A New Method to Reduce the Statistical and Systematic Uncertainty of Chance Coincidence Backgrounds Measured with Waveform Digitizers

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Abstract

A new method for measuring chance-coincidence backgrounds during the collection of coincidence data is presented. The method relies on acquiring data with near-zero dead time, which is now realistic due to the increasing deployment of flash electronic-digitizer (waveform digitizer) techniques. An experiment designed to use this new method is capable of acquiring more coincidence data, and a much reduced statistical fluctuation of the measured background. A statistical analysis is presented, and used to derive a figure of merit for the new method. Factors of four improvement over other analyzes are realistic. The technique is illustrated with preliminary data taken as part of a program to make new measurements of the prompt fission neutron spectra at Los Alamos Neutron Science Center. It is expected that these measurements will occur in a regime where the maximum figure of merit will be exploited.

Keywords: chance coincidence background, waveform digitizer, statistical uncertainty, systematic uncertainty

1. Introduction and Motivation

The impact of backgrounds on the statistical significance of an experiment is of such importance that Knoll discusses it in his seminal book, *Radiation Detection and Measurement*, just as soon as sufficient statistical background is presented to understand the subject[1]. One of the compromises in performing an experiment is to reduce the amount of time spent taking foreground data

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27 (and losing counts and statistics), to take background data. If too much time is
 28 spent on the background measurement, then valuable foreground statistics are
 29 lost. On the other hand, if insufficient background data are obtained, the quality
 30 of the foreground data is compromised by a poor background subtraction. In
 31 the book, Knoll presents an expression for the optimal division of time between
 32 the two stages of data collection. The question of systematic changes between
 33 the two separate measurements is not considered — but is often apparent, for
 34 example if a background normalization factor has to be applied during the data
 35 analysis. The complications introduced by backgrounds warrant all reasonable
 36 attempts to remove the background.

37 Over the years the coincidence technique has proven to be an powerful
 38 method to reduce or even remove large backgrounds. Knoll also discusses the
 39 nature of chance coincidences as a source of background in coincidence exper-
 40 iments, and presents a simple formula to estimate the chance coincident rate.
 41 The formula is traditionally used to help design layouts for experiments, and
 42 provide estimates for the quality of the data when setting up and experiment.
 43 The formula for rate of chance coincidences, r_γ , between two detectors, counting
 44 at rates r_a and r_b , during a coincidence time window with width, Δ_t , is

$$r_\gamma = r_a r_b \Delta_t. \quad (1)$$

45 A key concept in the derivation of eq. (1) is the dead time, t_d , of the detector,
 46 electronics and the data acquisition system (DAQ), even though t_d is not explicit
 47 in the formula. Eq. (1) is only valid if $r_a t_d \ll 1$ and $r_b t_d \ll 1$.

48 While eq. (1) is well known, we are not aware of any analyzes using it to
 49 measure the detailed background shape in a coincidence measurement, presum-
 50 ably due to the difficulty in measuring r_a and r_b reliably. High singles rates
 51 impose challenges in acquiring, saving and analyzing complete data sets using
 52 an event-triggered DAQ with very real dead-time concerns.

53 The current work is motivated by the increasing use of flash digitizers run-
 54 ning semi-continuously (waveform digitizers), with waveform analysis capabil-
 55 ities on board, to build high throughput DAQs with no contribution to dead

time (see e.g. [2–4]). When combined with computers with large storage areas it is now realistic to record complete data sets of all the signals from all the detectors with dead time due only to signal overlap (pileup) in the detector and preamps. Coincidences are identified, in software, later in the analysis.

With such a DAQ we show that Knoll’s expression to estimate chance coincidence rates can now be used to determine a bin-by-bin measurement of the background obtained simultaneously with the foreground. In addition to reducing systematic uncertainties, this allows to obtain the maximal signal statistics, and it will be shown yields a very small statistical uncertainty on the background measurement. The question of how much time to spend on the foreground and background measurements becomes trivial — do both all the time!

