schematic of the preamplifier was modeled using Qucs, an open-source circuit
 331 simulator [34], and the output was compared to the experimental results. The
 332 simulation produced a table of time and induced current values. This data table
 333 was processed by Qucs that then produced a simulated output preamplifier
 signal (i.e., voltage) as a function of time. For ease of viewing and comparison,

Figure 6: A: Plot of multiple signals collected from a ULBPC using 3 atm P-10. The black line shows the mean signal based on a point-by-point Gaussian fit of the signals. B: Simulated signals using the same conditions.

334 the experimental induced signals from the ULBPC and the simulated waveforms
 335 were normalized to a baseline of zero using the average of their respective pre-
 336 trigger baseline. The pulse rise time and amplitude of signals produced via
 337 simulations show similar characteristics to the experimental measurements. The
 338 mean amplitude of 1000 experimental waveforms was 1560 ± 109 ADC channels
 339 for the copper K_{α} peak and 250 simulated pulses from 8.05 keV photons had a
 340 mean amplitude of 1502 ± 87 ADC channels. Therefore, the mean amplitudes of
 341 the induced waveforms agree within uncertainty and also show the same (lower)
 342 trend of simulated gain compared to experimental.

5. Conclusions

A simulation procedure was successfully developed to model the ULBPC manufactured at PNNL. First, a CAD model was imported into ANSYS Maxwell (version 16) to calculate the electric potential map. This solution was then imported into G++ to simulate transport and avalanche phenomena in the gas volume. In order to accomplish this work, new classes were added to G++ to properly import the Gmsh-formatted files. An entirely new point-location search algorithm was introduced involving a “stochastic walk” that requires far fewer steps to locate tetrahedra of interest. This new method lead to a large reduction in simulation time on the order of three orders of magnitude compared to the original algorithm.

The gas gain of the ULBPC was found to be ≈ 1200 through simulations at a voltage of +2150 V and 3 atm P-10 with a Penning ratio of 0.24. The deviations between the simulated and experimental gas gain are likely due to the use of a fixed Penning ratio in the simulation, uncertainty in the first Townsend coefficient and the mechanical model used in the simulation not encompassing all fine mechanical details of the experimental apparatus (the anode wire surface) are responsible. The experimental and simulated gas gain was compared using a Diethorn plot. Within the statistical uncertainty, the Diethorn parameters are in agreement but there is a clear systematic offset between the experimental data (8 keV and 59.5 keV) and more significantly, between the simulated and experimental measurements. The offset comparing the experimental and simulated values could be due to the constant Penning ratio used in the simulation, or the use of a constant W-value for the two photon energies (or a combination of these factors). The disagreement between the two experimental data sets can be explained by an electric field screening due to space charge effects of the 59.5 keV data. Future work is expected to address items that can allow the simulations to correlate with experimental results of ULBPCs. Furthermore, the signal output of the preamplifier coupled to an ULBPC was modeled using the induced current from G++ and the Qucs program. The gas gain of the signals generated by Qucs was calculated by sampling the amplitude (largest bin) and back-calculating the number of secondary electrons. This distribution exhibited the same (lower) trend that the microscopic avalanche simulations suggest in comparison to the experimental gas gain data.

Utilizing G++ for the ULBPCs is a powerful toolkit for the future development of the low-background program at PNNL. This capability is envisioned as being able to predict detector response to geometrical changes in the detector volume eliminating costly trial and error experimental implementation. Examples include mechanical changes to the dielectric components, varying the anode wire diameter and changing the detector volume to support all applications covered by the ULBPCs. Future work includes adding random noise to the simulated waveforms at a level represented by the experimental apparatus, and analyzing the G++ induced waveforms with the current ULBPC software to fine-tune the simulation parameters. Following the inclusion of (electrical) noise to the Qucs waveforms, the energy resolution of each simulated induced signal

could be measured and compared to the experimental energy resolution. Efforts are currently underway to estimate the fiducial volume of the full proportional counter using the tools described in this work.

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