Before proceeding, we rewrite eq. (1) in terms of the counts observed in and experiment, which also allows us to easily quantify the statistical uncertainty. The numbers of counts, a and b , obtained in each detector, and summed over N measurements, are then related to the measured singles rates by $a = r_a N \Delta_t$ and $b = r_b N \Delta_t$, and the number of chance coincidences, γ , in the time-difference window, Δ_t , is then

$$\gamma \pm \sigma_\gamma = \frac{ab}{N} \pm \sqrt{\frac{ab(a+b)}{N^2}}, \quad (2)$$

where σ_γ , the statistical uncertainty on γ , assuming Gaussian statistics for a and b .

In the next two sections we briefly describe an experiment being developed to measure the prompt fission neutron spectrum (PFNS) on ^{239}Pu [5], and then in more detail, two possible analysis procedures which use eq. (2) to estimate backgrounds for this experiment. In the following section we contrast this approach with more traditional methods for measuring backgrounds — resulting in the derivation of a figure of merit for the new method of analysis. There then follows a comment on the data-acquisition requirements to obtain a data set which is complete enough to apply the current technique (and a numerical validation of $r_a t_d \ll 1$ and $r_b t_d \ll 1$ for the example experiment). Further formulas derived from eq. (2), and which are needed to implement a full analysis,

Figure 1: Conceptual layout for a two-arm time of flight $^{239}\text{Pu}(n; f, xn)$ experiment, detecting fission fragments in a parallel-plate avalanche counter(PPAC), (which implicates the timing of an incoming neutron, with respect to the neutron production time signal from the t_0 pick off), and fission neutrons are detected in the lithium glass neutron detectors.

are presented in a short series of appendices.

2. Example Experiment With Complicated Backgrounds

Data for outgoing fission neutrons from neutron induced fission of ^{239}Pu are used to illustrate the techniques discussed here. The coincidence nature of the experiment arises from detecting one of the fission fragments together with a neutron generated in the fission process. The detection of a fission fragment implicates the timing properties of the incoming neutron.

The data were acquired as part of the development for the PFNS measurements currently being performed at the WNR/LANSCE neutron spallation source[6]. A conceptual figure of the experiment is show in fig. 1. Beam delivery was structured on two time scales (fig. 2): micro pulses of protons (each making their own timing signal, t_0 , and forming their source of spallation neutrons) occur at nominal $1.8\mu\text{s}$ intervals; and macro pulses, being groups of 347 micro

Figure 2: Structure of the proton beam to WNR.

98 pulses, separated by several milliseconds. Only two of every three macro pulses
99 were delivered to WNR.

100 Model ZT4441-DP-PXI digitizers[7] (sample rate 400Ms^{-1}) were used to
101 acquire 1ms long waveforms spanning the duration of complete macro pulses.
102 Triggers for most of the digitizers were distributed across a PXI bus from a
103 self-triggering digitizer receiving the t_0 signal. ZT1000PXI cards[7] enabled
104 distribution of the trigger and a stable reference clock between PXI crates.
105 Up to 22 lithium-glass detectors and digitizer channels were used to detect the
106 outgoing neutrons[8]. A further ten digitizer channels processed the signals from
107 a multi-segment parallel-plate avalanche counter (PPAC) for detecting fission
108 fragments[9]. Each waveform was analyzed on board the digitizer using custom
109 firmware, to generate a list of parameters such as a time stamp, the baseline
110 height, and two integrals of the peak area at different time offsets from the peak
111 position, for all the peaks found in the waveform. The lists of parameters were

Figure 3: Distribution of singles events within the macro pulses for one lithium-glass detector and one PPAC foil. Cuts to reduce backgrounds have been applied (see text).

112 read from each digitizer across the PXI bus into computer memory for storage
 113 and further processing, before preparing the digitizers to process another macro
 114 pulse.

115 Backgrounds from several sources, are present in the data. Some of the
 116 more significant sources are: 1) fast neutrons generated from interactions other
 117 than fission from a particular sample foil; 2) a "sea" of thermal and epithermal
 118 neutrons (see the rising and decaying baseline in the top panel of fig. 3; 3)
 119 sensitivity of the neutron detectors to gamma rays; and 4) a small residual
 120 sensitivity of the PPAC to the large alpha decay rate of the plutonium samples
 121 (fig. 3, lower panel).

122 These backgrounds remain significant, even after applying some simple back-
 123 ground reduction cuts, such as a cuts to separate neutrons from gamma rays in

Figure 4: Lithium glass detector (upper panel), and PPAC (lower panel) signal times within the micro-pulse structure.

the lithium-glass detectors (exploiting the +4.78 MeV Q-value of the ${}^6\text{Li}(n,t)\alpha$ reaction), and also PPAC pulse-height cuts to optimize fission-alpha-decay separation. We anticipate using a more complete set of cuts for a final analysis of the data from these experiments, but we still expect many components of the background to remain.

The PPAC and lithium-glass signal times within a macro pulse are denoted t_f , and t_n , respectively.

3. Coincidence Analysis

Two methods are available to fully identify the triplet of times $\{t_0, t_f, t_n\}$, of a fission event.

In the first method, the lists of $\{t_f\}$ and $\{t_n\}$, from which fig. 3 was made, are searched to form a list of coincidences $\{t_f, t_n\}$. At this point a histogram

Figure 5: Time of flight histogram (upper curve, black online) for coincidences from one lithium glass-PPAC combination, correlated on a common t_0 , and the measured background from chance coincidences (lower curve, red online).

136 of $t_n - t_f$ could be filled, and a measurement of the background due to chance
 137 coincidences could be derived using eq. (2). The quantities a and b would be
 138 taken from the singles histograms of fig. 3, and N would be the number of macro
 139 pulses used to acquire the data.

140 To complete the first method, pairs of $\{t_0, t_f\}$ are also identified, and then
 141 correlated with the $\{t_f, t_n\}$ pairs on a common t_f . Note that the background due
 142 to chance-coincidences is determined before the fission event time is correlated
 143 with the neutron production time. This makes it difficult to use the first method
 144 to study the background dependence on $t_f - t_0$. Therefore a second method was
 145 found, which although leading less directly to the $\{t_0, t_f\}$ coincidences, allows
 146 to measure the background even with complicated cuts.

147 For the second method, lists of $\{t_0, t_f\}$ between the fission-fragment and
 148 t_0 signals (the same list as used in the final stages of the first method), and

Figure 6: Time of flight histogram (upper curves, black online) for coincidences between many combinations of lithium glass-PPAC detectors correlated on a common t_0 , and the measured background due to chance coincidences (lower curves, red online), for four different ranges of $t_f - t_0$, and summed over all available lithium-glass-PPAC combinations.

149 between the neutron detector and t_0 signals $\{t_0, t_n\}$ are formed, and appropriate
 150 histograms filled (fig. 4). The t_0 times are not generated by a Poisson process
 151 and so eq. (2) is not yet applicable. A second search is then used to correlate
 152 pairs of $\{t_0, t_f\}$ and $\{t_0, t_n\}$ on a common t_0 , thus allowing to fill the $t_n - t_f$
 153 histogram (fig. 5). $\{t_0, t_f\}$ and $\{t_0, t_n\}$ are each driven by a (time-dependent)
 154 Poisson process, and so eq. (2) can be used to determine a background due to
 155 chance coincidences for the histogram of $t_n - t_f$ for a common t_0 (red curve on
 156 fig 5). The a and b values are taken from the histograms of fig. 4, and N is now
 157 the total number of t_0 signals. In contrast to the first method, the background
 158 can still be determined even with cuts applied to the fission time, $t_f - t_0$, by just
 159 applying cuts to the list of $\{t_0, t_f\}$ before filling the histograms of figs. 4 and 5.
 160 Fig. 6 shows such an analysis.

161 For the analyzes both with and without cuts (figs. 5 and 6), attention
 162 is brought to the smaller peak in the double coincidence data for $50\text{ns} <$
 163 $t_n - t_f < 100\text{ns}$, the tails of which are of interest (especially the slower times,
 164 i.e. $< 240\text{ keV}$ outgoing neutron energy)[5]. The large background under the
 165 low energy tail (larger $t_n - t_f$) makes this region especially suitable for the new
 166 background measurement techniques (see the figure-of-merit discussion below).
 167 The larger peak ($0\text{ns} < t_n - t_f < 50\text{ns}$) is dominated by fission photons, which
 168 are to be removed by cuts in a more complete analysis, but should also contain
 169 events from fission with higher energy outgoing neutrons. Features of the back-
 170 ground shapes are better understood once the full two-dimensional background
 171 is presented below, but it is important to note from fig. 6, firstly that the back-
 172 grounds are not flat and secondly that the background shape varies with the
 173 incident neutron energy.

174 As a practical matter, data processing for the background analysis may
 175 be performed either before or after adding independent subsets of the data
 176 set, depending on whether significant systematic changes in the count rates
 177 have occurred or not. Systematic changes between subsets require independent
 178 background analyzes, where as summing over independent subsets requires less
 179 resources (e.g. CPU time and memory) to process the data. Both approaches
 180 work (see Appendix A), or even a combination, depending on the details of
 181 the experiment. The current data were acquired in four hour subsets spanning
 182 about one hundred hours and one background analysis was performed on the
 183 summed results.

184 A second practical consideration is how to handle the two-dimensional (2D)
 185 nature of eq. (2). The values of a and b each depend on one or the other of
 186 the two times for the random processes, in this case, $t_f - t_0$ and $t_n - t_0$, so
 187 that $\gamma(t_a, t_b)$ is a dyadic function of these times. The one-dimensional (1D)
 188 background presented in fig. 5 is a projection along lines parallel to the primary
 189 diagonal of $\gamma(t_a, t_b)$ (see right hand side of fig. 7), just as the coincidence spec-
 190 trum itself is a projection along the same lines of a correlation plot between
 191 $\{t_f - t_0\}$ and $\{t_n - t_0\}$ (left hand side of fig. 7). Processing the complete 2D

Figure 7: Measured two-dimensional correlations (white indicates zero counts) between $\{t_f - t_0\}$ and $\{t_n - t_0\}$ (left), and the measured background quantity $\gamma(t_a, t_b)$ (right). Overlaid diagonal lines (red online) mark the limits used to project one dimensional distributions of fig. 5 (and 6).

background space is not always necessary. Firstly it is only necessary to process that section of the space near the coincidence diagonal (between the red lines in fig. 7). Secondly, for most analyzes a direct determination of the 1D distribution can proceed using the formulas of Appendix B. If a 2D analysis is still required (e.g. the next four paragraphs), a lower resolution will often suffice.

Understanding the 2D nature of the problem can provide insights into the results obtained, as features of the 1D background shape can be attributed to summing over different features in the 2D plot of fig. 7. These features will become more or less dominant depending on the regions summation used due to cuts (such as those on $t_f - t_0$ in fig. 6), and also due to the sensitivity of each detector to the physical processes contributing to the background.

For example, one feature of the 1D background in fig. 5 is a modestly peaked structure at small time differences. The 2D distribution shows that the small 1D peak arises from a very high and narrow peak, located at the intersection of the high count rate bands near $t_f - t_0 \approx 0$, and $t_n - t_0 \approx 0$, with tails following along each of the band. The peak is then a result of chance coincidences between fission neutrons (or other beam induced neutrons) and fission signals. The upper

209 left panel of fig. 6 shows a much stronger peak, now shifted to slightly negative
 210 $t_n - t_f$. In this case, the two most forward angle neutron detectors (not included
 211 in fig. 7) are very sensitive to direct scattering of neutrons from the beam and
 212 would show up in the two-dimensional analysis as a strong enhancement of
 213 the horizontal band near $t_n - t_0 \approx 0\mu\text{s}$, and the 1D peak arises from chance
 214 coincidences between the scattered beam neutrons and fissions[10]. Similarly,
 215 the step in the background in the lower right panel of fig. 6 arises from the
 216 scattering into these two detectors, now from beam neutrons from the next
 217 micro pulse at $t_n - t_0 \approx 1.8\mu\text{s}$.

218 On the other hand, the low background in the center of the 2D distribution
 219 sums over many channels in the 1D projection, and can therefore become com-
 220 parable in magnitude to the peak at $t_n - t_0 \approx t_f - t_0 \approx 0$. The instantaneous
 221 neutron beam intensity is reduced in this region. Therefore chance coincidences
 222 in this region are more likely to involve neutrons from the "sea" of thermal and
 223 epithermal neutrons, and/or the plutonium alpha decays events in the PPAC.

224 Another striking aspect of the background measurement in figs. 5 and 6 is
 225 how small the statistical fluctuations on the background measurements are. In
 226 fact, the one-sigma uncertainty bars are marked on the figure, and even on the
 227 log plot are hard to see. The origin of the small uncertainties derives from both
 228 the large number of counts available in the two input spectra, and also from
 229 dividing by a large value of N (in this case $N \approx 2.5 \times 10^9$). A full statistical
 230 analysis, including a derivation of the covariance matrix between elements of
 231 the background measurement is presented in Appendix C.

232 The method is extensible to more complicated analysis, such as calculating
 233 the background as a function of a singles parameter against the time difference
 234 of the coincidences; or even then summing over the time difference coordinate.
 235 For example, full analysis of the example experiment will apply a kinematic
 236 cut in the outgoing time-of-flight versus outgoing neutron pulse height space to
 237 remove the photon peak from figs. 5 and 6, requiring summing the background
 238 contribution only in the same region. Other possibilities, perhaps more suitable
 239 for other experiments, include projecting the background onto the axis of the

Figure 8: Upper plot — signal-to-noise ratio as a function of the background strength, ϕ , from the complete singles method (solid line), and traditional methods (dotted line). Lower plot — figure of merit, M , from eq. (9).

240 singles parameter (or combination of singles parameters); or to extend eq. 2 to
 241 higher multiplicity coincidences.

242 4. Figure of Merit

243 We now consider the impact of these results when deciding how much time
 244 to spend acquiring data, by contrasting the signal-to-noise ratio of traditional
 245 methods with that for the new method.

246 With a system that does not record all the singles data, it is often necessary
 247 to dedicate some fraction, τ , of the time to measuring the foreground (with
 248 background), and the remaining fraction $(1 - \tau)$ to measuring the background
 249 alone. If the strength of the background relative to the foreground is ϕ , then the
 250 number of counts, c_{f+b} and c_b obtained during the two parts of the measurement

251 are

$$c_{f+b} = C(1 + \phi)\tau \pm \sqrt{C(1 + \phi)\tau}, \quad (3)$$

$$c_b = C\phi(1 - \tau) \pm \sqrt{C\phi(1 - \tau)}, \quad (4)$$

252 where C is a characteristic constant of the experiment (representing the number
 253 of foreground counts if no time is dedicated to the background measurement,
 254 i.e. $\tau = 1$, any ϕ). The number of foreground counts, c_f , is obtained from these
 255 equations by normalizing c_b and subtracting from c_{f+b} :

$$\begin{aligned} c_f &= c_{f+b} - \frac{\tau}{1 - \tau} c_b \\ &= C\tau \pm \sqrt{\frac{C\tau(1 + \phi - \tau)}{1 - \tau}}. \end{aligned} \quad (5)$$

256 The signal-to-noise ratio, $S_t = c_f/\sigma_{c_f}$, for the measurement follows:

$$S_t = \sqrt{\frac{C\tau(1 - \tau)}{1 + \phi - \tau}}. \quad (6)$$

257 The optimal value of τ for such an experiment is obtained when the partial
 258 derivative of the signal-to-noise ratio, with respect to τ , is zero:

$$\tau = 1 + \phi - \sqrt{\phi(1 + \phi)}. \quad (7)$$

259 The small and large background limits of eq. (7) may be anticipated intu-
 260 itively. Firstly, if there is no background, all the time should be used to mea-
 261 sure the foreground, i.e. $\phi = 0$ implies $\tau = 1$. Secondly, at the other extreme,
 262 $\lim_{\phi \rightarrow \infty}(\tau) = 1/2$ implies to never spend more than half the time measuring
 263 the background. Further implications of eqs. (6) and (7) are discussed in [1].

264 We now contrast this with an experiment designed to acquire all the singles
 265 data from which the coincidences were identified. The maximum amount of
 266 time is dedicated to measuring the foreground, which gives increased foreground
 267 statistics; the background is truly measured in situ, which decreases systematic
 268 uncertainties in the background measurement; and the background is measured
 269 from singles data, which has relatively small statistical uncertainty.

270 If the singles rates are large, then the product ab of eq. (2) has a relatively
 271 small uncertainty, and if N is also large, then the statistical uncertainty on γ
 272 can be insignificant compared to the statistical uncertainty, $\sqrt{C(1+\phi)}$, of the
 273 measurement. The signal-to-noise ratio of a complete measurement analyzed
 274 according to eq. (2) is then

$$S_n = \sqrt{\frac{C}{1+\phi}}. \quad (8)$$

275 A useful figure of merit, M , for this new method is how much longer an
 276 incomplete singles data set and separate background measurement would take,
 277 to obtain the same uncertainty as a complete measurement with background
 278 derived from singles data, i.e.

$$M = \left(\frac{S_n}{S_t}\right)^2 = \frac{1+\phi-\tau}{(1+\phi)(1-\tau)\tau}. \quad (9)$$

279 M is greater than one for any appreciable background; and for large ϕ , where
 280 $\tau = 1/2$, M can easily approach four as $M_{\tau=1/2} = (2+4\phi)(1+\phi)$. The figure
 281 of merit from eq. (9), using the optimal τ from eq. (7), is graphed in fig. 8.

282 5. Validity of Chance Coincidence Equations

283 The effective live-time of a DAQ with significant readout or conversion times
 284 can be reduced by selectively triggering the system on a subset of events —
 285 perhaps even triggering on only the desired coincidence events. It is very difficult
 286 to apply eq. (2) to such data sets. Suitably complete singles data sets should not
 287 include biases from the embedded coincidences, and ideally will not need any
 288 significant dead-time corrections. Therefore suitable data sets must be acquired
 289 using a non-selective trigger — in direct contradiction to a coincidence DAQ
 290 system with a large dead time.

291 A practical system to acquire a complete data set requires detectors and
 292 analog electronics with a relatively fast decay time to avoid pile up; and a dead-
 293 timeless DAQ. Flash analog-to-digital converters, with buffered multi-sample
 294 readout are ideally suited to this technique. Self triggered, on-board processing
 295 capabilities to identify and characterize peaks are not strictly required, but for

Figure 9: Time difference between adjacent peaks in one lithium-glass detector. Upper curves (red online) show raw data, and lower curves (black online) show data after extra peaks are removed (see text).

many experiments will be the only practical way to obtain sufficient coincidence statistics.

It is also necessary to ensure that a data set is not over-complete. Fig. 9 shows the distribution of the time differences between each peak and the previous for one of the lithium-glass detectors, over two different time scales. The firmware was configured to search for 200ns wide peaks, which is then the dead time, t_d . The decay constant of the time between adjacent cleaned peaks gives a count rate in the range of 5ks^{-1} . The product of the rate and the dead time is then about 0.001, which is sufficiently less than one that eqs. (1) and (2) are valid. Peaks at 1.8, 3.6, $5.4\mu\text{s}$, ... arise from the real change in the count rate during micro pulses. The raw data curves (upper curves, red online) show other structures and extraneous data for time differences less than $1.5\mu\text{s}$. These arise

308 predominantly from the peak searching algorithm re-triggering on fluctuations
 309 in the tail of large peaks, and must be removed by a suitable set of cuts on
 310 the data before applying eq. (2) as in the lower curves (black online). The cuts
 311 should be applied before searching for coincidences, so that the background and
 312 signal are determined under the same conditions.

313 6. Conclusions

314 A method for measuring the background due to random coincidences in
 315 coincidence experiments has been presented. Although the method is based on
 316 a well known formula, we describe it as a new method, as it requires the use of
 317 a DAQ which can record all the singles data from which the coincidences are
 318 derived. The ability to take such data has only recently become practical as we
 319 now start to use waveform digitizers, which run continuously during time periods
 320 of interest. The dead times are now driven by detector response functions, rather
 321 than the digitization process or data recording process.

322 The method is very powerful, as it allows for true *in situ* background mea-
 323 surements, with a high degree of statistical certainty. The term *in situ* applies
 324 to the physical environment in which the detectors are located, and also to the
 325 application of the same data analysis cuts used to search for coincidences. The
 326 high statistical accuracy, even for experiments with a very small background,
 327 follows directly from an analysis procedure starting from sets of complete sin-
 328 gles data from which the coincidences have been identified. This contrasts with
 329 traditional background measurement techniques, in which a separate set of co-
 330 incidence measurements are made, with a change to the physical environment
 331 of the experiment, such that the possibility of forming a desirable coincidence
 332 has been removed.

333 A figure of merit for the method has been derived, based just on the statis-
 334 tical improvement compared to traditional methods of measuring backgrounds.
 335 Factors of up to four times more effective use of time are realistic. Impressive
 336 as this figure of merit is, it is still an underestimate of the power of the cur-

rent technique. The current technique removes the systematic uncertainty in normalizing the background measurement, as the conditions are now known to be truly the same as during the foreground measurement. This systematic improvement in the background measurement has not been included in the figure of merit.

Furthermore, the algorithms used to extract the background are a form of analytic data reduction. They make no prior assumptions about the shape of the background, and do not involve any free parameters (in particular they do not use any type of fitting). As such, they are capable of reducing the data to determine very complicated background shapes, even while maintaining large figures of merit.

It is anticipated that the method will be of use to a large class of coincidence experiments, in which backgrounds can not be fully eliminated. One such set of experiments is the measurement of the prompt fission neutron spectrum being performed at WNR/LANSCE. The method has been demonstrated using some data sets from this project.

Appendix A. Summing Over Similar Data Sets

As a practical matter, we must consider the application of eq. (2) under the scenario of summing two data sets, taken sequentially, but otherwise under the same conditions. We use subscripts 1 and 2 to identify the two data sets.

Adding the background histograms directly gives

$$\gamma_{1+2} = \left(\frac{a_1 b_1}{N_1} + \frac{a_2 b_2}{N_2} \right) \pm \sqrt{\frac{a_1 b_1 (a_1 + b_1)}{N_1^2} + \frac{a_2 b_2 (a_2 + b_2)}{N_2^2}}, \quad (\text{A.1})$$

whereas, adding the singles histograms, and then calculating the background gives

$$\gamma_{1+2} = \frac{(a_1 + a_2)(b_1 + b_2)}{N_1 + N_2} \pm \sqrt{\frac{(a_1 + a_2)(b_1 + b_2)(a_1 + a_2 + b_1 + b_2)}{(N_1 + N_2)^2}}. \quad (\text{A.2})$$

It is not immediately clear that eqs. (A.1) and (A.2) are the same result — indeed, at first glance they look very different. This dilemma is resolved by

362 recalling that the number of counts in a data set is proportional to the time spent
 363 acquiring the data. Defining the detector dependent constant of proportionality
 364 as k_a or k_b so that $a = k_a N_j$ and $b = k_b N_j$, we find that eqs. (A.1) and (A.2)
 365 both reduce to the same result,

$$\gamma_{1+2} = k_a k_b (N_1 + N_2) \pm \sqrt{k_a k_b (k_a + k_b) (N_1 + N_2)}, \quad (\text{A.3})$$

366 as desired.

367 Appendix B. One Bin of the Projected 1D Background

368 To determine a 1D time-difference histogram for the random coincidence
 369 background, it helps to make explicit the time dependence of the quantities in
 370 eq. (2),

$$\gamma(t_a, t_b) = \frac{a(t_a)b(t_b)}{N} \pm \frac{1}{N} \sqrt{a(t_a)b(t_b)(a(t_a) + b(t_b))}. \quad (\text{B.1})$$

371 A single bin of $\beta(\delta_t)$, is then a sum over independent elements of $\gamma(t_a, t_b)$
 372 along the diagonal line $t_b = t_a + \delta_t$ (upper diagonal line, dark blue online, in
 373 fig. C.10). The fluctuations from individual $\gamma(t_a, t_b)$ add in quadrature, so that

$$\begin{aligned} \beta(\delta_t) &= \sum_{t_a} \gamma(t_a, t_a + \delta_t) \\ &= \frac{1}{N} \sum_{t_a} a(t_a)b(t_a + \delta_t) \pm \frac{1}{N} \sqrt{\sum_{t_a} a(t_a)b(t_a + \delta_t)(a(t_a) + b(t_a + \delta_t))}. \end{aligned} \quad (\text{B.2})$$

374 Appendix C. Correlations in the Projected 1D Background

375 The background in a second bin of β , sums along a different set of γ values,
 376 such as the lower diagonal line (light blue online) in fig C.10. One term of
 377 $\beta(\delta_{t1})$ has the same value of $a(t_a)$ as one term of $\beta(\delta_{t2})$. These terms lie on the
 378 intersection of the diagonal lines (blue online) in fig. C.10 with the vertical line
 379 (red online) of constant $a(t_a)$. There is a similar correlation, with a different
 380 term of $\beta(\delta_{t1})$, now along the horizontal line (orange online), for a constant
 381 $b(t_b)$. In general, these two correlations occur for each term of $\beta(\delta_t)$, and lead
 382 to a full a covariance matrix, $\text{Cov}\beta(\delta_{t1}, \delta_{t2})$.

Figure C.10: Properties of γ and β . $\beta(\delta_{t1})$ and $\beta(\delta_{t2})$ are sums of γ along the upper and lower diagonal lines respectively (dark and light blue online). Correlations arise along vertical lines (red online) of constant $a(t_a)$ and horizontal lines (orange online) of constant $b(t_b)$.

383 Expressions for $\text{Cov}(\beta(\delta_{t1}), \beta(\delta_{t2}))$ can be derived using the methods com-
 384 monly described in text books (see e.g. [11]). However such an approach is overly
 385 cumbersome, as simplifications are easily found, reflecting the simple geometries
 386 of fig. C.10.

387 First we observe that in eq. (B.2), the partial derivatives $\partial\beta(\delta_t)/\partial\gamma(t_a, t_a +$
 388 $\delta_t)$ are one (or zero), reflecting that the vertical (red online) and horizontal
 389 (orange online) lines intersect the upper diagonal (dark blue online) line of
 390 fig. C.10 at only one point.

$$\text{Cov}(\beta(\delta_{t1}), \beta(\delta_{t2})) = \sum_{t_{a1}} \sum_{t_{a2}} \text{Cov}(\gamma(t_{a1}, t_{a1} + \delta_{t1}), \gamma(t_{a2}, t_{a2} + \delta_{t2})). \quad (\text{C.1})$$

391 Next we observe that frequently $\text{Cov}(\gamma(t_{a1}, t_{a1} + \delta_1), \gamma(t_{a2}, t_{a2} + \delta_2))$ is zero,
 392 reflecting that the vertical (red online) and horizontal (orange online) lines of
 393 fig. C.10 intersect the lower diagonal line (light blue online) at just two points.
 394 With these zeroes, the double summation is reduced to just one or two single
 395 summations:

$$\text{Cov}(\beta(\delta_{t_1}), \beta(\delta_{t_2})) = \begin{cases} \sum_{t_a=1}^{\max(t_a)} \text{Cov}(\gamma(t_a, t_a + \delta_{t_1}), \gamma(t_a, t_a + \delta_{t_2})) & \text{if } \delta_{t_1} = \delta_{t_2}, \\ \sum_{t_a=1}^{\max(t_a)} \text{Cov}(\gamma(t_a, t_a + \delta_{t_1}), \gamma(t_a, t_a + \delta_{t_2})) + \sum_{t_a=\delta_{t_2}-\delta_{t_1}}^{\max(t_a)} \text{Cov}(\gamma(t_a, t_a + \delta_{t_1}), \gamma(t_a + \delta_{t_1} - \delta_{t_2}, t_a + \delta_{t_1})) & \text{otherwise.} \end{cases} \quad (\text{C.2})$$

396 Finally we use

$$\text{Cov}(\gamma(t_{a_1}, t_{b_1}), \gamma(t_{a_2}, t_{b_2})) = \begin{cases} \frac{a(t_a)b(t_b)(a(t_a) + b(t_b))}{N^2} & \text{if } t_{a_1} = t_{a_2} \equiv t_a, t_{b_1} = t_{b_2} \equiv t_b, \\ \frac{a(t_{a_1})a(t_{a_2})b(t_b)}{N^2} & \text{if } t_{a_1} \neq t_{a_2}, t_{b_1} = t_{b_2} \equiv t_b, \\ \frac{a(t_a)b(t_{b_1})b(t_{b_2})}{N^2} & \text{if } t_{a_1} = t_{a_2} \equiv t_a, t_{b_1} \neq t_{b_2}, \\ 0 & \text{otherwise,} \end{cases} \quad (\text{C.3})$$

397 (obtained by standard techniques from eq. (B.1)) to get the desired covariance
398 matrix, in terms of the counts in the singles spectra:

$$\text{Cov}(\beta(\delta_{t_1}), \beta(\delta_{t_2})) = \frac{1}{N^2} \sum_{t_a=1}^{\max(t_a)} a(t_a)b(t_a + \delta_t)(a(t_a + \delta_{t_1} - \delta_{t_2}) + b(t_a + \delta_{t_1})) \quad (\text{C.4})$$

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